Noninstantaneous Wave-CISK for the Interaction between Convective Heating and Low-Level Moisture Convergence in the Tropics

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ABSTRACT

The interaction between tropical convective heating and thermally forced circulation is investigated using a global dry primitive-equation model with the parameterization of wave-conditional instability of the second kind (CISK). It is demonstrated that deep convective heating can hardly sustain itself through the moisture convergence at low levels regardless of the fraction of immediate consumption of converged moisture. In contrast, when the fraction is large, shallow convective heating and its forced circulation exhibit preferred growth of small scales. As the “CISK catastrophe” mainly comes from the instantaneous characters of moisture–convection feedback in the conventional wave-CISK, a noninstantaneous wave-CISK is proposed, which highlights the accumulation–consumption (AC) time scale for the convective heating accumulation and/or the converged moisture consumption. In the new wave-CISK, once moisture is converged, the release of latent heat takes place gradually within an AC time scale. In this sense, convective heating is not only related to the instantaneous moisture convergence at the current time, but also to that which occurred in the past period of the AC time scale. The noninstantaneous wave-CISK could guarantee the occurrence of convective heating and/or moisture convergence at larger scales, and then favor the growth of long waves, and thus solve the problem of CISK catastrophe. With the new wave-CISK and AC time scale of 2 days, the simulated convective heating-driven system bears a large similarity to that of the observed convectively coupled Kelvin wave.

1. Introduction

The majority of rainfall observed in the tropics occupies specific spectra of temporal and spatial scales by categories, of which the dry counterparts could be found in the wave solutions of linear equatorial beta-plane shallow-water equations (Wheeler and Kiladis 1999; Roundy and Frank 2004). These convectively coupled equatorial waves (CCEWs) play important roles in global climate system by modulating outgoing longwave radiation (OLR) and latent heat release, and then contribute greatly to the extended-range

predictability in the tropics as well as extratropics (e.g., Bechtold et al. 2008). Among these CCEWs, the convectively coupled Kelvin wave (CCKW), which usually bears a large horizontal scale and propagates eastward with a speed of $10\text{--}17\text{ m s}^{-1}$ (Yang et al. 2007; Roundy 2008), controls a considerable portion of not only synoptic variability but also that of longer time scales via interaction with the Madden–Julian oscillation (Zhang 2005).

Although some of the structures of CCEWs compare well with the set of solutions of the shallow-water model (Matsuno 1966; Lindzen 1967), corresponding mechanisms that lead to these observed characters still remain unknown, especially how these large-scale atmospheric waves couple with convections. The first attempt to explain the coupling process could date back to the 1970s, when the theory that combines wave activity

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and conditional instability of the second kind (CISK), termed wave-CISK, was proposed (Hayashi 1970; Lindzen 1974). The simplest form of the wave-CISK feedback is that large-scale atmospheric waves produce regions of low-level moisture convergence where vigorous convections develop; latent heat released by the convections induces upward motions and further strengthens low-level convergence. Numerical models with the convective parameterization (CP) based on the wave-CISK concept did reproduce the CCEWs with some success (e.g., Hayashi and Sumi 1986; Lau and Peng 1987), but they failed to resolve the finite zonal wavelength, with the fastest growth occurring at the smallest scale (Crum and Dunkerton 1992; Matthews and Lander 1999). This phenomenon is called “CISK catastrophe.”

Another way to look at the interaction between atmospheric waves and convections is the quasi-equilibrium (QE) thinking (Emanuel 2007), which was first introduced by Arakawa and Schubert (1974). QE states that the instability built by slowly varying large-scale forcing is removed by convection in such a short time that a quasi-equilibrium state would maintain in spite of some fluctuations. The temperature profile of the troposphere keeps moist adiabatic neutral (Xu and Emanuel 1989), which is largely controlled by the sub-cloud-layer moist entropy. In strict QE, it is predicted that unstable growth of convections cannot be seen as heating and temperature fields are strictly in quadrature under this circumstance (Emanuel et al. 1994). However, this phase relationship contradicts observation studies that heating and temperature are almost in phase (Straub and Kiladis 2003b; Kiladis et al. 2005; Benedict and Randall 2007), though the mechanism of wind-induced surface heat exchange may remedy the discrepancy to some extent (Emanuel 1987; Neelin et al. 1987).

These two perspectives disagree on what is the driving force and the causality of convection–circulation feedback. Nevertheless, it seems that an oversimplification of vertical structures has been made in both of their original forms, which may add to their conflicts. In most models based on CISK and QE concepts, the grand baroclinic mode or a simple deep heating profile with merely positive heating from cloud base to the top of troposphere was used (Emanuel 1987, 1993; Hendon 1988; Matthews and Lander 1999). Recent observations reveal the self-similar structures of CCEWs with shallow convection leading in the front and deep convection and stratiform precipitation in the tail (Kiladis et al. 2009). It is also found that, in order to explain the fine structure of CCEWs, shallow convective heating or stratiform heating must be included (Mapes 2000;

Straub and Kiladis 2003b; Wang et al. 2015a; Wang et al. 2016; Khouider and Majda 2006, 2007; Zhang and Song 2009; Lappen and Schumacher 2014). Wu (2003) proposed a combined mechanism of CISK and QE in different phases. In the shallow convection stage, released latent heat is less balanced by ascending-driven adiabatic cooling, which triggers the unstable growth of convection through CISK feedback. Once shallow convection penetrates the inversion layer and turns into deep convection, a quasi-equilibrium state can be reached because the deep heating signals propagate out rapidly. But the analysis of Wu (2003) is limited in the steady state and thus cannot explain the development of convections.

