Aircraft Observations of Tropical Cyclone Boundary Layer Turbulence over the South China Sea

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ABSTRACT

There have been no high-frequency aircraft observations of tropical cyclone (TC) eyewall boundary layer turbulence since two flights into Atlantic hurricanes in the 1980s. We present an analysis of the first TC boundary layer flight observations in the South China Sea by the Hong Kong Observatory comprising four eyewall penetrations. We derive the vertical flux of momentum and vertical momentum diffusivity from observed turbulence parameters. We observe negative (upward) vertical fluxes of tangential momentum near the eyewall consistent with a jet below the flight level near the radius of maximum wind. Our observations of vertical momentum diffusivity support a superlinear relationship between diffusivity and wind speed at the high wind speeds in the inner-core of TCs (power-law exponent of 1.73 ± 0.20) while the few existing boundary layer hurricane observations in the North Atlantic suggest a more linear relationship.

1. Introduction

The inner-core regions of tropical cyclones (TCs) are characterized by high wind speeds, turbulent activity, and convection. When an eye is present, the radius of maximum wind speed (RMW) is typically located within the eyewall. Standard measures of TC intensity are therefore sensitive to processes within the eyewall. Complex phenomena such as eyewall vorticity maxima and eyewall replacement cycles (Kossin 2015) have been observed within this region and are likely to be important factors in the development and maintenance of TCs. The turbulent activity in the inner-core boundary layer means that vertical fluxes of momentum, heat, and moisture have a significant effect on TC development (Ooyama 1969; Emanuel 1986; Rotunno et al. 2009; Smith and Montgomery 2010; Bryan 2012). Recently studies (J. A. Zhang et al. 2017; Lu and Wang 2019) showed that turbulent mixing in a numerical weather prediction (NWP) model was important in

forecasting both rapid intensification events and the properties of landfalling TCs (F. Zhang et al. 2017; Zhang and Pu 2017).

In NWP models turbulent transport is parameterized as a subgrid-scale flux-gradient process with an effective turbulent diffusivity. Simulated TCs have been shown to be sensitive to the choice of planetary boundary layer (PBL) parameterizations (Braun and Tao 2000; Nolan et al. 2009; Smith and Thomsen 2010; Kepert 2012). The PBL schemes employed in NWP are usually calibrated to observations of both fluxes and diffusivities directly, and to the resulting vertical profiles of, for example, wind speed, temperature, and moisture. There remains a scarcity of observed turbulence data in the eyewall of the TC boundary layer because the very conditions which require attention—high wind speeds and turbulent intensity—make the gathering of boundary layer, inner-core data very challenging and safety concerns have largely prohibited the required research.

The most significant existing data comes from research aircraft flights in the 1980s when two hurricane boundary layer eyewall penetrations were made. Over 20 years later, Zhang et al. (2011, hereafter Z11) presented an analysis of the collected data in which turbulent fluxes and diffusivities were derived. These observations

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have been used by, for example, [Zhang et al. \(2012\)](#), as a means of calibrating an NWP PBL scheme. However, some uncertainty in these existing data due to a relatively low sampling frequency of 1 Hz and only two data points at wind speeds above 50 m s^{-1} prompted the authors to appeal for further analyses of the turbulent characteristics in the high-wind region of TCs. [Zhang and Drennan \(2012\)](#) undertook an observational study of turbulent fluxes and diffusivity in the hurricane boundary layer restricted to relatively low wind speeds outside the core but found a dependence of flux and diffusivity on height.

Here we present new vertical turbulent flux and diffusivity estimates based on data obtained from research flights in TC boundary layers comprising four eyewall penetrations over the South China Sea (SCS) jointly carried out by the Hong Kong Observatory (HKO) and the Government Flying Service of Hong Kong. These data contain more observations at wind speeds above 50 m s^{-1} than in the previous study and are the first over the SCS and indeed the western North Pacific.

