The Effect of Turbulence on the Accretional Growth of Graupel^①

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ABSTRACT

Wind tunnel experiments were carried out to investigate the influence of turbulence on the collection kernel of graupel. The collection kernel defines the growth rate of a graupel accreting supercooled droplets as it falls through a cloud. The ambient conditions were similar to those occurring typically in the mixed-phase zone of convective clouds, that is, at temperatures between -7° and -16°C and with liquid water contents from 0.5 to 1.3 g m^{-3} . Tethered spherical collectors with radii between 220 and $340\text{ }\mu\text{m}$ were exposed in a flow carrying supercooled droplets with a mean volume radius of $10\text{ }\mu\text{m}$. The vertical root-mean-square fluctuation velocity, the dissipation rate, and the Taylor-microscale Reynolds number of the turbulent flow were determined as $u_{\text{rms}} = 0.13\text{ m s}^{-1}$, $\varepsilon = 0.13\text{ m}^2\text{ s}^{-3}$, and $R_{\lambda} = 48$, respectively. The collection kernels of tethered graupel grown under laminar and turbulent conditions revealed no measurable difference, indicating that turbulence has no effect on the growth of graupel in the investigated size range. A comparison of laminar collection kernels to theoretically calculated values from a continuous growth model showed that graupel growth is strongly dominated by the fast increase of the radius due to the accretion of rime ice with low density. It is assumed that, compared to a water drop growing by collision and coalescence, this causes a fast increase in the swept volume overcompensating all other effects such as the self-induced stochastic movements due to surface roughness and latent heat release, as well as the possible influence of the flow's small-scale turbulence.

1. Introduction

The most effective pathway to form precipitation sized ice particles in the mixed-phase zone of convective systems is riming, that is, the accretion of supercooled

droplets that freeze onto the surface of the ice particle upon collision. Ice particles growing in that way are termed as graupel as long as their maximum dimensions do not exceed 5 mm , beyond which they are called hail (according to the AMS Glossary). Compared to riming, the growth of ice particles by water vapor diffusion is very slow and of minor importance in the mixed-phase regime (Brdar and Seifert 2018).

In recent years efforts have been made to characterize the influence of turbulence on the collection kernel of cloud droplets. Grabowski and Wang (2009, 2013)

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performed direct numerical simulation (DNS) studies to solve the size-gap problem in warm rain initiation. They found that cloud turbulence considerably increases the collection kernel of cloud droplets from 10 to 50 μm while growing due to collision-coalescence in comparison to a laminar case. For larger drops the influence of turbulence becomes less dominant but is still significant. This was shown in the experimental wind tunnel study of [Vohl et al. \(1999\)](#), who demonstrated that the collection kernel of drops in a size range of 100- μm radius is still increased by up to 20% in turbulent flows.

However, very little experimental information about the influence of turbulence on the collection kernel of ice particles such as graupel is available. There are only a few model studies in literature that address the effect of turbulence on the collection kernel of rimed ice particles ([Naso et al. 2018](#); [Pinsky et al. 1998](#); [Pinsky and Khain 1998](#)). In the latter two studies, it was indicated that the collection kernels of the ice particles are enhanced due to a higher relative velocity between the ice particles and the supercooled drops in a turbulent airflow. This is especially true for ice particles with low density since their settling velocity and their inertia are smaller than those of solid frozen spheres. Hence, the relative velocities, and thus the collection kernels may be higher than in a gravity driven case. More recently, [Naso et al. \(2018\)](#) investigated the influence of turbulence on disk-shaped ice crystals and also found that turbulence enhances the collision rate with increasing values of the flow dissipation rate ε .

However, particularly because the tropospheric ice phase contributes very effectively to the formation of global precipitation (e.g., [Mülmenstädt et al. 2015](#)), it is crucial to determine such collection kernels of graupel in a turbulent flow. A better understanding of that process would improve the predictability of the microphysical structure of clouds as well as integral quantities such as ice water content, hailstone size, and surface precipitation by models. This motivated us to conduct experimental wind tunnel studies addressing the question, to what extend small-scale cloud turbulence contributes to the collection kernel of initially spherical ice particles during conical growth (see [Fig. 1](#)) in a dry growth regime.

2. Theory

The mass growth rate of a single solid sphere suspended within a cloud of a polydisperse droplet size distribution can be described by the continuous growth equation ([Pruppacher and Klett 2010](#)):

FIG. 1. (right) Schematic depiction of the experimental setup. Also shown are (top left) an unrimed spherical epoxy collector and (bottom left) a rimed collector after a typical riming time period of 120 s. Labeled in the right panel are the positions of the collector (A), turbulence producing ring obstacle (B), cloud droplet generation (C), and liquid water and temperature measurements (D).

$$\frac{dm(t)}{dt} = \frac{4\pi\rho_w}{3} \int_0^{r_{\max}} K[R(t), r] n(r) r^3 dr, \quad (1)$$

where m is the mass of the collector sphere changing with time t , R and r describe the radii of the collector and the colliding droplets (m), and $n(r)$ is the droplet size distribution. The gravitational collection kernel K ($\text{m}^3 \text{s}^{-1}$) is given by [Pruppacher and Klett \(2010\)](#):

$$K[R(t), r] = E_{\text{coll}} E_{\text{coal}} \pi [R(t) + r]^2 |V - v|, \quad (2)$$

where V and v are the corresponding settling velocities of the collector and the colliding droplets, respectively (m s^{-1}). The E_{coll} is the collision efficiency, and E_{coal} is the coalescence efficiency; E_{coal} is assumed to be unity here, which implies that each time a droplet hits, it freezes onto the hydrometeor.

According to [Ayala et al. \(2008\)](#) four effects may enhance the collisional growth rate of drops in a turbulent flow by increasing the collection kernel compared to the laminar case:

- 1) Inertia effect: The motions of the droplets are influenced by local flow accelerations that result in an increase of their relative motion.

- 2) Accumulation effect: In turbulent flows, regions of high droplet concentrations may form, that is, inertial droplets accumulate in regions of low vorticity. In this way, the collection kernel may be enhanced by a nonuniform droplet distribution.
- 3) Settling effect: The settling velocity of sedimenting droplets may be significantly enhanced due to downward flow motions around vortices.
- 4) Hydrodynamic effect: When droplets approach the collector sphere, their relative velocities change in orientation and magnitude leading to a significant reduction of the radial relative velocities.

In our wind tunnel experiments we were not able to distinguish among all these effects but rather observed an overall impact. However, one can define characteristic numbers for estimating the influence of flow turbulence on the motion of the collector and the droplets by a scale analysis. The Stokes number (St) is defined as the ratio of the drop's relaxation time and a characteristic time describing the motion of the drop due to turbulent flow accelerations. A strong interaction between the flow and the particles occurs when the Stokes number is on the order of 1 (Ayala et al. 2008). Thus, particles that satisfy this criterion have enough inertia to accumulate in regions of preferential concentrations. Values of St much larger or much lower than 1 do not contribute much to the accumulation effect. Another important number is the dimensionless velocity (S_v). By definition it is the ratio of the eddy turnover time to the time it takes for a drop to sediment across that eddy (e.g., Devenish et al. 2012; Wang and Maxey 1993). Thus, it is a measure for the interaction time of drops that fall under gravity with turbulence. Hence, a value of S_v much larger than 1 denotes that the drop sediments fast across an eddy. In that case the eddy does not influence the motion of the drop. In contrast, a value much lower than 1 denotes that sedimentation does not play any significant role in reducing the time of drop–eddy interaction (Ayala et al. 2008).

