On the Computation of Apparent Direct Solar Radiation[✉]

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ABSTRACT

Near-forward-scattered radiation coming from the vicinity of the sun's direction impacts the interpretation of measurements of direct solar radiation by pyrheliometers and sun photometers, and it is also relevant for concentrating solar technology applications. Here, a Monte Carlo radiative transfer model is employed to study the apparent direct solar transmittance $t(\alpha)$, that is, the transmittance measured by an instrument that receives the radiation within a half-field-of-view (half-FOV) angle α from the center of the solar disk, for various ice cloud, water cloud, and aerosol cases. The contribution of scattered radiation to $t(\alpha)$ increases with increasing particle size, and it also depends strongly on ice crystal morphology. The Monte Carlo calculations are compared with a simple approach, in which $t(\alpha)$ is estimated through Beer's law, using a scaled optical depth that excludes the part of the phase function corresponding to scattering angles smaller than α . Overall, this optical depth scaling approach works very well, although with some degradation of the performance for ice clouds for very small half-FOV angles ($\alpha < 0.5^\circ$ – 1°), and in optically thick cases. The errors can be reduced by fine-tuning the optical depth scaling factors based on the Monte Carlo results. Parameterizations are provided for computing the optical depth scaling factors for water clouds, ice clouds, aerosols, and for completeness, Rayleigh scattering to allow for a simple calculation of $t(\alpha)$. It is also shown that the optical depth scaling used in delta-two-stream approximations is inappropriate for simulating the direct solar radiation received by pyrheliometers.

1. Introduction

For a plane-parallel horizontally homogeneous atmosphere, the direct-beam transmittance t_{dir} of solar radiation can be computed trivially using Beer's law

$$t_{\text{dir}} = \exp\left(-\frac{\tau}{\cos\theta_0}\right), \quad (1)$$

where τ is the optical depth of the atmospheric column and θ_0 is the solar zenith angle.¹ While both τ and t_{dir} depend on wavelength, the wavelength index λ is omitted here for simplicity. However, Eq. (1)

is not compatible with measurements of direct solar radiation made with pyrheliometers. These measurements typically cover a half-field-of-view angle (half-FOV angle; aka opening half-angle) of $\alpha = 2.5^\circ$ – 3° around the sun (Blanc et al. 2014), which is considerably larger than the half width of the solar disk (on average, 0.267°). Some of the radiation received by such instruments is actually scattered radiation rather than direct radiation (i.e., solar radiation not modified by atmospheric processes). The issue is also relevant for concentrating solar technology (CST) applications (e.g., Blanc et al. 2014; Reinhardt et al. 2014), and for the retrieval of cloud or aerosol optical depth with sun photometers (Shiobara and Asano 1994; Kinne et al. 1997; Sinyuk et al. 2012; Segal-Rosenheimer et al. 2013). For these applications, α is usually smaller than for pyrheliometers. Blanc et al. (2014) quote a range of 0.7° – 2.3° for concentrating solar power systems, while Segal-Rosenheimer et al. (2013) consider a range of 0.6° – 1.85° for sun photometers. Several papers have discussed the contribution of scattering by ice clouds and aerosols to the radiation received from directions near the sun (Shiobara and Asano 1994; Kinne et al. 1997; Gueymard 2001; Arola and Koskela 2004; Russell et al. 2004;

¹ In fact, this equation is not exact for a finite solar disk, but the error is negligible for practical purposes (e.g., $< 2 \times 10^{-6}$ for $\theta_0 = 45^\circ$).

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Sinyuk et al. 2012; Segal-Rosenheimer et al. 2013; Reinhardt 2013; Reinhardt et al. 2014; Sun et al. 2016; Haapanala et al. 2017). Less attention has been paid to water clouds, in part because their optical depth is often too large to allow the operation of CST plants (Reinhardt et al. 2014).