2. Data overview

The HKO has worked in partnership with the Government Flying Service of the Hong Kong Special Administration Region since 2009 in equipping a fixed-wing aircraft, the BAe Jetstream 4100 (JS41), with an Aircraft Integrated Meteorological Measuring System 20 Hz (AIMMS20). The collaboration has resulted in research flights into TCs in the northern SCS (e.g., [Chan et al. 2014](#)). The AIMMS20 system provides measurements of the three components of the wind, temperature, relative humidity, and pressure at a frequency of 20 Hz. An overview of the AIMMS20 can be found in [Beswick et al. \(2008\)](#) and more details of the aircraft-AIMMS20 setup are reported in [Chan et al. \(2011\)](#).

The JS41 was routinely used for search and rescue (SAR) missions over the SCS region often including low-altitude surveillance flights under adverse weather conditions (including but not limited to TCs). The aircraft and its onboard instrument underwent routine inspection and maintenance by qualified engineers. No evidence was found that the data probe might be contaminated or degraded by sea salt. Furthermore, the JS41 performed routinely data comparison flights (up to three times every week) where the wind measurements by the AIMMS-20 probe were validated against concurrent Doppler lidar measurements at the Hong Kong International Airport.

Four flights through four TCs (Kalmaegi, Linfa, Mujigae, and Nida) over the period 2014–16 were chosen for this

analysis as they contain legs in the boundary layer in the TC inner core. In [Fig. 1](#) we show the evolution of the minimum pressure $P_{\min}$ of the TCs observed as estimated by four TC agencies and $P_{\min}$ at the time of the flight derived from flight-level data. Flights 1 and 4 coincide with the approximate lifetime $P_{\min}$ of the respective cyclones, before they make landfall and dissipate. Linfa is in the process of making landfall and has possibly started to dissipate at the time of flight 2. Mujigae is undergoing a period of rapid intensification that continues for some time after flight 3.

The flight paths, dates, and times and observed wind vectors are shown in [Fig. 2](#). All flights begin in Hong Kong and head out into the SCS with typical airspeeds of 100 m s^{-1} .

Turbulent characteristics were calculated for selected legs (or flux runs) of each flight chosen according to quality control criteria detailed in [section 4](#). Radial profiles of the total wind speed for each of these selected runs from the four flights are shown in [Fig. 3a](#) with radius normalized by the observed RMW. [Figure 3b](#) shows the azimuthal and radial location of each run relative to the translation of the cyclone center. It shows sampling of all quadrants with a small bias to the right front.

3. Data analysis method

Turbulent vertical fluxes of tangential and radial momentum were calculated using the eddy-covariance method given by

$$F_t = \overline{-v'_t w'}, \quad (1)$$

$$F_r = \overline{-v'_r w'}, \quad (2)$$

respectively, where v_t is the tangential wind speed, v_r is the radial wind speed, primes signify turbulent fluctuations from the mean, and an overbar represents time averaging. From these components the total momentum flux is given by

$$|F| = (F_t^2 + F_r^2)^{1/2}. \quad (3)$$

As vertical wind shear data were not available, following [Z11](#) we calculate the vertical momentum diffusivity K_m using two “indirect” methods. The first, as used by [Hanna \(1968\)](#), is given by

$$K_m = c l_w \sigma_w, \quad (4)$$

where σ_w is the standard deviation of vertical wind velocity, l_w is the mixing length scale given by $l_w = \sigma_w^3/\varepsilon$, where ε is the eddy dissipation rate (EDR), and c is

FIG. 1. The evolution of minimum pressure of the four TCs studies as estimated by four agencies. The red square is the minimum flight-level pressure observation extrapolated to sea level.

a constant taken as $c = 0.41$. The second method, known as the turbulent kinetic energy (TKE) method, is given by

$$K_m = c_2 e^2 / \varepsilon, \quad (5)$$

where c_2 is an empirical parameter given the value 0.03 and e is the TKE calculated using the turbulent fluctuations of the three wind components

$$e = (1/2)(\overline{v_t^2} + \overline{v_r^2} + \overline{w^2}). \quad (6)$$

Both the Hanna and TKE method for calculating K_m require the calculation of the EDR. A commonly used spectral method (e.g., Večenaj et al. 2010; Zhang 2010) is given by

$$\varepsilon = \alpha^{-3/2} \frac{2\pi f}{U_a} [f S_u(f)]^{3/2}, \quad (7)$$

where f is the frequency, U_a is the airspeed relative to the aircraft, S_u is the power spectral density of the along mean wind velocity component, and α is the one-dimensional Kolmogorov constant taken to be 0.5 following Z11. We evaluate the above using the compensated power spectrum method following Sreenivasan (1995).