For the understanding of interaction between large-scale circulation and convection within wave-CISK, the CISK catastrophe is the stumbling block that has to be removed. Since the 1970s, several mechanisms have been introduced to damp the fast growth of small scales (e.g., Davies 1979; Lau and Peng 1987; Wang 1988; Zhao and Weare 1994), but the cause of this phenomenon remains obscure. It is proposed here that the shrinking tendency of the zonal scale of convection is produced by the interaction of high-frequency waves generated at model grids.

In this study, we will investigate the pure dynamics of convective heating–circulation interaction in a dry multilevel numerical model. The multilevel model enables us to quantify the different efficiencies of specified heat sources with different vertical structures in driving low-level moisture convergence, which are hardly achievable using a two-layer model with the grand baroclinic-mode assumption. We put our focus on low-level moisture convergence, for it is the dominant moisture source of the convective system of the CCEWs. Another focus we placed is on the initial adjustment, because the coupling mechanism of convections and large-scale circulation is the major concern of this study. Overall, the approach we used here is still within an idealized simulating framework.

The paper is arranged as follows. In section 2, the model settings and wave-CISK parameterization used in this study are introduced. Section 3 presents the simulations with conventional wave-CISK and the analysis of potential cause of CISK catastrophe. The noninstantaneous wave-CISK is developed in section 4. Comparison between the simulated structures of convectively coupled Kelvin wave with the noninstantaneous wave-CISK and observations is also provided herein. Discussion and summary are offered in sections 5 and 6.

2. Numerical model and experiment design

All the experiments are conducted using the dynamic core of the Weather Research and Forecasting (WRF) Model, version 3.7.1 (Skamarock et al. 2008). The domain covers the entire globe. Sensitivity tests show that the main results do not change with horizontal resolution from 1° to 4° . The default resolution in our simulations is 2° (unless otherwise specified), for 2° is fine enough to partially resolve large-scale circulation below 2000 km. Polar filtering, with which zonal wavenumbers above the threshold (the wavenumber corresponding to horizontal resolution of 2°) are truncated, is applied when the latitude is above the critical value of 45° . The model top is set to be at 50 km, and the number of vertical layers is 100 (68 layers below 10 km) so that large vertical wavelengths and an enough number of vertical structures can be resolved. To remove the equivalent barotropic component and avoid the spurious reflection of wave energy, a rigid lid is placed at the top and a sponge layer with a large damping rate is applied at the upper 10 layers. No topography is included in the model.

Simulations start from the initial condition of an isothermal (300 K) atmosphere in a quiescent state. To simplify dynamic process and to see how the atmosphere responds to heating directly, the boundary layer, cumulus, and microphysics parameterization schemes of the WRF Model are turned off. Though surface fluxes and radiation have been proposed to be important to the organization or aggregation of tropical convections (Bretherton et al. 2005; Emanuel et al. 2014; Arnold and Randall 2015), they are not included in the present model to exclude other instability mechanisms. In spite of condensation (precipitation), the moisture profile we used keeps constant during the simulation, which may partly represent the effect of surface moisture fluxes. Rayleigh damping and Newtonian cooling with the same time scale of 8 days are applied everywhere in the atmosphere. The time step of simulations is 10 min.

Two types of convective heating are used in the simulations: 1) a prescribed heating that does not change with time and 2) a positive-only wave-CISK-type CP in which heating interacts with large-scale wave activities. The prescribed heating with Gaussian shape in zonal and meridional directions is given as

$$Q_s = A \exp[-(\lambda - \lambda_0)^2/\lambda_L^2] \exp(-\varphi^2/\varphi_L^2) J(z), \quad (1)$$

where A denotes the heating amplitude; λ and φ represent the longitude and latitude; $\lambda_0 = 90^\circ\text{E}$ is the

longitude of the center of heat source; and λ_L and φ_L are the e -folding longitude and latitude of heating away from the center, which are prescribed as $\lambda_L = 20^\circ$ and $\varphi_L = 10^\circ$ to mimic the convective heating at the synoptic scale in equatorial region. The vertical profile of heating $J(z)$ is given as

$$J(z) = \begin{cases} 0, & z \geq z_t \\ e^{z/2H} \sin\left(\frac{z - z_b}{z_t - z_b} \pi\right), & z_b < z < z_t \\ 0, & z \leq z_b, \end{cases} \quad (2)$$

where $H = 8785$ m is the scale height and z_b and z_t denote the heights of the heat base and top. Here, the heat base is set to be $z_b = 0.9$ km, which approximately represents the height of the lifting condensation level in tropics. The factor $e^{z/2H}$ in Eq. (2) arises from the presence of the vertically decaying density profile $e^{-z/2H}$ (Wu et al. 2000).

In this study, two heating profiles with the heat tops at $z_t = 5.0$ and 14.0 km are adopted for shallow convective heating (SCH) and deep convective heating (DCH), respectively, according to radar observations during the Dynamics of the Madden–Julian Oscillation (DYNAMO; Yoneyama et al. 2013) field campaign (Xu and Rutledge 2014; Rowe and Houze 2015; Ruppert and Johnson 2015). The heating amplitude A in Eq. (1) is given a moderate value of 3.5 K day^{-1} . Additional tests indicate that this value is not crucial, for the results are essentially linear if A is no larger than 17 K day^{-1} . The shallow and deep heating profiles described above are shown in Fig. 1a.