3. Methodology

a. Wind tunnel

In the Mainz vertical wind tunnel single hydrometeors with radii from tens of microns up to several centimeters can be observed while suspended at their terminal velocities for long times under simulated atmospheric conditions. The tunnel air temperature covers the range of the tropospheric temperature profile relevant for mixed-phase clouds (i.e., from -30° to 0°C). A key for investigating turbulence effects in the wind tunnel is that the airflow is laminar with a residual turbulence

intensity (i.e., the root-mean-square fluctuation velocity over the mean vertical tunnel velocity) less than 0.5% (Vohl et al. 1999). This allows one to artificially generate a turbulent flow by placing obstacles into the airflow upstream of the location where the hydrometeors are levitated. A more detailed description of the wind tunnel design and characteristics can be found in the review articles of Szakáll et al. (2010) and Diehl et al. (2011).

b. Experimental procedure

Freely levitating ice particles in a turbulent environment over a time period of minutes is nearly impossible. Therefore, we decided to suspend the collector spheres in a size range from 220 to $340\ \mu\text{m}$ on a thin copper wire of $20\ \mu\text{m}$ in diameter. This setup was mounted in a way that the collectors could be positioned in the center of the turbulent flow (see Fig. 1). Then the wind tunnel was operated at speeds that are equivalent to the suspended object's terminal settling velocity. As can be estimated from the scale analysis, even the integral scales in the turbulent flow of the wind tunnel would affect the motion of an ice collector only weakly ($S_v = 17$ for typical experimental conditions; see Table 1). It is emphasized that the collectors were not tethered in a fixed position. By using a very thin copper wire, they could move in a horizontal plane that encompasses a diameter of about 15 mm. Thus, the limitation due to the tethering disables only the motion in the vertical direction. Even though the motion of the collector due to turbulence is expected to be weak, at least the horizontal components of the turbulent velocity fluctuations, which contain two-thirds of the turbulence energy, could affect the motion of the collector, and thus the collection kernel in our experiments. The initial collector spheres were produced out of epoxy because it was not possible to let a water drop freeze directly on the wire. The epoxy spheres had an overall bulk density of $1.09\ \text{g cm}^{-3}$, which is close to pure ice density. Test experiments with epoxy spheres previously coated with ice yielded no different values of the collection kernel compared to those with pure epoxy spheres. This indicates that the riming of an epoxy surface evolves similarly to that of an ice surface. Prior to each individual measurement the size of the collector sphere was determined using a microscope coupled to a commercial digital single-lens reflex camera (D5100 16MP, Nikon Inc., Tokyo, Japan). To cover a broader collector size range the radii of the spheres were varied between 220 and $340\ \mu\text{m}$. Knowing the size, the corresponding terminal velocity for an equivalent ice sphere was calculated by applying the parameterization of Beard and Grover (1974, hereafter BG74). The resulting velocities were in the range from 1.6 to $2.3\ \text{m s}^{-1}$.

TABLE 1. Juxtaposition of the flow and particle characteristic quantities of the experiments with typical values prevailing in a well-developed convective cloud. All experimental values were calculated for a mean tunnel velocity of 2.1 m s^{-1} , which corresponds to the terminal fall velocity of a mean collector size of $282 \mu\text{m}$. The values for cloud conditions were calculated for a dissipation rate of $0.04 \text{ m}^2 \text{ s}^{-3}$ and a root-mean-square fluctuation velocity of 2 m s^{-1} (e.g., Grabowski and Wang 2013; Shaw 2003; Siebert et al. 2007).

Quantity	Symbol	Unit	Experiments	Cloud	Equation
Root-mean-square velocity	u_{rms}	m s^{-1}	0.13	2	$u_{\text{rms}} = c\bar{u} + d$
Dissipation rate	ε	$\text{m}^2 \text{ s}^{-3}$	0.13	0.04	$\varepsilon = a\bar{u}^b$
Kolmogorov length	η_K	m	3.4×10^{-4}	4.7×10^{-4}	$\eta_K = (v^3/\varepsilon)^{1/4}$
Kolmogorov time	τ_K	s	9.3×10^{-3}	1.8×10^{-2}	$\tau_K = (v/\varepsilon)^{1/2}$
Kolmogorov velocity	v_K	m s^{-1}	3.7×10^{-2}	2.7×10^{-2}	$v_K = (v\varepsilon)^{1/4}$
Taylor length	λ	m	4.6×10^{-3}	1.4×10^{-1}	$\lambda = \left(\frac{15vu_{\text{rms}}^2}{\varepsilon}\right)^{1/2}$
Taylor-microscale Reynolds number	R_λ	—	4.8×10^1	2.2×10^4	$R_\lambda = u_{\text{rms}}(\lambda/v)$
Integral length	l	m	1.5×10^{-2}	2.0×10^2	$l = u_{\text{rms}}/\varepsilon$
Integral time	τ_l	s	1.2×10^{-1}	1.0×10^2	$\tau_l = l/u_{\text{rms}}$
Mean collector radius	a_c	m	282×10^{-6}	—	—
Mean collector settling velocity	v_c	m s^{-1}	2.1	—	—
Mean volume droplet radius	a_d	m	10×10^{-6}	—	—
Mean droplet settling velocity	v_d	m s^{-1}	1.3×10^{-2}	—	$v_d = \frac{2a_d^2 g(\rho_d - \rho_a)}{9\eta_a}$
Stokes droplet radius	$a_{d,\text{Stokes}}$	m	26×10^{-6}	36×10^{-6}	$a_{d,\text{Stokes}} = (9\rho_a v \tau_K / 2\rho_d)^{1/2}$
Relaxation time collector	τ_c	s	2.2×10^{-1}	—	$\tau_c = \left(\frac{2\rho_d a_d^2}{9\rho_{av}}\right) \left(\frac{24}{C_D N_{\text{Re}}}\right)$
Relaxation time droplets	τ_d	s	1.2×10^{-3}	—	$\tau_d = \frac{2\rho_d a_d^2}{9\rho_{av}}$
Stokes number collector (Kolmogorov)	$\text{St}_{c,K}$	—	24	12	$\text{St}_{c,K} = \tau_c/\tau_K$
Stokes number collector (integral)	$\text{St}_{c,l}$	—	1.9	2.2×10^{-3}	$\text{St}_{c,l} = \tau_c/\tau_l$
Stokes number droplets (Kolmogorov)	$\text{St}_{d,K}$	—	0.12	7.2×10^{-2}	$\text{St}_{d,K} = \tau_d/\tau_K$
Dimensionless velocity collector (Kolmogorov)	$\text{Sv}_{c,K}$	—	57	78	$\text{Sv}_{c,K} = v_c/v_K$
Dimensionless velocity collector (integral)	$\text{Sv}_{c,l}$	—	17	1.1	$\text{Sv}_{c,l} = v_c/u_{\text{rms}}$
Dimensionless velocity droplets (Kolmogorov)	$\text{Sv}_{d,K}$	—	0.35	0.48	$\text{Sv}_{d,K} = v_d/v_K$
Mean Reynolds number collector	$\overline{N_{\text{Re},c}}$	—	100	—	$\overline{N_{\text{Re},c}} = \frac{2a_c v_c}{v}$

