Formulation of Three-Dimensional Quasi-Residual Mean Flow Balanced with Diabatic Heating Rate and Potential Vorticity Flux

TAKENARI KINOSHITA

Japan Agency for Marine-Earth Science and Technology, Kanagawa, Japan

KAORU SATO

Department of Earth and Planetary Science, The University of Tokyo, Tokyo, Japan

KENTARO ISHIJIMA,^a MASAYUKI TAKIGAWA, AND YOUSUKE YAMASHITA

Japan Agency for Marine-Earth Science and Technology, Kanagawa, Japan

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ABSTRACT

Three-dimensional (3D) quasi-residual mean flow is derived to diagnose 3D dynamical material transport associated with stationary planetary waves. The 3D quasi-residual mean vertical flow does not include the vertical flow due to tilting of the potential temperature caused by stationary waves, which is apparent but not seen in the mass-weighted isentropic mean state. Thus, the quasi-residual mean vertical flow is balanced with the term of diabatic heating rate. The 3D quasi-residual mean horizontal flow is balanced with the sum of the forcing due to transient wave activity flux divergence and the forcing associated with fluctuation of the potential vorticity due to stationary waves (defined as the effective Coriolis forcing). The zonal mean of the effective Coriolis forcing corresponds to the divergence of stationary wave activity flux. Thus, the zonal mean of derived 3D quasi-residual mean flow is exactly equal to the traditional residual mean flow. To demonstrate the usefulness of this quasi-residual mean flow, we analyze material transport of atmospheric sulfur hexafluoride (SF₆) by using an atmospheric chemistry transport model. Comparison between the derived 3D quasi-residual mean flow and traditional residual mean flow shows that the zonal mean of advection of SF₆ associated with the 3D quasi-residual mean flow derived is almost equal to that of the traditional residual mean flow. Next, it is confirmed that the horizontal structure of advection of SF₆ associated with the 3D quasi-residual mean flow is balanced with the transport because of the nonlinear, nonconservative effects of disturbances. This relation is similar to the results for traditional residual mean flow in the zonal-mean state.

1. Introduction

The transformed Eulerian mean (TEM) equation derived by Andrews and McIntyre (1976, 1978) can be used to diagnose relations between wave activities and zonal-mean flow. The traditional residual mean flow ($[v_a]^*$, $[w_a]^*$) is expressed as the sum of the Eulerian mean flow and quasi-Stokes correction under

the assumptions of small amplitude and is written as follows:

$$\begin{aligned} [v_a]^* &\equiv [v_a] - \rho_0^{-1} \frac{\partial}{\partial z} \left(\rho_0 \frac{[v^\dagger \theta^\dagger]}{\theta_{0z}} \right), \\ [w_a]^* &\equiv [w_a] + \frac{\partial}{\partial y} \left(\frac{[v^\dagger \theta^\dagger]}{\theta_{0z}} \right), \end{aligned} \quad (1.1)$$

where v_a and w_a are the respective meridional and vertical ageostrophic flows, v is the meridional geostrophic flow, θ is the potential temperature, $\theta_0(z)$ is a reference potential temperature, $\rho_0(z)$ is the basic density, $[\]$ expresses the zonal mean, and $\dagger$ is the deviation from the zonal mean. The suffix z denotes the partial derivative in the vertical direction. Note that Eq. (1.1) is expressed under the quasigeostrophic (QG) assumption. Thus, the

^a Denotes content that is immediately available upon publication as open access.

^a Current affiliation: Meteorological Research Institute, Tsukuba, Japan.

Corresponding author: Takenari Kinoshita, t-kinoshita@jamstec.go.jp

traditional residual mean flow is approximately equal to the zonal-mean Lagrangian mean flow and has been widely used for examining material transport, as in Brewer–Dobson circulation (Brewer 1949; Dobson 1956). On the other hand, in the steady state, these traditional residual mean meridional and vertical flows are balanced with forcings because of the wave activity flux divergence and diabatic heating rate divided by $\theta_0(z)$, respectively:

$$-f_0[v_a]^* = \rho_0^{-1} \nabla \cdot \mathbf{F}, \quad [w_a]^* = \frac{[Q]}{\theta_{0z}}, \quad (1.2)$$

where f_0 is the Coriolis parameter, $\nabla \cdot \mathbf{F}$ denotes the wave activity flux divergence, and Q is the diabatic heating rate. The relation between the residual mean meridional flow and wave activity flux divergence is used for the concept of downward control (Haynes et al. 1991) and is useful for examining material transport driven by specific types of waves (e.g., McLandress and Shepherd 2009; Okamoto et al. 2011). Dunkerton (1978) showed that the mean meridional material circulation of the stratosphere and mesosphere can be inferred by using $[w_a]^* = [Q]/\theta_{0z}$ and the continuity equation of residual mean flow. Next, we show that the zonal-mean potential vorticity equation can be modified to the continuity equation of the traditional residual mean flow. The potential vorticity equation is written as follows:

$$\begin{aligned} \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} + v \frac{\partial q}{\partial y} &= \frac{\partial q}{\partial t} + \frac{\partial(uq)}{\partial x} + \frac{\partial(vq)}{\partial y} \\ &= -X_y + Y_x + f_0 \rho_0^{-1} \frac{\partial}{\partial z} \left(\rho_0 \frac{Q}{\theta_{0z}} \right), \end{aligned} \quad (1.3)$$

where $q \equiv f_0 + \beta y - u_y + v_x + \rho_0^{-1} \partial/\partial z (\rho_0 \theta/\theta_{0z})$ is the QG potential vorticity, β is the beta effect, u is the zonal geostrophic flow, and X and Y are the unspecified horizontal components of friction, or nonconservative forcing. When the terms X and Y are neglected, the zonal-mean potential vorticity equation in the steady state is expressed in the following:

$$\frac{\partial}{\partial y} [v^\dagger q^\dagger] = f_0 \rho_0^{-1} \frac{\partial}{\partial z} \left(\rho_0 \frac{[Q]}{\theta_{0z}} \right). \quad (1.4)$$

Using the relation between the potential vorticity flux and wave activity flux divergence (e.g., Edmon et al. 1980), the potential vorticity equation is expressed as follows:

$$\frac{\partial}{\partial y} (\rho_0^{-1} \nabla \cdot \mathbf{F}) = f_0 \rho_0^{-1} \frac{\partial}{\partial z} \left(\rho_0 \frac{[Q]}{\theta_{0z}} \right). \quad (1.5)$$

Substituting Eq. (1.2) into Eq. (1.5), Eq. (1.5) is modified as follows:

$$-f_0 \frac{\partial}{\partial y} [v_a]^* = f_0 \rho_0^{-1} \frac{\partial}{\partial z} (\rho_0 [w_a]^*). \quad (1.6)$$

Thus, the zonal-mean potential vorticity equation can be modified to the continuity equation of the residual mean flow. Note that the conception of Eqs. (1.2)–(1.6) are important to derive the 3D quasi-residual mean horizontal flow.