Assuming, for simplicity, an ideal instrument that receives all radiation coming from angles smaller than α from the center of the solar disk and no radiation from larger angles, the measured apparent direct transmittance can be written as

$$t(\alpha) = t_{\text{dir}} + t_{\text{sc}}(\alpha). \quad (2)$$

Here, $t_{\text{sc}}(\alpha)$ is the contribution of scattered radiation to the transmittance $t(\alpha)$ for the half-FOV angle α , and it includes both the circumsolar radiation and the scattered radiation coming from the direction of the solar disk (Blanc et al. 2014; Haapanala et al. 2017). The question addressed in this paper is the following: Given knowledge of the atmospheric properties, in particular the physical characteristics of scattering particles such as ice crystals, cloud droplets, or aerosols, how does one compute $t(\alpha)$ or $t_{\text{sc}}(\alpha)$? We are seeking here a simple solution that would be affordable for routine use in atmospheric models, for example, for diagnosing apparent direct solar radiation in numerical weather prediction (NWP) forecast products, or for making climate model projections of solar energy availability.

Recently, Sun et al. (2016) developed a parameterization for the contribution of circumsolar radiation to the direct solar radiation measured by pyrheliometers, for the case of mineral dust aerosols (Cusack et al. 1998). The parameterization is expressed as a polynomial of both τ and θ_0 , for two spectral intervals. They also noted that in current solar radiation schemes in NWP and climate models, the direct solar flux is computed either without including the contribution of scattered radiation, or by including it in terms of the delta scaling of optical depth for two-stream approximations (e.g., Joseph et al. 1976). They found that compared to clear-sky measurements of direct solar radiation made at Tamanrasset in the Sahara Desert, the use of delta scaling resulted in substantial positive errors [a relative mean bias (RMB) of 18% for the Edwards and Slingo (1996) radiation scheme (ES) and 7% for the Sun (2011) scheme (SES2)], while the biases were smaller without delta scaling (RMBs of 5% and -4% for the ES and SES2 schemes, respectively).

In this paper, however, we argue that the concept of scaled optical depth (aka. apparent optical depth) provides a good and physically transparent basis for the parameterization of $t(\alpha)$. The key point is how to choose

the scaling factor. It has been suggested previously (Shiobara and Asano 1994; Mauno et al. 2011; Segal-Rosenheimer et al. 2013) that the apparent direct solar radiation measured by instruments with a half-FOV angle α can be approximated to a good degree using Beer's law, if the optical depth τ is replaced by a scaled optical depth $k_0\tau$:

$$t(\alpha) \approx t_{\text{scaled}}(\alpha) = \exp\left(-\frac{k_0\tau}{\cos\theta_0}\right), \quad (3)$$

where

$$k_0 = 1 - \omega \int_0^\alpha P_{11}(\theta_s) \sin\theta_s d\theta_s. \quad (4)$$

Here, ω is the single-scattering albedo, θ_s the scattering angle, and P_{11} the scattering phase function (normalized so that the integral over the range 0° – 180° equals 1). In fact, these equations are exact under the assumption that the sun is a point source and only single scattering is considered.

In studies highly relevant for the present work, Reinhardt (2013) and Reinhardt et al. (2014) employed the scaled optical depth for the parameterization of circumsolar radiation. Reinhardt (2013) developed a lookup table approach for the optical depth scaling factor for ice clouds (as a function of the half-FOV angle and the ice crystal effective radius r_e , for several assumptions about ice crystal habits) and for aerosols (as a function of the half-FOV angle and relative humidity, for several aerosol types). Reinhardt et al. (2014) then applied the ice cloud parameterizations to the retrieval of circumsolar radiation from geostationary satellite observations. In these studies, the scaling factors were derived by fitting the transmittances to Monte Carlo radiative transfer calculations (Mayer 2009). However, Reinhardt et al. (2014) noted that the values derived for ice clouds differed less than 5% from those computed directly from the single-scattering properties [Eq. (4)], when the slant optical depth was $\tau_s < 3$ and the half-FOV angle was $\alpha > 0.5^\circ$.

