A Two-Dimensional Dynamical Model for the Subseasonal Variability of the Asian Monsoon Anticyclone

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ABSTRACT

The Asian monsoon anticyclone, which develops in the upper troposphere and lower stratosphere during boreal summer, exhibits significant subseasonal variability with a characteristic spatial structure. The dynamics of this variability is investigated using a nonlinear β-plane shallow-water model. The equivalent depth is estimated using reanalysis data to relate the three-dimensional dynamics in isentropic coordinates to the shallow-water model. Composite analysis reveals the resemblance of the horizontal structures between the Montgomery streamfunction and thickness on the 360-K level. However, the coefficients of the linear regressions between those two variables are strongly dependent on latitude. The estimated equivalent depths of the northern region are more than 2 times greater than those of the southern region. This is attributable to the background thermal structure around the tropopause. Based on this, a latitude-dependent mean depth is incorporated into the shallow-water model to numerically investigate responses to a steady localized forcing in the subtropics. With the inclusion of the latitudinal dependence of the mean depth, the vortex shedding state is able to have a longitudinally confined structure, which differs from the conventional case of constant mean depth. The spatial structure of this numerical solution corresponds to the observed structure, in which low-PV air is largely confined to finite longitudes within the Asian monsoon anticyclone. This suggests the possible role of dynamical instability and the interaction with the subtropical jet in determining the characteristic structure of the Asian monsoon anticyclone.

1. Introduction

The large-scale circulation in the subtropics is characterized by monsoons, which are primarily driven by land–ocean heat contrast and the seasonal migration of the tropical convergence zone. In boreal summer, the Asian monsoon is the largest monsoon circulation and is accompanied by strong tropical convection over South and Southeast Asia (Webster et al. 1998).

In the upper troposphere and lower stratosphere (UTLS) over South Asia, a large-scale anticyclonic circulation persists during boreal summer. In this study, we refer to it as the Asian monsoon anticyclone (AMA), as used in the UTLS research community, although other names such as the Tibetan high (Liu et al. 2007) and South Asian high (Zhang et al. 2002; Nützel et al. 2016) are also popular. The AMA has a large spatial scale and a characteristic thermal structure in which the thermal tropopause is locally lifted by approximately 20 K in potential temperature (Ploeger et al. 2015). The existence of an upper-level anticyclone have been understood as a steady Rossby wave forced by strong convective heating in Southeast Asia as a first-order approximation (Hoskins and Rodwell 1995; Highwood and Hoskins 1998), though nonlinear dynamics have an essential role (Hsu and Plumb 2000, hereafter HP00; Plumb 2007).

This characteristic spatial structure near the tropopause is known to be an effective transport pathway that allows minor atmospheric constituents to move between the troposphere and the stratosphere (Dunkerton 1995; Dethof et al. 1999; Gettelman et al. 2004). The role of the AMA in the cross-tropopause transport of minor constituents has been intensively studied in recent years using chemical transport models (Park et al. 2009; Bergman et al. 2013; Vogel et al. 2014, 2016; Pan et al. 2016), satellite measurements (Randel et al. 2010; Luo et al. 2018), and in situ measurements by radiosondes (Bian et al. 2012) and aircrafts (Gottschaldt et al. 2018).
The AMA exhibits strong subseasonal variability in its intensity, location, and horizontal structure. The AMA center has two preferred locations over the Tibetan Plateau and western Asia, defined as the Tibetan mode and the Iranian mode, respectively (Zhang et al. 2002). The east–west oscillation between these locations is important because it is related to summer rainfall variability in East Asia (Zhang et al. 2002; Nützel et al. 2016), and to the chemical properties in the UTLS (Yan et al. 2011). Pan et al. (2016) classified the temporal states of the anticyclone into four phases, which include the double-center phase and the zonally elongated phase in addition to those that correspond to the Tibetan and Iranian modes. The transition from one phase to the other has been found to be important for the horizontal transport and irreversible mixing of the air uplifted from the boundary layer (Pan et al. 2016; Gottschaldt et al. 2018). The dominant time scale of this variability is 10–20 days and is often labeled as quasi-biweekly (Zhang et al. 2002; Ortega et al. 2017).

Despite its importance, the dynamics of the subseasonal variability of the AMA is not well understood. The AMA variability can be considered the response to temporal variations of convective heating over South and Southeast Asia. The subseasonal variability of the monsoonal convection coupled with low-level circulation is itself a significantly important topic, and there have been innumerable studies on the topic, which has been extensively investigated in the literature. There are two main modes of variability with time scales of 30–60 and 10–20 days (Krishnamurti and Bhalme 1976; Annamalai and Slingo 2001). Their relationship to the dynamical variability in the UTLS remains a topic of ongoing discussion. Previous analyses have shown a positive lag correlation between the intensity of the AMA and convection (Randel and Park 2006; Garny and Randel 2013). It has also been shown that strong convection tends to be followed by westward-propagating anticyclonic anomalies (Garny and Randel 2013; Nützel et al. 2016).

Dynamical instability can also play a role in the AMA variability. The vorticity balance in the upper troposphere around the AMA indicates the importance of nonlinear eddy transport (Sardeshmukh and Held 1984), which is related to spontaneous anticyclonic vortex shedding (Plumb 2007). The dynamics leading to vortex shedding is considered to be two dimensional. Using a nonlinear β-plane shallow-water model with a steady localized forcing and a uniform linear relaxation, HP00 successfully demonstrated the transition from a finite-amplitude steady anticyclone to a state of spontaneous periodic vortex shedding.