This technique implicitly assumes the Kolmogorov $-5/3$ power-law relationship holds over some frequency range. Bounds on the inertial range vary in the literature and are a function of the airspeed and scale of the largest eddies amongst other things. Instead of attempting to calculate a universal ideal inertial range for our data, we find the range of frequencies over which a least squares fit best matches the theoretical $-5/3$ gradient for each flight leg in the range 0.1–10 Hz with a width of half a decade. Deviations in the power spectrum fit gradient, $m_{\text{err}} = m + 5/3$, are typically smaller than 0.02.

The position of the cyclone center is required to calculate the radial distance, R , and to transform rectilinear wind velocity components u and v into tangential and radial components u_t and u_r . We found best track estimates insufficiently accurate for this purpose and so a method based on Willoughby and Chelmon (1982) was used to find the circulation center at the time of closest approach. A longer sampling window of 500 s was used to reduce the effect of asymmetric flow in the eye biasing the center location estimate. We derived the translation velocity from best track position estimates and combined it with our estimated center location at time of closest approach to give the center location as a function of time during the flight.

FIG. 2. Path of flights segments below 1 km (dark blue) overlaid on satellite imagery. Orange symbols are arbitrarily scaled observed mean wind vectors, turquoise segments starting with black dots show the flight legs selected by quality control filters used in the analysis, and magenta is the trajectory of the cyclone during the flight interpolated from best track data. The plot titles give the cyclone name, flight segment start date and time (UTC), and duration (min).

The RMW was then estimated using the observed 1-min average wind speed.

4. Leg selection and quality control

We used an algorithm which scans through the flight-level data identifying legs which satisfy a set of quality control criteria. As momentum flux and diffusivity are known to vary with height, we constrain the analysis to legs with altitudes between 500 and 700 m. Aircraft attitude data are examined to ensure legs are level and straight. Following Zhang et al. (2009) absolute pitch and roll are required to be less

than 5° . Legs with altitude variation greater than 30 m and heading variation exceeding 30° were rejected. We excluded legs with a mean wind speed below 10 m s^{-1} . We also reject legs where an inertial range could not be found according to $|m_{\text{err}}| > \alpha_m$ with α_m set to 0.1.

We apply two flux stationarity tests to the legs. First, a linear fit to the cumulative sum of $v'_i w'_i$ and $v'_r w'_r$ (Affre et al. 2000) must have an r^2 value above 0.8 to ensure that flux is accumulated approximately evenly across the duration of the leg and is not dominated by relatively short-lived, isolated events. In addition, we check that the leg duration and sampling frequency are such that the range of frequencies contributing to the flux are adequately

FIG. 3. (a) Mean observed wind speed U against mean radius normalized by RMW (R/RMW) for each selected leg and (b) spatial distribution of legs relative to storm center with direction of storm translation at top and radial distance normalized by RMW.

captured by examining the smoothness of the cospectrum ogive tails. Smooth, horizontal upper and lower ogive tails signify vanishing contributions to the flux at the extremes of the captured frequency range (Zhang et al. 2009; Z11). This is implemented in the quality control algorithm by least squares linear fitting to the upper and lower quartiles of the ogive and rejecting legs with large tail variability.

The above filtering resulted in 34 successful legs, corresponding to 33% of the flight time in the prescribed altitude range and includes five observations at wind speeds above 50 m s^{-1} . This compared to the Z11 analysis where 24 legs were identified for turbulence analysis, two of which were above 50 m s^{-1} .

Figure 4 shows analysis of a selection of legs with both high and low wind speeds passing the quality control criteria. Power spectra for v'_r , v'_θ , and w' , exhibit a wide $-5/3$ slope showing that a satisfactory inertial range exists. Cospectra, ogives and cumulative sum for $v'_r w'$ and $v'_\theta w'$ show that the flux contributions are largely from the center of the sampled frequency range and that flux is accumulated roughly evenly throughout the leg duration. Table 1 contains summary data and turbulence calculations for each leg passing the quality control filters.