In this work, we utilize a Monte Carlo model to study the contribution of scattered radiation $t_{\text{sc}}(\alpha)$ to the apparent direct transmittance $t(\alpha)$. This model and the assumptions about cloud and aerosol properties are introduced in section 2. In section 3 we discuss, based on selected ice cloud, water cloud, and aerosol cases, the general behavior of $t(\alpha)$ and $t_{\text{sc}}(\alpha)$. We also evaluate the accuracy of the optical depth scaling approach as represented by Eqs. (3) and (4), demonstrating its generally good performance but also including a novel analysis of when and why it fails. In section 4, the dependence of the optical depth scaling factors on the

microphysical properties of clouds and aerosols is discussed. We also demonstrate, for a much more extensive set of cases than discussed by Sun et al. (2016), the inapplicability of the optical depth scaling factor used in delta-two-stream approximations for estimating the direct solar radiation measured by pyrheliometers. In section 5, parameterizations are developed for the optical depth scaling factor. Similar to Reinhardt (2013) and Reinhardt et al. (2014), the parameterizations are based on scaling factors optimized through Monte Carlo calculations. However, in addition to ice clouds and aerosols considered in these studies, we also include water clouds, and for consistency, the (admittedly tiny) effect of Rayleigh scattering. Finally, the main findings are summarized in section 6.

2. Model

A forward Monte Carlo radiative transfer model was employed to study the angular distribution of downwelling solar radiation at the surface. This model was introduced as a monochromatic Monte Carlo model by Barker (1992, 1996) and was updated by Räisänen et al. (2003) for broadband solar radiation calculations. For the present study, an option was added to account for the finite width of the solar disk and limb darkening (see appendix A). Also, additional diagnostics were included in the Monte Carlo model to keep track of the distribution of downwelling photons at the surface with respect to the angular distance from the center of the solar disk, the number of scattering events, and the largest scattering angle occurring during the photon trajectory.

Gaseous absorption and molecular scattering were treated using the scheme of Freidenreich and Ramaswamy (1999), which divides the solar spectrum into 25 bands. The single-scattering properties of clouds and aerosols were averaged over the same bands. The water cloud single-scattering properties were computed using Mie theory (de Rooij and van der Stap 1984; Mishchenko et al. 1999) and refractive index data from Hale and Querry (1973), assuming a gamma distribution of droplet sizes with an effective variance ν_e of either 0.1 or 0.2. For ice clouds, we primarily use the bulk scattering model (version 3.6) for a general habit mixture (GHM) provided by Baum et al. (2014). Some results are also presented for the Baum et al. (2014) solid columns (SC) and aggregates of solid columns (AggSC) models, as well as for individual habits in the Yang et al. (2013) database. For the latter, a lognormal distribution with respect to the particle maximum dimension $D_{\max}$ was assumed, with a geometric standard deviation of 1.5 and the median of

$D_{\max}$ adjusted for each value of effective radius r_e considered. Finally, for aerosols, we employed refractive index and size distribution data from the Optical Properties of Aerosols and Clouds (OPAC) database (Hess et al. 1998) to compute the single-scattering properties using Mie theory, for eight values of RH between 0% and 99%. Recomputing the single-scattering properties was necessary in order to obtain the scattering phase functions at a higher angular resolution than those readily available in the OPAC database.