However, the actual spatial structure of the western part of the anticyclonic vortex during the daily evolution of the AMA is different from the result of the idealized two-dimensional model experiments in HP00. Specifically, the longitudinal structure in which the air is mostly confined within the anticyclone is not explained by the conventional shallow-water model, as discussed in detail in section 5. Therefore, an improvement in the dynamical model framework is necessary for a better understanding of the AMA variability. This study extends the two-dimensional dynamical model by considering the vertical and horizontal structure revealed by observations and taking into account the latitudinal variations of the background fields. The purpose of this treatment is to clarify the processes of vortex shedding generation with realistic spatial structures, which is important for characterizing horizontal tracer transport and mixing in the UTLS around the AMA.

The remainder of this article is organized as follows. Section 2 describes the data and method used in this study. In section 3, the basic features of the subseasonal variability of the AMA are described using reanalysis data. In section 4, an analysis is conducted to investigate the spatial characteristics of the subseasonal variability and their relationship to two-dimensional dynamics. In section 5, we propose an extension of the conventional shallow-water model by introducing a latitudinally dependent mean depth and examining its impact on the reproduced variability of the anticyclone. Section 6 presents the conclusions and discussion.

2. Data and methods

2a. Reanalysis and OLR data

Dynamical variables are obtained from the ERA-Interim data (Dee et al. 2011). Using gridded pressure level data with 1.5° × 1.5° horizontal resolution, variables in isentropic coordinates are calculated at every 5 K. The time period analyzed is June–August from 1979 to 2016.

As a proxy of convective activity, which is the dominant source of thermal forcing, the daily data of the outgoing longwave radiation (OLR) from the National Oceanic and Atmospheric Administration (NOAA) (Liebmann and Smith 1996) from 1979 to 2016 is used. The OLR data have been interpolated to a 2.5° × 2.5° horizontal grid.

2b. Isentropic coordinates

The momentum and continuity equations in isentropic coordinates (Andrews et al. 1987) are used in this study, as follows:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \theta \frac{\partial u}{\partial \theta} - f v = -\frac{\partial M}{\partial x}, \tag{1} \]
\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \theta \frac{\partial v}{\partial \theta} + f u = -\frac{\partial M}{\partial y}, \tag{2} \]
\[ \frac{\partial \sigma}{\partial t} + \frac{\partial (\sigma u)}{\partial x} + \frac{\partial (\sigma v)}{\partial y} + \frac{\partial (\sigma \theta)}{\partial \theta} = 0, \tag{3} \]

where \( f \) is the Coriolis parameter, \( \theta \) corresponds to the diabatic heating rate, and
\[ \sigma = -\frac{1}{g} \frac{\partial p}{\partial \theta} \tag{4} \]
is the equivalent thickness. The Montgomery stream-function \( M \) is defined as
\[ M = C_p T + g z \tag{5} \]
and satisfies a relationship with \( p \) that can be written as
\[ \frac{\partial M}{\partial \theta} = C_p \left( \frac{p}{p_0} \right)^{R_c/C_p}, \tag{6} \]
where \( C_p \) and \( R_c \) are the specific heat at constant pressure and the gas constant of dry air, respectively. From Eqs. (4) and (6), \( M \) can be determined from \( \sigma \) given boundary conditions.

The Ertel's potential vorticity (PV), which is defined in isentropic coordinates as
\[ q = \frac{f + \zeta}{\sigma}, \tag{7} \]
where \( \zeta \) is relative vorticity, is used in this study to describe the dynamical structure of the AMA. PV maps on an isentropic level have been widely used for studies on tracer transport and mixing, as PV is approximately conserved and therefore used as a proxy of passive tracer.

c. \( \beta \)-plane shallow-water model

The numerical experiments in section 5 are performed using a \( \beta \)-plane shallow-water model similar to the one used in HP00 but with an extension regarding the mean depth. The governing equations are given as follows:
\[ \frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} + (f_0 + \beta y) u - g \frac{\partial h}{\partial x} + \nu_m \nabla^2 u, \tag{8} \]
\[ \frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - (f_0 + \beta y) v - g \frac{\partial h}{\partial y} + \nu_m \nabla^2 v, \tag{9} \]
\[ \frac{\partial h}{\partial t} = -\frac{1}{\partial x} [u(H + h)] - \frac{1}{\partial y} [v(H + h)] + \nu_h \nabla^2 h - \frac{h}{T} + F_h, \tag{10} \]

where \( \nu_m \) is the diffusion coefficient.

The mean depth is denoted by \( H \). In this study, \( H \) is treated as a function of latitude. The PV in this shallow-water system is defined as
\[ q = \frac{f_0 + \beta y + \zeta}{H + h}, \tag{11} \]

where \( \zeta \) is the relative vorticity.

The steady mass source \( F_h \), which corresponds to a vertical gradient of the diabatic heating in the real atmosphere, is given as follows:
\[ F_h = F_{ho} \exp \left( -\frac{x^2 + y^2}{b^2} \right), \tag{12} \]

where \( b \) is the half-width of the forcing. Note that the origin of the coordinate system is set to be the forcing center. This means that setting the \( f_0 \) value is equivalent to specifying the latitude of the forcing center.

d. The approximate relationship between isentropic coordinates and shallow-water equations

In this study, we attempt to relate the shallow-water equations to the three-dimensional dynamical field of the AMA variability by introducing an approximate relationship between them. The basic idea is that the equations in isentropic coordinates in Eqs. (1)–(6) can be related to the shallow-water equations, when a linear relationship between the two variables \( \sigma \) and \( M \) is approximately satisfied. It is assumed that \( \sigma \) can be decomposed into a steady base state \( \sigma \) and a deviation from it \( \sigma' \), which is assumed to be small, so that Eq. (3) can be linearized as follows:
\[ \frac{\partial \sigma'}{\partial t} + \sigma' \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \theta}{\partial \theta} \right) = 0. \tag{13} \]