Figure 1 shows a schematic depiction of the experimental setup. The collector spheres were positioned in the center of the upper experimental section (labeled A). Afterward, the extendable turbulence producing ring obstacle was moved into the center of the lower experimental section (labeled B). The cloud of super-cooled droplets was generated in the lower part of the tunnel by four spraying nozzles (Air Atom 1/4 J, Spraying Systems Deutschland GmbH) (labeled C). During the subsequent riming period of 120 s the adjusted terminal velocity was kept constant and corresponded to that of the initially unrimed collector. Over that period of time, for a spherical collector of $310\text{-}\mu\text{m}$ initial radius, the final velocity (2.21 m s^{-1}) differs only 5% compared to the initial velocity (2.32 m s^{-1}). However, the terminal velocity of a freely levitated graupel initiated by a frozen drop passes through a velocity minimum (2.15 m s^{-1}) and then increases again (Heymsfield et al. 2018). Hence, on average, the terminal velocity of a

freely levitated graupel is lower than in our case but differs by less than 8% at most. To compare the present results with earlier wind tunnel studies on the growth of graupel, a growth time of 120 s was chosen. (A video of a growing graupel particle in the wind tunnel with an initial embryo size of 1 mm and an ambient air temperature of -10°C is provided in the online supplemental material.)

For the experimental determination of the collection kernel the knowledge of the rate of change of mass and the liquid water content during a certain growth time were necessary [see Eq. (5)]. The liquid water content was obtained by measuring the dewpoint of air containing the evaporated droplets and of air without droplets (measurement position is labeled D in Fig. 1). For that, the air and air-droplet system was isokinetically sucked through a heated copper pipe that was coupled to a dewpoint hygrometer (DP3-D/SH, MBW Elektronik AG, Switzerland). The liquid water content was calculated

from the difference of the two dewpoint measurements. Values ranged from 0.5 to 1.2 g m^{-3} with most frequent values around 0.9 g m^{-3} . After the riming period the rimed particle was removed from the tunnel and the rime ice of the collector was melted at the interface of two immiscible liquids, one with a higher density than water (1,1,2-trichlorotrifluoroethane, $\rho = 1.6 \text{ g cm}^{-3}$) and one with a lower density than water (paraffin oil with low viscosity, $\rho = 0.9 \text{ g cm}^{-3}$). The mass of the rime ice was determined from the spherical drop of the molten rime ice using a microscope coupled to a digital camera. The ambient temperatures during the measurements ranged from -15° to -9°C with most frequent values of about -11°C . The experiments were performed in an alternating manner, that is, one growth experiment in turbulent flow and the ensuing one in laminar flow. This allowed us to quantify the influence of turbulence on the collection kernel and directly compare it to the measurements in laminar flow.

c. Characteristics of droplet population and riming conditions

Size spectra were measured 11 times during the set of experiments under typical experimental conditions, that is, at a velocity of 2.1 m s^{-1} and at a temperature of about -11°C . The top panel of Fig. 2 shows the mean distribution of the droplet number concentration measured by a Classical Scattering Aerosol Spectrometer Probe Electronics (Particle Measurement Systems Inc.) at the position where the collectors were located during the experiments. In the bottom panel of Fig. 2 the normalized mass distribution is shown, which was obtained by dividing the water mass of one bin by the total mass of the distribution. The arithmetic mean diameter of the droplet number distribution was $8.3 \mu\text{m}$, while the mean volume diameter was $21 \pm 11 \mu\text{m}$. The errors due to count statistics ranged from 10% for the highest concentrations (small droplet sizes) to 70% for the lowest concentrations (largest droplet sizes). The large error for the largest droplets was caused by the small number of detections in that size range. The average standard deviation due to the averaging of the 11 droplet size measurements is 25%. According to the Schumann–Ludlam limit the growth conditions corresponded to the dry growth regime meaning that the surface temperatures of the riming graupel did not exceed 0°C . This was confirmed from calculations balancing the latent heat released from the freezing droplets and the heat transported to ambient air (Macklin and Payne (1967), Eq. (1) as well as from measurements of von Blohn et al. (2009, hereafter VB09).

FIG. 2. (top) Averaged number concentration distribution of the rime droplets and (bottom) liquid water content normalized per bin size measured in situ within the observation section of the wind tunnel. Error bar of each bin indicates the standard deviation due to the averaging of the 11 different droplet size measurements.

d. Flow and particle characteristics

The turbulent flow was produced by employing the same setup as in a previous study of Vohl et al. (1999). The level of turbulence in the wind tunnel was characterized by means of a hot wire instrument (TSI Inc., Shoreview, Minnesota, IFA-100). The measured turbulence energy spectra agreed well with the empirical one-dimensional energy spectra given by Townsend (1976) and Grover and Pruppacher (1985) and was consistent with atmospheric observations (Vohl et al. 1999). It was found that the dissipation rate ε depends strongly on the mean air velocity (Vohl et al. 1999). As the three discrete values of the vertical velocities measured by Vohl et al. (1999) were in most cases lower than the velocities adjusted in the present study (PS), the energy dissipation rate was extrapolated for higher mean vertical velocities $\bar{u}$ according to the following fit function (see Fig. 3a):

$$\varepsilon = a\bar{u}^b, \quad (3)$$

where $a = 9.3 \times 10^{-3}$ and $b = 3.52$ are fit coefficients. In a similar way a continuous relationship of the root-mean-square velocity u_{rms} , and the mean tunnel air velocity was obtained by applying a linear regression on the measured data given in Vohl et al. (1999) and Vohl (2000) (see Fig. 3b):

$$u_{\text{rms}} = c\bar{u} + d, \quad (4)$$

FIG. 3. (left) Measured dissipation rates at three different mean wind tunnel air velocities. (right) Measured root-mean-square velocity fluctuations at different mean wind tunnel air velocities. Black data points are from Vohl et al. (1999) and Vohl (2000); solid lines are data fits as given in Eqs. (3) and (4). The red and blue data points are determined from the COMSOL simulations and are discussed in section 4.

where $c = (6.5 \pm 0.9) \times 10^{-2}$ and $d = (-1.1 \pm 1.7) \times 10^{-2} \text{ m s}^{-1}$ are fit coefficients with their 95% confidence intervals. The error bars of the black data points in Fig. 3b were estimated as 5% (Vohl 2000). Flow and particle characteristic numbers for the experiments as well as for the ones typically observed in convective clouds are listed in Table 1. The quantities with indices K , λ , and l define the length and time scales associated with the Kolmogorov scale, the Taylor microscale, and the integral scale, respectively. The characteristic quantities for the collectors are indicated with index c and those for the droplets with index d . Apparently, the small-scale turbulence of the flow in the wind tunnel, defined by the Kolmogorov microscale, is in good agreement with that of a natural cloud. That is, the rime droplets are in both cases affected in a similar way. However, the Taylor microscales as well as the integral scales in the wind tunnel differ considerably from a real cloud. This deviation is caused by the wind tunnel cross-sectional area of $17 \text{ cm} \times 17 \text{ cm}$, which poses experimental limits to the largest scales of the turbulence that can be simulated in the wind tunnel.

