A Comprehensive Observational Study of Graupel and Hail Terminal Velocity, Mass Flux, and Kinetic Energy

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ABSTRACT

This study uses novel approaches to estimate the fall characteristics of hail, covering a size range from about 0.5 to 7 cm, and the drag coefficients of lump and conical graupel. Three-dimensional (3D) volume scans of 60 hailstones of sizes from 2.5 to 6.7 cm were printed in three dimensions using acrylonitrile butadiene styrene (ABS) plastic, and their terminal velocities were measured in the Mainz, Germany, vertical wind tunnel. To simulate lump graupel, 40 of the hailstones were printed with maximum dimensions of about 0.2, 0.3, and 0.5 cm, and their terminal velocities were measured. Conical graupel, whose three dimensions (maximum dimension 0.1–1 cm) were estimated from an analytical representation and printed, and the terminal velocities of seven groups of particles were measured in the tunnel. From these experiments, with printed particle densities from 0.2 to 0.9 g cm⁻³, together with earlier observations, relationships between the drag coefficient and the Reynolds number and between the Reynolds number and the Best number were derived for a wide range of particle sizes and heights (pressures) in the atmosphere. This information, together with the combined total of more than 2800 hailstones for which the mass and the cross-sectional area were measured, has been used to develop size-dependent relationships for the terminal velocity, the mass flux, and the kinetic energy of realistic hailstones.

1. Introduction

According to the AMS Glossary of Meteorology and the World Meteorological Organization International Cloud Atlas, graupel particles are heavily rimed ice particles that are “often indistinguishable from very small soft hail except for the size convention that hail must have a diameter greater than 5 mm” (American Meteorological

Society 2015). Hail is precipitation in the form of balls or irregular lumps of ice with diameters generally in the range 5–50 mm, which can be spheroidal, conical, or irregular in shape, and which can be either transparent or partly or completely opaque.

A knowledge of the properties of graupel and hail during their growth and early melting stages is an essential component necessary for numerical modeling applications and for characterizing their radar signature. The pathways that an ice particle follows from its initial formation through to hail can vary widely. Warmer cloud-base temperatures ($T > 9^{\circ}\text{C}$) generally favor graupel embryos that are from frozen drops, whereas colder cloud-base temperatures ($T < 9^{\circ}\text{C}$) favor snow

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crystals as graupel embryos (Knight 1981). The process by which the initial ice particle forms strongly influences the embryo shape: lump graupel generally develop from frozen drops and conical graupel from snow.

The density of graupel particles varies widely, depending on their formation mechanism, time of growth from the initial embryo, accreted sizes of the droplets, impact velocity of the drops on the graupel, liquid water content, whether there is wet growth or not—a function of the particle temperature—and the air temperature (e.g., Pflaum and Pruppacher 1979). Typical graupel densities are 0.4 g cm^{-3} . Prodi (1970) measured mean densities of hailstones that ranged between 0.82 and 0.87 g cm^{-3} , with the interesting but expected result that there is a general decrease of density from the hailstone periphery toward the center. The high densities of hailstones are attributable in part to the process whereby, as hail melts from the 0°C dewpoint level to the surface, water soaks into hollows inside the particle, raising the density. However, Knight et al. (2008) concluded that very low-density growth of hail is more common than has been suspected.

Oblate spheroidal hail is common (Browning and Beimers 1967; Barge and Isaac 1973; Matson and Huggins 1980; Knight 1986). Most hailstones have axis ratios (minimum/maximum dimension) of ≈ 0.8 (Knight 1986), becoming more oblate with increasing size, with ratios decreasing from about 0.95 for 5-mm stones to 0.6–0.7 for 5-cm stones. This is an important property of hailstones, since measurements of the terminal velocity of hailstones often assume that they are spherical and solid ice (Laurie 1960; Young and Browning 1967; Mroz et al. 2017). Clearly, the masses of typical hailstones are not the same as those of solid ice spheres.

There have been a few studies that have measured the terminal velocities of hailstones. Roos and Carte (1973) used replicas of 10 nonspheroidal hailstones that spanned a wide range of masses. Terminal velocities were measured in a vertical wind tunnel, a mine shaft, and in free fall, from which the associated drag coefficients were calculated. Roos (1972) derived the drag coefficient of the very large Coffeyville, Kansas, hailstone from measurements during drops of replicas of the stone from a helicopter and in a vertical wind tunnel. Matson and Huggins (1980) directly measured the terminal velocities of more than 600 hailstones in the diameter range from 0.5 to 2.5 cm while in free fall, from which they calculated their drag coefficients.

Microphysical processes have a direct impact on convective buoyancy and hence convective fluxes, because of condensate loading and latent heating/cooling resulting from phase changes. Two recent studies highlight the need for better and more reliable representations of graupel and hail density, mass, and terminal

velocity. Morrison and Milbrandt (2011) performed three-dimensional simulations of a supercell in the Weather Research and Forecasting (WRF) Model using two different two-moment bulk microphysics schemes. Despite general similarities in these schemes, the simulations were found to produce distinct differences in the storm structure, precipitation amounts, and strength of the cold pool. They concluded that there is a critical need for further observational studies of the density and fall velocities of ice particles, highlighting graupel and hail. More recently, a squall line event during the Mid-latitude Continental Convective Clouds field campaign in Oklahoma in 2011 was simulated by three bin (spectral) microphysics scheme coupled to WRF. Their study showed that different bin microphysics schemes—an improvement over two-moment schemes—simulated a wide spread of microphysical, thermodynamic, and dynamic characteristics of the squall line. Differences in the simulations were partially attributed to the density–size and terminal velocity–size relationships.

The goal of this article is to develop a better understanding of the properties of graupel and hail particles by reviewing relevant literature on their properties and by using a unique set of data derived from natural hailstones. It builds upon the work presented in Heymsfield et al. (2014, hereafter H14) with an even larger and more robust hail measurement catalog. The unique data used here were obtained from three-dimensional (3D) laser scans of the three-dimensional structure of hailstones. Such a dataset facilitates deriving the aspect ratios, masses, and terminal velocities more accurately than has been possible in the past. In section 2, the methodology used to derive the terminal velocities of graupel and hail in the laboratory is described. In section 3, properties of graupel and small hail measured near the 0°C level reported in past literature and previously unpublished data are presented. The information may be very useful for modeling applications and improving graupel and hail growth parameterizations. In section 4, the focus is on hail observed near the surface, drawing on earlier studies and the laboratory experiments. The results from sections 3 and 4 are discussed and summarized in section 5.

2. Experimental study of hail and graupel properties

a. Hail

The Insurance Institute for Business and Home Safety (IBHS), through their annual hail field research program from 2012 to 2017, has compiled an extensive database of natural hailstone measurements (H14; Brown et al. 2015; Giammanco et al. 2017). The catalog contains over 3000

hailstones in which the maximum diameter D , the minimum diameter, the intermediate dimension, and the mass were physically measured in the field following the passage of a hail-producing thunderstorm. In addition, each hailstone was photographed. In 2016, IBHS pioneered the use of a handheld, 3D laser scanning unit to create high-resolution digitized models of natural hailstones (Giammanco et al. 2017). Over 100 hailstone computer-aided design (CAD) models of natural hailstones have been created. The detailed datasets enable the application of 3D printing technology to model natural hailstones in great detail to properly capture their unique shapes and further explore their aerodynamic properties.