Under this condition, the dynamics on a specific isentropic level can be approximated by a set of shallow-water equations with a mean depth \( H \) using the estimated value of the equivalent depth \( H_e \), with which a linear relationship between \( \sigma' \) and \( M \) is described as follows:
\[ M = M_0 + g H_e \frac{\sigma'}{\sigma_0}. \tag{14} \]

In the analysis in section 4, we allow latitudinal dependence of \( \sigma_0 \), \( M_0 \), and \( H_e \), assuming they vary slowly with latitude so that the relationship to the shallow-water equations still holds. This corresponds to introducing a latitudinal dependence in the mean depth \( H \) and a steady background zonal wind jet in the shallow-water system described in Eqs. (8)–(10).
3. Basic characteristics of the Asian monsoon anticyclone and subseasonal variability

Figure 1 shows the seasonal-mean (June–August) PV and Montgomery streamfunction on the 370-K isentropic level averaged over 1979–2016. The 370-K level is chosen as a typical level to describe the AMA, where the contrast of PV is clear (Ortega et al. 2017). The seasonal-mean location of the AMA can be explained by a classical linear model as a response to a localized thermal forcing in the tropics (Gill 1980; Highwood and Hoskins 1998; Park et al. 2007). However, the structure of PV near the tropopause with a persistent negative latitudinal gradient in the southern part of the AMA indicates the importance of nonlinear dynamics, as stated in section 1. In the following, the characteristics of the subseasonal variability of the AMA are described and compared to the conceptual vortex shedding reproduced by the conventional two-dimensional model in HP00.

Figure 2 shows daily PV and Montgomery streamfunction $M$ on the 370-K level from 0000 UTC 1 July to 0000 UTC 6 July 2016. The anticyclone exhibits frequent deformation and splitting in its daily evolution, and its center, which is seen in both $M$ and PV, moves between eastern and western locations. There are also less frequent cases in which a small portion of the low-PV area is cut off and detached from the main vortex. This process is important in the long-range transport of tracers of tropospheric origin (Vogel et al. 2014, 2016).

Aside from this process, the air remains trapped within the anticyclone. In Fig. 2, a low-PV area [less than 1.0 PVU ($1 \text{ PVU} = 10^{-6} \text{K kg}^{-1} \text{m}^2 \text{s}^{-1}$)] is elongated (Figs. 2a–c), and most of it is shed to the west (Figs. 2c,d). This westward migration of low PV occurs several times in one summer, as described in previous studies showing time–longitude plots (Randel and Park 2006; Nützel et al. 2016; Ortega et al. 2017). Thereafter, the western portion of the low-PV air drifts slightly northward and turns eastward, seemingly following the anticyclonic advection (Figs. 2d–f). This example shows that the air with low PV largely stays inside and does not escape from the AMA during vortex shedding. The confinement of the air within the AMA has also been indicated by trajectory analyses (Garny and Randel 2016).

While horizontal transport out of the AMA is generally limited, the variability is particularly important for the irreversible mixing. As low PV corresponds to air of tropospheric origin, the PV distribution is similar to that of the mixing ratio of passive tracers. Several studies have shown the resemblance between PV and the chemical tracer distribution on daily time scales based on satellite measurements (Randel and Park 2006; Ploeger et al. 2015). The deformation of low-PV areas leads to stirring and mixing with entrained air from outside (Gottschaldt et al. 2018).

4. Vertical structure of the anticyclone and its relationship to the shallow-water system

a. Composite analysis

In this section, we examine the three-dimensional structure of the AMA near the tropopause to find its relationship to the shallow-water model based on the method described in section 2d. First, we classify daily data according to the AMA center longitude to examine possible differences in the vertical structure. Figure 3 shows the occurrence frequency of the center longitude. The ERA-Interim data in isentropic coordinates for June–August from 1979 to 2016 are used. The data are regridded to $2.5^\circ \times 2.5^\circ$ to compare the results with similar analyses in previous studies (Zhang et al. 2002; Nützel et al. 2016). The center longitude is defined as the maximum location of the Montgomery streamfunction at 370 K. It was confirmed that the calculation based on the geopotential height in pressure coordinates produces a similar outcome [which corresponds to Fig. 5a in Nützel et al. (2016)].

The distribution has two broad peaks near $60^\circ$ and $90^\circ$E, as noted first by Zhang et al. (2002). Note that the two peaks in Fig. 3 appear less isolated from each other than their result based on the previous generation of the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis data. The dependence of the longitudinal distribution on the choice of reanalysis data has been recently demonstrated by Nützel et al. (2016).

In this study, the AMA is labeled as the Tibetan and Iranian modes, when the center of the AMA lies within $45^\circ$–$65^\circ$E and $85^\circ$–$105^\circ$E, respectively. These ranges are defined based on the result shown in Fig. 3 and are slightly different from those used in previous studies. The Tibetan and Iranian modes show total occurrences of 1372 (39.2%) and 1391 (39.8%), respectively, out of 3496 in the daily data for June–August from 1979 to 2016.
Figure 4 shows composite maps of $M$ and $\sigma$ for each mode on the 360-, 370-, and 380-K isentropes. First, it is worth mentioning that the maximum values in $\sigma$ are large, that is, over twice the values of their surrounding area. This is reflective of higher tropopause around the AMA.

The longitudinal location of the $\sigma$ maximum coincides with that of the $M$ maximum for each mode. In contrast, the latitudinal location correspondences between the peaks of the two quantities depend on the isentropic levels. The latitudes of the $M$ and $\sigma$ peaks coincide at 360 K. Whereas the peaks in $M$ are located at approximately 30\degree N for all three levels, those in $\sigma$ shift northward at higher levels.