The calculations reported here assumed a midlatitude summer atmosphere, a solar zenith angle of $\theta_0 = 45^\circ$, and a Lambertian surface albedo of 0.2. Water clouds were located at a height of 1–2 km, ice clouds at 9–11 km, and aerosols at 0–3 km. Sensitivity tests indicated that choices regarding the atmospheric profile, surface albedo, and cloud or aerosol height have only minor impacts on the results presented in this paper. Changing θ_0 has one obvious effect: increasing (decreasing) θ_0 increases (decreases) the slant optical depth $\tau_s = \tau/\cos\theta_0$ corresponding to a given vertical optical depth τ . Therefore, when the apparent direct transmittance or the contribution of scattered radiation are plotted as a function of τ , the curves shift toward smaller τ with increasing θ_0 . Apart from the change in τ_s , the impacts of changing θ_0 are rather small. For brevity, the sensitivity tests are not reported in detail.

Twenty-five million photons were used in the broadband simulations. This yields a standard error of at most $\approx 10^{-4}$ in broadband $t(\alpha)$.

3. Contribution of scattered radiation to the apparent direct transmittance $t(\alpha)$

a. General features

We first consider the effect of scattered radiation on the apparent direct transmittance for a half-FOV angle $\alpha = 2.5^\circ$ representative of pyrheliometers, based on Monte Carlo calculations. Figure 1 displays the broadband solar transmittance $t(2.5^\circ)$, the actual direct-beam solar transmittance t_{dir} , and the transmittance t_{scaled} computed using scaled optical depth [Eqs. (3) and (4)], for three cases: a water cloud with an effective droplet radius of $r_e = 10 \mu\text{m}$ (Fig. 1a), an ice cloud following the GHM size–shape distribution (Baum et al. 2014) ($r_e = 30 \mu\text{m}$) (Fig. 1b), and the transported mineral aerosol type in the OPAC database, which consists of relatively large aerosol particles ($r_e = 1.85 \mu\text{m}$) (Fig. 1c). The impact of gaseous absorption and Rayleigh scattering is included in these transmittances. If needed, the transmittances can be converted into

FIG. 1. Broadband transmittance of direct solar radiation (black lines) and the total broadband transmittance computed at a half-FOV angle of $\alpha = 2.5^\circ$ with the Monte Carlo model (red lines) and with the optical depth scaling approach [Eqs. (3) and (4)] (dashed blue lines) for (a) a water cloud with an effective droplet radius $r_e = 10 \mu\text{m}$ and a droplet size effective variance $v_e = 0.10$, (b) an ice cloud with a size–shape distribution following the GHM model with $r_e = 30 \mu\text{m}$, and (c) the transported mineral aerosol type ($r_e = 1.85 \mu\text{m}$). A solar zenith angle of $\theta_0 = 45^\circ$ is assumed.

radiative fluxes on a plane perpendicular to the sun by multiplying them by the total solar irradiance $S_0 \approx 1361 \text{ W m}^{-2}$, or on a horizontal surface by multiplying them by $S_0 \cos \theta_0 \approx 962 \text{ W m}^{-2}$, assuming an Earth–sun distance of exactly 1 astronomical unit.

In the ice cloud and water cloud cases (Figs. 1a,b), the total transmittance $t(2.5^\circ)$ is substantially larger than the actual direct transmittance t_{dir} , and it approaches zero much slower than t_{dir} with increasing optical depth. In the aerosol case, the difference between $t(2.5^\circ)$ and t_{dir} is much smaller but not negligible. It is further seen that the optical depth scaling approach [Eqs. (3) and (4)] reproduces $t(2.5^\circ)$ very well, in fact within 0.002 in all three cases.

Figure 1 indicates that the contribution of scattered radiation to the “direct” radiation measured by pyrheliometers is largest when the scattering particles are large. This can be understood by considering the phase function in near-forward-scattering directions. Figure 2a shows the spectrally averaged phase functions for the water cloud, ice cloud and aerosol cases. The forward-scattering peak is much stronger and sharper in the ice cloud case than in the water cloud case and especially than in the aerosol case. In the exact forward direction, the ice cloud phase function is larger than the water cloud (aerosol) phase function by a factor of 48 (1140), but it falls below the water cloud (aerosol) phase function at scattering angles larger than 0.45° (1.2°) (see the insert in Fig. 2a). The corresponding phase function integrals

$$I(\theta_s) = \int_0^{\theta_s} P_{11}(\theta'_s) \sin \theta'_s d\theta'_s \quad (5)$$

(normalized to 1 over the range 0° – 180°) are displayed in Fig. 2b. In the ice cloud and water cloud cases, about 49% and 39% of the scattered energy goes to angles smaller than $\theta_s = 2.5^\circ$, but in the aerosol case, this fraction is only 9%.