In the positive-only wave-CISK, the convective heating is given as follows (e.g., Lau and Peng 1987; Hendon 1988; Matthews and Lander 1999):

$$Q_{wc} = \begin{cases} -\alpha LMJ(z)/(C_p D), & M < 0 \\ 0, & M \geq 0, \end{cases} \quad (3)$$

where α is the moisture factor, which is defined as the ratio of water vapor condensed to the total converged moisture in subcloud layers, it determines how much of the accumulated latent heat could be released; $L = 2.43 \times 10^6 \text{ J kg}^{-1}$ is the latent heat of evaporation; $C_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$ is the specific heat at constant pressure of air; M is the integrated moisture divergence in the subcloud layer (from the surface to z_b); and D is the vertical integration of the air mass weighted by the vertical distribution of heating rate:

FIG. 1. (a) Normalized profiles of shallow (solid line) and deep (dashed line) convective heating. (b) Background moisture profile as used in Wang and Chen (2017).

$$M = \int_0^{z_b} \rho \left[\frac{\partial}{\partial x}(uq) + \frac{\partial}{\partial y}(vq) \right] dz$$

$$= \int_0^{z_b} \rho q \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz, \quad (4)$$

$$D = \int_{z_b}^{z_t} \rho J(z) dz, \quad (5)$$

where q is a globally uniform profile of specific humidity in deep tropics as used by Wang and Chen (2017) (Fig. 1b), and ρ is the density of air. It should be noted that there is no moisture equation in our model; we only use the moisture profile to calculate the divergence of moisture content as shown in Eq. (4). Since the moisture profile is steady and horizontally homogeneous, the advection term of moisture in Eq. (4) is ignored. The moisture divergence integration is from the surface to z_b for moisture is the most abundant in this layer, and it makes the greatest contribution to convective heating. Simulation results with wave-CISK do not change greatly when the upper level of integration is lifted.

3. Conventional wave-CISK and CISK catastrophe

a. Simulations of conventional wave-CISK

To start with, the responses of atmosphere to the prescribed constant heating are examined. The model is run for 30 days with the heating Q_s described in Eq. (1). Figure 2 shows the temporal evolution of the integrated moisture divergence in the subcloud layers M on the equator for SCH and DCH. In the SCH (Fig. 2a), low-level moisture converges to the heating center during the first day and retreats soon later. The large moisture convergence with the comparable amplitude shows up

again after 2 days while it has migrated to the east of the heat source. From then on, a part of the moisture convergence and the accompanying divergence to the east keep propagating eastward and dissipating at the same time. Meanwhile, there is also plenty of moisture convergence in the heating area, which evolves to the steady state gradually. The eastward-propagation speed of the moisture convergence in the first few days is nearly 13 m s^{-1} . In the DCH, the adjustment process when heating is turned on can also be seen but with shorter time period and much smaller amplitude of moisture convergence (Fig. 2b). The propagation speed is more than twice of that in the SCH. The above results are quite consistent with previous studies (Lau and Peng 1987; Hendon 1988).

This eastward-propagation speed of moisture convergence is a reflection of the dominant vertical mode onto which a given heating profile mostly projects. Following the vertical mode decomposing analysis of Wu et al. (2000), the spectral energy density against vertical wavelength for the baroclinic modes and the corresponding gravity wave speeds are shown in Fig. 3. The spectral energy for SCH concentrates greatly to the mode with vertical wavelength of about 4 km, of which the gravity wave speed (also the Kelvin wave speed) is about 13 m s^{-1} . In contrast, the DCH projects onto more dispersed bands of vertical wavelength and the averaged gravity wave speed should be over 30 m s^{-1} . These estimated speeds match those detected in Fig. 2 for both heating profiles.

Figure 2 shows that the external heating induces distinct adjustment in the low-level atmosphere, especially in the first 4 days. Moreover, the Kelvin wave structure first builds up to the east of the heat source on day 3. In the following experiments, we turn off the prescribed heating at the end of day 3 and switch on the

FIG. 2. Time-longitude Hovmöller diagrams of integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) on the equator with prescribed heating profiles: (a) shallow and (b) deep convective heating.

positive-only wave-CISK CP to see if convective heating can be self-sustaining through the interaction with large-scale circulations. Unlike the traditional method with balanced initial fields (Lau and Peng 1987; Matthews and Lander 1999), here, the wave-CISK parameterization is turned on during the adjustment process to represent a more realistic situation. The wave-CISK is applied only in the tropical region (30°S – 30°N) in all experiments.

For the conventional wave-CISK, a critical value of moisture factor α_c could separate the stable and unstable regimes, which differs from each other among previous studies (Lau and Peng 1987; Chang and Lim 1988; Matthews and Lander 1999). Model configurations, heating profiles, and background states can all contribute to the variation of α_c . Based on the simulations with SCH, we find that $\alpha_c = 0.6$ is the marginal value above which the model goes unstable. Figure 4 shows the evolution of integrated moisture divergence in the subcloud layer M for two different moisture factors, which fall into the stable ($\alpha = 0.5$) and unstable ($\alpha = 1.0$) regimes, respectively. In the stable mode (Fig. 4a), moisture convergence in the heat source splits into two parts after the CP is turned on, which can be related to the Kelvin and Rossby waves. The eastward-propagating Kelvin wave can sustain itself via the positive feedback of convective heating and large-scale circulation. Its zonal scale is decreasing at the same time. The number of grid cells with negative M at the equator in the region of the Kelvin wave has shrunk to five on day 24, which is the effective

resolution of the WRF Model because of implicit numerical diffusion. This number remains unchanged in later days. On the other hand, the westward-propagating Rossby waves emerge rather periodically, but all of them are damped dramatically and disappear soon within 3 days. The horizontal distribution of moisture divergence M on day 17 is shown in Fig. 5a, where the Kelvin wave structure can be clearly seen. However, in the unstable mode (Fig. 4b), moisture convergence (negative M) grows exponentially on grids and propagates out quickly after day 4. At the end of day 17, the whole domain of the tropics is filled with isolated

FIG. 3. Spectral energy density vs vertical wavelength for shallow (thick solid line) and deep (thick dashed line) convective heating and the corresponding gravity wave speed (thin solid line) for baroclinic modes.