Figure 5 shows composite maps of $M$ and PV for each mode. The longitudinal structures correspond well with each other for all modes. The latitude of minimum PV nearly coincides with that of $M$ on the 370- and 380-K levels. In contrast, at 360 K, the PV minimum is located between 25\degree and 30\degree N, several degrees south of the maximum $M$. This can be understood as a manifestation of the background PV structure at 360 K, where a low PV value indicates tropospheric air is dominant.

As described in section 2d, it is considered that an approximate linear relationship between $M$ and disturbances of $\sigma$ implies a relationship between the dynamics on that isentropic level and the shallow-water system. The northward shift of $\sigma$ on higher levels reflects the background thermal structure in the tropical and extratropical UTLS, with the highest static stability near the tropical lower stratosphere [see Fig. 2 in Gettelman et al. (2011)]. Nevertheless, there is a good correspondence between $M$ and $\sigma$ at 360 K. This implies that the 360-K level is the reasonable choice to examine its relationship to the shallow-water system, even though the structure is not purely barotropic. The relationship in Eq. (14) is regarded as a model empirically expressing the latitudinal dependence.

b. Estimation of the equivalent depth

Equivalent depth $H_e$ is estimated from reanalysis data on the 360-K isentrope for both the Iranian and Tibetan
modes by Eq. (14) in section 2d. Note that this is a crude approximation because perturbations in $s$ can be comparable to $\sigma$. Therefore, we restrict ourselves to the qualitative implications of the results.

Scatterplots are made between the gridpoint values of $M$ and $s$ over the AMA ($10^\circ$–$45^\circ$N, $30^\circ$–$120^\circ$E) on the 360-K isentrope for each mode. The data values used for the scatterplots are sampled from every eight grids ($12^\circ$) in longitude, two grids ($3^\circ$) in latitude, and approximately 8 days in time. These intervals are determined as the values beyond which autocorrelation in $s$ is below 0.3.

The specification of $s$ in Eq. (14) is not trivial, as disturbances in $s$ have a finite amplitude. In the

![Fig. 4. Composite of Montgomery streamfunction (contours) and thickness (color) on isentropic surface of (a),(b) 380; (c),(d) 370; and (e),(f) 360 K. (a),(c),(e) The Iranian mode and (b),(d),(f) the Tibetan mode. The contour interval is 500 m$^2$ s$^{-2}$ in all panels. Color scales on the right are for panels in the corresponding row.](image)

![Fig. 5. As in Fig. 3, but for Montgomery streamfunction (contours) and PV (color).](image)
following, $\sigma$ is a function of latitude and is defined as the value averaged over the Western Hemisphere at $180^\circ$–$360^\circ$E, so that it can be used as the “background” state that is free from the influence of the AMA. The value of $\sigma$ at $45^\circ$N is approximately 70% of that at $15^\circ$N calculated at $360^\circ$K. This difference does not significantly affect the following outcome.

Figure 6 shows the scatterplots for each of the two modes. The horizontal axis denotes the normalized thickness $s/s_0$, and the vertical axis denotes $M/g$. Therefore, the gradient of the lines in Fig. 6 obtained by linear regression corresponds to the equivalent depth $H_e$. The values from grid points to the north and south of $35^\circ$N, for example, are shown in blue and red, respectively. It is found that the estimated equivalent depths are significantly different for the northern and southern parts of the AMA. For the two groups of points in Fig. 6 illustrated in different colors, the estimated $H_e$ from the grid points to the north of $35^\circ$N is nearly 3 times larger than that to the south. This result suggests that $H_e$ should be treated as a function of latitude rather than as a constant. Therefore, the estimation of $H_e$ at each latitude (every $3^\circ$) is performed. The term $H_e$ has a large positive gradient approximately to the north of $30^\circ$N (Fig. 7). At higher latitudes, $H_e$ is greater than twice the value at low latitudes. There is little difference between the results for the Tibetan and Iranian modes, which suggests that the outcome does not depend on the longitudinal location of the AMA. The implication of the estimated latitudinal dependence of $H_e$ is discussed in section 4c.

c. Interpretation of the latitudinal dependence of $H_e$

The result of the larger $H_e$ at higher latitudes may be counterintuitive, as the time-averaged thickness to the north of the subtropical jet along the northern flank of the AMA is greater than that in the south at $360^\circ$K (Figs. 4e and 4f). This can be explained by considering the discontinuous thermal structure around the tropopause. Note that values of $M$ at high latitudes are comparable to those at low latitudes when $s/s_0$ is large but are significantly smaller when $s/s_0$ is close to unity, which results in the steeper gradient of the fitted lines in Fig. 6. This corresponds to the tightly packed isolines in $M$ over approximately $30^\circ$–$40^\circ$N in Figs. 4e and 4f, indicating the subtropical jet and associated thermal structures of the nearby latitudes. To illustrate the contrast of the meridional structures between the cases of large $\sigma$ at $360^\circ$K at higher latitudes and those at lower latitudes, composites of $u$ are obtained based on $s$ values at $24^\circ$ and $36^\circ$N, as shown in Fig. 8. The large $M$ and $\sigma$ values at the higher latitudes are caused by occasional northward intrusions of upper-tropospheric air with low PV (e.g., see $40^\circ$–$60^\circ$E in Fig. 2e). This can be regarded as the horizontal deformation of the latitudinal boundary between the southern region with larger thickness and the northern region with smaller thickness. However, when described from an Eulerian perspective, which is necessary in relation to the shallow-water model with the latitudinal dependence of the background parameters fixed with time, this results in large effective values of $H_e$ at northern latitudes. Northward movements of