e. Analysis methods

The collection kernels K were calculated by the discretized version of the continuous growth model equation (Pflaum and Pruppacher 1979, hereafter PP79; VB09):

$$K = \frac{\overline{dm}/dt}{\overline{w}_L}, \quad (5)$$

where $\overline{dm}$ is the measured rimed mass of accreted droplets on the surface of the collector sphere during the

growth time dt , and $\overline{w}_L$ corresponds to the liquid water content measurements averaged over the growth period. The momentum p corresponding to the kernel of a certain collector was determined by multiplying the weighted averaged graupel mass during a 120 s growth period $\overline{m}_{\text{gr}}$ with the constant initial velocity corresponding to that graupel $\overline{v}_{\text{term}}$:

$$p = \overline{m}_{\text{gr}} \overline{v}_{\text{term}}. \quad (6)$$

Weighted averaging was applied because the graupel grew conically (see Fig. 1) at a constant flow velocity, which resulted in a quasi-quadratic mass increase with time. Therefore, the power-law relationship for conical graupel given by Cober and List [1993, their Eq. (11)] was used:

$$x = a_x t^{b_x}, \quad (7)$$

where x is the accreted mass, a_x is a factor describing the riming rate (mg min^{-1}), t is the time, and the exponent b_x is a fit parameter depending on the liquid water content and the impact velocity. Before using this equation, the riming rates for each graupel were determined. Afterward, the mass increase was calculated for each second of the growth time. The average $\overline{m}_{\text{gr}}$ was obtained by adding the initial mass of an equivalent ice sphere to the arithmetic mean of the 120 data points. According to the calculation given in PP79, the growth rate $(dm/dt)_{\text{diff}}$ due to diffusional uptake of water vapor was in all cases in the range of 1% or lower. Therefore, it was neglected in the computation of the collection kernel and the momentum.

FIG. 4. High-resolved computational mesh, which consists of 31×10^6 elements. (a) The element sizes on a horizontal x - y slice at the vertical location of the measurement points. (b) Vertical view of the locally refined mesh in the turbulence area with elements filtered up to the size of 2.2×10^{-3} m.

4. Simulations of the turbulent flow with COMSOL

Since the characterization of the turbulent flow relies solely on the extrapolations of the measurements from Vohl et al. (1999) we decided to additionally simulate the flow field inside the wind tunnel with the partial differential equation software COMSOL Multiphysics (COMSOL 2015). The simulated flow characteristics, such as the RMS velocity, dissipation rate, and turbulence spectrum were then compared to the ones obtained from the extrapolations derived from the measurements of Vohl et al. (1999). For the simulations, the laminar flow module of COMSOL and direct numerical simulations via incompressible Navier–Stokes equations were applied. First-order finite elements (P1 + P1) were used for velocity as well as for pressure discretization. To resolve the eddies and also to limit the computational costs a locally refined computational mesh in the turbulence area was used. The 4.8×10^6 mesh elements with element sizes between 4.07×10^{-4} and 1.27×10^{-2} m (and a chosen time step of 5 ms) were distributed in such a way that the spatial resolution in the turbulence region was as high as possible. The mesh was locally refined in the cylindrical area where turbulence appears, that is, around (with element sizes of 0.4–0.8 mm) and behind (with element sizes of 0.4–5.9 mm) the ring obstacle, and thus, corresponding partially to the Kolmogorov scale there. To investigate the impact of the spatial discretization scales on the numerical simulation results, a further numerical experiment with a higher spatial resolution was conducted by two additional mesh refinements inside the cylindrical area around the obstacle, resulting in 31×10^6 elements. The final mesh of this experiment is illustrated in Fig. 4. Here we received about 4-times-smaller element sizes. Analyzing the second simulation, we found a

negligibly small difference in the validated RMS velocity in comparison to our first model (see the blue dot in Fig. 3b).

The corresponding inflow boundary condition of the model (inflow velocity) was set in a way, that the mean vertical velocity in the experimental section of the wind tunnel was 2.0 m s^{-1} . Figure 5 shows the modeled wind tunnel flow around the ring obstacle, centered at the inlet of the experimental section. It can be seen that the velocity downstream of the ring decreases, and the streamlines indicate that eddies develop in the center of the experimental section. They decrease in intensity with increasing distance from the ring obstacle.

A one-dimensional energy spectrum $S(k)$ was determined for the vertical velocity fluctuations w' by utilizing a discrete fast Fourier transformation $F_w(k_n) = \sum_{j=0}^{N-1} [w'(t_j)/N] e^{-ik_n[\bar{w}/(2\pi)]t_j}$ to the simulated data, where N is the finite number of data points, k_n denotes the single wavenumbers for $n = 1, \dots, N/2$, i the imaginary unit, $\bar{w}$ the mean vertical velocity, and j the j th data point with $t_j = t_0 + j\Delta t$, where Δt is the sampling interval. The discrete time series were calculated up to 3 s with a time step of 0.5 ms. In this way the eddy turnover times (see Table 1) were resolved. Before applying the discrete fast Fourier transformation, the data were multiplied by a bell taper function (Stull 1988) in order to ensure a periodic record. After calculating the square of the norm of the Fourier transformation and taking the Nyquist frequency into account, the discrete energy spectrum $E_w(k_n) = 2|F_w(k_n)|^2$ was obtained. The spectral energy density $S(k)$ was calculated by dividing the discrete energy spectrum $S_w(k_n) = E_w(k_n)/\Delta k$ by the wavenumber interval $\Delta k = k_{i+1} - k_i$. Analogously to the experiments of Vohl et al. (1999)

FIG. 5. Simulation of the wind tunnel flow with COMSOL. Shown is the velocity field for the vertical component (depicted by the color code) at a central cross section of the tunnel and streamlines (pink), i.e., tangents to an instantaneous vector field. For a better representation of the 3D geometry, the wind tunnel model was tilted. Also shown are the positions of the ring obstacle and the measurement points.

and Vohl (2000) we used one time series for the spectral analysis.

Figure 6 shows a comparison of the experimentally derived energy density spectrum of Vohl et al. (1999) (at 1.77 m s^{-1}) and the one obtained from our numerical simulation (at 2.00 m s^{-1}). In accordance with a higher mean tunnel velocity the simulated spectrum shows higher energies for wavenumbers lower than about 900 m^{-1} . We attribute the larger oscillation amplitudes for wavenumbers lower than about 400 m^{-1} also to the larger velocity fluctuations that originate from the higher mean tunnel velocity. For wavenumbers larger than about 900 m^{-1} , the features of the measured spectrum are represented well by the simulation. Even though the slopes in the inertial as well as in the viscous subrange differ between the two spectra, the derived quantities such as the root-mean-square velocity and the dissipation rate agree well with the parameterizations obtained from the data of Vohl et al. (1999) and Vohl (2000). The results for these two parameters from the COMSOL simulations were included in Fig. 3 as red dots (and one blue dot for comparison in Fig. 3b). The root-mean-square velocity was determined by averaging over different time intervals. To exclude the impact of the chosen position of the collector point, we additionally averaged the obtained root-mean-square velocity over four different off-center measurement points distributed evenly in x and y directions at a distance of 2 cm from the center (see Fig. 5). The corresponding

error bar was obtained by calculating the maximum error, that is, by taking the difference of the maximum and the minimum value of the root-mean-square velocity.