FIG. 4. Time-longitude Hovmöller diagrams of integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) on the equator for the cases of conventional wave-CISK parameterization with different moisture factors: (a) $\alpha = 0.5$ and (b) $\alpha = 1.0$. All parameterized heating is projected onto shallow heating profile herein.

convections at single grids or lines (Fig. 5b). The model blows out on day 18. In contrast, the DCH with the same moisture factor $\alpha = 1.0$ produces fast damping mode (not shown). Moisture convergence keeps decreasing

after day 3 and is nearly zero a couple of days later. The unstable mode does not show up until tuning α to be unrealistically larger than 3.8. This means that the DCH cannot be self-sustaining through the feedback, which

FIG. 5. Horizontal structure of integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) on day 17 for the cases of conventional wave-CISK parameterization with different moisture factors: (a) $\alpha = 0.5$ and (b) $\alpha = 1.0$. All parameterized heating is projected onto shallow heating profile.

FIG. 6. Normalized wave amplitude of moisture divergence M as a function of zonal wavenumber for the stable mode ($\alpha = 0.5$, red line) and unstable mode ($\alpha = 1.0$, black line) of conventional wave-CISK parameterization with shallow heating. Moisture divergence is averaged in tropics (5°S – 5°N). Average time ranges are days 28–30 for stable mode and days 15–17 for unstable mode.

also indicates that deep tropical waves may hardly be convectively coupled by themselves.

The CISK catastrophe does happen in both the stable and unstable runs. Fourier transform is applied to moisture divergence M in longitude for grids within 5°S – 5°N . The relative amplitude of waves in zonal wavenumber space is displayed in Fig. 6. In the unstable mode, amplitude increases with wavenumber especially in the tail (wavenumbers above 15). Wavenumbers around 10 dominate the stable mode, which is still too large for planetary-scale waves. This problem is the focus of criticism concerning the CISK mechanism, and many efforts have been devoted to solve it. Lau and Peng (1987) argued that the positive-only convective heating would cause upscale transfer of energy and suppress the unstable growth of small-scale waves because of the nonlinearity. But the analytic analysis of Crum and Dunkerton (1992) and the numerical simulation of Matthews and Lander (1999) agreed that CISK catastrophe could be modified but not averted by the positive-only convective heating. In the 2.5-layer model derived by Wang (1988), an additional portion of moisture convergence induced by a frictional boundary layer was also included to feed back to diabatic heating. This extra heating helped to amplify the growth rate of planetary-scale waves. In a rather similar way, Brenowitz et al. (2016) considered the contribution of large-scale averaged moisture convergence to convective heating, and thus, large-scale waves were preferred. The conclusion may be drawn from the above discussion that moisture convergence in a broader

region should be included to remedy the CISK catastrophe, but the reason to do so is not readily apparent. In the next subsection, we will discuss the cause of unstable growth at the smallest scale by checking the wave activities on single grids.

b. Causes of CISK catastrophe

The simplest way to understand how conventional wave-CISK leads to unstable growth is to think of the competition between parameterized convective heating and updraft-induced adiabatic cooling (Chang and Lim 1988; Hendon 1988). These two components almost cancel each other in the situation of marginal value of moisture factor α_c . So if convective heating is smaller than adiabatic cooling ($\alpha < \alpha_c$), the stable mode appears (Fig. 4a), and if the reverse case happens ($\alpha > \alpha_c$), it is the unstable mode (Fig. 4b). Since we are using a grid-point model, it is foreseeable that energy on the grids will grow exponentially through the positive feedback in unstable wave-CISK modes. As shown in Fig. 3, the average propagation speed of waves in the SCH is about 13 m s^{-1} . Considering the grid length of about 220 km (2° resolution), it takes at least 4–5 h for the wave signal to cover the distance from one grid to the next. Recall that the time step of the simulation is only 10 min. Obviously, waves cannot reach the next grid within such a short time. As a result, convergence would amplify locally on single grids and high-frequency fluctuations with half-cycle period of 4–5 h will be generated. Since low-level convergence is accompanied by updraft, gravity waves will also be generated at the same time. According to the tropical wave dispersion relationship, high-frequency gravity waves must have very short wavelengths.

Figure 4b shows how gravity waves break up the large-scale wave envelope. The transition from large scale to grid scale occurs on the fifth day (one day after the parameterized heating is turned on) and first appears in the Kelvin wave region. Then the gridscale waves propagate eastward and westward rather symmetrically and interfere with each other. Moisture convergence on grids is enhanced or cancelled by the interfering waves, depending on their phase relationship. As a result, only the waves to the front of the envelope can maintain their structures and keep growing all the time. The blowout happens first on these waves. Figure 7a shows the evolution of low-level moisture divergence M during day 17 on two adjacent grids on the equator. As we have estimated, the moisture divergence on the farther-east grid (103°E ; red line) leads that on the other grid (101°E ; black line) by about 4 h, indicating a westward propagation. The plot also shows that the waves on the two grids are almost out of phase and the period of the fluctuation is about 10 h. A further estimation can be

FIG. 7. (a) Evolution of integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) on the equator at two adjacent grids of 101° (black line) and 103°E (red line) for the unstable mode ($\alpha = 1.0$) with shallow convective heating on day 17. (b) Normalized amplitude of moisture divergence M as a function of frequency on tropical grids (5°S – 5°N averaged) in cases with horizontal resolutions of 2° (black), 3° (blue), and 4° (red).

made that if the horizontal grid length increases (decreasing the resolution), the period of the fluctuation on single grids will increase accordingly, since the propagation speed would not change. The Fourier transform is applied to moisture divergence M in time series on tropical grids (5°S – 5°N). Relative amplitude as a function of frequency/period in cases of different resolutions is displayed in Fig. 7b. The dominant periods are 10, 16, and 24 h for the resolution of 2° , 3° , and 4° , respectively, which are almost twice of the time a signal takes to travel across one grid length with the speed of 13 m s^{-1} .