5. Results and discussion

a. Momentum flux

The vertical flux of total horizontal momentum $|F|$ is shown as a function of total wind speed U in Fig. 5a.

Momentum flux tends to increase with wind speed. The maximum flux observed here is approximately $2.5 \text{ m}^2 \text{ s}^{-2}$ in the eyewall of Linfa at a speed of 55 m s^{-1} . This compares to a value of $4.5 \text{ m}^2 \text{ s}^{-2}$ observed in hurricane Hugo at a similar speed as reported in Z11. The vertical flux of tangential momentum F_t is plotted against distance from the storm center normalized by RMW in Fig. 5b. Values range from approximately -0.79 to $1.35 \text{ m}^2 \text{ s}^{-2}$. This is a similar range to that observed by Zhang and Drennan (2012) over a range of heights in the boundary layer. There is a strong dependency on normalized radial distance from the storm center with fluxes beyond twice the RMW all positive (downward) and relatively small (less than $0.5 \text{ m}^2 \text{ s}^{-2}$). Near the RMW fluxes exhibit a larger range with a tendency toward large negative values (upward fluxes). We have no observations of the vertical variation in v_t for these cyclones but we propose the change in sign of F_t around the RMW must coincide with a change in sign of the vertical gradient of v_t at flight level ($\sim 600 \text{ m}$). This is consistent with the height of maximum tangential wind speed reducing from above flight level to below flight level as RMW is approached. This is in agreement with previously observed wind structure as observed, for example, by Zhang et al. (2013) in a dropsonde composite study (their Fig. 3) where the jet height lowers from above 1 km to below 500 m as RMW is approached from outside the inner core.

Figure 5c shows the vertical flux of radial momentum F_r against R/RMW . Fluxes are mostly positive (downward) varying from 0 to $2 \text{ m}^2 \text{ s}^{-2}$. There is a dependence on R/RMW , with observations in the eyewall tending to

FIG. 4. Example analysis of (a),(b) two high- and (c),(d) two low-wind-speed legs. (top to bottom) Power spectra of v_t , v_r , and w are in black, blue, and orange respectively, and cospectra, ogives (O), and cumulative sums (CS) of $v_t w'$ (black) and $v_r w'$ (blue).

be much larger. This is consistent with increased vertical momentum diffusivity K_m in the high wind speeds in the eyewall. Steepening of the vertical gradient of v_r at flight level in the eyewall may also contribute to this effect and, again, this agrees with the dropsonde composite study of Zhang et al. (2013, their Fig. 4.).

b. Vertical momentum diffusivity

Values of K_m calculated using the Hanna method varied from below $10 \text{ m}^2 \text{ s}^{-1}$ at low wind speeds to above $200 \text{ m}^2 \text{ s}^{-1}$ at high wind speeds in the core. In Z11 it was shown that U is one covariate of K_m and a logarithmic dependence was proposed. Here we propose an empirically based power-law model,

$$K_m = aU^b, \quad (8)$$

as with this form we can readily explore linearity through the b exponent. This formulation also enables us to compare the relationship between K_m and U across different sets of observations. We are not proposing this relationship is valid beyond the scope of the TC boundary layer. To determine the coefficients a and b we apply linear regression to the log of the

variables. We perform this to both the new data analyzed here and the Z11 data for comparison using the Hanna method calculation. Figure 6 shows log-log plots of K_m against U and estimates and standard errors of a and b coefficients for the Z11 data and the current study. The exponent in the power law b was found to be 1.14 ± 0.23 for the Z11 data, not significantly different from 1 therefore consistent with a linear relationship. However, the new data support a significantly superlinear relationship with a b exponent of 1.73 ± 0.20 . The coefficients are not in agreement to within their standard errors. The new data contain three more entries (5) for wind speeds above 50 m s^{-1} than the Z11 data (2) giving more confidence that the K_m - U relationship holds at the very high wind speeds encountered in TC eyewalls.