FIG. 6. One-dimensional energy spectrum as a function of wavenumber determined for the vertical velocity component with COMSOL (blue). The mean tunnel velocity of the simulation was set to 2.00 m s^{-1} , equivalent to the mean velocity during the experiments. The red curve represents the spectrum derived from the measurements of Vohl et al. (1999) at a mean tunnel velocity of 1.77 m s^{-1} .

FIG. 7. Experimental results of the collection kernel for freely and tethered grown graupel particles in laminar flow as a function of the average collector momentum. PS denotes the data of the present study. All sizes are given as radii. The symbols and the solid lines correspond to the experimental data points and parameterizations, respectively. The dashed lines are the collection kernels that are parameterized from theory when applying Eq. (8).

5. Results of the riming experiments

a. Laminar flow

The results of the present experiments on the growth of graupel tethered in the wind tunnel in a laminar flow were compared to freely levitated laminar cases from earlier wind tunnel studies (PP79; VB09). Figure 7 shows the experimentally obtained collection kernels K of the graupel as function of their average momentum p [Eq. (6)]. Typical experimental errors for each dataset are only shown at some data points for clarity. Also given in Fig. 7 are calculated collection kernels of drop–drop collisions and of ice–drop collisions. These were determined as described in VB09 by the following experimentally derived parameterization:

$$K = ap^b, \quad (8)$$

TABLE 2. Parameters a and b used in Eq. (8) to calculate the collection kernel of collector–drop collisions for various combinations of collector (ice particles and liquid drops) and riming drop sizes. Parameter b was the same for drop–drop and ice–drop collisions. All values in the table correspond to laminar cases.

Study	Collector radius (μm)	Modal drop radius (μm)	a (drop–drop)	a (ice–drop)	b	Change in a (%)
PP79	300	6	7.98	7.72 ± 0.29	0.768	–3
PP79	150	10	9.18	10.60 ± 0.44	0.739	16
VB09	350	13	9.57	12.73 ± 1.29	0.735	33
PS	280	10	9.18	16.57 ± 0.82	0.739	81

with constant parameters a and b . The sizes of the collector–drop pairs together with the corresponding values of a and b are summarized in Table 2. All sizes of riming drops are given as mean volume radii. The parameterization given in Eq. (8) is based on calculations according to Eq. (2) by using the collision efficiencies of BG74.

For a better comparison of the calculated values of liquid drop and ice collection kernels, the exponents b were kept constant for the same riming droplet sizes. Thus, the difference in the liquid drop and ice collection kernels were characterized solely by varying the factor a . The relative change of a when comparing drop and ice collectors is also provided in Table 2. The decrease of a in the 300- μm –6- μm case is within the confidence interval. For the other cases an increase of a was found for ice–drop collisions; that is, the ice collection kernel was enhanced in comparison to the liquid drop collection kernel (VB09).

The 150- μm –10- μm and the 300- μm –6- μm datasets of PP79 and VB09, respectively, were reanalyzed. Data points describing wet-growth conditions (i.e., for graupel surface temperatures close to 0°C) were not considered in the 150- μm –10- μm cases. For the original 350- μm –13- μm cases the final graupel mass was considered for the calculations, while for all other cases and the present case the time-averaged graupel mass was used. This reanalysis resulted in a more pronounced increase of the ice collection kernels as it was found by VB09.

The ice collection kernel measured in the present study with tethered graupel is significantly enhanced when compared to the ice collector kernel with the same size of riming droplets (10 μm) for freely levitated graupel of PP79 (see Table 2). This enhancement might have two reasons: First, owing to the tethering, the graupel in the present study grew conically (see Fig. 1) with a rather high total cone angle of approximately 120° . That angle can be attributed to the oscillatory movements of the graupel during riming. Therefore, the total rimed mass accreted at the base of the graupel resulting in a much faster increase in the cross-sectional area. Hence, the swept volume has also

been increased compared to a freely levitated graupel, which can freely rotate and, therefore, the accreted mass is homogeneously distributed across the whole surface of the collector. However, this is only valid for the initial growth phase of freely falling graupel. At later times, they fall with a preferred direction and also grow conically. Second, the terminal velocity of a graupel initiated by a frozen drop, which is favored when cloud-base temperatures are higher than $+9^{\circ}\text{C}$ (Knight 1981), passes through a minimum during the first stages of riming (see also Heymsfield et al. 2018). In contrast, the terminal velocities of the graupel in the present experiments were kept constant at the initial terminal velocity of a pure ice sphere. Thus, the relative velocities between ice collector and droplets have been increased in the tethered cases yielding higher values of the collection kernel.

b. Turbulent flow

Figure 8 shows the measured collection kernels obtained from the growth experiments of this study using tethered graupel in laminar and turbulent flows under dry growth conditions. The solid lines represent fits applying Eq. (8) where parameter b was set equal to the value of drop–drop collisions ($b = 0.739$); see section 5a. The resulted fit parameters are $a = 16.14 \pm 0.82$ for the turbulent case and $a = 16.57 \pm 0.82$ for the laminar case. Considering the measurement errors, no difference between laminar and turbulent flow conditions could be observed. This implies that the flow turbulence had no significant influence on the collection kernel of graupel in the investigated momentum range. Thus, turbulence effects as found for drop–drop collisions (Vohl et al. 1999) were not confirmed for ice–drop collisions (i.e., the growth of graupel). This is further discussed in the next section.

6. Discussion

a. Scaling analysis

In this subsection we use the previously introduced Stokes number (St) and dimensionless velocity (Sv) (see section 2) to estimate the influence of the turbulence effects on the collection kernel in the present experiments.

1) Inertia effect: The high values of the Stokes number ($St_{c,K} = 24$) and the dimensionless velocity ($Sv_{c,K} = 58$) (see Table 1) imply only weak collector–eddy interactions with respect to the Kolmogorov scale. Hence, the motions of the collectors in the wind tunnel were rather influenced by the larger scales of the flow; therefore we calculated St and Sv for the integral scale. Even though the Stokes number for

FIG. 8. Comparison of graupel growth in laminar flow (green) and in turbulent flow (blue) under dry growth conditions. The lines represent the parameterizations when applying Eq. (8) to the data.

the integral scale is in a range where it is assumed to influence the motion of the graupel ($St_{c,I} = 1.9$), the dimensionless velocity is quite high ($Sv_{c,I} = 17$). This indicates that the motion of the collector was strongly determined by gravity so that the inertia effect contributed little if at all, to the collection kernel. In contrast, the cloud droplets have a Stokes number ($St_{d,K} = 0.12$) and a dimensionless velocity number ($Sv_d = 0.35$) clearly below 1, which suggests that the droplets were affected by the flow turbulence.