FIG. 6. Scatterplot between $\sigma$ and $M$ sampled from daily snapshots of the (a) Iranian mode and (b) Tibetan mode. Red and blue points are grid values south and north of $35^\circ$N, respectively. Values in bottom right of each panel are the estimated equivalent depths (m) and are calculated as the slope of fitted line for corresponding parts/colors of the domain.
large-amplitude disturbances in $\sigma$ are accompanied by large displacements of isentropes of 330 and 340 K (Fig. 8b). In contrast, large $\sigma$ values at low latitudes are accompanied by moderate vertical displacements of isentropes above 350 K (Fig. 8a). Therefore, the range of isentropes that contribute to $M$ through the integration of Eq. (6) is larger at higher latitudes. As a result, the sensitivity of $M$ with respect to $\sigma$ is apparently greater in the northern region. The latitudinal difference in the sensitivity of $M$ to $\sigma$ will be described in the shallow-water model as a latitudinally varying mean depth $H$ (i.e., larger values at higher latitudes).

Another implication of Fig. 6 is related to the different values in $M$ for the different latitudes when $\sigma$ is close to normal, which indicates that both the $H_c$ and $M_0$ of Eq. (14) depend on latitude. This implies the necessity of including a background westerly wind in the shallow-water model. Although this may be important for the characteristic behavior of the anticyclone in the shallow-water system, this study mainly focuses on the effect of the latitudinally varying $H_c$. A discussion of this point is provided in section 6.

5. Numerical experiments with the shallow-water model

Numerical experiments are performed using the shallow-water model described in section 2c [Eqs. (8)–(10)]. First, the behavior of the model with a constant mean depth $H$, which has been theoretically studied by HP00, is examined with a realistic choice of parameters. Second, the model is extended to include the latitudinal dependence of $H$, and the results of the experiments are discussed.

a. Experiments with constant $H$ and realistic values of parameters

The first set of experiments is performed with a constant mean depth $H$, similar to HP00. In the present study, the parameter values in Eqs. (8)–(10) with which the experiments are performed are estimated from the reanalysis data.
The mean depth is prescribed as \( H = 100 \, \text{m} \), which is comparable to the result shown in Fig. 7. The relaxation time scale is set to \( \tau \approx 23 \, \text{days} \) to mimic the radiative relaxation toward the equilibrium temperature near the tropopause. A similar value (25 days) has been used in studies of the AMA using a global mechanistic model (Hoskins and Rodwell 1995; Liu et al. 2007). The diffusion coefficients \( v_m \) and \( v_h \) are specified as sufficiently large to stabilize calculation and suppress unwanted nonaxisymmetric instability (see section 3 in HP00) but are also kept small not to alter the essential dynamics of the system. In the following experiments, \( v_h = v_m \approx 3 \times 10^4 \, \text{m}^2 \, \text{s}^{-1} \) is used.

The estimation of the realistic values of the amplitude \( F_{h0} \) and the spatial scale \( b \) of the forcing in the shallow-water model is difficult, considering the significant spatial and temporal variability of convective heating. In this study, we estimate the reasonable range of amplitude from the reanalysis data.

Figures 9a and 9b show horizontal divergence averaged for June–August from 1979 to 2016 on the 360- and 340-K levels, respectively. The area of large horizontal divergence in the subtropics at 360K lies at approximately 20°–25°N, 90°–100°E. This area partly corresponds to the minimum of seasonal-mean OLR, indicating that the divergence on this level is caused by the vertical gradient of deep convective heating. At 340K, strong divergence is localized over the Tibetan Plateau, corresponding to large convective and sensible heating below. The roles of the Tibetan Plateau and the Himalayan Mountains on the Asian summer monsoon have been discussed for decades. Recent modeling studies (Boos and Kuang 2010, 2013; Ma et al. 2014) have suggested the importance of the mechanical effect of Himalayan topography rather than the heating over the plateau, in contrast to the previous idea that the heating is essential in driving the monsoon circulation in South Asia (Wu et al. 2012). As seen in Fig. 9a, the contribution of the heating over the Tibetan Plateau to horizontal divergence is not as clear as that of deep convection at 360K. This study focusing on the dynamics on the specific isentropic level of 360K treated the role of the horizontal divergence over the Tibetan Plateau as only secondary in driving the AMA, which is consistent with the view of Boos and Kuang (2010).

Using the value in Fig. 9a, \( F_{h0} \) is estimated based on the following approximate relationship:

\[
F_h \sim H \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right),
\]

assuming the balance within the forcing region (see section 3b of HP00). The width of the forcing is prescribed as \( b = 600 \, \text{km} \) to be comparable with the spatial scale seen in Fig. 9a. Therefore, the amplitude of the forcing is estimated to be \( F_{h0}/H \sim 0.5 \, \text{day}^{-1} \). When the time scale, height, and horizontal length are normalized using \( f_0 \), \( H \), and Rossby’s deformation radius \( \sqrt{gH/f_0} \), respectively, this corresponds to a normalized area-integrated forcing amplitude \( \tilde{F}_{h0} \sqrt{b^2} \sim O(10) \). In the following experiments, values of \( F_{h0} \) corresponding to the range of \( \tilde{F}_{h0} \sqrt{b^2} \) from 1 to 50 are used. The central latitude of the forcing is set to approximately 24°N, where \( f_0 = 6 \times 10^{-5} \, \text{s}^{-1} \).