Since K_m is known to vary with height, comparison of the two datasets should be qualified by the broader range of heights spanned by the Z11 data (422–847 m) than the new HKO data (583–653 m). However, most of the measurements from both sets are contained within the height range 450–650 m and limiting the data to this range gives similar power-law exponents (Z11: $b = 1.06 \pm 0.28$; HKO: $b = 1.65 \pm 0.28$). We also

TABLE 1. Summary data and calculations for flight legs. Columns show flight number (1 = Kalmaegi, 2 = Linfa, 3 = Mujigae, 4 = Nida), time and date, duration (s), altitude z (m), radius R (km), total wind speed U (m s^{-1}), F_t ($\text{m}^2 \text{s}^{-2}$), F_r ($\text{m}^2 \text{s}^{-2}$), EDR ε ($\text{cm}^2 \text{s}^{-3}$), σ_w (m s^{-1}), TKE e ($\text{m}^2 \text{s}^{-2}$), K_m (Hanna) ($\text{m}^2 \text{s}^{-1}$), and K_m (TKE) ($\text{m}^2 \text{s}^{-1}$).

Flight	Time and date	Duration	z	R	U	F_t	F_r	ε	σ_w	e	K_m (Hanna)	K_m (TKE)
1	0019:40 UTC 16 Sep 2014	355	638	68	41.0	0.08	0.16	32.5	1.02	2.17	137.8	43.4
2	2330:55 UTC 8 Jul 2015	315	609	165	15.6	0.01	0.03	13.2	0.37	0.43	5.9	4.2
2	2341:00 UTC 8 Jul 2015	180	607	108	24.7	0.04	0.10	36.2	0.51	0.81	7.6	5.4
2	2354:55 UTC 8 Jul 2015	180	610	34	56.1	-0.47	1.39	284.7	1.59	7.04	91.5	52.2
2	2358:25 UTC 8 Jul 2015	180	617	26	55.6	-0.28	0.52	75.5	1.10	2.91	79.9	33.7
2	0012:10 UTC 9 Jul 2015	185	621	33	53.5	-0.57	0.91	178.3	1.43	5.43	96.4	49.5
2	0015:30 UTC 9 Jul 2015	260	621	27	54.0	-0.44	1.05	234.1	1.88	8.12	219.9	84.4
2	0019:50 UTC 9 Jul 2015	180	622	29	55.0	-0.79	1.64	423.2	2.21	13.22	230.8	123.9
2	0032:35 UTC 9 Jul 2015	180	643	68	33.1	0.31	0.74	112.0	1.08	3.03	49.9	24.6
2	0038:50 UTC 9 Jul 2015	205	644	99	26.2	0.11	0.11	16.3	0.75	1.18	79.5	25.6
2	0047:40 UTC 9 Jul 2015	360	611	88	32.1	0.09	0.13	18.4	0.57	0.90	22.9	13.2
2	0053:40 UTC 9 Jul 2015	180	612	81	34.3	0.08	0.11	31.2	0.56	1.17	12.5	13.3
3	1021:00 UTC 3 Oct 2015	180	605	235	25.2	0.28	0.65	92.3	1.09	5.93	61.9	114.4
3	1024:00 UTC 3 Oct 2015	235	600	213	26.1	0.29	0.78	100.7	1.24	3.53	96.2	37.2
3	1027:55 UTC 3 Oct 2015	180	597	191	27.1	0.19	0.22	50.9	0.84	1.72	40.2	17.5
3	1030:55 UTC 3 Oct 2015	205	598	170	27.7	0.47	0.23	36.2	0.92	1.83	82.1	27.7
3	1039:00 UTC 3 Oct 2015	180	613	121	30.1	0.64	0.48	78.9	1.25	3.34	125.6	42.4
3	1042:00 UTC 3 Oct 2015	180	605	106	31.4	0.37	0.47	59.3	1.18	2.69	136.0	36.7
3	1049:35 UTC 3 Oct 2015	195	608	64	34.0	0.22	0.56	98.4	0.96	2.54	35.4	19.6
3	1054:20 UTC 3 Oct 2015	325	612	46	29.3	0.06	0.44	34.8	0.95	2.00	96.7	34.6
3	1104:35 UTC 3 Oct 2015	180	622	60	22.6	-0.11	0.24	12.2	0.68	1.56	72.3	59.8
3	1139:00 UTC 3 Oct 2015	180	602	27	37.0	-0.17	0.32	113.7	1.12	3.36	56.5	29.9
3	1145:40 UTC 3 Oct 2015	180	605	49	42.7	0.12	0.23	107.0	0.89	2.15	23.8	12.9
3	1148:40 UTC 3 Oct 2015	180	599	64	41.6	0.92	0.73	73.7	1.22	3.60	122.4	52.7
3	1151:40 UTC 3 Oct 2015	180	595	80	39.4	0.24	0.77	262.2	1.30	3.77	44.0	16.2
3	1154:40 UTC 3 Oct 2015	185	594	95	37.1	0.94	1.59	156.1	1.42	5.96	107.1	68.2
3	1158:10 UTC 3 Oct 2015	355	590	117	34.5	0.46	0.35	45.0	0.97	2.98	80.7	59.2
4	0712:00 UTC 1 Aug 2016	180	653	286	10.8	0.03	-0.05	2.2	0.25	0.63	6.6	52.9
4	0715:25 UTC 1 Aug 2016	265	646	255	14.9	0.03	-0.02	3.7	0.33	0.30	13.6	7.3
4	0742:30 UTC 1 Aug 2016	360	611	121	35.0	0.20	0.81	105.2	1.17	4.86	73.0	67.3
4	0827:25 UTC 1 Aug 2016	265	583	107	33.7	0.54	0.16	40.6	1.00	4.16	100.7	127.5
4	0841:55 UTC 1 Aug 2016	180	621	101	39.1	1.35	1.79	485.1	2.25	13.53	218.5	113.3
4	0845:05 UTC 1 Aug 2016	180	628	108	39.1	0.38	1.32	324.7	1.79	7.40	129.3	50.6
4	0848:05 UTC 1 Aug 2016	180	632	118	38.1	-0.36	0.66	293.8	1.28	5.67	37.8	32.8