Advances in 3D printing allow for the use of acrylonitrile butadiene styrene (ABS) plastic at specified densities. Hailstone models based on the 3D scans of natural hailstones were printed and color coded according to the density of the printing material (Fig. 1) at five different densities ρ : 0.3, 0.4, 0.5, 0.7, and 0.9 g cm^{-3} . A total of 53 hailstones, ranging in size from 1.0 to 6.7 cm (maximum diameter), were studied. The masses of the hailstones, ranging from 2.0 up to 41.3 g, were obtained by an analytical balance (Sartorius), and their cross-sectional areas were determined using two-dimensional (2D) photographic imaging. From the knowledge of these derived parameters, the drag coefficient and other related parameters were derived.

In addition, ice spheres—2 of 0.5-cm diameter, 6 of 1.0-cm diameter, 7 of 1.6-cm diameter, and 11 of 2.6-cm diameter—were investigated. These had solid ice density. The purpose of using different solid ice spheres but with the same size, shape, and mass was to determine whether the same results were obtained for different particles. As noted later, the velocities of the same size particles differed only slightly.

The terminal velocity measurements were carried out in the vertical wind tunnel facility at Johannes Gutenberg University in Mainz, Germany. The facility allows for the hydrodynamic properties of single cloud and precipitation particles from diameters of several tens of micrometers up to a few centimeters to be investigated under flow conditions very close to those in the atmosphere. [See, for example, Szakáll et al. (2010) for details about the wind tunnel construction and flow characteristics.] In the current series of experiments, a 3D observation section of $10 \times 10 \text{ cm}^2$ (for hail) or $17 \times 17 \text{ cm}^2$ (for graupel) cross-sectional area was used, in which the flow speed can be quickly and accurately set to any values from 1 to 40 m s^{-1} by a sonic-controlled valve. The larger cross-sectional area for graupel was chosen because they were more difficult to keep away from the wind tunnel walls than for hail. The airspeed in the wind tunnel at several positions was calibrated for different opening positions of the sonic valve by a hot-wire probe

FIG. 1. Examples of the 3D-printed hailstones used in the experiments. Color coding represents density of the printing material—blue: 0.3 g cm^{-3} ; purple: 0.4 g cm^{-3} ; red: 0.5 g cm^{-3} ; orange: 0.7 g cm^{-3} ; and white: 0.9 g cm^{-3} .

(TSI, Inc.). The measurements were carried out at ambient temperature ($\sim 20^\circ\text{C}$) and pressure ($\sim 1013 \text{ hPa}$).

For each experiment, the synthetic hailstone was introduced into the wind tunnel and exposed to the laminar tunnel airstream. When the investigated hailstone was levitated in the vertical airstream (i.e., the gravitational force was balanced by the aerodynamic drag), the airspeed was recorded and considered as the hailstone terminal fall velocity. Two different sets of experiments were carried out for each hailstone. In a first set of experiments, the hailstones were tethered on one side by thin nylon fibers in order to prevent them from touching the tunnel walls. In this arrangement, the flow was increased up to the levitating velocity. By carefully and systematically increasing and decreasing the airspeed around this velocity, we were able to determine the terminal velocity. In a second set of experiments, the hailstones were allowed to move freely in the divergent part of the wind tunnel, slightly above the straight observation section. Because of the irregular shapes of the hailstones, their motions were at times irregular, with fluttering. The motion hindered the stabilization in the straight wind tunnel section. However, the free particle motion within the divergent section improved the stabilization so that the terminal fall velocity could be determined with higher precision. In the divergent section of the wind tunnel, the airspeed decreases continuously with height; thus, the hailstones levitated at a certain level where their terminal velocity was equal to the airspeed. The deviation between the velocities determined from the two sets of experiments was on average less than 10%, which has also been considered as the measurement uncertainty.

TABLE 1. Properties of graupel and hail reported in previous studies.

Study	No. of points	Diam range (cm)	Density (g cm^{-3})	Mass	Area	V_t	Collection temperature ($^{\circ}\text{C}$)
Auer (1972) graupel	50	0.05–0.50	N	Fit to Auer (1972) $m(D)$ vs D	N (assumed circular)	Y	$T \sim +20^{\circ}$
Locatelli and Hobbs (1974)	185	0.055–0.42	Y	Y	Y	Y	$T < 0^{\circ}$
Braham (1963)	63	0.049–0.94	Y	N	N	N	$-10^{\circ} < T < 0^{\circ}$
Kajikawa (1975) graupel	243	0.050–0.50	N	Y	Y	Y	$\sim -5^{\circ} < T < 2^{\circ}$
Heymsfield (1978)	125	0.13–0.91	N	Y	Y	N	$-1^{\circ} < T < -32^{\circ}$
Kajikawa (1975) hail	33	0.51–0.89	N	Y	Y	Y	$\sim -5^{\circ} < T < 2^{\circ}$
KH	40	0.85–1.6	Y	Y	Y	Y	$T = 2^{\circ}$
Matson and Huggins (1980)	548	0.5–2.5	N	Estimated from photographs	Y	Y	$T \sim 25^{\circ}$
Auer (1972) hail	13	0.60–2.4	N	N	N	Y	$T \sim +20^{\circ}$
Willis et al. (1964)	3	7.0	Y	Y (different densities)	Y	Y	Spherical hailstones, $T < 0^{\circ}$
Young and Browning (1967)	16	10	N	Y	Y	Y	Spherical hailstones, $T < 0^{\circ}$
List et al. (1969)	5 with different roughness	12.0	N	Y (solid ice)	Y (circular)	Y	Wind tunnel $T \sim +25^{\circ}$
Roos (1972)	10	15.5	N	Y (different densities)	Y	Y	Coffeyville hailstone $T \sim 25^{\circ}\text{C}$

The ice spheres ($\rho \approx 0.9 \text{ g cm}^{-3}$) showed a comparatively stable flow pattern; therefore, they could easily be levitated in the straight observation section of the wind tunnel.

b. Graupel

The terminal velocities of graupel particles—lump and conical—were measured in the vertical wind tunnel. For both types of graupel, the purpose of deriving their terminal velocity was not to provide exact replicas of natural graupel but rather to develop relationships that would apply to graupel in general. The methodology is discussed in section 3.

Lump graupel was simulated by using 40 of the scanned hail images and printing them with maximum diameters from 2 to 5 mm. Their volumes were measured from the 3D computer images, scaled to the correct dimensions.

To simulate conical graupel, the analytic representation of conical graupel draws upon the work of Kubicek and Wang (2012). They developed the equation

$$x^2/[a \cos^{-1}(z/lc)]^2 + y^2/[a \cos^{-1}(z/lc)]^2 + z^2/lc^2 = 1, \quad (1)$$

where x and y are the semimaximum and minimum horizontal lengths, z is the vertical dimension, and “ lc ” determines the sharpness of the apex of the cone. These were printed with $x = y = 2\text{--}5 \text{ mm}$. Based on direct collections of graupel particles in cumulus congestus clouds, Heymsfield (1978) reported that the cone apex angles of conical graupel ranged from 30° to 80° , with a

mean of 60° . For that reason, an apex angle of 60° was chosen; this determined the z dimension.

3. Observed properties of graupel and hail

In this section, we draw on the properties of graupel and hail from earlier measurements (Table 1, arranged approximately in order of increasing particle size).

a. Graupel/small hail near the melting layer

The most accurate measurements of the density ρ of graupel that have developed in natural clouds from near to above the 0°C level have used immersion methods (Braham 1963; Knight and Heymsfield 1983, hereafter KH), while less accurate measurements were obtained from careful measurements of their masses and estimates of their approximate 3D dimensions (Locatelli and Hobbs 1974), and reported graupel densities in Heymsfield and Kajikawa (1987). The above references suggest that graupel densities are generally in the range of $0.2\text{--}0.7 \text{ g cm}^{-3}$, with considerable scatter (large symbols, Fig. 2a).