The short-wave-selecting mechanism as discussed above in the case of the unstable wave-CISK mode can also be applied in the stable mode. The fluctuation also occurs on grids, and the high-frequency waves interfere with each other. However, since extra energy produced by wave-CISK with moderate moisture factor ($\alpha = 0.5$) can be balanced by the numerical diffusion because of discretization, no unstable growth is coupled, and large-scale waves narrow down to smaller scales rather steadily. As can be seen in Fig. 4a, the wavelength shrinks gradually in the rear of the Kelvin wave envelope. An example of how this shrinking happens is shown in Fig. 8, which depicts the evolution of moisture divergence M and the perturbation of geopotential height at 850 hPa in the region of the dashed box in Fig. 4a. Within the second band of eastward-propagating moisture convergence (dark line in the middle of Fig. 8a) are some convergence centers emerging periodically. The amplitude of the convergence is damped greatly after day 18.5. At that time, the gravity waves generated by the major convection band (bottom-right corner of Fig. 8b) propagate westward and run into the second band at 50°W . The interfering of these waves diminishes the moisture convergence

and finally shuts it down. At last, only the waves in the front of the envelope remain.

Note that it is the immediate feedback of converged moisture to convective heating, which is how convection is expressed in conventional wave-CISK, that causes the CISK catastrophe. This misrepresentation of moisture–convection feedback triggers unstable growth of small-scale waves if there is no large-enough numerical dissipation, though different behaviors may appear in numerical models with gridded and spectral discretization. When projecting positive-only heating onto discrete wave spectrums, wave leaking is inevitable. In their simulations with a spectral model, Lau and Peng (1987) showed that wave energy was redistributed in the wavenumber domain, for the nonlinearity of positive-only heating brought out responses in other wavenumbers. On the other hand, analytical study of Crum and Dunkerton (1992) verified the tendency of positive-only wave-CISK to narrow down the horizontal scale of heating. The final scale in the spectral model is a result of the balance between the shrinking tendency of wave-CISK and the expanding tendency of spectral truncation (Matthews and Lander 1999).

4. Noninstantaneous wave-CISK

a. New parameterization scheme design

The discussion in the last section indicates that wave signals propagate so slowly (speed determined by the shallow heating profile) that convective heating is not able to force moisture convergence on nearby grids in time, and then the moisture–convection feedback occurs locally. It is natural to extend the time scale of this feedback to let heating influence the adjacent region.

FIG. 8. Time-longitude Hovmöller diagrams for (a) integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) and (b) perturbation of geopotential height at 850 hPa in the region shown by the dashed box in Fig. 4a. In (a), positive and negative values are denoted by gray and dark lines with intervals of 0.07 and $0.1 \text{ g s}^{-1} \text{ m}^{-2}$, respectively. In (b), gray and dark lines indicate values above and below -0.60 m with an interval of 0.02 m .

Basically, this idea is close to the concept of considering the time lag between the maximum low-level moisture convergence and convective heating (e.g., Davies 1979; Chang and Lim 1988; Cho et al. 1994). It is necessary to extend the time scale of moisture-convection feedback in new wave-CISK for the following reasons: 1) observations do support this phase lag that the boundary layer moisture convergence leads the precipitation (e.g., Chen and Wang 2017); 2) it takes at least 2–3 days for the atmosphere to response to convective heating and build up the low-level moisture convergence related to the Kelvin and Rossby waves (Fig. 2); and 3) in order to guarantee the numerical stability of the gridpoint model and inhibit the fast growth of high-frequency gravity waves, enough time should be given so that waves can propagate to the next grid smoothly.

The noninstantaneous wave-CISK is introduced here. The basic truth upon which the new wave-CISK is constructed is that once moisture is converged it releases latent heat gradually; that is, the latent heat releasing lasts for an extended period. This may be related to the time it takes for large-scale convective systems to be built, which includes mesoscale processes of clouds formation. A time scale could be defined as τ for the amount of converged moisture to be totally consumed (by a factor of α). As a result, convective heating related

to the moisture convergence in the past period of τ constitutes the heating at the current moment. In this sense, τ is also the time scale of convective heating accumulation. We call τ the accumulation-consumption (AC) time scale. A time-dependent weighting function is given for the gradual release of heating related to the consumption of converged moisture, which, of course, is also the weighting function for convective heating accumulation:

$$r(t, t_i) = 1 - \left[\frac{t_i - (t - \tau/2)}{\tau/2} \right]^2, \quad \text{when } t - \tau \leq t_i \leq t, \quad (6)$$

where t is the current time and t_i represents each time step before time t . Equation (6) maximizes at the time of $t - \tau/2$ and goes down to zero at present time t and the time of $t - \tau$. The shape of the weighting function is similar to that of Davies (1979). The parameterized heating based on the noninstantaneous wave-CISK at the current time t is thus the weighted mean of heating provided by moisture convergence in the past period of τ :

$$Q^*(t) = \frac{\sum_{t-\tau}^t r(t, t_i) Q_{\text{wc}}(t_i)}{\sum_{t-\tau}^t r(t, t_i)}, \quad \text{when } t - \tau \leq t_i \leq t, \quad (7)$$

FIG. 9. Schematic diagrams for (a),(c) the moisture–convection feedback loops and (b),(d) the way that moisture convergence M feeds back in (a),(b) conventional wave-CISK and (c),(d) noninstantaneous wave-CISK. In (b) and (d), vertical columns denotes the moisture converged at each time step. Within the red dashed box is the amount of moisture that actually feeds back to heating at time step t_0 . The dark blue blocks in (d) represent the total amount of moisture that will feed back to heating if the converged moisture at time step t_0 is all consumed. The accumulation–consumption time scale τ is also indicated.

where $Q_{wc}(t_i)$ is calculated using Eq. (3) with the integrated moisture convergence $M(t_i)$ at time step t_i . The calling interval of this parameterization t_i is set to 1 h.