Many studies have generalized the TEM equations to three dimensions (3D) to examine local wave activity and material transport. Hoskins et al. (1983), Trenberth (1986), and Plumb (1986) extended the TEM equations to 3D by using the time mean instead of the zonal mean under the QG approximation. Takaya and Nakamura (1997, 2001) derived a phase-independent 3D wave activity flux and quasi-Stokes correction applicable to quasi-stationary Rossby waves in zonally asymmetric flow by using the wave activity density and wave energy conservation laws.

Regarding the primitive equation, Miyahara (2006) and Kinoshita et al. (2010) extended the TEM equations to 3D by using the time mean under the assumption that the Coriolis parameter is constant. They used the dispersion relation for the inertia-gravity wave in the terms included in the horizontal component of the 3D quasi-Stokes correction. Thus, their quasi-Stokes correction is applicable only to inertia-gravity waves. Kinoshita and Sato (2013a,b) derived time-mean 3D residual mean flow and wave activity flux applicable to both Rossby waves and gravity waves. Kinoshita and Sato (2014) extended the 3D TEM equations to equatorial waves. Finally, Noda (2010, 2014) formulated a generalized 3D TEM equation for a plane wave under the Wentzel–Kramers–Brillouin (WKB) approximation.

In the ocean dynamics, McDougall and McIntosh (1996, 2001) formulated the temporal-residual-mean (TRM) flow using the conservation equations of mean density and density variance. The TRM flow includes not only the horizontal eddy flux of density but also the effects of advection of density variance by the mean flow corresponding to the second order of quasi-Stokes corrections in the vertical. The horizontal component of the TRM flow is equivalent to the thickness-weighted mean velocity in density coordinates. Plumb and Ferrari (2005) discussed the conception of residual mean circulation on a zonal-mean flow. They focused on the quasi-Stokes streamfunction included in the residual mean circulation and

residual buoyancy flux in the TEM buoyancy equation. They argued that the quasi-Stokes streamfunction should be chosen in such a way as to eliminate the skew component of the raw eddy flux. That is, the residual buoyancy flux on the right-hand side of the TEM buoyancy equation depends only on the diapycnal component of the raw buoyancy flux. In this conception, the residual vertical flow and residual buoyancy flux in the TEM buoyancy equation are zero in the adiabatic condition.

Almost all of these previous studies derived the 3D TEM equations by proving that the residual mean flow is equal to the sum of the mean flow and quasi-Stokes correction and that the wave activity flux is a product of the group velocity and wave activity density under the small-amplitude assumption. Relations similar to those expressed by Eq. (1.2) and especially the relation between the residual mean vertical flow and diabatic heating rate were investigated for transient waves in 3D. However, effects of stationary waves in those relations have not been confirmed in 3D.

On the other hand, [Sato et al. \(2009\)](#) examined the horizontal structure of ozone increase processes after a mature stage of the Antarctic ozone hole in spring by using ozonesonde observations at Syowa Station and observation by the satellite-based Improved Limb Atmospheric Spectrometer II (ILAS-II) in 2003. They found that the observed downward transport of ozone has significant longitudinal dependency. They also showed that the longitudinal structure of the downward transport corresponds not to a quasi-stationary planetary wave's vertical component but to isentropes modified by a quasi-stationary planetary wave, with the amplitude and phase varying on a seasonal time scale. Thus, it appears that dynamical material transport at high latitudes has longitudinal dependency and relates to stationary wave activities.

Based on the above considerations, the purpose of this study is to derive the 3D quasi-residual mean flow to diagnose 3D dynamical material transport including the effects of stationary waves. The obtained 3D quasi-residual mean flow corresponds to the 3D residual mean flow of [Plumb \(1986\)](#) with the tilting effects of potential temperature and potential vorticity surfaces due to stationary waves removed. Then, for transient waves, the conception of skew eddy flux is established. On the other hand, for stationary waves, the conception is established when the zonal mean is applied. Therefore, the derived flow is named "3D quasi-residual mean flow." The 3D quasi-residual mean flow is balanced with diabatic heating

rate and potential vorticity flux. Next, to demonstrate the usefulness of the derived 3D residual mean flow, the dynamical material transport of sulfur hexafluoride (SF₆) is examined by using both the traditional and derived residual mean flows.

The rest of the paper is organized as follows. In [section 2](#), the vertical component of the 3D residual mean flow is derived. The horizontal components are then derived in [section 3](#). In [section 4](#), the treatment of gravity wave forcing is discussed. [Section 5](#) shows the usefulness of the derived 3D residual mean flow through the analysis of SF₆ transport. A summary and the concluding remarks are given in [section 6](#).

2. 3D quasi-residual mean vertical flow balanced with diabatic heating rate

Let the overbar symbol ($\overline{\cdots}$) denote a variable as a time-mean state, with its deviation denoted by a prime ($'$). Then we can write the thermodynamic equation including the 3D residual mean flow of [Plumb \(1986\)](#) in the time-mean state as follows:

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u} \frac{\partial \bar{\theta}}{\partial x} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} + \bar{w}_a^* \theta_{0z} = \bar{Q}, \quad (2.1)$$

where u and v are the geostrophic zonal and meridional flows, respectively; $\bar{w}_a^* \equiv \bar{w}_a + (\bar{u}'\bar{\theta}'/\theta_{0z})_x + (\bar{v}'\bar{\theta}'/\theta_{0z})_y$ is the 3D residual mean vertical flow of [Plumb \(1986\)](#); and the suffixes x and y denote the respective partial derivatives in the zonal and meridional directions. Note that the time derivative is retained in the equation to denote time variation with a longer time scale than the time mean. Because the 3D residual mean vertical flow is derived using the time mean, it includes the quasi-Stokes correction of transient waves.