The calculations are performed in the spectral space with the cutoff wavenumbers of 512 and 128 in the \( x \) and \( y \) directions, respectively. The boundary conditions are periodic in the \( x \) direction and rigid in the \( y \) direction. The model width in \( x \) is given to be sufficiently large so that the disturbances propagating westward can be

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**Fig. 9.** Seasonal-mean horizontal divergence on (a) 360- and (b) 340-K isentropic surfaces averaged for June–August 1979–2016. Blue contours in (a) show climatological OLR (W m\(^{-2}\)) averaged for the same period. The area with height above 3000 m is hatched.
suppressed by linear relaxation before they return to the forcing region. In the northwestern and southwestern one-fourth of the model domain, sponge layers are used to suppress the velocity and height tendencies to minimize the impact of artificial reflection.

The resultant states after sufficiently long integration show either a steady anticyclone or a state of periodic vortex shedding, depending on the amplitude of the forcing. Figure 10 is an example of the vortex shedding, where PV and the normalized height disturbance \( h'/H \) are shown in color and contour, respectively. The normalized amplitude of the forcing is 40.0, which is well above the threshold \( F_{b0} \tau b^2 \approx 1.1 \) for unsteady solutions found in this set of experiments.

Note that the range of the amplitude used in this set of experiments is beyond that in the theoretical assumption in HP00 (their section 3b), in which an inequality equivalent to \( F_{b0} \tau b^2 \ll 1 \) is required. This means that the amplitude of the forcing in this experiment is so large that the advection of the height disturbance [the first two terms in Eq. (10)] is as important as the relaxation inside the anticyclone. Therefore, the transition of the response from a steady anticyclone to an unsteady vortex shedding state, with increasing amplitude of the forcing, does not follow their result based on nondimensional parameters [Eq. (8) in HP00]. The violation of the assumption in the previous study is inevitable if the realistic value of the relaxation time scale much larger than that of inertial oscillation is used, such as \( \tau = 120 f_0^{-1} \approx 23 \) days in this case. This reduces the relative importance of the linear relaxation in dynamical balance inside the anticyclone and leads to a different critical amplitude of the forcing, which is beyond the limit in the scale analysis by HP00. Nevertheless, the spatial structures of the vortex shedding in PV and height \( h' \) in Fig. 10 correspond well with Fig. 12 in HP00, even though the large amplitudes may change the absolute vorticity balance in their scaling argument.

However, the vortex shedding state shown in Fig. 10 has a notable difference from the observed behavior of the AMA in its longitudinal structure described in section 3. In Fig. 2, an anticyclonic vortex moving westward turns around at the longitude near 30°E. In contrast, the vortices with low PV in Fig. 10 propagate westward until they eventually decay.

b. Experiments with latitude-dependent \( H \)

The results in section 4 show that the estimated equivalent depth has a large positive gradient in the northern part of the AMA. Therefore, the latitudinal dependence of the mean depth \( H \) is introduced to the shallow-water model, and its impact on the behavior of the anticyclone is examined.

Experiments are performed using the same model described in section 5a except for the mean depth \( H \), which is prescribed as a function of latitude, as follows:

\[
H(y) = \begin{cases} 
\gamma H_0 & (y \geq y_c + y_w) \\
A(y) H_0 & (y_c - y_w \leq y \leq y_c + y_w), \quad \text{with} \\
H_0 & (y \leq y_c - y_w) 
\end{cases}
\]

where \( y_w \) and \( y_c \) are the width and the center latitude of the transition region, respectively; and \( \gamma \) is defined as the ratio of the mean depth at the northern part of the model to the southern part where \( H \) is uniform and \( H_0 \) is a constant. We examine the cases of \( 1 \leq \gamma \leq 3 \) and \( y_c > 0 \), which correspond to the transition latitude located on the north of the forcing center. The mean depth in the southern part is fixed as \( H_0 = 100 \) m.

A new type of unsteady solution is found in specific ranges of the parameters. A typical example of such a solution is shown in Fig. 11 as a time sequence of PV and \( h \) field of an experiment with parameters \( \gamma = 2.5 \), \( y_w = 1000 \) km, and \( y_c = 600 \) km. This solution exhibits a vortex shedding behavior with a different spatial structure from that of the case of constant \( H \) shown in Fig. 10. For example, a vortex with low PV (at \( X = -12 \), \( Y = -1 \) in nondimensional coordinates in Fig. 11a) is detached from the elongated main vortex and propagates westward. Then it turns eastward at approximately \( X = -15 \) (Figs. 11b and 11c), and merges into the main vortex. During this cycle, the anticyclonic air mass mostly remains inside the anticyclone, in contrast to the case in Fig. 10 where the air mass repeatedly leaks westward from the main anticyclone. This behavior is quasi-periodic, such that smaller-scale low-PV anomalies are detached every few days, while the macroscopic structure is nearly stationary. The spatial structure and time evolution of height disturbance in Fig. 11 are similar to those of \( M \) in the real atmosphere shown in Fig. 2, although the oscillation period is shorter.
The parameter sensitivity of the solution types is examined. Experiments are performed with various sets of parameter values $g$, $y_c$, and $F_{h0}$. Other parameters are fixed as follows: $f_0 = 6.0 \times 10^{-5} \text{s}^{-1}$ (equivalent to the forcing center location at $\sim 24^\circ \text{N}$), $\tau = 23 \text{ days}$, $H_0 = 100 \text{ m}$, and $b = 600 \text{ km}$, the same as the first set of experiments in section 5a. The width of the $H$ slope is also prescribed based on the result in Fig. 7 as $y_w = 1000 \text{ km}$. Figure 12a shows the types of solutions in the two-dimensional parameter space of $g$ and the normalized spatially integrated forcing amplitude $F_{h0} \tau b^2$ when $y_c = 600 \text{ km}$. Note that the horizontal axis is logarithmic. Only the results for $y_c = 600 \text{ km}$ are shown, as the experiments with $y_c = 200$–1000 km did not change the essential results. The solutions are classified into four types: a steady anticyclone to the east of the forcing region ($\checkmark$), a steady anticyclone to the west of the forcing region ($\times$), a quasi-periodic state with unbounded westward vortex shedding ($\bullet$) as shown in Fig. 10, and a quasi-periodic state with a longitudinally bounded vortex shedding ($\star$) as shown in Fig. 11. The change in solution types with increasing amplitude of