The schematic diagrams for the moisture–convection feedback in conventional and noninstantaneous wave-CISK are given in Fig. 9. In conventional wave-CISK, heating-induced moisture convergence feeds back to convective heating immediately (Figs. 9a and 9b). At each time step (e.g., t_0), converged moisture is represented by the light blue column, which is also the exact one that fuels heating (encompassed by the red dashed box in Fig. 9b). However, in the noninstantaneous wave-CISK (Figs. 9c and 9d), the induced moisture convergence is divided into several parts, and only one of them feeds back immediately (though the corresponding weighting is zero at the moment). At the time step of t_0 , part of moisture convergence of the past (time steps t_{-1} and t_{-2}) is also included in the moisture content (within the red dashed box) that fuels heating, with the weighting function given by Eq. (6). The converged moisture at t_0 also takes a longer time (time steps t_0 , t_1 , and t_2) to release latent heat completely, and during this period, much more moisture has fed back to heating (shown with blocks in dark blue). In noninstantaneous

wave-CISK, not only the time scale of convective heating accumulation has been extended; the associated heating also lasts for a longer time. This means both two legs of moisture–convection feedback have been prolonged (Fig. 9c).

b. Comparison with observations

Different AC time scales τ of 6 h, 12 h, 1 day, 2 days, 3 days, and 4 days are adopted to test the sensitivity of the noninstantaneous wave-CISK. In these six different cases, the prescribed heating Q_s linearly increases its amplitude A to the maximum (3.5 K day^{-1}) at the end of 6 h, 12 h, 1 day, 2 days, 3 days, and 4 days, respectively, to mimic the gradual development of convective clusters, and then noninstantaneous wave-CISK is turned on. As anticipated, the growth rates of convection systems highly depend on the moisture factor α . As the time scale gets larger, α increases accordingly because a larger time scale means converged moisture releases latent heat more slowly, under which circumstance more moisture can be condensed. Here, α is tuned in each experiment to guarantee the steady growth with the similar rate (Table 1). All the experiments show a smoothly eastward-propagating Kelvin wave (Fig. 10). CISK catastrophe is removed here as the steadily growing

TABLE 1. Moisture factors α corresponding to different AC time scales τ used in Fig. 10.

τ	6 h	12 h	1 day	2 days	3 days	4 days
α	0.68	0.78	1.00	1.43	2.05	2.95

patterns with large scales could be maintained even with large moisture factors ($\alpha > 1.0$). Increasing AC time-scale τ extends the zonal scale and decreases the propagation speed (from 14.2 m s^{-1} in Fig. 10a to 11.2 m s^{-1} in Fig. 10f), which can be explained in terms of the synthesis of waves. The projection of shallow heating profile composes a wide spectrum of vertical wavelengths (corresponding to different propagation speeds) with the peak at about 4 km. The small time scale selects fast

waves out because only with fast waves can the atmosphere adjust itself in such a short time. Lengthening τ introduces more components of slow waves, which will definitely slow down the average speed and broaden the zonal coverage of the convection envelope. Interestingly, when the AC time scale is long enough (Fig. 10f), Rossby waves could be also selected out. Further experiments indicate that an extremely large time scale (above 6 days) enhances Rossby waves and suppresses Kelvin waves because Kelvin waves cannot feed back in time (not shown). In this sense, the weighted mean of convective heating, as shown in Eq. (7), is like a low-pass filter, which removes high-frequency noises and can be seen clearly in the frequency response functions shown in Fig. 11.

FIG. 10. Time-longitude Hovmöller diagrams of integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$) on the equator with the noninstantaneous wave-CISK parameterization for different accumulation-consumption time scales: (a) 6 h, (b) 12 h, (c) 1 day, (d) 2 days, (e) 3 days, and (f) 4 days.

FIG. 11. Frequency response functions for different accumulation–consumption time scales: (a) 6 h, (b) 12 h, (c) 1 day, (d) 2 days, (e) 3 days, and (f) 4 days. The unit of frequency is h^{-1} .

We have also tested the sensitivity of horizontal structure to the initial zonal heating scale ϕ_L . Almost no sensitivity is found (not shown), which means these simulated wave structures are the intrinsic modes of atmosphere.

The above results illustrate that the new wave-CISK is applicable in models to get the plausible patterns of convectively coupled Kelvin waves. We choose the case with AC time-scale $\tau = 2$ days to be compared with observations, for the close spatial scale (about 3000 km) and propagation speed (12.4 m s^{-1}) to observations (Roundy 2008; Kiladis et al. 2009). It should be noted first that only shallow heating is conducted at present. In the DCH, as has been tested before, induced moisture convergence is far less than enough to support the development of convections. The deep Kelvin wave decouples with convections and dissipates soon. After the convective parameterization is turned on, both the low-level convergence and the heating amplitude drop dramatically and approach zero rapidly (not shown). Therefore, the analysis here is confined to the shallow convection stage, which dominates the wave structures in the lower atmosphere. More realistic simulations with shallow-to-deep convection transition will be the focus of our future study.