- 2) Accumulation effect: Even though the Stokes number of the mean mass radius (St_d) is relatively low ($St_d = 0.12$), there is a significant part of the droplet size distribution (right side of the distribution in the lower panel of Fig. 2) where values of St_d reach about 0.5. Hence, the accumulation effect contributed, at least weakly, to the overall turbulence effect.
- 3) Hydrodynamic effect: The magnitude and the direction of the droplets' motions were influenced by the flow turbulence, which might enhance the collision efficiency by reducing the hydrodynamic forces acting between the collector and the droplets. Moreover, in a turbulent flow, even droplets outside the gravitational collection volume have a finite probability to collide with the collector, which increases the collision statistics, and thus the efficiency (De Almeida 1976, 1979a; De Almeida 1979b).
- 4) Settling effect: The motion of the collector in the vertical direction was restricted by the mounted copper wire and the constant wind tunnel velocity.

FIG. 9. Velocity chart of a graupel particle freely levitated in the laminar vertical airflow of the wind tunnel (unpublished data from the experiments of VB09). The velocity was recorded at 0.5 Hz.

Thus, this effect did not contribute to an increase of the collection kernel in the present experiments.

Even though the hydrodynamic and accumulation effects should have increased the collection kernel in the present experiments, the results do not imply such an enhancement. In the subsequent discussion we provide a possible explanation for the results based on earlier wind tunnel studies on the growth of freely levitated graupel in laminar flow (PP79; VB09).

b. New analysis and computation of the collection kernel in laminar flow

Even in laminar flow, the collection kernel of graupel increases substantially with increasing colliding droplet size when compared to liquid collector drops (see Fig. 7, Table 2). VB09 speculated that this enhancement might be due to the increasing inertia of the larger droplets in combination with the surface protrusions; that is, larger droplets could be captured more readily by those protrusions. With respect to the present results we investigate the enhancement of the collection kernel with droplet size in more detail because we assume that the enhancement overcompensates the effect of the flow turbulence.

Figure 9 shows the vertical velocity recorded for a freely levitated graupel with an initial frozen-drop radius of $320\ \mu\text{m}$ (unpublished data from the experiments of VB09). During the first 16 s the measurement represents the velocity of the freely levitated frozen drop without riming. At $t = 16\text{ s}$, liquid droplets were injected into the tunnel and riming started (see Fig. 1). During

the time period between 16 and 27 s the droplets were slowly transported with the airstream from the initial injection position to the position of the collector. The disturbed motion of the collector during this time period was likely due to some droplet evaporation in the lower part of the wind tunnel. This generated local temperature, density, and water vapor perturbations in the flow, and destabilized the freely levitated frozen ice sphere. Afterward, the graupel grew in an environment with a constant liquid water content until it was removed from the wind tunnel at about 87 s. It can be clearly seen that in the growth phase after 27 s the graupel moved with much more fluctuations as before. This is also reflected by the root-mean-square velocities showing a value of $u_{\text{rms}} = 0.02\ \text{m s}^{-1}$ in the nonriming case ($t < 16\text{ s}$) and $u_{\text{rms}} = 0.10\ \text{m s}^{-1}$ in the riming case ($t > 27\text{ s}$). That is, the velocity fluctuations owing to the unsteady flow induced by the vortex shedding of a freely levitated growing graupel itself are of comparable magnitude as the flow turbulence generated by the ring obstacle in the present experiments ($u_{\text{rms}} = 0.13\ \text{m s}^{-1}$). This indicates that even in a laminar flow the flow field around a growing graupel is not steady anymore, which would be the case for a nongrowing rigid sphere. Such self-induced fluctuations might be responsible for the increase of the ice collection kernel in comparison to the drop collection kernel. As shown in Fig. 9 these stochastic motions might originate from a combination of graupel surface roughness and a local release of latent heat because of freezing (VB09). To examine whether this was the case during the experiments of VB09 the growth rates of the graupel were calculated according to Eqs. (1) and (2), which do

FIG. 10. Comparison of calculated collection kernels (symbols) to experimentally obtained values of freely levitated graupel from earlier studies (solid lines). Dashed lines indicate fits to the simulated datasets when applying Eq. (8) (see Table 3).

not account for any other motion of the collector than a straight vertical fall. The mass increase dm of graupel with sizes used in the experiments of PP79 and VB09, was calculated using the experimental parameters such as initial frozen drop radius, average terminal velocity, growth time, average liquid water content, and mass distribution of the droplets. The collision efficiencies were determined using the interpolation scheme of BG74, while the coalescence efficiency was set to 1. The time step dt was 1 s. After each time step, from the incremental mass increase dm , the volume increase was computed using the average density of the rime ice from the experiments. Under the assumption of a spherical geometry the new size of the graupel and, from that, the new collection kernel for the next time step was determined. Afterward, the computed data were parameterized according to Eq. (8) with the same constant b as in the corresponding experimental cases.

Figure 10 shows a comparison of measured and calculated collection kernels as function of the graupel momentum. The solid lines represent the experimental

data, the symbols the computed ones, and the dashed lines the fits of the computed data. The data of the present study are not included because the graupel grew conically, while the applied calculations presume a spherical shape. In Table 3 the fit parameters and the corresponding 95% confidence intervals of the computed kernels are compared to the experimentally derived ones. The best agreement between the measured and mean computed collection kernels was obtained in the 350- μm –13- μm case (VB09). Here the fit parameters overlap very well within their 95% confidence interval.

In the 150- μm –10- μm case of PP79 the mean computed collection kernels lie slightly below the experimental ones, but with respect to the confidence intervals, still indicating a fairly good agreement.

The computation of the 300- μm –6- μm case (PP79) clearly shows a considerable overestimation compared to the measured kernels. Here we assume that the measured droplet size distribution of PP79 underestimated the small droplets in their experiments due to their size determination method. With the applied rod impactation method it is very difficult to measure the smaller droplet sizes since they are evaporating quite fast. A clue for this is that they observed rear capture in their experiments, which is not typical for a size of 6 μm in radius. A sensitivity test showed that the computed kernels match very well with the experimental curve when assuming monodisperse droplets of 5 μm in radius. Therefore, we assume that their real spectrum contained more of the small droplets and the mass weighted spectrum would be shifted to smaller sizes. Also, in that size range the collection efficiency is very sensitive; that is, a small difference in droplet sizes causes a large difference in collection efficiency.

Nevertheless, for the spectra that are relatively robust in terms of the collection efficiency, the comparison shows that the experimentally measured collection kernels in laminar flow can be reproduced by the continuous growth equation [Eqs. (1) and (2)]. This indicates that the self-induced fluctuations caused by the surface roughness of the graupel and the local latent heat release were not responsible for the increased ice collection kernels. Another possible reason is the influence of the continuously increasing collector size, which overcompensates the other potential enhancement effects.

TABLE 3. Fit values of a (errors corresponding to the 95% confidence intervals) resulting from the fits when applying Eq. (8) to the simulated and experimental ice–liquid kernels.

Study	Collector radius (μm)	Modal drop radius (μm)	a (experimental)	a (simulated)	b
PP79	300	6	7.72 ± 0.29	11.29 ± 0.13	0.768
PP79	150	10	10.60 ± 0.44	9.97 ± 0.39	0.739
VB09	350	13	12.73 ± 1.29	12.63 ± 0.16	0.735

Sensitivity computations indicated that, within the range of approximately constant collection efficiencies, the ice collection kernel is dominated by the increasing collector size. In comparison to a liquid drop collector, the size of a graupel increases faster because the density of the accreted ice is low during dry growth conditions (since the droplets freeze at once when they are deposited on the ice surface). This faster growth enhances the sweep-out volume, and thus the ice collection kernel when compared to a liquid drop collection kernel at the same momentum. The enhancement of the ice collection kernel with the droplet size is affected by (i) enhanced mass accretion in the same time period, and (ii) lower density of rimed ice. Thus, the sweep-out volume increases faster for larger droplets than for smaller ones.