The exception is for the measurements by Braham (1963). In those measurements, fluids with different densities, ranging from 0.71 to 0.94 g cm^{-3} , were used, and the snow or ice pellets collected either sank (the particles had a higher density) or floated (lower density). The “minimum possible” densities of those particles are plotted in Fig. 2a. Given the convective clouds from which the observations were obtained and the relatively warm cloud bases, it seems likely that the graupel

FIG. 2. Graupel and small-hail (a) density, (b) mass, and (c) mass/area for particles sampled close to or at temperatures below 0°C. These earlier studies, from which these data are derived, are indicated in each panel with more detail given in Table 1. In (a), the number of graupel particles in each study is the first number, and the number of hailstones is the second number. In (a), arrows point to the data from Braham (1963). In (b) and (c), percentiles for increments of diameter are shown.

originated from frozen drops, thereby resulting in high densities. Also note that the diameter used for the KH particles is the equivalent spherical diameter D_e , not the maximum diameter D .

The particles for which the densities were estimated (small symbols, Fig. 2a) show a wide scatter, either because of uncertainties in the measurement of the particle dimensions or because many of the particles were rimed to heavily rimed snow.

Several relationships have been developed that relate the density accreted on an ice particle to the mean radius of the droplets, the impact velocity of the droplets on the graupel, and the graupel surface temperature (e.g., Pflaum 1978; Heymsfield and Pflaum 1985). Using a

reasonable mean radius of $6 \mu\text{m}$, a typical liquid water content of 1 g m^{-3} , a graupel surface temperature of -5°C , and impact velocities of 1.0 and 3.0 m s^{-1} would predict accretional densities of about 0.3 and 0.5 g cm^{-3} , respectively. These values are similar to the observations, both in the trend with size and in the observed densities.

In convective clouds with warm bases, a significant fraction of the graupel is likely to develop from frozen drops. A question relevant to the present study then is how their density and mass change as they grow. Drawing on the data presented in Pflaum (1978), we can presume that, on average, a $470\text{-}\mu\text{m}$ -diameter frozen drop growing at a temperature of -15°C over a 180-s

period to a 1.1-mm-diameter graupel attains a mean density of 0.21 g cm^{-3} and an accretional density of 0.15 g cm^{-3} . At warmer temperatures, $\sim -5^\circ\text{C}$, where there is still dry growth, accretional densities are much larger, $\sim 0.5 \text{ g cm}^{-3}$, and therefore, the mean graupel densities are much larger.

The measurements of the ice particle mass m as a function of D show a somewhat more regular dependence, with a mean bulk density (mass/volume of sphere of the same D ; rather than the instantaneous accretional density) of about 0.1 g cm^{-3} (small symbols in Fig. 2a). The apparent exceptions are for the collections of the hail-size particles by KH. The bulk density of these particles is 0.44 g cm^{-3} , only slightly higher, on average, than for the graupel-size particles (Fig. 2a). The primary difference is that the diameters of the KH particles were expressed in terms of D_e , not D .

For modeling purposes, it is important not only to estimate the mean mass of the particles but also their distribution about the mean. For this reason, the mass is shown as a function of diameter given in percentiles in Fig. 2b. The percentiles in this figure and in the figures that follow are derived in the following way. The diameter along the abscissa was divided into equal numbers of particles, typically 20–30. Within each diameter bin, percentiles of the value along the ordinate were derived in five intervals: 10th, 25th, 50th, 75th, and 90th percentiles. In some instances where there was only one particle in a given percentile, the percentile was taken as that value in that bin. Because of the relatively few very large hail-size particles, percentiles are not given for particles larger than 2 cm.

The percentiles in Fig. 2b show that the mass varies by a range from 1/5 to 5 about the mean curve. In future modeling of graupel, this variability should be taken into account. To facilitate such an effort, power laws were fitted to the relationship between D and m , with the coefficients given for each of the percentiles in Table 2. Unsurprisingly, the coefficient a in the m – D relationship increases with increasing percentile; however, the slope b decreases appreciably. If only an average m – D relationship is needed, the relationship shown in italic text should be used.

As a means of showing the sphericity of graupel and small hail observed near the melting layer, the ratios of their mass to cross-sectional area have been derived from the earlier studies. Perfectly spherical ice would have a mass-to-area ratio of $(2 \times 0.91/3)D$. Graupel observed near the melting layer becomes less spherical with increasing size, whereas small hail evidently becomes more spherical as it develops (Fig. 2c). An exponential curve appears to fit the data quite well (given in Fig. 2c and summarized in Table 3), as well as the percentiles shown in the figure (see fits to the percentiles

TABLE 2. Relationships for graupel and hail in terms of fractional occurrence. Relationships based on the median values in each category are shown in italic text. Units for variables are as follows: m (g), D (cm), m/A (cgs), V_t (m s^{-1}), KE (J), and MF (g cm s^{-1}).

Fractional occurrence/percentile	a	b
Coefficients in mass–diameter power-law relationships (from Fig. 2b); graupel and small hail near melting layer ($m = aD^b$)		
0.1	0.0096	2.49
0.25	0.0191	2.61
<i>0.50</i>	<i>0.0349</i>	<i>2.62</i>
0.75	0.0454	2.33
0.90	0.0574	2.25
Coefficients in mass/area–diameter power-law relationships (from Fig. 2c); graupel and small hail near melting layer ($m/A = aD^b$)		
0.1	0.0756	1.33
0.25	0.0984	1.28
<i>0.50</i>	<i>0.135</i>	<i>1.18</i>
0.75	0.187	1.04
0.90	0.219	0.92
Coefficients in mass–diameter power-law relationships (from Fig. 6a); hail ($m = aD^b$)		
0.1	0.19	2.78
0.25	0.27	2.72
<i>0.50</i>	<i>0.37</i>	<i>2.69</i>
0.75	0.47	2.73
0.90	0.53	2.82
Coefficients in mass/area–diameter power-law relationship (from Fig. 6b); hail ($m/A = aD^b$)		
0.1	0.32	0.88
0.25	0.44	0.91
<i>0.50</i>	<i>0.59</i>	<i>0.83</i>
0.75	0.75	0.82
0.90	0.81	0.94
Coefficients in graupel/hail Best number–Reynolds number power-law relationships (from Fig. 10b) ($\text{Re} = aX^b$)		
0.1	2.53	0.43
0.25	3.35	0.43
<i>0.50</i>	<i>3.03</i>	<i>0.44</i>
0.75	2.23	0.47
0.90	1.55	0.49
Coefficients in terminal velocity–diameter power-law relationships (from Fig. 13); hail ($V_t = aD^b$; $P = 1000 \text{ hPa}$)		
0.1	3.74	0.83
0.25	4.83	0.74
<i>0.50</i>	<i>6.16</i>	<i>0.65</i>
0.75	8.10	0.53
0.90	9.59	0.44
Coefficients in kinetic energy–diameter power-law relationships; hail, ($\text{KE} = aD^b$; $P = 1000 \text{ hPa}$)		
0.1	0.0064	3.75
0.25	0.0121	3.74
<i>0.50</i>	<i>0.0244</i>	<i>3.54</i>
0.75	0.0400	3.62
0.90	0.0511	3.80
Coefficients in mass flux–diameter power-law relationships (from Fig. 16); hail ($\text{MF} = aD^b$; $P = 1000 \text{ hPa}$)		
0.1	79.0	3.54
0.25	127.6	3.40
<i>0.50</i>	<i>198.4</i>	<i>3.32</i>
0.75	266.1	3.33
0.90	329.2	3.37

TABLE 3. Comparison of graupel and hail property relationships developed in different studies.