Figure 12 shows the horizontal structures of winds, geopotential height (or pressure), heating, and integrated moisture divergence on day 20. Though horizontal winds are Kelvin wave dominated with nearly no meridional components, Rossby wave signals can still be seen in the geopotential height fields to

the west of heating (Fig. 12a). Heating is almost in phase with easterly winds and negative perturbation of geopotential height at 850 hPa, which seems to be inconsistent with the results of composite analyses that horizontal winds below heating are mostly westerly (e.g., Roundy 2008; Kiladis et al. 2009). In fact, the convective heating centers denoted by OLR or the brightness temperature in their analyses are mainly the centers of deep convections. Considering the westward tilt with height in cloud morphology of CCKWs (Kiladis et al. 2009), the shallow convective heating located to the east of deep convection should collocate with easterly winds in the lower troposphere. Another important feature is the boundary layer moisture convergence lying about 10° of longitude to the east of shallow heating (Fig. 12b). This tilt structure in the lower atmosphere is the key to the propagation of the convection system.

Figure 13 shows the phase relationship of convective heating, boundary layer moisture convergence, and surface zonal winds. Westerly winds prevail right below the convective heating and to the rear of it. In the eastern edge, the transition from westerlies to easterlies contributes to the strong subcloud layer moisture convergence, which leads heating by about one-eighth of a cycle. It is foreseeable that the low-level convergence to the east of the heating will provide moisture and nourish the new birth of convections, while the divergence to the tail will cut off the moisture supply and suppress convection activities. This phase relationship thus leads to the eastward propagation of CCKWs, which is similar to observations in Straub and Kiladis (2003a,b).

FIG. 12. The simulated horizontal structure on day 20 with noninstantaneous wave-CISK for an accumulation–consumption time scale $\tau = 2$ days. (a) Horizontal structures of heating (K day^{-1} , shading), geopotential height perturbation (interval of 0.5 m, contours), and horizontal winds (vectors) at 850 hPa. (b) Integrated moisture divergence in subcloud layers M ($10^{-2} \text{ g s}^{-1} \text{ m}^{-2}$, shading), pressure perturbation (interval of 5 Pa, contours), and horizontal winds (vectors) at the surface.

The longitude–pressure cross sections along the equator of horizontal zonal wind, vertical velocity, horizontal divergence, and temperature perturbation on day 20 are shown in Fig. 14. At this time, shallow heating is located at about 100°W (Fig. 12a). The shallow heating profile maximizes at the height of 3 km (approximately 700 hPa), which turns out to be the critical layer for the vertical tilt. Below 700 hPa, the horizontal wind fields show coherent westward-tilt structures, while above 700 hPa, both horizontal winds and temperature perturbation tilt to the east (Fig. 14). This boomerang-like vertical structure can be seen in almost all the observations of CCEWs with reanalysis or sounding data (e.g., Kiladis et al. 2009). However, the observed critical layer where the transition of tilting direction locates is usually at the height of 300 hPa. This discrepancy is due to the shallow heating profile we used here and can be eliminated by including a deep heating stage.

The positive temperature perturbation is confined to the levels between 600 and 800 hPa (Fig. 14d), which is a little narrower than the profile of shallow heating (0.9–5 km). This is because the adiabatic cooling due to upward motions cancels convective

heating in the upper and lower boundaries of the heating profile. In spite of this, the positive correlation between temperature and convective heating still holds at most of the heating levels, which generates

FIG. 13. Zonal distribution of integrated moisture convergence in subcloud layers (blue line), surface zonal wind (black line), and heating (red line) on the equator on day 20 for the non-instantaneous wave-CISK with $\tau = 2$ days, as shown in Fig. 10d.

FIG. 14. Longitude–pressure cross section along the equator for the simulated vertical structure on day 20 with the noninstantaneous wave-CISK for $\tau = 2$ days for (a) zonal wind u (interval of 0.5 m s^{-1}), (b) vertical velocity w (intervals of 0.050 and 0.025 cm s^{-1} for positive and negative values, respectively), (c) horizontal divergence (interval of $4 \times 10^{-7} \text{ s}^{-1}$), and (d) temperature perturbation (interval of 0.4 K). The red and blue lines denote positive and negative values, respectively.

kinetic energy to support the unstable growth of the convection system.

5. Discussion

As shown in section 3, the immediate consumption of converged moisture in the conventional wave-CISK limits the time scale of moisture–convection feedback to be the integration time step of model. The response to convective heating could not propagate out in time, and thus, direct feedback on grids causes the instability locally. Considering boundary layer (Wang 1988) or large-scale forcing (Brenowitz et al. 2016) influences appears close to including convergence in a broader region at each time step, which implicitly helps the response propagate out. While in the noninstantaneous wave-CISK, both the accumulation of convective heating and the consumption of converged moisture (the upper and lower legs of moisture–convection feedback in Fig. 9c) have been rearranged to last for a given time scale. These two extended processes can also help the

response propagate out. Here, two sensitivity experiments are conducted to verify their respective effects more directly.

In the first experiment, with different calling intervals of CP, the accumulation of moisture convergence in conventional wave-CISK is delayed to ensure that convective heating has enough time to drive large-scale circulation in the numerical model. The calling interval of conventional wave-CISK is extended to 6 h, 12 h, 1 day, 2 days, 3 days, and 4 days, within which heating stays unchanged with the value of the last call. When calling the conventional wave-CISK, convective heating is calculated [Eq. (3)] with the moisture convergence at that moment. Other settings are the same as the unstable mode shown in Fig. 4b. It can be clearly seen that increasing the CP calling interval can suppress high-frequency gravity waves and select large-scale waves (Fig. 15). A too-short interval (less than 1 day) still suffers the problem of fast growth at small scales (Figs. 15a and 15b), though they have already been damped quite a lot. If the interval is too long, low-level

FIG. 15. As in Fig. 4b, but for the different calling interval of conventional wave-CISK: (a) 6 h, (b) 12 h, (c) 1 day, (d) 2 days, (e) 3 days, and (f) 4 days.

moisture convergence cannot feed back to convection in time before it is damped (Figs. 15e and 15f), so moisture convergence decays continuously. Only the experiments with moderate calling interval show growing wave with finite horizontal scale (Figs. 15c and 15d), although there are some unreal periodic oscillations due to abrupt changing of heating structures when calling the parameterization. Relative amplitudes of zonal wavenumbers (after Fourier transform is applied to moisture divergence in longitude, as done in Fig. 6) are shown later (see Fig. 17a). As the interval prolongs, the maximum amplitude shifts to the wavenumber of 3 rapidly, and small-scale waves are damped greatly.