Then, as will be shown in [section 5](#), this 3D residual mean vertical flow is not balanced with the diabatic heating rate from upper troposphere to stratosphere in winter extratropical regions, where stationary planetary waves are dominant. Note that although [Takaya and Nakamura \(1997, 2001\)](#) and [Sato et al. \(2013\)](#) introduced the 3D residual mean flow and quasi-Stokes correction applicable to stationary waves, it is not confirmed whether their 3D residual mean vertical flow is balanced with the diabatic heating rate. In fact, we calculate the 3D residual mean vertical flow derived by [Sato et al. \(2013\)](#) in [section 5](#) and find that the horizontal distribution of the 3D residual mean vertical flow is different from that of diabatic heating rate. Note that although the analysis method of [Sato et al. \(2013\)](#), which uses an extended Hilbert transform, is

not applicable to the dynamical material transport associated with stationary waves, it is applicable to those waves' activities and group velocities. We suspect that this difference is due to the terms including first-order stationary waves in the thermodynamic equation. Then we define waves as transient waves ($'$) and stationary waves ($\overline{(\cdots)'}^{\dagger}$) obtained from the deviation from the time-mean zonal-mean state. That is, the background state is the time-mean zonal-mean field. A new 3D quasi-residual mean vertical flow including first-order stationary waves is derived as the following:

FIG. 1. A schematic figure of zonal-log pressure cross section of the tilt of potential temperature and zonal wind; Δx is the longitudinal displacement, and Δz is the vertical displacement of isentropic surface.

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x} (\bar{u} \bar{\theta}) + \frac{\partial}{\partial y} (\bar{v} \bar{\theta}) + \bar{w}_a^* \theta_{0z} &= \bar{Q}, \\ \frac{\partial \bar{\theta}}{\partial t} + \frac{\partial}{\partial x} ([\bar{u}][\bar{\theta}] + [\bar{u}]\bar{\theta}^{\dagger} + \bar{u}^{\dagger}[\bar{\theta}] + \bar{u}^{\dagger}\bar{\theta}^{\dagger}) + \frac{\partial}{\partial y} (\bar{v}^{\dagger}[\bar{\theta}] + \bar{v}^{\dagger}\bar{\theta}^{\dagger}) + \bar{w}_a^* \theta_{0z} &= \bar{Q}, \\ \frac{\partial \bar{\theta}}{\partial t} + \left\{ \bar{w}_a^* + \frac{\partial}{\partial x} \left(\frac{[\bar{u}]\bar{\theta}^{\dagger} + \bar{u}^{\dagger}[\bar{\theta}] + \bar{u}^{\dagger}\bar{\theta}^{\dagger}}{\theta_{0z}} \right) + \frac{\partial}{\partial y} \left(\frac{\bar{v}^{\dagger}[\bar{\theta}] + \bar{v}^{\dagger}\bar{\theta}^{\dagger}}{\theta_{0z}} \right) \right\} \theta_{0z} &= \bar{Q}. \end{aligned} \quad (2.2)$$

Here, we define the new 3D quasi-residual mean vertical flow as follows:

$$\begin{aligned} \bar{w}_a^{**} &= \bar{w}_a + \bar{w}^{\dagger} \equiv \bar{w}_a + \left(\frac{[\bar{u}]\bar{\theta}^{\dagger} + \bar{u}^{\dagger}[\bar{\theta}] + \bar{u}^{\dagger}\bar{\theta}^{\dagger} + \bar{u}'\bar{\theta}'}{\theta_{0z}} \right)_x \\ &+ \left(\frac{[\bar{v}]\bar{\theta}^{\dagger} + \bar{v}^{\dagger}[\bar{\theta}] + \bar{v}^{\dagger}\bar{\theta}^{\dagger} + \bar{v}'\bar{\theta}'}{\theta_{0z}} \right)_y, \end{aligned} \quad (2.3)$$

where $\bar{w}^{\dagger}$ is the vertical component of the 3D quasi-Stokes correction applicable to both transient and stationary waves. Thus, the thermodynamic equation is modified as follows using the 3D quasi-residual mean vertical flow:

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{w}_a^{**} \theta_{0z} = \bar{Q}. \quad (2.4)$$

This formulation is derived by extracting effects of stationary waves from time-mean advection after dividing the thermodynamic equation into transient eddy flux and time-mean advection. The quasi-residual mean vertical flow [Eq. (2.2)] is balanced with $\bar{Q}/\theta_{0z}$ when the time derivative of the time-mean potential temperature is negligibly small. The zonal mean of the quasi-residual mean vertical flow is exactly equal to the traditional two-dimensional (2D) residual mean vertical flow, because it includes the effects of stationary waves. Figure 1 schematically illustrates the physical meaning of newly added terms, in log-pressure coordinates. The black line represents the tilt of potential

temperature surface resulting from stationary planetary waves. The yellow arrow represents the zonal-mean flow. The vertical flow due to the tilt of potential temperature is expressed with $[\bar{u}]\Delta z/\Delta x = [\bar{u}](\partial \bar{\theta}^{\dagger}/\partial x)/[\partial(\bar{\theta}^{\dagger} + \theta_0)/\partial z] \approx [\bar{u}](\partial \bar{\theta}^{\dagger}/\partial x)/\theta_{0z}$. This is included in the second term on the right-hand side of the above definition of the residual mean vertical flow [Eq. (2.2)]. The same consideration is applicable to other terms of the residual mean vertical flow. Thus, the newly added terms express the effects of the potential temperature surface's tilt due to stationary planetary waves. Note that McDougall and McIntosh (2001) introduced this effect as a part of Eulerian-mean vertical flow in section 9 and that the derived vertical flow in this study corresponds to the 3D residual mean vertical flow of Plumb (1986) with the potential temperature surface's tilt due to stationary waves removed. Comparison between $\bar{w}_a$ and $\bar{w}^{\dagger}$ is shown in section 5 (Fig. 6).