	Parameter	Laurie (1960), Waldvogel et al. (1978)	Heymsfield and Wright (2014)	H14	This study
Graupel and small hail ($T < 3^{\circ}\text{C}$)	Density	—	$\rho = 0.18D^{0.33}$	—	—
	Mass	—	$m = 0.0944D^{3.35}$	—	$m = 0.034D^{2.59}$ ($D < 0.47$ cm) $m = 0.111D^{4.13}$ ($D > 0.47$ cm)
	Area	—	—	—	—
	Mass/area	—	—	—	$m/A = 0.018D$ ($D < 0.29$ cm) $m/A = 0.306D^{2.27}$ ($D > 0.27$ cm)
	Reynolds number–Best number	—	—	—	$\text{Re} = 0.286X^{0.594}$
	Terminal velocity	—	—	—	$V_t = 6.35D^{0.87}$
Graupel and small hail ($P = 1000$ hPa)	Density	—	—	—	—
Graupel–hail composite	Mass	—	—	—	—
	Area	—	—	—	—
	Mass/area	—	—	—	—
	Reynolds number–Best number	—	$\text{Re} = 0.106X^{0.693}$ ($X < 6.77 \times 10^4$) $\text{Re} = 0.55X^{0.545}$ ($X > 6.77 \times 10^4$)	—	$\text{Re} = 1.16X^{0.49}$
	Terminal velocity–diameter ($P = 1000$ hPa)	—	$V_t = 488D^{0.84}$	$V_t = 12.3D^{0.57}$ ($D > 0.13$ cm)	—
	Kinetic energy–diameter	—	—	$\text{KE} = 0.038D^{3.33}$ ($D > 0.13$ cm)	—
Hail	Mass	$m = 0.91(\pi/6)D^3$	—	$m = 0.33D^{3.01}$	$m = 0.37D^{2.69}$
	Area	$A = \pi/4D^2$	—	$A = 0.62D^{1.96}$	—
	Mass/area	$0.91(2/3)D$	—	$m/A = 0.50D^{1.15}$	$m/A = 0.64D^{0.72}$ ($D < 1.2$ cm)
	Reynolds number–Best number	—	—	—	—
	Reynolds number–Best number spheres	$\text{Re} = 1.19X^{0.50}$ (fit to Laurie data)	—	—	$\text{Re} = 1.164X^{0.49}$
	Terminal velocity–diameter ($P = 1000$ hPa)	$V_t = 13.95D^{0.51}$ (fit to Laurie data)	—	—	$V_t = 6.1D^{0.72}$
	Kinetic energy–diameter	$\text{KE} = 0.0406D^{4.02}$ (fit to Laurie data)	—	—	$\text{KE} = 0.0243D^{3.55}$
	Mass flux	—	—	—	$\text{MF} = 188D^{3.35}$

in Table 2). If only an average m/A – D relationship is needed, the relationship shown in italic text should be used.

A generalized approach toward deriving the terminal velocity V_t of water droplets and ice particles has been used in numerous studies dating back to Davies (1945) and Best (1950). In this approach, Reynolds number (Re)–Best number (X) relationships have been developed for calculation of V_t for different assumptions about the particle mass, the density, the cross-sectional area, and the height in the atmosphere. The Reynolds number is derived from

$$\text{Re} = V_t D / \nu, \quad (2)$$

where the variable ν is the kinematic viscosity of air. Given that Re should represent the fluid flow state, it is appropriate to use D . The nondimensional Davies or Best number X is given from

$$X = C_d \text{Re}^2 \quad (3)$$

or

$$X = 2mD^2g/(\rho_f \nu^2 A), \quad (4)$$

where ρ_f is the air density, A is the cross-sectional area of the particle, and g is the acceleration due to gravity. Given the particle characteristics and the air density, the relationship between Re and X can be used to calculate V_t . The mass-to-area curve is particularly useful given that the Best number is directly proportional to the mass divided by the area (see Fig. 2c).

A caveat needs to be mentioned with respect to (2), (3), and (4). The dimension D and area A , and therefore terms Re and X , have been derived in different ways. For example, KH and Matson and Huggins (1980) used the measured masses and/or the exterior particle dimensions from which they represented D and A as the equivalent spherical diameter D_e and the equivalent cross-sectional area A_e , respectively. The drag coefficient is similarly affected. For this reason, the Matson and Huggins (1980) data have been reformulated so as to convert their (equivalent) spherical diameter to a physical diameter (see appendix for details). This was done by taking the size-dependent mean ratio of minimum to maximum diameter as a function of the spherical diameter (Table 2; Matson and Huggins 1980) and using these mean ratios and measured masses to estimate the physical diameter. Using the 3D scans from the IBHS data, we estimate at most that the error in D would be 10%.

Heymsfield and Westbrook (2010, hereafter HW) reported on the estimated drag coefficients C_d as a function of Re for 400 graupel and unrimed ice particles using data from Locatelli and Hobbs (1974), KH, and Heymsfield and Kajikawa (1987). From theoretical and laboratory fluid dynamics studies, augmented with observations, they developed an Re – X relationship that takes into consideration the area ratio A_r , the ratio of the particle area to the area of a circle of the same maximum dimension (Fig. 3a). Their curve fits the data quite well. Some of the aforementioned graupel and small hail data, augmented by the data reported in Pflaum (1978), are also shown in Fig. 3a. For the KH data, rather than using D_e and A_e , the physical dimensions and areas of the particles were estimated from $D = D_e \times (1/m)^{1/3}$, and then A_e was derived from these values of D^2 . All other datasets use the maximum dimension to derive the Re – X relationship. Over the range $1 < Re < 1000$, the HW relationship captures the observations quite well. Their relationship also agrees well with the theoretical treatment for conical graupel reported by Wang and Kubicek (2013), where $Re > 20$. The relationship derived by Böhm (1989) is also plotted. Note that in Böhm (1992), the effect of turbulence along the surface of graupel particles is considered in the development of a relationship for the drag coefficient. That study shows that improved agreement with the Heymsfield and Kajikawa (1987) graupel data is obtained by this

consideration of the effect of turbulence. It does worsen the agreement with the observations of KH and is not shown here. The zoomed view covering the ranges $10^3 < X < 10^5$ and $10 < Re < 500$ (Fig. 3b) suggests that 0.8 times the HW values might provide a slightly better fit to the observations.