![Fig. 11. A time series from $\Delta t = (a) 0$ to (d) 6.9 days of PV (color) and height disturbance (contours) of the shallow-water model with a latitude-dependent mean depth, with the time interval of $12 f_0^{-1} \approx 2.3 \text{ days}$. The $X$ and $Y$ axes are scaled by Rossby's deformation radius ($L_R = 527 \text{ km}$). The mean depth $H$ as a function of latitude is shown to the right of (a). See section 5b for detailed configurations.](image-url)
the forcing is illustrated in Figs. 12b–12e, each of which corresponds to cases with parameter sets marked with rectangles in Fig. 12a.

When the ratio of the mean depth \( \gamma \) is small, there are two types of solutions: a steady anticyclone to the west (\( s \)) and unbounded vortex shedding (\( d \)) as shown in HP00. For \( \gamma \) greater than 2.0, a new type of unsteady state with longitudinally trapped vortex shedding (\( w \)) appears. The critical value of \( \gamma \) for such transition is between 2.0 and 2.25, which is comparable to the ratio of the estimated \( H \) in section 4 between the high and low latitudes. When the amplitude of the forcing is increased further, the resultant state transitions to an unbounded vortex shedding state. Additionally, large \( \gamma \) and small amplitudes of the forcing also lead to another new type of steady solution (\( 3 \)) in which the anticyclone is located to the east of the forcing region.

c. Interpretation of the solution types

As discussed above, a new unsteady state (\( \star \)) with a longitudinally bounded structure is observed when \( \gamma \) is greater than 2.0. The structure of such a state is similar to that of the observed variability of the AMA (Fig. 2). This subsection attempts to give an explanation of how these different states can emerge as a result of incorporating a latitudinally varying mean depth, from the perspective of PV.

First, the vortex shedding reproduced by the conventional model with constant mean depth (Fig. 10) is understood as the beta effect. The steady response to a finite-amplitude localized mass source in the sub tropics is an axisymmetric anticyclone in nonlinear balance. The inclusion of the beta effect results in the modified structure of the anticyclone elongated westward. Such an elongated anticyclonic vortex becomes unstable, as the instability criterion based on the meridional gradient of zonally uniform PV is verified. The subsequent westward propagation of isolated vortices with low PV can be explained through PV inversion. Suppose there is an isolated synoptic-scale area with a PV value close to zero on the \( \beta \) plane with a constant mean depth \( H \). A peak in \( h' \) should be formed in the north of the low-PV area, because of the higher values of \( f \) there. PV inversion indicates that the latitude of maximum height should also be to the north of the low PV, which roughly corresponds to that of the relative vorticity. Therefore, the resultant balanced anticyclonic flow is easterly at the latitude of the low-PV area, leading to westward advection. Such a configuration of PV and height fields is clearly observed in Fig. 10. A similar pattern is also found in \( M \) and PV in the real atmosphere during the westward propagation of low PV (Fig. 1c).

Second, the structure change of the unsteady solution caused by introducing latitudinally dependent \( H \) is discussed. With sufficiently large \( \gamma \), the state of the steady anticyclone (\( O \)) to the west of the forcing region appears for larger forcing amplitudes, compared to the previous case with \( \gamma = 1 \). The steady anticyclone has a larger spatial extent and a characteristic PV structure such that the low PV takes an annular shape. This can be understood as the result of the modification of the background PV near the forcing region by introducing a positive gradient in \( H \). In other words, the large-scale effective beta around the forcing region becomes close to zero, resulting in the steady state similar to the \( f \) plane.

Note that the anticyclone center in \( h' \) is located in the middle of the low-PV annulus (Fig. 12d), unlike for the vortex shedding states (Fig. 12e). From the PV inversion
perspective, the southern part of this low-PV disturbance contributes to the height maximum in the north as previously shown. In contrast, for the northern part, larger $H$ leads to weaker negative relative vorticity. Therefore, an isolated low-PV area should correspond to an area of negative relative vorticity whose peak is shifted southward, leading to a shifted positive height disturbance. As a result, a circular low-PV area creates a height maximum in the middle of the area and keeps the PV structure steady.

As the amplitude of the forcing increases, the PV and height structures of the steady solution are elongated to the west. When the amplitude exceeds a certain limit, the solution eventually generates unsteady behavior with westward vortex shedding (●). However, such vortices have a smaller spatial scale than those of the anticyclone seen in $h'$. Therefore, when the amplitude of the forcing is not too strong, disturbances with low PV are mostly advected by a large-scale anticyclonic flow. When the amplitude is further increased, disturbances grow so large that they can break the bounded structure of the anticyclone itself in $h'$, leading to a state of unbounded vortex shedding (●).

6. Summary and discussion

In this study, we investigated the structure of the subseasonal variability of the AMA using reanalysis data on isentropic coordinates, and we introduced a simple shallow-water model with latitudinally dependent mean depth. The dynamics of the subseasonal variability, especially the oscillatory behavior of its central location and vortex shedding with low-PV air, are crucial for understanding the impact of the AMA on the stratosphere–troposphere exchange of chemical tracers.