3. 3D quasi-residual mean horizontal flow balanced with potential vorticity flux

In this section, we formulate the quasi-residual mean horizontal flow. Considering the continuity equation, the quasi-residual mean horizontal flow needs to have the terms including first-order stationary waves. We focus on the fact that the potential vorticity equation can be modified to the continuity equation of residual mean flow in the 2D TEM equations [Eqs. (1.4)–(1.6)] and derive the quasi-residual mean horizontal flow from the

time-mean potential vorticity equation and horizontal momentum equation by using the concept of the relation between potential vorticity flux and wave activity flux divergence (e.g., Edmon et al. 1980) and the relation between 2D residual mean meridional flow and wave activity flux divergence in the TEM zonal momentum equation [Eq. (1.2)]. The time-mean potential vorticity equation is written as follows when only the diabatic heating rate is considered:

$$\begin{aligned} \frac{\partial \bar{q}}{\partial t} + \frac{\partial}{\partial x}(\bar{u}\bar{q} + \overline{u'q'}) + \frac{\partial}{\partial y}(\bar{v}\bar{q} + \overline{v'q'}) \\ = \rho_0^{-1} f_0 \frac{\partial}{\partial z} \left(\frac{\rho_0 \bar{Q}}{\theta_{0z}} \right). \end{aligned} \quad (3.1)$$

Here, the term $\bar{u}\bar{q}$ includes a square of zonal-mean components $[\bar{u}][\bar{q}]$ whose zonal derivative is zero. Then we rewrite the time-mean potential vorticity equation as follows to eliminate the effect of the square of zonal-mean time-mean components:

$$\begin{aligned} \frac{\partial \bar{q}}{\partial t} + \frac{\partial}{\partial x}(\bar{u}\bar{q} - [\bar{u}][\bar{q}] + \overline{u'q'}) + \frac{\partial}{\partial y}(\bar{v}\bar{q} + \overline{v'q'}) \\ = \rho_0^{-1} f_0 \frac{\partial}{\partial z} \left(\frac{\rho_0 \bar{Q}}{\theta_{0z}} \right). \end{aligned} \quad (3.2)$$

From the concept of relation between potential vorticity flux and wave activity flux divergence, the terms $\bar{u}\bar{q} - [\bar{u}][\bar{q}] + \overline{u'q'}$ and $\bar{v}\bar{q} + \overline{v'q'}$ are rewritten as follows:

$$\begin{aligned} \bar{u}\bar{q} - [\bar{u}][\bar{q}] + \overline{u'q'} &= (f_0 + \beta y)\bar{u} + (\bar{u}\bar{v} + \overline{u'v'})_x + (\bar{v}^2 + \overline{v'^2} - \bar{S})_y + f_0 \rho_0^{-1} \left(\rho_0 \frac{\bar{u}\bar{\theta} - [\bar{u}][\bar{\theta}] + \overline{u'\theta'}}{\theta_{0z}} \right)_z, \\ \bar{v}\bar{q} + \overline{v'q'} &= (f_0 + \beta y)\bar{v} - (\bar{u}^2 - [\bar{u}]^2 + \overline{u'^2} - \bar{S})_x - (\bar{u}\bar{v} + \overline{u'v'})_y - f_0 \rho_0^{-1} \left(\rho_0 \frac{\bar{v}\bar{\theta} + \overline{v'\theta'}}{\theta_{0z}} \right)_z, \end{aligned} \quad (3.3)$$

where $\bar{S} \equiv (1/2)(\bar{u}^2 - [\bar{u}]^2 + \bar{v}^2 - \{g_0(\bar{\theta}^2 - [\bar{\theta}]^2)\}/\theta_{0z})$, g_0 is the gravitational acceleration, and the relation $[\bar{v}] = 0$ is used. On the other hand, the time-mean zonal

and meridional momentum equation can be written as follows when $[\bar{u}]^2 = 0$ is added to the zonal momentum equation:

$$\begin{aligned} \bar{u}_t - f_0 \bar{v}_a - (f_0 + \beta y)\bar{v} + \bar{\Phi}_x &= -(\bar{u}^2 - [\bar{u}]^2 + \overline{u'^2})_x - (\bar{u}\bar{v} + \overline{u'v'})_y, \\ \bar{v}_t + f_0 \bar{u}_a + (f_0 + \beta y)\bar{u} + \bar{\Phi}_y &= -(\bar{u}\bar{v} + \overline{u'v'})_x - (\bar{v}^2 + \overline{v'^2})_y. \end{aligned} \quad (3.4)$$

From Eqs. (3.3) and (3.4), replacing the right-hand side of time-mean zonal and meridional momentum equations to $\bar{v}\bar{q} + \overline{v'q'}$ and $-(\bar{u}\bar{q} - [\bar{u}][\bar{q}] + \overline{u'q'})$, respectively, and adding the residual terms by the replacement to the

time-mean ageostrophic meridional and zonal flows yield the transformed time-mean zonal and meridional momentum equations including the effects of stationary wave forcing:

$$\begin{aligned} \bar{u}_t - f_0 \bar{v}_a^{**} + \bar{\Phi}_x &= \bar{v}\bar{q} + \overline{v'q'} = \bar{v}^\dagger([\bar{q}] + \bar{q}^\dagger) + \overline{v'q'}, \\ \bar{v}_t + f_0 \bar{u}_a^{**} + \bar{\Phi}_y &= -(\bar{u}\bar{q} - [\bar{u}][\bar{q}] + \overline{u'q'}) = -[\bar{u}]\bar{q}^\dagger - \bar{u}^\dagger([\bar{q}] + \bar{q}^\dagger) + \overline{u'q'}, \\ \bar{u}_a^{**} &\equiv \bar{u}_a + \left(\frac{\bar{S}}{f_0} \right)_y - \rho_0^{-1} \left(\rho_0 \frac{[\bar{u}]\bar{\theta}^\dagger + \bar{u}^\dagger[\bar{\theta}] + \overline{u'\theta'} + \overline{u'\theta'}}{\theta_{0z}} \right)_z, \\ \bar{v}_a^{**} &\equiv \bar{v}_a - \left(\frac{\bar{S}}{f_0} \right)_x - \rho_0^{-1} \left(\rho_0 \frac{[\bar{v}]\bar{\theta}^\dagger + \bar{v}^\dagger[\bar{\theta}] + \overline{v'\theta'} + \overline{v'\theta'}}{\theta_{0z}} \right)_z. \end{aligned} \quad (3.5)$$