Some physical insight into the phase function behavior in the near-forward directions can be obtained by noting that in the limit of geometrical optics (i.e., particles large compared to the wavelength), the phase function can be decomposed into the ray tracing part P_{ray} and the diffraction part P_{diff} (e.g., Macke et al. 1996):

$$P_{11}(\theta_s) = \frac{2\omega - 1}{2\omega} P_{\text{ray}}(\theta_s) + \frac{1}{2\omega} P_{\text{diff}}(\theta_s). \quad (6)$$

The ray tracing part (i.e., rays passing through the scattering particles) can contribute substantially to the near-forward scattering for ice crystals with completely smooth surfaces (see the discussion in section 4), but in the cases considered in Fig. 2, diffraction dominates the near-forward scattering. According to the quasi-small-angle approximation of the Fraunhofer diffraction theory (van de Hulst 1981; Hogan 2006), the diffraction phase function of a single particle is

$$P_{\text{diff}}(\theta_s) = P_0 \left[\frac{2J_1(2\pi\theta_s r_p/\lambda)}{2\pi\theta_s r_p/\lambda} \right]^2. \quad (7)$$

Here, J_1 is the Bessel function of the first kind, r_p is the projected-area-equivalent radius of the particle, and P_0 is determined by the normalization condition $\int_0^\pi P_{\text{diff}}(\theta_s) \sin \theta_s d\theta_s = 1$. A key feature of Eq. (7) is that

FIG. 2. (a) Spectral-mean scattering phase functions (P_{11} weighted by the cloud or aerosol extinction efficiency, single-scattering albedo, and the direct solar irradiance for a cloud-free and aerosol-free atmosphere) for the water cloud (black), ice cloud (red), and mineral aerosol (ochre) cases considered in Fig. 1. The insert highlights the near-forward-scattering directions $0^\circ \leq \theta_s \leq 5^\circ$. (b) Corresponding phase function integrals $I(\theta_s)$. The numerical values indicate $I(2.5^\circ)$. The dotted lines represent the contribution of diffraction to $I(\theta_s)$ approximated by Eqs. (6) and (7), using the projected-area-equivalent radius r_p integrated over the size distribution ($8.5 \mu\text{m}$ in the water cloud case, $50.8 \mu\text{m}$ in the ice cloud case, and $0.89 \mu\text{m}$ in the aerosol case).

the width of the main diffraction lobe is inversely proportional to r_p/λ . Furthermore, Eqs. (5)–(7) indicate that when $\theta_s \gg \lambda/(2\pi r_p)$, the contribution of diffraction to $I(\theta_s)$ asymptotically approaches to values close to 0.5, at wavelengths with $\omega \approx 1$. It is demonstrated in Fig. 2b that the use of Eqs. (5)–(7) along with r_p integrated over the size distribution, and with P_{ray} set to zero, predicts well the behavior of spectrally averaged $I(\theta_s)$ in the ice cloud and water cloud cases. In the aerosol case, $I(\theta_s)$ is underestimated severely, both because of the failure of the geometrical optics approximation for relatively small particles and because of the inaccuracy of describing

the diffraction pattern with a single r_p averaged over a wide size distribution.