Finally, the influence of ice collector and riming droplet sizes together with the low density of rimed ice during dry growth seems to be responsible for the fast growth of graupel even in laminar flow. It is surmised that this effect overcompensates the one originating from the self-induced fluctuations.

c. Collection kernel in turbulent flow

In the tethered case the stochastic movements of the graupel were restricted compared to a freely levitated graupel. Even so, the flow turbulence of the relevant scales prevailing during our experiments had no observable effect on the collection kernel. Thus, the collection kernel of graupel is strongly affected by the fast growth in size due to the low density of the acquired ice mass. In case of a conical graupel this effect is even more pronounced compared to a spherical graupel. That is, the majority of the droplets collide at the upstream side of the conical graupel and are not equally distributed across the whole surface as it is the case for a spherical one. This causes a faster increase of the cross-sectional area, and thus the sweep-out volume compared to a spherical graupel. Finally, it is likely that this increase in the sweep-out volume of the graupel predominates over all other possible effects that might enhance the collection kernel such as surface roughness, accelerations of the graupel by the latent heat release, and small-scale flow turbulence.

This finding is in conflict with the results of Pinsky et al. (1998), who found that turbulence increases the collection kernel of graupel substantially compared to liquid drops. However, they used graupel particles with a density of 0.3 g cm^{-3} (in our experiments the average graupel density was about 0.5 g cm^{-3}), which increases the interaction with the turbulent flow. Furthermore, Pruppacher and Klett (2010) pointed out that the authors overestimated the influence of turbulence on the collection rate in their statistical approach, since they estimated the inertial accelerations two

orders of magnitude too high compared to typical values. Moreover, they added the root-mean-square graupel velocity difference in the calculation of the collection kernel, which does not have a strong physical basis.

We note here that the present results are only valid for graupel that are initiated by frozen drops, that is, with an initially high mean graupel density. However, graupel are also formed by the riming of ice crystals, which serve as collectors for the riming droplets. This yields graupel with lower densities and, therefore, lower inertia. For these low-density graupel the interactions with the turbulent flow increase, which may lead in turn to an enhancement of the turbulence effects on their collection kernels (Naso et al. 2018; Pinsky and Khain 1998).

7. Summary and conclusions

Wind tunnel experiments were carried out to investigate the influence of flow turbulence on the collection kernel of spherical ice particles with initial radii between 220 and $340 \mu\text{m}$. The collectors were tethered on a thin wire to maintain them in the vertical airstream. To quantify the influence of turbulence, the experiments were performed alternating in laminar and turbulent flows. The turbulent flow was characterized by a vertical root-mean-square fluctuation velocity of 0.13 m s^{-1} and a dissipation rate of $0.13 \text{ m}^2 \text{ s}^{-3}$, whereas the laminar flow had a rest turbulence of less than 0.5%. Liquid water contents w_L and ambient temperatures T_a corresponded to typical dry growth conditions (i.e., w_L ranging from 0.5 to 1.3 g m^{-3} and T_a ranging from -16° to -7°C). Each graupel was exposed to supercooled droplets with a mean mass radius of $10 \mu\text{m}$ for 120 s. From the ratio of the mass growth rate and the liquid water content the collection kernel was determined.

The results from the laminar case confirmed previous findings that the ice collection kernel is enhanced in comparison to the liquid drop collection kernel. However, an additional increase of the ice collection kernel in a turbulent flow could not be observed. The comparison of the ice collection kernels with tethered graupel during laminar and turbulent flow conditions revealed no measurable difference between these two cases within the experimental errors.

A scaling analysis indicated that the generated turbulence contributed only to a minor degree to the motion of the collector and, thus the collection kernel. In contrast, an enhanced collection kernel should have been expected due to an influence of flow turbulence on the motion of the riming droplets.

To speculate why this was not the case we reanalyzed results from previous wind tunnel experiments of PP79 and VB09 with freely levitated graupel in laminar airflow.

This issue was investigated in more detail by elaborating that freely levitated graupel growth is never purely turbulent-free. Owing to the rough surface and the release of latent heat the graupel were stimulated to perform stochastic movements with u_{rms} velocities comparable to the generated flow turbulence in the present experiments. To decide whether these induced movements were responsible for the enhancement of the collection kernels of graupel compared to drops, calculations applying the continuous growth equation were performed. The results showed that the calculated collection kernels agreed well with the experimental data. This indicates that the self-induced stochastic motion was not responsible for the increase of the collection kernels. Sensitivity tests revealed that the dominant factor in the increase of the collection kernel is the fast radius growth due to the accretion of rime ice with low density. From these considerations it could be surmised that this increase in the sweep-out volume predominates over all other effects that might enhance the collection kernel such as surface roughness and accelerations of the graupel due to the release of latent heat (which both enhance the self-induced movements of the graupel) as well as small-scale flow turbulence.

The results of the present study suggest that neglecting turbulence effects on the initial phases of graupel growth under dry growth conditions in cloud models is justified. However, this might change if one considers the collisional growth of low-density graupel initiated by ice crystals (e.g., needles, columns, or hexagonal plates). In this case the influence on the collection kernel might be significant due to a lower inertia of the graupel and the related interaction with the turbulent flow. This issue should be addressed in future experimental and theoretical studies.