Using long-term reanalysis data, composites were obtained for two modes, the Tibetan and Iranian modes, which are defined according to the AMA center longitudes. For each mode, the longitudinal structures in Montgomery streamfunction $M$ and thickness $\sigma$ coincided well with each other at 360, 370, and 380 K. In contrast, the correspondence of the peaks’ latitudes in $M$ and $\sigma$ was found only at 360 K, while $\sigma$ peaks at a higher latitude than $M$ at 370 and 380 K. The approximate linear relationship between $M$ and $\sigma$ at 360 K was used to estimate the equivalent depth $H_e$. The coefficients of the linear relationship between them were found to be strongly dependent on latitude near the subtropical jet. The estimated $H_e$ values to the north of 30°N had a large positive gradient with respect to latitude.

Based on this result, numerical experiments were performed using a shallow-water model that is regarded as an extension of the model by HP00. The experiments with a constant mean depth and a realistic relaxation time scale of approximately 23 days show the possibility of vortex shedding with large-amplitude height disturbances, which is consistent with the large $\sigma$ deviation observed on the isentropes around 370 K in the real atmosphere. However, the vortex shedding state with a longitudinally confined structure was not reproduced by the conventional model. Thus, another set of experiments were performed using a shallow-water model that featured a mean depth as a function of latitude, implemented based on the observations. In some experiments, a longitudinally bounded structure of an anticyclone with quasi-periodic shedding of low-PV areas was observed. Such behavior has been confirmed as possible to generate when the latitudinal dependence of the mean depth and the amplitude of the forcing are comparable to the values estimated from the reanalysis data.

The reproduced temporal variability, in which the low-PV area is shed westward and migrates clockwise inside the anticyclone toward the east, bears similarity to the observation. In reality, the AMA has a distinct western boundary almost throughout the summer, beyond which low-PV air rarely leaks infrequently, as evident in the isentropic PV map in the longitude–time section in previous studies [Fig. 6a in Ortega et al. (2017)]. These facts suggest that the bounded structure found in the shallow-water experiments in this study can serve as a relevant model for the AMA variability. Specifically, in some experiments showing the bounded unsteady anticyclone, a moderate maximum in height is found in the western part of the anticyclone. This can be compared to the Iranian mode in the real atmosphere, and the spontaneous generation of vortex shedding can lead to the east-west oscillation of the height maximum location. This implies the possibility that the characteristic longitudinal structure of the AMA is reproduced without imposing any additional longitudinal constraints.

However, it is still difficult to find a sufficient explanation for the observed variability of the AMA only from the results of the experiments with the shallow-water model. The reproduced behavior may correspond to only a part of the important features of the AMA, because the forcing given to the model is steady and fixed in location. For example, the model in this study was not able to explain the detailed temporal variation of low-PV area, such as the double-center phase and the zonally elongated phase defined in Pan et al. (2016), and the events of eastward shedding of anticyclonic vortices from the AMA, which are suggested to contribute to long-range transport (Vogel et al. 2014, 2016). Further discussion of these behaviors requires the consideration...
of the temporal and/or spatial variability of convective heating.

There are also other external factors that can affect the spatial characteristics of the AMA in the real atmosphere. First, the subtropical jet can modify the AMA through various processes. As stated at the end of section 4, the results of our analysis showed a negative latitudinal gradient in $M_0$, which corresponds to a steady background westerly jet in the shallow-water system. The inclusion of a westerly jet provides a smaller or even reversed effective beta on the southern flank of the jet, which can modify the behavior of the anticyclone in a similar way to varying mean depth examined in this study. Moreover, the westerly jet enhances the eastward advection in the north of the anticyclone. From the perspective of the background PV largely controlling the dynamics of the anticyclone, the former effect alone could lead to the similar behavior of a bounded anticyclone given the sufficient intensity of the jet. It is interesting to perform additional experiments with a modified shallow-water model to examine the behavior of the anticyclone under the combined effects of westerly advection and varying mean depth. Kosaka et al. (2009) showed that localized barotropic or baroclinic energy conversion along the subtropical jet is essential in forming a quasi-stationary wave pattern, which may contribute to the bimodal structure. A stationary wave pattern downstream of the North American monsoon forced by convective heating (Hoskins and Rodwell 1995) may also be important in forming the western limit of the AMA. The relative importance of these effects should be investigated in future studies.

The time scale is also an important characteristic of the variability of the AMA. Figure 13 shows the period of the unsteady states in days as estimated from each experiment shown in Fig. 12. The dominant period is estimated as the time lag that gives the first isolated positive peak of autocorrelation greater than 0.1. A set of three asterisks corresponds to the experiments that give steady solutions, and a set of three dashes corresponds to unsteady solutions without clear characteristic time scales. In the case of the unbounded vortex shedding (black numerals), the estimated dominant time scales are approximately 1–3 weeks. Increasing the amplitude of the forcing usually leads to shorter time scales. The bounded unsteady solutions shown in red numerals generally have shorter time scales, which vary from 1 to 2 weeks. Therefore, the reproduced variability of the shallow-water model in this study has realistic time scales. The implication of this parameter dependence is an interesting topic for future work.

The possibility of the spontaneous generation of the quasi-periodic variability of the AMA on the subseasonal scale as shown in the present study also has important implications for the variability in the troposphere. The coexistence of dynamical variabilities in the UTLS and in the lower troposphere associated with convection has been examined from various perspectives (Annamalai and Slingo 2001; Fujinami and Yasunari 2004; Wang and Duan 2015; Nützel et al. 2016; Ortega et al. 2017). Although the causal relationship between the AMA and tropospheric circulation or convection is not yet fully explained, a better understanding of the role of the UTLS variability in this relationship would improve our understanding of the topic.

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REFERENCES
Annamalai, H., and J. Slingo, 2001: Active/break cycles: Diagnosis of the intraseasonal variability of the Asian summer