Note that the relation $\overline{AB} - [\bar{A}][\bar{B}] = [\bar{A}]\bar{B}^\dagger + \bar{A}^\dagger([\bar{B}] + \bar{B}^\dagger)$, where A and B are arbitrary physical variables,

and $[\bar{v}] = 0$ are used in Eq. (3.5). Note also that the zonal mean of the right-hand side of the zonal

momentum equation is equal to the divergence of wave activity flux [e.g., $\overline{v'q'} = \nabla \cdot \mathbf{F}_{(P)}$, where $\mathbf{F}_{(P)}$ is the transient wave activity flux of Plumb (1986)]. Since the term $\bar{S}$ is equal to $\bar{u}^2 - [\bar{u}]^2 + \bar{u}'^2 + \bar{v}^2 + \bar{v}'^2 - \bar{E}$, $\bar{E} \equiv (1/2)(\bar{u}^2 - [\bar{u}]^2 + \bar{v}^2 + \{g_0(\bar{\theta}^2 - [\bar{\theta}]^2)\}/\theta_0\theta_{0z})$, the obtained quasi-residual mean horizontal flow $\bar{u}_a^{**}$ and $\bar{v}_a^{**}$ is similar to Eqs. (4.4) and (4.5) of Plumb (1986) and includes terms for first- and second-order stationary waves. Thus, the zonal mean of quasi-residual mean meridional flow is exactly equal to the traditional 2D residual mean meridional flow. The new 3D quasi-residual mean flow satisfies the continuity equation. Moreover, by substituting Eqs. (2.3) and (3.5) into Eq. (3.2), the time-mean potential vorticity equation can be modified to the continuity equation of 3D quasi-residual mean flow as follows:

$$\begin{aligned} \frac{\partial \bar{q}}{\partial t} - \frac{\partial}{\partial x}(f_0 \bar{u}_a^{**} + \bar{\Phi}_y + \bar{v}_t) - \frac{\partial}{\partial y}(f_0 \bar{v}_a^{**} - \bar{\Phi}_x - \bar{u}_t) &= \rho_0^{-1} f_0 \frac{\partial}{\partial z} \left(+\rho_0 \bar{w}_a^{**} + \frac{\rho_0 \bar{\theta}_t}{\theta_{0z}} \right), \\ -\frac{\partial}{\partial x}(f_0 \bar{u}_a^{**}) - \frac{\partial}{\partial y}(f_0 \bar{v}_a^{**}) &= \rho_0^{-1} f_0 \frac{\partial}{\partial z} (+\rho_0 \bar{w}_a^{**}). \end{aligned} \quad (3.6)$$

Focusing on Eq. (3.5) again, the term $\bar{v}^t([\bar{q}] + \bar{q}^t)$ has the local phase structure of stationary waves. The first term $\bar{v}^t[\bar{q}]$ is almost equal to $f_0 \bar{v}^t$, which is balanced with pressure gradient forcing. The second term $\bar{v}^t \bar{q}^t$ is the Coriolis-like forcing due to a fluctuation of the potential vorticity of stationary waves. Then we regard the term $\bar{v}^t([\bar{q}] + \bar{q}^t)$ as the effective Coriolis forcing. This interpretation is also applicable to the meridional momentum equation. Figure 2 schematically illustrates the relation among the residual mean meridional flow, pressure gradient, and effective Coriolis forcing. For this figure, we assume that the sum of the effective Coriolis forcing and the divergence of transient wave activity flux, $\bar{v}^t([\bar{q}] + \bar{q}^t) + \overline{v'q'}$, is balanced with the sum of pressure gradient forcing and $f_0 \bar{v}_a^{**}$. Note that Eq. (3.5) includes the pressure gradient forcing term since the Coriolis forcing, which is different from that of Plumb (1986). However, there is no problem with removing the terms including Coriolis forcing from potential vorticity equation [Eq. (3.1)]. Then Eq. (3.1) can be modified as follows, which is a similar expression to Eq. (4.3) in Plumb (1986):

$$\begin{aligned} \bar{u}_t - f_0 \bar{v}_a^{**} &= \bar{v}^t(\bar{q} + \bar{q}^t) + \overline{v'q'}, \\ \bar{v}_t + f_0 \bar{u}_a^{**} &= -[\bar{u}]\bar{q}^t - \bar{u}^t(\bar{q} + \bar{q}^t) + \overline{u'q'}, \\ \bar{q} &= [\bar{q}] - f_0. \end{aligned} \quad (3.7)$$

FIG. 2. A schematic diagram of horizontal cross section of effective Coriolis forcing, pressure gradient forcing, and residual mean meridional flow.

4. Nonconservative terms and gravity wave forcing

In section 3, the 3D quasi-residual mean horizontal flow is derived under the assumption that the term of friction or nonconservative forcing is neglected and is balanced with the sum of pressure gradient forcing, effective Coriolis forcing, and transient Rossby wave activity flux divergence. However, the 3D quasi-residual mean horizontal flow can also be derived in the condition including the terms of friction or nonconservative forcing as follows:

$$\begin{aligned} \bar{u}_t - f_0 \bar{v}_a^{**} + \bar{\Phi}_x &= \bar{v}^t([\bar{q}] + \bar{q}^t) + \overline{v'q'} + \bar{X}, \\ \bar{v}_t + f_0 \bar{u}_a^{**} + \bar{\Phi}_y &= -[\bar{u}]\bar{q}^t - \bar{u}^t([\bar{q}] + \bar{q}^t) \\ &\quad - \overline{u'q'} + \bar{Y}, \end{aligned} \quad (4.1)$$

Thus, considering that the right-hand side of Eq. (4.1) encompasses all forcings, including gravity wave forcing, it can be said that the new 3D quasi-residual mean flow is balanced with the sum of pressure gradient forcing, effective Coriolis forcing, transient Rossby wave activity flux divergence, and nonconservative forcing including gravity wave forcing. The 3D quasi-residual mean flow in this study was derived to establish an analysis method for material transport due not only to transient waves but also to stationary waves. As an analysis example for such dynamical transport driven by gravity waves and other waves, we introduce the following method.