examined the residuals of the K_m-U model and found no significant dependence on height for either the Z11 or the HKO observations. This is likely due the relatively narrow range of altitudes sampled and gives some confidence that the difference in exponents between datasets is not purely an artifact of height difference.

There is a large scatter in the K_m-U plot reflected by an r^2 value of 0.43. Values of K_m at around 30 m s^{-1} vary by over a factor of 10 suggesting the importance of influences other than U in determining K_m . As the scatter around the K_m-U model is not explained by variation in altitude we expect that other factors (e.g., stability, wind shear) which can influence turbulent mixing are responsible and acknowledge that U is only one of many potential covariates of K_m .

The TKE method of calculating K_m yields a closer agreement with the Z11 data. We find b coefficient of 0.91 ± 0.30 with the corresponding value in the Z11 data

1.13 ± 0.21 . However, we have lower confidence in using this method as the scatter was much larger ($r^2 = 0.16$) and the b value was very sensitive to quality control filters applied. This is possibly because the stationary tests used here do not directly test for the stationarity of the components of TKE (v'_t , v'_r , and w'). We have therefore focused our analysis on the results from the Hanna method.

The power-law formulation and exponent provides a method for comparing observed diffusivity behavior with wind speed to those generated by parameterizations in numerical models. To demonstrate this, we present preliminary results from idealized TC simulations using the Weather Research and Forecasting (WRF) model (version 3.7.1; Skamarock et al. 2008) configured as described in Wang and Toumi (2019). The two most widely used parameterizations are examined: 1) the Yonsei University (YSU; Hong et al. 2006)

FIG. 5. Vertical flux of (a) total momentum against wind speed, (b) tangential momentum against radius normalized by RMW, and (c) radial momentum against radius normalized by RMW.