It is interesting and potentially important to examine the change in V_t of a graupel particle subsequent to the freezing of its parent drop in a convective cloud with a warm base. For that reason, we draw on the observations of such growth from past research using a vertical wind tunnel at the University of California, Los Angeles (Pflaum 1978). Interestingly, but not surprisingly, variable V_t for a 470- μm frozen drop decreases relative to when it was a liquid drop, and the result is a graupel particle of 1.1 mm with a V_t that is 25% lower than the initial velocity (Fig. 4a). It is possible to use the complete Pflaum dataset where only the beginning and ending points were recorded to compare the initial C_d based on solid ice spheres of the same diameter to the values derived at the end of the experiment. In Fig. 4b, the C_d for the initial frozen drops and for the graupel at the end of growth are plotted. The C_d for solid ice spheres of the same diameter as for the Pflaum experiments (indicated along the abscissa) are derived from the formulation for spheres by Abraham (1970). The final values of C_d are considerably higher than the initial ones.

b. Hail

It has become quite clear that hailstones that fall to the ground become more oblate in their cross sections with size. From collections of hail at the ground in Oklahoma, Colorado, and Alberta, Canada, Knight (1986) derived the minimum to maximum diameters in their approximate fall orientation for the hailstones as functions of their D (Fig. 5a). These ratios—the minimum to maximum diameters taken from thin sections sliced from the hailstones—decrease with increasing size. From that study of 6208 particles, Knight (1986) noted that there was a general tendency toward decreasing sphericity with increasing size. For the IBHS data, where the area ratio is available rather than the aspect ratio, the trend with size is essentially flat (Fig. 5a). We suggest that the measurements of small hail may be subject to their more appreciable melting because of their relatively low mass, which is why they depart from the Knight (1986) data at small sizes. Given this point, it is therefore reasonable to suggest that variable A_r decreases from about 1.0 at ~ 0.2 cm to about 0.7 at ~ 6 cm. The particle cross-sectional area therefore would decrease by about 30% over that size range.

One caveat should be noted. From the structure of hailstones, Knight and Knight (1970) deduced that

FIG. 3. Reynolds number vs Best number relationship for graupel and hail sampled near the 0°C level from earlier studies. (a) Data over the full range of Reynolds and Best numbers from numerous studies. (b) Zoomed view over a restricted range to show the variability. In each panel, the relationship derived by HW is plotted, and plotted as well with the corresponding Re reduced from 1 to 0.8. Also shown is the Böhm (1989) relationship.

hailstones larger than 2 cm in diameter tumble rapidly as they fall. If so, then the area would vary somewhere between the maximum and the minimum cross-sectional areas of the hailstones. However, Roos and Carte (1973) found from their measurements that although hailstones with single or multiple spikes tended to fall with the spikes horizontal and to spin at a rate up to a few rotations per second, the most stable stones were the large oblate ellipsoids that fell with their maximum cross section horizontal. We observed this effect in the wind tunnel; the large, flat stones were very stable, especially as they started to rotate. Given that ellipsoids are most common, it is reasonable to assume that the maximum

cross-sectional area should provide a good estimate of the terminal velocity. This result is supported by the work of Böhm (1989), who found that the use of the maximum cross-sectional area provided a good comparison of theoretical and observed hail terminal velocities.

For the 63 data points from the 2016 season where 3D digital models are available, there is a weak trend for decreasing hailstone volume relative to spheres with increasing size (Fig. 5b), a trend that agrees with Giammanco et al. (2017), who used the first pilot datasets of 3D scanned hail to examine this topic. It is also useful to point out that as a hailstone falls, and tumbling motions are not too extreme, melting will occur on the

FIG. 4. (a) Change in V_t of an initially frozen drop of diameter $470\mu\text{m}$ with time based on 11 growth experiments reported in Pflaum (1978). (b) The change in the derived C_d from the beginning to the end of growth experiments reported in Pflaum (1978) as a function of C_d of solid ice spheres of the diameter. The experiments were conducted over shorter time periods than in (a). The results for the particle used in (a) are plotted with red symbols. For the black symbols, C_d estimated, D , ρ , and V_t at the end of growth were known, but it was necessary to assume that the particles were spherical. The approximate initial and final diameters are shown near where each group of particles is plotted. Boxes and arrows are used to indicate the change in C_d during growth. Shown on the bottom of (b) is D_e .

underside of the stone and less so on the top, decreasing its volume (Rasmussen and Pruppacher 1982). The effect should be more pronounced as the stone size increases, thereby increasing V_t . This point, and the Knight (1986) and IBHS data, support the view that hailstone sphericity relative to that of spheres (of 1) decreases with increasing size.

There have been relatively few measurements of the mass and the ratio of mass to area as a function of the

hail maximum dimension D of naturally grown hailstones that fall to the surface. Here, we draw on an augmented set of measurements reported in H14, derived during the IBHS multiyear field research program conducted in the Great Plains of the United States. As noted in the H14 study, the large dataset continued to illustrate that the masses decrease relative to solid ice spheres as the D increases (Fig. 6a). This can be readily seen from the plotted curve for a density of solid ice

FIG. 5. (a) Hail area ratio or aspect ratio as derived from the IBHS study and those by Knight (1986) from three geographical areas. The flatness of the curve at sizes below 1 or 2 cm in the IBHS study may be due to a more nonuniform melting of these smaller stones. These are mostly from new data. (b) A measure of the hail sphericity from a sample of a recent IBHS dataset where volume was derived directly from the 3D scanned particles.

spheres and also from the right ordinate, which shows variable D_e . The best-fit power law to the data is shown in the figure. In addition, given percentiles of the population in size bins are shown in the figure, and power-law fits to those values are given in Table 2. If only the average $m-D$ and $m/A-D$ relationships are needed, the relationship shown in italic text should be used.

Note that Fig. 6 indicates some points above the line for solid ice spheres. We looked carefully at the data; 7% of the particles have a density (mass/equivalent spherical volume) $>1.0 \text{ g m}^{-3}$, and 10% have a density of between

0.91 and 1.0 g m^{-3} . Some of the latter stones may have water inclusions. We attribute the 7%–10% of the particles above the solid ice sphere line to be the result of measurement uncertainties, including the maximum dimension.

We suggest a reason for the increasing sphericity with decreasing size (Figs. 5b and 6a). Hail of all sizes falling into the melting layer (outside of updrafts) is generally dry as indicated by radar polarimetric data (Ryzhkov et al. 2013) and may be relatively nonspherical. Based on the polarimetric data, the signatures of dry hail aloft gradually transform into the ones typical for rain closer

FIG. 6. Hail (a) mass and (b) mass-to-area relationships as a function of diameter (maximum dimension) as derived from earlier and recent IBHS studies. The number of particles used in each analysis are shown. In (a) small points show data and larger symbols show the values for given percentiles within ranges of sizes. Best-fit power laws to the data are shown in each panel. The right vertical axes describe the equivalent spherical diameter.

to the ground. This is both indicative of decreasing hail size with melting and a tendency toward increased sphericity. Note that raindrops may not be perfectly spherical; raindrop axial ratios become more oblate with increasing size.

c. Mainz experiments

The test particles used in the wind tunnel study were generated by printing 3D computer-aided design models of the hailstones (section 2). The models printed with a white-colored plastic were supposed to have the same dimensions as the actual particle with a density of about 0.91 g cm^{-3} . Figure 7 shows the dimensional relationships derived from measurements of the printed (test) particles relative to the dimensions from the scanner

(measured). Although the lengths (maximum dimensions) of the particles are about the same, the widths of the test particles seem to be about 0.5 cm larger than the actual measured particles (Fig. 7a). The result may be tied to the limits of 3D printing using ABS plastic and any expansion or contraction during the printing process. In addition, printed hailstones were produced in two solid halves to ensure that the surface was well captured. At the time of the study, the capability to produce quality printed hailstones as one solid body had not been successful. The supporting plastic structures needed for this altered the surface too much to be considered useable. The oversizing, of course, shows up as an oversizing of the cross-sectional area of the particle (Fig. 7b). There is no systematic trend noted in the mass

Comparison, Measured vs Test Particles

FIG. 7. Comparison of the measured test particles vs those of the actual 3D images of the hailstones. (a) Length and width; (b) area; (c) mass; and (d) ratio of mass to area, a quantity that is directly proportional to the Best number. In all panels, only the data from particles with printed densities of 0.91 g cm^{-3} (white stones) are shown. In (c) and (d), filled squares are for particles that are very flat and thus would be expected to have different properties than the majority of particles.

measured for the actual particles versus those that were printed (Fig. 7c). Note that there were some hailstones that were far from spherical; these are noted as “very flat particle” in the figure. On average, the ratio of the printed hailstone mass/area to the measured values

shows quite a bit of scatter (Fig. 7d), but this variance would have little effect on our estimates of variable C_d and the Reynolds number–Best number relationships from the wind tunnel measurements. Although the properties of the printed hailstones are not identical to the

original ones, the range of printed particle parameters is within the range of the measured particles and allows us to determine values for C_d , Re , and X for the particles.