The quasi-residual mean meridional flow $\bar{v}^{**}$ in this study and the effective Coriolis forcing divided by the Coriolis parameter $\bar{v}^i([\bar{q}] + \bar{q}^i)/f_0$ are calculated using the zonal and meridional flows and potential temperature. The difference between the quasi-residual mean meridional flow and the sum of the pressure gradient force divided by the Coriolis parameter and the term $\bar{v}^i([\bar{q}] + \bar{q}^i)/f_0$ corresponds to the quasi-residual mean flow driven by forcings due to transient Rossby waves and nonconservative forcing, including gravity wave forcing as follows:

$$\bar{v}^{**} - \frac{\bar{\Phi}_x}{f_0} + \frac{\bar{v}^i([\bar{q}] + \bar{q}^i)}{f_0} \approx -\frac{\bar{v}'q' + \bar{X}}{f_0}. \quad (4.2)$$

Nonconservative terms are divided into gravity wave forcings, which are expressed with wave activity flux divergence, and other nonconservative effects:

$$\bar{X} = \rho_0^{-1} \nabla \cdot \mathbf{F}_{\text{GW}} + \tilde{X}, \quad (4.3)$$

where $\rho_0^{-1} \nabla \cdot \mathbf{F}_{\text{GW}}$ is wave activity flux divergence of gravity waves and $\tilde{X}$ is the other nonconservative effects. When gravity waves are defined as the components having total horizontal wavenumbers larger than 21 (i.e., wavelengths below 1800 km), the gravity wave forcing can be calculated using the wave activity flux divergence of Kinoshita et al. (2010):

$$\mathbf{F}_{\text{GW}} \equiv \rho_0 \begin{pmatrix} \overline{u'^2_{\text{GW}}} - \overline{S_{\text{GW}}} + \overline{u_y \frac{S_{\text{GW}}}{f}} - \overline{u_z \frac{u'_{\text{GW}} \theta'_{\text{GW}}}{\theta_z}} \\ \overline{u'_{\text{GW}} v'_{\text{GW}}} - \overline{u_x \frac{S_{\text{GW}}}{f}} - \overline{u_z \frac{v'_{\text{GW}} \theta'_{\text{GW}}}{\theta_z}} \\ \overline{u'_{\text{GW}} w'_{\text{GW}}} + \overline{u_x \frac{u'_{\text{GW}} \theta'_{\text{GW}}}{\theta_z}} + (\overline{u_y} - f) \frac{\overline{v'_{\text{GW}} \theta'_{\text{GW}}}}{\theta_z} \end{pmatrix}, \quad (4.4)$$

$$\overline{S_{\text{GW}}} \equiv \frac{1}{2} \left(\overline{u'^2_{\text{GW}}} + \overline{v'^2_{\text{GW}}} - \frac{g \overline{\theta'^2_{\text{GW}}}}{\theta \theta_z} \right),$$

where the subscript GW indicated waves with a total horizontal wavenumber larger than 21. Then, by using $\bar{v}_a^{**}$, $\bar{v}^i([\bar{q}] + \bar{q}^i)/f_0$, and $\rho_0^{-1} f^{-1} \nabla \cdot \mathbf{F}_{\text{GW}}$, we can compare the flow driven by gravity waves and Rossby waves from the 3D quasi-residual mean meridional flow including the effects of all waves. Note that gravity wave forcing can also be calculated using the formulas of Miyahara (2006) or Noda (2010, 2014) when the Coriolis parameter is constant.

5. Case study

Using the newly derived 3D quasi-residual mean flow, we investigate the 3D structure of dynamical transport of

minor constituents. The minor constituent used in this study was SF₆, which is industrially produced as an electrical insulator and almost entirely an anthropogenic gas. Moreover, because its lifetime is almost 3200 years, SF₆ can be used as a passive tracer. By using the SF₆ mixing ratio, a comparison analysis among the 2D residual mean flow, Plumb's (1986) 3D residual mean flow, and our derived 3D quasi-residual mean flow was conducted.

a. Data

To prepare atmospheric SF₆ data, we used the Center for Climate System Research/National Institute for Environmental Studies/Frontier Research Center for Global Change (CCSR/NIES/FRCGC) an atmospheric general circulation model-based chemistry transport model (ACTM) (Numaguti et al. 1997; Ishijima et al. 2010). The horizontal resolution is T42 (corresponding to almost 300 km), and the vertical resolution is 67 sigma-coordinate layers (Ishijima et al. 2010). The model top is at almost 90 km. The analysis period was January 2013, after a spinup run from 2001 to 2012. The sea surface temperature was prescribed by ERA-Interim data. Convection and vertical diffusion schemes based on Arakawa and Schubert (1974) and Moorthi and Suarez (1992) were used for subgrid scales. The model uses orographic gravity wave forcing of McFarlane (1987).

b. Meridional cross section of residual mean flow

First, Fig. 3 shows the monthly mean, zonal-mean, residual mean flow and mass-streamfunction results obtained in this analysis. Comparison between the traditional residual mean flow (Fig. 3a) and that of Plumb (1986) (Fig. 3b) shows indirect circulation in the Northern Hemisphere's stratospheric polar region around 60°N in Fig. 3b, unlike in Fig. 3a. This difference is because the quasi-Stokes correction of Plumb (1986) includes the effects of only transient waves. Thus, in the region where the transient waves are small, such as the Northern Hemisphere's stratospheric polar region, the residual mean flow of Plumb (1986) is nearly equal to the Eulerian-mean flow. On the other hand, the quasi-residual mean flow and mass streamfunction in Fig. 3c, obtained using Eqs. (3.2) and (3.5), are almost the same as those in Fig. 3a, since the zonal mean of quasi-Stokes correction derived in this study does include the effects of stationary waves. Note that we confirm that transient waves are dominant below 100 hPa, quasi-stationary waves are dominant above 30 hPa around 60°N, and the stratospheric sudden warming does not occur in this analysis period (not shown).

Figure 4 shows meridional cross sections of the monthly mean, zonal-mean, residual mean vertical flow (Fig. 4a: traditional; Fig. 4b: this study), together with $\bar{Q}/\theta_{0z}$ (Fig. 4c).

FIG. 3. Meridional cross sections of monthly mean, zonal-mean residual mean flow (arrows) and mass streamfunction (color shading) in January 2013: (a) traditional 2D TEM equations, (b) 3D residual mean flow of [Plumb \(1986\)](#), and (c) 3D quasi-residual mean flow of this study.