nonlocal PBL scheme coupled with the MM5 similarity surface layer scheme (Zhang and Anthes 1982) and 2) the local Mellor–Yamada–Janjić (MYJ) PBL scheme with the Janjić Eta Monin–Obukhov surface-layer scheme (Janjić 2002). The simulated TCs were allowed to mature, then for 2 days, hourly instantaneous values of K_m and U at approximately 570 m were azimuthally averaged (Zhang et al. 2012). Figure 7 shows the simulated U and K_m values for the two simulations with power-law fit coefficients. The exponents for the YSU and MYJ simulations were 0.89 ± 0.01 and 4.25 ± 0.02 , respectively. Our observed exponent of 1.73 ± 0.20 sits between these values and is compatible with neither to within standard error. We note that at, for example, 40 m s^{-1} , the

diffusivities are similar between the parameterizations but the variation with wind speed is dramatically different making the case for determining the exponent. A thorough examination is beyond the scope of this paper instead we present this analysis to demonstrate the value of the power-law exponent as a means of comparison between observations and models. A full analysis could examine spatial and temporal scales, TC development stages, the height, and the role of the surface layer.

c. Power law relations of EDR, TKE, and σ_w to U

The K_m – U relationship can be thought of in terms of the power-law behavior of the components of both the Hanna and TKE K_m formulas. In Fig. 8 we show the

FIG. 6. Vertical momentum diffusivity K_m calculated using the Hanna method against wind speed found by (a) Z11 and (b) this study. Coefficients refer to $K_m = aU^b$ determined through linear regression of $\log_{10}K_m$ onto $\log_{10}U$.

FIG. 7. Azimuthally averaged vertical momentum diffusivity K_m against wind speed U for idealized TC simulations using the YSU (blue squares) and MYJ (orange circles) PBL schemes. Estimates of coefficients a and b refer to the power-law model $K_m = aU^b$. Data from within the eye and outside 10 times the RMW were excluded.

log-log scatters of EDR, σ_w , and TKE against U revealing their power-law nature. Applying the same power-law model as above for $\epsilon-U$ gives $b_\epsilon = 2.77$. Similarly, for σ_w-U we find $b_{\sigma_w} = 1.13$. Because K_m given by the Hanna method is proportional to σ_w^4/ϵ , $b_{K_m} = 4b_{\sigma_w} - b_\epsilon = 1.73$ as in Fig. 6b. Večenaj et al. (2010) derive theoretical values of $b_{\sigma_w} = 1$, $b_\epsilon = 3$, and $b_{\text{TKE}} = 2$ which combine to give a theoretical value of $b_{K_m} = 1$ for both Hanna and TKE methods. Our observations suggest that in TC boundary layer conditions, b_{σ_w} is slightly larger than the theoretical

value of 1 and b_ϵ is less than 3 leading to our observed value of b_{K_m} differing significantly from 1 for the Hanna method. The observed b_{TKE} is lower than the theoretical 2 which leads to the b_{K_m} values for the TKE method becoming significantly lower than for the Hanna method. The relatively low r^2 for the TKE- U relation (0.58) may be due to quality control issues as discussed above and leads us to have less confidence in the TKE method for establishing a K_m-U relationship.

6. Conclusions

Observations of vertical momentum flux and diffusivity in the high-wind boundary layer region of TCs are scarce. We present new data from four eyewall penetration flights into TC boundary layers in the SCS jointly carried out by the HKO and Government Flying Service of Hong Kong. This paper shows the first direct observations of negative (upward) vertical fluxes of tangential momentum in the eyewall of the TC boundary layer. These are consistent with the current knowledge of TC wind structure.

We also demonstrate a robust, approximately super-linear dependence of K_m on U across the range of observed wind speeds. This finding disagrees with the only previous example of diffusivity measurements at very high wind speeds in the Hurricane boundary layer which found the relationship to be approximately linear. However, the new data has more points at very high wind speeds, increasing confidence that the K_m-U relationship we present holds in the eyewall of TCs.

The new observational data presented here may be useful in helping constrain the parameterization schemes in numerical models dealing with subgrid-scale mixing

FIG. 8. Log-scale plots of (a) EDR, (b) σ_w , and (c) TKE against U . The parameters a and b refer to $y = ax^b$, which were calculated using linear regression as described in the text. Markers are as in Fig. 5.

in extreme wind regimes, potentially improving their forecast skill.

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