Before discussing the results of the experiments, it is useful to mention the tests of similar-size particles. Differences in the terminal velocities of the ice spheres were small: the mean and standard deviations of the terminal velocities of the 11 stones of 2.6-cm diameter were $22.3 \pm 0.25 \text{ cm s}^{-1}$; for the 7 stones of 1.6-cm diameter, $18.5 \pm 0.22 \text{ cm s}^{-1}$; the 6 stones of 1.0-cm diameter, $14.6 \pm 0.14 \text{ cm s}^{-1}$; and for the 2 stones of 0.5-cm diameter, $11.6 \pm 0.15 \text{ cm s}^{-1}$.

The Re – X relationship found from our studies, color coded according to the printed densities (except for the graupel, which all had the same density), shows a linear distribution on a log–log plot (Fig. 8a). The measurements included hailstones that were suspended with a thin wire as well as those freely suspended, with little differences observed. The relationships noted conform quite closely to that developed by Heymsfield and Wright (2014) using data from earlier experiments (see supplementary information); their curve, however, is somewhat lower than that for the graupel Re and somewhat higher than that for the hailstones. In section 4, the Re – X relationship from this study and a number of earlier studies are combined and characterized in terms of both the physical diameter and the equivalent spherical diameter.

Ideally, for the same particle, changing its mass (density) would reveal how, in general, variable C_d would change with the Re , as would be the case, for example, for ice spheres (see asterisk in Fig. 8b). For the individual hailstones, increasing the mass decreases variable Re , all else being equal. However, because of the complexity of the hail shapes, no single relationship between changes in the Re and changes in the C_d are evident for the hailstones. The graupel particles follow the curve developed for spheres, but with variable C_d that are about 25% larger (Fig. 8b). On average, it is noted that increasing the Re leads to a decrease in the C_d .

4. Discussion

In this section, the data for graupel and hail are synthesized, and composite relationships describing the observations are developed. Given the wide range of hail shapes and cross-sectional areas as a function of particle size, it is important to characterize the size dependence of these and associated properties as they strongly influence their Re – X relationship and the resulting values for V_t and kinetic energies. For those reasons, and as previously noted for graupel and hail mass, the size-dependent relationships developed here are presented in ranges of percentiles in Table 2.

Although the data are based on relatively small sample sizes, nonetheless, it is important to show variability in this way. The relationships shown in italic text in Table 2 compose a more extensive dataset as they are based on the median values per size range for all of the particles.

a. Drag coefficients

The relationship between variables C_d and Re derived from the earlier observations (Table 1) and experimental wind tunnel results is plotted in Fig. 9a. Note that the values of C_d and Re are based on the particles' maximum dimensions D rather than D_e . Not surprisingly, there is considerable scatter in the data, a result primarily due to the complex shapes of graupel and hail particles, but also to experimental uncertainties. For example, it is very difficult to determine the cross-sectional area of the particles as they fall; gyrating or tumbling would obviously affect the cross sections. The values of C_d follow a similar trend to that followed by the curve for smooth spheres. This is especially noticeable in the case for the graupel and hail data from our wind tunnel observations and the wind tunnel observations of Pflaum (1978), although the values of C_d are about 25% larger. Our observations conform closely with those Roos and Carte (1973) selected as their most reliable estimates. Because the C_d – Re relationship is not used explicitly to derive values of V_t (these are derived later), no curve relating variable C_d to Re is derived, nor are percentiles given. Note that it was necessary to adjust the Matson and Huggins (1980) data to conform to a definition of C_d and Re in terms of D (see appendix).

Curves showing the results of theoretical and analytical representations of the C_d – Re relationships based on the assumption of solid ice spheres (except where noted) are plotted in Fig. 9b. The reference curve for spheres has the lowest values of variable C_d , although the data (curve fitted here and plotted in the figure) for conical graupel recently derived by Chueh et al. (2018) have relatively low values of C_d as well. The wind tunnel observations for spheres (Fig. 9b) line up nicely with the curve developed by Böhm (1989), for reasons discussed later. Also plotted in Fig. 9b are the C_d – Re values from the experimental data, where the values are derived from the assumption of equivalent diameter spheres. [See the appendix for how the Matson and Huggins (1980) data were derived.] The experimental data line up nicely with the representation developed by Böhm (1989).

b. Reynolds number–Best number

It is important to show the distinction between the Re – X relationships for the idealized spherical hail, represented in terms of D_e , and for natural hailstones, which is shown in Fig. 10a. It is noted that the Re for ice

Best Number, Drag Coefficient $f(\text{Reynolds Number})$

FIG. 8. (a) Reynolds number vs Best number for the test particles. Different colors represent their printed densities. No colors are assigned to the graupel because they were all of the same density. (b) Derived C_d as function of Re . Lines connect the data for the same hail-size particles, with color coding indicating the printed density. Also shown by asterisks in both panels are test particles for spherical solid ice. A curve representing the density for spherical particles from Cheng (2009) is plotted.

spheres with roughness follows a curve where C_d is much closer to 0.4 rather than the expected 0.6. In contrast, the data for natural hail represented in terms of D_e are much closer to values of C_d of 0.8–1.0. The Böhm (1989) curve cuts across the range of C_d from higher to lower values.

A composite of the Re – X data for natural graupel and hailstones represented in terms of the value of D shows a relatively linear relationship on a log–log plot and for graupel alone is also relatively linear (Fig. 10b), indicating that single power laws for graupel and hail combined and for graupel only, respectively,

$$Re = 1.16X^{0.49} \quad (\text{graupel and hail}) \quad \text{and} \quad (5a)$$

$$Re = 0.29X^{0.59} \quad (\text{graupel only}), \quad (5b)$$

are well suited for the true characteristics (mass, cross-sectional area) of these particles. With a constant value of C_d , the exponent in (5a) and (5b) would be 0.5 and >0.5 if the ratio of the particle mass to the area remained the same. The exponent in (5a) is close to 0.5 but slightly lower, reflecting both changes in the drag coefficient and mass-to-area ratio. Power-law relationships are developed

FIG. 9. Drag coefficient vs Reynolds number for graupel and hail. (a) Earlier measurements and those derived from our current wind tunnel experiments are shown. A reference curve for smooth spheres based on [Cheng \(2009\)](#) is plotted. See [Table 1](#) for more details on the earlier graupel and hail studies. (b) Curves representing theoretical and empirical studies (from [Böhm 1989](#); [HW](#); [Wang and Kubicek 2013](#); [Chueh et al. 2018](#)) together with the reference curve for smooth spheres are plotted.

from the data subdivided in percentiles ranging from the 10th to 90th percentiles in [Table 2](#). With increasing percentile, both the coefficient and the exponent in the $Re-X$ relationship increase. If only average $Re-X$ relationships are needed, the relationship shown in italic text should be used.