The figure shows that the structures of traditional residual mean vertical flow ([Fig. 4a](#)) and of this study ([Fig. 4b](#)) are consistent with the structure of $\bar{Q}/\theta_{0z}$ ([Fig. 4c](#)).

c. Horizontal structure of residual mean flow

[Figure 5](#) shows the 3D quasi-residual mean meridional flows of this study, together with the term $-\{\bar{v}^i([\bar{q}] + \bar{q}^i) + \bar{v}'q' + \bar{X} - \bar{\Phi}_x\}/f_0$, at 30 hPa in January 2013. Note that all horizontal maps in this study represent the 30-hPa surface. Since the quasi-residual mean meridional flow of this study ([Fig. 5a](#)) is equal to the term $-\{\bar{v}^i([\bar{q}] + \bar{q}^i) + \bar{v}'q' + \bar{X} - \bar{\Phi}_x\}/f_0$ ([Fig. 5b](#)) when the time derivative of the time-mean zonal flow is negligibly small in [Eq. \(4.1\)](#), [Fig. 5a](#) exhibits almost the same distribution as [Fig. 5b](#).

Next, [Fig. 6](#) shows horizontal maps of the time-mean vertical flow vertical component of quasi-Stokes correction of this study, residual mean vertical flows of [Sato et al. \(2013\)](#), quasi-residual mean vertical flow, and $\bar{Q}/\theta_{0z}$ at 30 hPa in January 2013. The time-mean vertical flow ([Fig. 6a](#)) and the quasi-Stokes correction ([Fig. 6b](#)) almost cancel out, which is consistent with the relation

between the Eulerian-mean flow and quasi-Stokes correction in the 2D TEM equations. As in [Fig. 4](#), the quasi-residual mean vertical flow ([Fig. 6d](#)) is balanced with $\bar{Q}/\theta_{0z}$ ([Fig. 6e](#)), while the residual mean vertical flow of [Sato et al. \(2013\)](#) ([Fig. 6c](#)) is different. Because the residual mean flow of [Sato et al. \(2013\)](#) does not include the term including first-order stationary waves, we suggest that the large upward and downward flows in [Fig. 6a](#) are caused by stationary wave components, which are tilts of the potential temperature surface associated with stationary planetary waves.

d. Dynamical transport of SF_6

In this section, we discuss the atmospheric transport of SF_6 . The zonal-mean tracer transport equation using the traditional residual mean flow is expressed as follows:

$$\begin{aligned} \frac{\partial[\chi]}{\partial t} = & -[v_a]^* \frac{\partial[\chi]}{\partial y} - [w_a]^* \frac{\partial[\chi]}{\partial z} \\ & + [S_{(\chi)}] + \rho_0^{-1} \nabla \cdot \mathbf{M}, \end{aligned} \quad (5.1)$$

FIG. 4. Meridional cross sections of monthly mean, zonal-mean residual mean vertical flows: (a) traditional 2D TEM equations, (b) this study, and (c) $\bar{Q}/\theta_{0z}$.

FIG. 5. Horizontal maps of (a) meridional components of monthly mean 3D quasi-residual mean flow and (b) $-\bar{v}^i([\bar{q}] + \bar{q}^i) + \bar{v}^i \bar{q}^i + \bar{X} - \Phi_x / f_0$ at 30 hPa in January 2013.

where χ is the tracer mixing ratio and $S_{(\chi)}$ is a term including not only chemical production and loss but also the effects of turbulent diffusion by the unresolved scale of motion. For the last term, we define $\mathbf{M} \equiv -\rho_0([\bar{v}^i \chi^i] - [\bar{v}^i \theta^i][\chi]_z / \theta_{0z}, [\bar{w}^i \chi^i] + [\bar{v}^i \theta^i][\chi]_y / \theta_{0z})$. The residual mean flow of Eq. (5.1) includes the effects of both transient and stationary waves. Note that $\mathbf{M} = 0$ when disturbances are linear, steady, adiabatic, and conservative (Andrews et al. 1987).

Figure 7 shows a meridional cross section of the monthly mean, zonal-mean SF_6 and its time derivative. The distribution is almost symmetric at the equator in the stratosphere and large in the Northern Hemisphere troposphere. The time derivative of SF_6 is positive in the Southern Hemisphere, and its distribution is along the contour line of monthly mean SF_6 . The increase, however, is about 1.0×10^{-3} ppt day $^{-1}$ and thus small.

Next, from the distribution of SF_6 and the residual mean flow, the advection of residual mean flow $(-[v_a]^* \partial \chi / \partial y - [w_a]^* \partial \chi / \partial z)$ can be considered positive from the tropical to

subtropical region where the upward flow is dominant and negative around the extratropical region where the downward flow is dominant. Figure 8 shows the monthly mean, zonal-mean advection of the residual mean flow, the difference between that advection and the time derivative of SF_6 , $\partial \chi / \partial t - (-[v_a]^* \partial \chi / \partial y - [w_a]^* \partial \chi / \partial z) = ([S_{(\chi)}] + \rho_0^{-1} \nabla \cdot \mathbf{M})$ and the term $\rho_0^{-1} \nabla \cdot \mathbf{M}$. The advection (Fig. 8a) is positive from the tropical to subtropical region and negative around the extratropical region. Because the time derivative of SF_6 is small, the term $([S_{(\chi)}] + \rho_0^{-1} \nabla \cdot \mathbf{M})$ (Fig. 8b) and the advection of residual mean flow (Fig. 8a) almost cancel out. Moreover, the term $\rho_0^{-1} \nabla \cdot \mathbf{M}$ (Fig. 8c) exhibits a similar distribution to the term $([S_{(\chi)}] + \rho_0^{-1} \nabla \cdot \mathbf{M})$ (Fig. 8b). Thus, the transport of SF_6 has a structure in which advection of the residual mean flow is balanced with transport because of the nonlinear, non-conservative effects of disturbances.