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REFERENCES

- Ayala, O., B. Rosa, L.-P. Wang, and W. W. Grabowski, 2008: Effects of turbulence on the geometric collision rate of sedimenting droplets. Part 1. Results from direct numerical simulation. *New J. Phys.*, **10**, 075015, <https://doi.org/10.1088/1367-2630/10/7/075015>.
- Beard, K. V., and S. N. Grover, 1974: Numerical collision efficiencies for small raindrops colliding with micron size particles. *J. Atmos. Sci.*, **31**, 543–550, [https://doi.org/10.1175/1520-0469\(1974\)031<0543:NCEFSR>2.0.CO;2](https://doi.org/10.1175/1520-0469(1974)031<0543:NCEFSR>2.0.CO;2).
- Brdar, S., and A. Seifert, 2018: McSnow: A Monte-Carlo particle model for riming and aggregation of ice particles in a multi-dimensional microphysical phase space. *J. Adv. Model. Earth Syst.*, **10**, 187–206, <https://doi.org/10.1002/2017MS001167>.
- Cober, S. G., and R. List, 1993: Measurements of the heat and mass transfer parameters characterizing conical graupel growth. *J. Atmos. Sci.*, **50**, 1591–1609, [https://doi.org/10.1175/1520-0469\(1993\)050<1591:MOTHAM>2.0.CO;2](https://doi.org/10.1175/1520-0469(1993)050<1591:MOTHAM>2.0.CO;2).
- COMSOL, 2015: COMSOL Multiphysics. COMSOL.
- De Almeida, F. C., 1976: The collisional problem of cloud droplets moving in a turbulent environment—Part I: A method of solution. *J. Atmos. Sci.*, **33**, 1571–1578, [https://doi.org/10.1175/1520-0469\(1976\)033<1571:TCPOCD>2.0.CO;2](https://doi.org/10.1175/1520-0469(1976)033<1571:TCPOCD>2.0.CO;2).
- , 1979a: The collisional problem of cloud droplets moving in a turbulent environment—Part II: Turbulent collision efficiencies. *J. Atmos. Sci.*, **36**, 1564–1576, [https://doi.org/10.1175/1520-0469\(1979\)036<1564:TCPOCD>2.0.CO;2](https://doi.org/10.1175/1520-0469(1979)036<1564:TCPOCD>2.0.CO;2).
- , 1979b: The effects of small-scale turbulent motions on the growth of a cloud droplet spectrum. *J. Atmos. Sci.*, **36**, 1557–1563, [https://doi.org/10.1175/1520-0469\(1979\)036<1557:TEOSST>2.0.CO;2](https://doi.org/10.1175/1520-0469(1979)036<1557:TEOSST>2.0.CO;2).
- Devenish, B. J., and Coauthors, 2012: Review article: Droplet growth in warm turbulent clouds. *Quart. J. Roy. Meteor. Soc.*, **138**, 1401–1429, <https://doi.org/10.1002/qj.1897>.
- Diehl, K., S. K. Mitra, M. Szakáll, N. von Blohn, S. Borrmann, and H. R. Pruppacher, 2011: The Mainz vertical wind tunnel facility: A review of 25 years of laboratory experiments on cloud physics and chemistry. *Wind Tunnels, Aerodynamics, Models, and Experiments*, J. D. Pereira, Ed., Nova Science, 69–92.
- Grabowski, W. W., and L.-P. Wang, 2009: Diffusional and accretional growth of water drops in a rising adiabatic parcel: Effects of the turbulent collision kernel. *Atmos. Chem. Phys.*, **9**, 2335–2353, <https://doi.org/10.5194/acp-9-2335-2009>.
- , and —, 2013: Growth of cloud droplets in a turbulent environment. *Annu. Rev. Fluid Mech.*, **45**, 293–324, <https://doi.org/10.1146/annurev-fluid-011212-140750>.
- Grover, S. N., and H. R. Pruppacher, 1985: The effect of vertical turbulent fluctuations in the atmosphere on the collection of aerosol particles by cloud drops. *J. Atmos. Sci.*, **42**, 2305–2318, [https://doi.org/10.1175/1520-0469\(1985\)042<2305:TEOVTF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1985)042<2305:TEOVTF>2.0.CO;2).
- Heymsfield, A. J., M. Szakáll, A. Jost, I. Giammanco, and R. Wright, 2018: A comprehensive observational study of graupel and hail terminal velocity, mass flux and kinetic energy. *J. Atmos. Sci.*, **75**, 3861–3885, <https://doi.org/10.1175/JAS-D-18-0035.1>.
- Knight, N. C., 1981: The climatology of hailstone embryos. *J. Appl. Meteor.*, **20**, 750–755, [https://doi.org/10.1175/1520-0450\(1981\)020<0750:TCOHE>2.0.CO;2](https://doi.org/10.1175/1520-0450(1981)020<0750:TCOHE>2.0.CO;2).
- Macklin, W. C., and G. S. Payne, 1967: A theoretical study of the ice accretion process. *Quart. J. Roy. Meteor. Soc.*, **93**, 195–213, <https://doi.org/10.1002/qj.49709339606>.
- Mülmenstädt, J., O. Sourdeval, J. Delanoë, and J. Quaas, 2015: Frequency of occurrence of rain from liquid-, mixed-, and ice-phase clouds derived from A-Train satellite retrievals. *Geophys. Res. Lett.*, **42**, 6502–6509, <https://doi.org/10.1002/2015GL064604>.
- Naso, A., J. Jucha, E. Lveque, and A. Pumir, 2018: Collision rate of ice crystals with water droplets in turbulent flows. *J. Fluid Mech.*, **845**, 615–641, <https://doi.org/10.1017/jfm.2018.238>.

- Pflaum, J. C., and H. R. Pruppacher, 1979: A wind tunnel investigation of the growth of graupel initiated from frozen drops. *J. Atmos. Sci.*, **36**, 680–689, [https://doi.org/10.1175/1520-0469\(1979\)036<0680:AWTIOT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1979)036<0680:AWTIOT>2.0.CO;2).
- Pinsky, M. B., and A. P. Khain, 1998: Some effects of cloud turbulence on water–ice and ice–ice collisions. *Atmos. Res.*, **47–48**, 69–86, [https://doi.org/10.1016/S0169-8095\(98\)00041-6](https://doi.org/10.1016/S0169-8095(98)00041-6).
- , —, D. Rosenfeld, and A. Pokrovsky, 1998: Comparison of collision velocity differences of drops and graupel particles in a very turbulent cloud. *Atmos. Res.*, **49**, 99–113, [https://doi.org/10.1016/S0169-8095\(98\)00073-8](https://doi.org/10.1016/S0169-8095(98)00073-8).
- Pruppacher, H. R., and J. D. Klett, 2010. *Microphysics of Clouds and Precipitation*. 2nd ed. Springer, 954 pp.
- Shaw, R. A., 2003: Particle-turbulence interactions in atmospheric clouds. *Annu. Rev. Fluid Mech.*, **35**, 183–227, <https://doi.org/10.1146/annurev.fluid.35.101101.161125>.
- Siebert, H., K. Lehmann, and R. A. Shaw, 2007: On the use of hot-wire anemometers for turbulence measurements in clouds. *J. Atmos. Oceanic Technol.*, **24**, 980–993, <https://doi.org/10.1175/JTECH2018.1>.
- Stull, R. B., 1988. *An Introduction to Boundary Layer Meteorology*. Kluwer Academic, 666 pp.
- Szakáll, M., K. Diehl, S. K. Mitra, and S. Borrmann, 2010: Shapes and oscillations of falling raindrops—A review. *J. Atmos. Res.*, **97**, 416–425, <https://doi.org/10.1016/j.atmosres.2010.03.024>.
- Townsend, A. A., 1976. *The Structure of Turbulent Shear Flow*. Cambridge University Press, 440 pp.
- Vohl, O., 2000: An experimental investigation of the effects of turbulence on the scavenging of aerosol particles by rain drops, and on the growth of cloud drops by collision. Ph.D. dissertation, Johannes Gutenberg University, 105 pp.
- , S. K. Mitra, S. C. Wurzler, and H. R. Pruppacher, 1999: A wind tunnel study of the effects of turbulence on the growth of cloud drops by collision and coalescence. *J. Atmos. Sci.*, **56**, 4088–4099, [https://doi.org/10.1175/1520-0469\(1999\)056<4088:AWTSOT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1999)056<4088:AWTSOT>2.0.CO;2).
- von Blohn, N., K. Diehl, S. K. Mitra, and S. Borrmann, 2009: Riming of graupel: Wind tunnel investigations of collection kernels and growth regimes. *J. Atmos. Sci.*, **66**, 2359–2366, <https://doi.org/10.1175/2009JAS2969.1>.
- Wang, L.-P., and M. R. Maxey, 1993: Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence. *J. Fluid Mech.*, **256**, 27–68, <https://doi.org/10.1017/S0022112093002708>.