Along the bottom of [Fig. 10b](#) are the diameters associated with the graupel particles, assuming a density of 0.1 g cm^{-3} . Also shown are diameters for hail, where the mass is taken from [Fig. 6](#):

$$m = 0.37D^{2.69}. \quad (6)$$

Because the hailstones used in the Mainz tunnel had masses that were generally higher than those of natural hailstones, the diameters do not apply to those particles.

In addition to the curve given by (5a) and (5b), the $Re-X$ curve by [Böhm \(1989\)](#) is plotted. This curve provides an excellent fit to the data across the full range of variable X .

To provide more detail for the $Re-X$ relationship derived for graupel and hail particles, [Fig. 11](#) shows the data from [Fig. 10b](#) subdivided by intervals of variable X ranging from 10^3 to 10^9 , with reference lines showing C_d ranging from 0.4 to 1.0. The data for graupel in

FIG. 10. Reynolds number vs Best number for (a) hail and (b) graupel and hail combined. (a) Based on earlier studies, for spherical ice, in some cases with a roughened surface. (b) Earlier results and those from our current wind tunnel measurements.

Figs. 11a and 11b suggest that the values of C_d are surprisingly low, although the Pflaum (1978) data line up nicely with the results from the Wang and Kubicek (2013) study. The graupel data from the wind tunnel look quite reasonable, with values of C_d that are well predicted by the Wang and Kubicek (2013) study (Figs. 11c,d). Likewise, the wind tunnel data for hail have their C_d comparable to what would be expected for spheres (i.e., ~ 0.5). The Böhm (1989) curve nicely fits the data across the full range of Re-X space.

Figure 12a shows a plot similar to Fig. 10a, with only those data where the D_e was derived or could be derived from the data. That included studies by Matson and Huggins (1980),

IBHS, KH, and by Roos (1972), for the Coffeyville hailstone, and Roos and Carte (1973). Also shown are the Mainz tunnel data where the particle mass and volume could be measured, which are used to develop the Re-X relationship (see the fit listed in Fig. 10a and in Table 2). Across the range of variable X shown in the figure, the median ratio of Re derived from the fit to the Laurie (1960) data to the fit of the data is 0.88. This indicates that V_t from the Laurie curve would be overestimated by about 12%.

The distribution of Re as a function of variable X is plotted for the 10th, 25th, 50th, 75th, and 90 percentiles in Fig. 12b. It indicates a relatively narrow range of Re at a given value of X . Also plotted is the Re-X relationship for

FIG. 11. As in Fig. 10, but plotted in intervals of the Best number X . Median values and associated error bars are shown. Also plotted is the curve for graupel sizes developed by Wang and Kubicek (2013).

solid ice spheres from the HW study. Through most of the range of X , Re conforms quite closely to the data. Although the HW relationship begins to deviate for $X > 10^8$, the Böhm (1989) relationship provides a good fit to the median values of the Re – X data over the full range of Re . Given the latter, the HW relationship would appear to work for $X < 10^8$ for graupel and small hail, and the Böhm relationship works well for $X > 10^8$ and not as well as the HW relationship for $X < 10^8$.

c. Terminal velocity

For each of the 2690 hail-size particles collected by the IBHS group, V_t was calculated from (5a) using the measured m , D , and the estimated cross-sectional area, measured from the scanned image when available, or

otherwise from the measured largest and smallest dimensions (Fig. 13). Although hail can fall to the ground located at any altitude above sea level, some standard altitude needs to be used. It was therefore assumed that the hailstones are falling at a pressure level of 1000 hPa. The results presented in Fig. 13 can be adjusted for any pressure altitude using the Re – X relationship and the mass and the area diameter relationships developed here.

There is about a $\pm 50\%$ variation in the terminal velocities around the median values for a given value of variable D . Power-law fits to the V_t – D relationship for hail indicate that, with increasing percentile (Table 2), the coefficient in the relationship increases, not surprisingly, whereas the exponent decreases. The relationship derived for hail from the median value,

FIG. 12. Reynolds number vs Best number based on the equivalent-sphere definition from a number of data sources, including those from the wind tunnel. (a) Data points and fit to the data. (b) Represented as a pseudo-PDF, with 10th, 25th, 50th, 75th, and 90th percentiles shown. Also plotted in (b) is a curve for solid ice spheres based on the HW and Böhm (1989) studies.

$$V_t = 6.1D^{0.72}, \quad (7)$$

where V_t is in meters per second and D is in centimeters, has an exponent that is usually considered to be 0.50, which would be the case for solid ice spheres with a constant drag coefficient. The terminal velocities for all of the particles are significantly below those derived from the Laurie (1960) curve (Fig. 13). To estimate the terminal velocities for the different pressure levels P , (7) can be multiplied by $(1000/P)^{0.55}$ (from HW). With increasing percentile, the coefficient in the relationship increases and the slope decreases (Table 2).

Equation (5b), the Re – X relationship for graupel particles, was used to calculate the V_t for the graupel particles

using their measured mass and cross-sectional areas, at the observed pressure and the temperature levels, and the results were compared to the measured values. The mean and median ratio of the calculated to measured values were 0.91 ± 0.31 and 0.89, respectively, indicating that the fit overall is quite good. Application to sea level conditions, 1000 hPa, results in the following V_t – D relationship:

$$V_t = 6.35D^{0.97}. \quad (8)$$

d. Kinetic energy

The size-dependent hailstone kinetic energy—the hailstone kernel— $KE(D)$, can readily be found from

FIG. 13. Terminal velocity for particles of hail sizes as derived from the IBHS mass and cross-sectional area data and the Re – X relationship. A least squares curve fit to the median values, the subdivision of velocities by percentile, and the Laurie (1960) curve are shown. The blue open squares are for the Coffeerville, Kansas; Vivian, South Dakota; and Aurora and Potter, Nebraska, giant hailstones discussed in H14.

$$KE = (1/2)mV_t^2. \quad (9)$$

As with V_t , KE (J) was calculated using the IBHS measured m , D , and estimated cross-sectional area. Note that horizontal wind will add to KE in actual hailfall conditions.

In Fig. 14, the calculated KE are plotted as a function of their D , along with a fit to those data. Interestingly, the fitted curve,

$$KE = 0.0243D^{3.55}, \quad (10)$$

deviates considerably from the Laurie (1960) curve. The latter is identical to that derived by Waldvogel et al. (1978), because it draws on the same Bilham and Relf (1937) dataset. This deviation is presumably because the smallest hailstones that fall to the ground are nearly spherical and solid ice. The equation by Schmid et al. (1992),

$$KE = 0.0458D^4, \quad (11)$$

is still used in Europe.

Also shown in Fig. 14 are the percentiles derived in each of the size ranges plotted in the figure, with fits to those percentiles presented in Table 2, along with the representative 90th-percentile curve highlighted in red. The 90th percentiles line up relatively close to the

Laurie (1960) curve. Note that the uncertainty in KE plotted values is unknown, because V_t for each of the particles had to be estimated, whose uncertainties are compounded because KE is proportional to variable V_t^2 . Unfortunately, the printed particles used in the Mainz wind tunnel experiments could not be used to derive the KE kernels corresponding to natural hailstones because 1) we deliberately used a range of densities such that we could reliably derive the Re – X relationship rather than a density of solid ice and 2) the dimensions of the tested particles differed from their natural counterparts (Fig. 7).