Next, the tracer transport equation using the 3D residual mean flow derived in the present study is given by the following:

$$\begin{aligned}
 \frac{\partial \bar{\chi}}{\partial t} = & -[\bar{u}_a^{**} + \bar{u}] \frac{\partial \bar{\chi}}{\partial x} - [\bar{v}_a^{**}] \frac{\partial \bar{\chi}}{\partial y} - (\bar{v}_a^{**\dagger} + \bar{v}) \frac{\partial [\bar{\chi}]}{\partial y} - [\bar{w}_a^{**}] \frac{\partial \bar{\chi}}{\partial z} \\
 & - \bar{w}_a^{**\dagger} \frac{\partial [\bar{\chi}]}{\partial z} + \bar{S}_{(\chi)} + \rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}, \\
 \rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})} = & -\frac{\partial (\bar{u}' \bar{\chi}' + \bar{u}^i \bar{\chi}^i)}{\partial x} - \frac{\partial (\bar{v}' \bar{\chi}' + \bar{v}^i \bar{\chi}^i)}{\partial y} - \rho_0^{-1} \frac{\partial \rho_0 (\bar{w}' \bar{\chi}' + \bar{w}^i \bar{\chi}^i)}{\partial z} \\
 & - [\bar{u}^s] \frac{\partial \bar{\chi}}{\partial x} - [\bar{v}^s] \frac{\partial \bar{\chi}}{\partial y} - \bar{v}^{s\dagger} \frac{\partial [\bar{\chi}]}{\partial y} - [\bar{w}^s] \frac{\partial \bar{\chi}}{\partial z} - \bar{w}^{s\dagger} \frac{\partial [\bar{\chi}]}{\partial z},
 \end{aligned} \tag{5.2}$$

FIG. 6. Horizontal maps of monthly mean (a) vertical flow, (b) vertical component of quasi-Stokes correction, (c) 3D residual mean vertical flow of [Sato et al. \(2013\)](#), (d) quasi-residual mean vertical flow, and (e) Q/θ_{0z} at 30 hPa in January 2013. The contours express the monthly mean potential temperature. Contour intervals are 10 K.

where $\overline{v}_a^{**\dagger}$ and $\overline{w}_a^{**\dagger}$ are the respective meridional and vertical components of the deviation of zonal-mean 3D quasi-residual mean flow and $\overline{v}^\dagger$ and $\overline{w}^\dagger$ are the respective meridional and vertical components of the deviation of zonal-mean 3D quasi-Stokes correction. [Figure 9](#) shows meridional cross sections of the advection of 3D residual mean flow of this study and $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$. The distributions of the above advection ([Fig. 9a](#)) and term $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$ ([Fig. 9b](#)) are almost the same as those of the traditional residual mean flow ([Fig. 8a](#)) and the term $\rho_0^{-1} \nabla \cdot \mathbf{M}$ ([Fig. 8c](#)), respectively. This is because the 3D quasi-residual mean flow of this study includes the quasi-Stokes correction associated with not only transient and stationary waves.

[Figure 10](#) shows horizontal maps of the monthly mean SF_6 ([Fig. 10a](#)), the advection of monthly mean 3D quasi-residual mean flow of this study ([Fig. 10b](#)), and the term $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$ ([Fig. 10c](#)) at 30 hPa in January 2013. Note that the component of advection of the geostrophic flow is omitted from [Figs. 10b and 10c](#). The panels show that the advection of 3D quasi-residual mean flow is positive in eastern Siberia, northern America, and a part of Greenland, where the poleward flow is large and downward flow is small. Similarly, with [Figs. 8a and 8c](#), these results

confirm that the advection of 3D quasi-residual mean flow is balanced with the term $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$.

Therefore, the 3D quasi-residual mean flow, which includes the quasi-Stokes correction associated with transient and stationary waves and corresponds to the diabatic heating rate and potential vorticity flux in the monthly mean state, satisfies the required characteristics

FIG. 7. Meridional cross section of monthly mean, zonal-mean SF_6 (contours) and its time derivative (color shading).

FIG. 8. Meridional cross section of monthly mean, zonal-mean (a) advection of residual mean flow ($-[v] \partial[\chi]/\partial y - [w] \partial[\chi]/\partial z$), (b) difference between time derivative of SF_6 and advection of residual mean flow ($[S_{(x)}] + \rho_0^{-1} \nabla \cdot \mathbf{M}$), and (c) $\rho_0^{-1} \nabla \cdot \mathbf{M}$.

of the traditional 2D residual mean flow. That is, under the condition that effects of source/sink of SF_6 and time variation of SF_6 with longer than monthly mean are small, the advection by 3D quasi-residual mean flow is balanced with nonlinear, nonconservative effects of disturbances.

6. Concluding remarks

In this study, the 3D quasi-residual mean flow has been extended to investigate dynamical transport associated with both stationary and transient waves. Note that the 3D quasi-residual mean flow was derived as the sum of Plumb's (1986) residual mean flow and terms due to the effects of stationary waves, which were derived from the time-mean flow and potential temperature. Therefore, this theory is consistent with the previous theory applicable to transient waves. To confirm the usefulness of the derived residual mean flow, we analyzed the

dynamical transport of SF_6 through an ACTM simulation. The results showed that the zonal-mean 3D quasi-residual mean flow has the same distribution as the traditional 2D residual mean flow. It was also shown that the horizontal structure of the 3D quasi-residual mean flow is balanced with the term $\rho_1^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$, which is a relation similar to that between the traditional 2D residual mean flow and the term due to the nonlinear, nonconservative effects of disturbances ($\rho_0^{-1} \nabla \cdot \mathbf{M}$). Thus, the derived 3D quasi-residual mean flow can diagnose dynamical transport associated with both transient and stationary waves.

In the future, we will extend this theory to the equatorial region. Almost all theories deriving 3D TEM equations, including the one in this study, are not applicable to the equatorial region, because the Coriolis parameter is included in the denominators of the formulas for Stokes drift. The formulas of Kinoshita and Sato (2014), however, are applicable to the equatorial region but cannot express

FIG. 9. Meridional cross section of monthly mean, zonal-mean (a) advection of 3D quasi-residual mean flow and (b) $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$.

FIG. 10. Horizontal maps of (a) monthly mean SF_6 (color shading) and horizontal component of 3D quasi-residual mean flow (arrows), (b) advection of 3D residual mean flow, and (c) the term of $\rho_0^{-1} \nabla \cdot \mathbf{M}_{(\text{new})}$ at 30 hPa in January 2013. The contour line is monthly mean SF_6 .

the effects of stationary waves. Thus, we will derive a 3D quasi-residual mean flow applicable to the equatorial region by combining this study and those formulas.

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