In another abrupt way, the calling interval is unchanged, but the moisture convergence on each grid is

equally redistributed to the nearby 3×3 , 5×5 , and 7×7 grids before it is consumed at each time step. By doing so, the total amount of moisture for the feedback stays the same as the unstable mode shown in Fig. 4b, but its horizontal coverage has been expanded artificially. Figure 16 shows the evolution of moisture divergence with the different redistribution grid numbers. The growth rate of small-scale waves declines with the increasing of redistribution grid number, which can also be deduced from the amplitude of moisture divergence at different wavenumbers (Fig. 17b).

Though unrealistically designed, the above two sets of sensitivity experiments suggest that the CISK catastrophe could be removed as long as the convergence region is expanded and local feedback is

FIG. 16. As in Fig. 4b, but for moisture convergence ($M < 0$) on each grid equally redistributed to the nearby (a) 3×3 , (b) 5×5 , and (c) 7×7 grids at each time step.

prevented. The proposed noninstantaneous wave-CISK meets the prerequisite in that (i) moisture convergence in the past time steps also feeds back to heating at current time and (ii) once moisture is converged, it keeps releasing heat for a time, and thus, the response can propagate out. This time lag between moisture convergence and convective heating may relate to the fact that it takes time for convective clouds to moisten the environment and then morphological structures could build up on the basis of humidity change (Powell and Houze 2013; Johnson and Ciesielski 2013). Observations that directly confirm this moisture–convection feedback have also been

documented recently (Hannah et al. 2016; Ahmed and Schumacher 2018).

The notion that a phase lag between convective heating and large-scale forcing leads to effective damping for high-frequency waves in noninstantaneous wave-CISK agrees well with the moist convective damping (MCD) mechanism discussed by Emanuel et al. (1994), though differences still exist. The phase lag in noninstantaneous wave-CISK consists of two parts basically (not mutually exclusive): one that the release of latent heat lags moisture convergence and the other that the atmospheric response lags convective heating. MCD occurs in the case that convection lags upward motion and falls into the cold

FIG. 17. As in Fig. 6, but for two sensitivity experiments. (a) Experiments with different extended calling intervals of conventional wave-CISK, as shown in Fig. 15. Line colors represent different intervals. (b) Experiments with redistributed moisture convergence, as shown in Fig. 16. Line colors represent different numbers of redistribution grids. Average time ranges are days 18–20.

phase of waves (Fig. 6b in Emanuel et al. 1994), which will damp the wave perturbation, especially for short wavelength modes. Obviously, MCD is closer to the first part of our phase lag.

The major deficiency of the wave-CISK type CP is the lack of guarantee of energy conservation. Notice that α is larger than 1.0 when $\tau > 1$ day (Table 1), which means more energy than the total latent heat of converged moisture is needed to support developing convections. This part of the energy deficit may come from the removed energy at higher frequency (Fig. 11) and also the low-level moisture convergence but above the heat base of 0.9 km. Modified noninstantaneous wave-CISK should be designed to conserve energy or any other form of it in the future.

6. Summary

Numerical simulations of shallow convection with the positive-only wave-CISK are conducted with a gridpoint model. Both stable and unstable modes for the conventional wave-CISK show the fast growth at small scales. It is found that slow propagating speed related to shallow heating profile makes low-level convergence linger at grids for a couple of hours. Local feedback of moisture–convection causes fluctuations at discrete grids, which generates high-frequency gravity waves that interfere with each other. As a result, a large-scale wave in unstable modes ($\alpha = 1.0$) breaks into grid scale, and only convection in the front can be amplified continuously through the positive feedback of wave-CISK. Meanwhile, in stable modes ($\alpha = 0.5$), convection is mitigated in the tail of the envelope, leaving narrowed waves propagating eastward. In contrast, converged moisture is not large enough to maintain the growth of deep convection, and a damping mode is always found.

Considering the time needed for the adjustment of atmosphere and moistening of environment, it is necessary to increase the time scale of the moisture–convection feedback. A noninstantaneous wave-CISK, which includes the collective effects of heating in the past, is proposed to remedy the conventional wave-CISK. With an accumulation–consumption time scale of 2 days in noninstantaneous wave-CISK, the simulated Kelvin waves bear similar propagation speeds and three-dimensional structures to observations. A coherent westward tilt of wind fields in the vertical direction is found in the lower atmosphere. Sub-cloud-layer convergence leads convective heating by about one-eighth of a cycle, which helps to trigger new convections to the east and thus the eastward propagation of CCKWs. The positive correlation between shallow convective heating and temperature perturbation generates kinetic energy and guarantees the unstable growth.

Two sets of sensitivity experiments verify that delayed accumulation of heating-driven moisture convergence or redistribution of converged moisture to a broader region can remedy the CISK catastrophe. The proposed noninstantaneous wave-CISK synthesizes both processes with a preset AC time scale. The AC time scale of 2 days for CCKWs is a rough guess from simulation tests at present. It could be estimated more accurately by diagnosis with a cloud-resolving simulation or field campaign observation results (e.g., Wang et al. 2015b; Li et al. 2018). It is also our goal to incorporate a transition to deep convection in the present model in a future study.

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