This new fitted relationship can be compared to the KE relationship given in H14, and it differs primarily in the use of a more reliable Re – X relationship than was used to derive V_t . Note that the information presented in Fig. 3a of H14 inadvertently showed two relationships for deriving KE over different size ranges. The second relationship, $KE = 12.6943D^{4.69}$, was due to an error in plotting their figure and should not have been included, and therefore, only the first one, $KE = 0.0379D^{3.525}$, should be used. The ratio of the relationship for KE from that equation, $2.0/0.96$, covers the size range 1–7 cm.

It is also useful to compare the terminal velocities and kinetic energies, represented in terms of D_e , to those derived from the Laurie (1960) study. As noted in Fig. 15, the observations for natural hail fall considerably below

FIG. 14. (a) Kinetic energy as a function of hail diameter (kinetic energy kernel). The IBHS hail collections—diameter, area, mass, and terminal velocity calculated from the $Re-X$ relationship developed here—are used to derive $KE(D)$. Also shown are percentiles (listed) derived from the data between each of the midpoints between the size bins. The Laurie (1960) and Waldvogel et al. (1978) curves—which are essentially identical—are plotted. A best-fit line to the data is shown. (b) Summary of the results shown in (a) for clarity.

those from the Laurie study. Presumably, this is due to the likelihood that roughening on the surfaces of natural hail increases the value of C_d relative to those of smooth hail.

e. Mass flux

The precipitation rate from hailstones are rarely measured or derived from hail pads and other sensors. Toward such an effort, the mass flux kernel (MF)—or, equivalently, the momentum—can be calculated from

$$MF = mV_t. \quad (12)$$

Multiplication of the MF times the concentration as a function of size and conversion constants yields the hail precipitation rate. The data are presented in Fig. 16, and the resulting relationship representing the median percentile of the data,

$$MF = 188D^{3.35}, \quad (13)$$

is very close to what would be expected from the use of the product of the terms in (10) based on (6) and (7). Also, the relationships derived from the percentiles are what would be expected from the product of the individual terms (Table 2).

FIG. 15. Properties of hail as represented by equivalent spherical diameter: (a) terminal velocity and (b) kinetic energy. Data are from the same sources identified in Fig. 12.

5. Summary and conclusions

Several properties of falling hail, covering the size range from about 0.5 to 7 cm, and the drag coefficients of lump and conical graupel, have been estimated in this study. The analysis draws on three-dimensional printed models of hailstones and graupel particles, whose terminal velocities were measured in the Mainz vertical wind tunnel, as well as data presented in earlier studies. The Reynolds number–Best number relationship allowed us to estimate the size-dependent terminal velocity, kinetic energy, and mass flux of realistic hailstones. The utility of this approach is that it does not depend on the specific density of the hailstones. Table 3 shows relationships that have been developed here and during the past few years to calculate various size-dependent and bulk properties of graupel and hail and

compares them to widely used relationships developed by Laurie (1960) and Waldvogel et al. (1978) that draw on the work of Bilham and Relf (1937). To summarize, the mass of hailstones falling at the surface can be represented as

$$m = 0.37D^{2.69}, \quad (14)$$

their terminal velocities as

$$V_t = 6.1D^{0.72}, \quad (15)$$

and their kinetic energies as

$$KE = 0.0243D^{3.55}. \quad (16)$$

Albeit with a smaller dataset and without measurements of the drag coefficients and Reynolds numbers of

FIG. 16. As in Fig. 13, but for hail mass flux or, alternatively, momentum as a function of D .

graupel and hail, Heymsfield and Wright (2014) also showed that natural hailstones fall much more slowly than those of solid ice spheres. It also adds to the H14 study, which focused on the kinetic energy of natural hailstones but without the detailed information on the Reynolds numbers and the drag coefficients derived here. A detailed comparison of the differences between those studies and that presented here is given in the supplementary information.

What is most surprising to us is how well the Böhm (1989) Reynolds number–Best number relationship fit the observations across most of the range of particle sizes from small graupel to large hail, especially for graupel when turbulence (Böhm 1992) is considered. With the use of the Böhm (1989) relationship and that developed here between the ratio of the particle mass to the cross-sectional area and the particle maximum or the equivalent spherical diameter, it is now justifiable to calculate the terminal velocities and the kinetic energies of rimed ice particles over a wide range of particle sizes and heights in the atmosphere on a rigorous theoretical basis. For graupel, the HW relationship, with the Re reduced by 20%, fits the observations extremely well.

One of the primary findings of the present study is the large variability we found in the terminal velocities of graupel and hail. The underlying finding suggests that models should take into account the statistical variability in the terminal velocity of natural graupel and hail in order to realistically model their growth. Also, there is an important need to properly account for the decrease

in the terminal velocity 1) of frozen drops that rim to become conical or lump graupel (Fig. 4a, based on Pflaum 1978), 2) as graupel fall to temperatures above 0°C (Kajikawa 1975), and 3) due to sublimation in subsaturated regions slightly above to below 0°C . Although most of the available graupel data reported in the literature are presented here, there is still a strong need for comprehensive measurements of graupel densities and cross-sectional areas under a wide range of conditions, such as warm and cold cloud-base temperatures, to facilitate more progress in this area.

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APPENDIX

Modifications to the Matson and Huggins (1980) Data

Matson and Huggins (1980) compiled a comprehensive, detailed set of hail data collected in Colorado. Their study included the relationship between the terminal velocity V_t and the particle equivalent spherical diameter D_e (their

Fig. 11), the relationship between the drag coefficient C_d and the Reynolds number Re (their Fig. 16) and the mean relationship between D_e and the ratio of the minimum $D_{\min}$ to the maximum diameter D (their Table 2). The diameter D_e was derived from the measured particle mass m .

Because of the way they derived C_d and Re and our desire to represent their data in terms of the X and Re values using both D_e and D , it was necessary to modify the data presented in their Figs. 11 and 16. Unfortunately, their original data are no longer available (A. Huggins 2017, personal communication).

Because their V_t and D_e values were directly measured, these parameters needed only minor modifications to make them more useful. The V_t values were derived for a pressure P of about 835 hPa rather than the more standard 1000 hPa. For the 548 data points picked off from Fig. 11, V_t was modified by the fraction $(835/1000)^{0.5} = 0.91$. The Re was derived accordingly. Furthermore, it was necessary to adjust the kinematic viscosity ν for its value at 835–1000 hPa (decrease of 15%). The values of $D_{\min}$ and D of each hailstone were estimated from the value of D_e by using the dependence of $D_{\min}/D$ as a function of D_e in Table 2. The particle cross-sectional area A was derived from the $D_{\min}$ and D values. Re was derived using an area-weighted diameter rather than a mass-weighted diameter. All the parameters were now available to derive an Re – X relationship based on the value D_e but also to derive it from D . The kinetic energy KE was derived as a function of variable D from the values of m and V_t .

The C_d -versus- Re data were modified in the following ways. Because variable D_e is based on the particle mass but was used in the calculation of Re , it was necessary to adjust Re using an area-weighted diameter rather than a mass-weighted diameter. Adjustments to the Re were made to compensate for the value of P . The C_d values were modified [see their (11)] to account for pressure, and an area-weighted D rather than variable D_e was used. An Re – X and KE – D relationship could be derived both as a function of the values of D_e and D .

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