In agreement with previous studies (Shiobara and Asano 1994; Mauno et al. 2011; Segal-Rosenheimer et al. 2013), Fig. 1 suggests that the simple optical depth scaling approach [Eqs. (3) and (4)] works quite well for computing the apparent direct solar radiation. But what are the limits of validity of this approximation? To explore this question, we consider in Fig. 3 the same water cloud, ice cloud and aerosol cases as in Fig. 1 but for the range $\alpha = 0.267^\circ$ – 15° . The lower limit equals the width of the solar disk, while the upper limit goes far beyond the half FOV of pyrheliometers [which is mostly below 3° , but near 5° for one of the instruments listed in Table 1 of Blanc et al. (2014)]. Also, at optical depths close to the upper limit considered here ($\tau = 10$ in the water cloud and ice cloud cases; $\tau = 5$ in the aerosol case), the transmittance is too small for CST applications or for optical depth retrievals with sun photometers. The main motivation for considering such an extensive range of α and τ is sensitivity testing. We focus on the contribution of scattered radiation t_{sc} to the apparent direct transmittance, as the true direct transmittance t_{dir} can be computed simply using Beer's law.

The values of t_{sc} in the Monte Carlo simulations (Figs. 3a–c) consistently peak at $\tau \approx 0.8$ – 1 . As θ_0 equals 45° , this corresponds to a slant optical depth $\tau_s \approx 1.1$ – 1.4 . However, as already noted above, t_{sc} depends strongly on the type of the particles considered, especially their size. These differences are most striking at small half-FOV angles, consistent with the differences in the phase function integrals shown in Fig. 2b. To quote a few numerical values, for $\alpha = 0.267^\circ$, the maximum in t_{sc} is 0.077 in the ice cloud case, 0.0077 in the water cloud case, and 0.0004 in the aerosol case. The corresponding maxima are 0.154, 0.064, and 0.0047 for $\alpha = 1^\circ$, and 0.172, 0.127, and 0.023 for $\alpha = 2.5^\circ$. The values for $\alpha = 0.267^\circ$ imply that in the presence of ice clouds (and only then), there can be a substantial amount of scattered radiation even within the area of the solar disk. The exact amount depends considerably on the ice crystal properties (Haapanala et al. 2017), and it can be much larger for smooth ice crystals than for the severely roughened ice crystals assumed in the GHM (e.g., for smooth plates with $r_e = 30 \mu\text{m}$, $t_{\text{sc}}(0.267^\circ)$ reaches ~ 0.20). The values for $\alpha = 1^\circ$ suggest that in the presence of optically thin water clouds and, especially, ice clouds, scattered radiation can make a substantial contribution to the energy received by concentrating solar power collectors and sun photometers. In contrast, for aerosols, this contribution is very small, except if the aerosol particles are unusually large

FIG. 3. Contribution of scattered radiation to apparent direct transmittance [$I_{sc}(\alpha)$ in Eq. (2)] computed at half-FOV angles in the range $\alpha = 0.267\text{--}15^\circ$, for (a) a water cloud with $r_e = 10 \mu\text{m}$ and $\nu_e = 0.10$, (b) an ice cloud with a size–shape distribution following the GHM model with $r_e = 30 \mu\text{m}$, and (c) the transported mineral aerosol type ($r_e = 1.85 \mu\text{m}$). (d)–(f) As in (a)–(c), but for the error associated with the optical depth scaling approach [Eqs. (3) and (4)]. (g)–(i) As in (a)–(c), but for the relative error (%) associated with the optical depth scaling approach. Note that the shading intervals are nonuniform, as indicated by the color bars, while the contours are drawn with constant intervals: 0.02 in (a)–(c), 0.005 in (d)–(f), and 20% in (g)–(i).

(see also Kinne et al. 1997; Russell et al. 2004; Sinyuk et al. 2012). Finally, we note that the ice cloud value for $\alpha = 2.5^\circ$ translates into a difference of $\sim 234 \text{ W m}^{-2}$ between direct radiation measured by a typical pyrheliometer and the true direct radiation on a plane perpendicular to the sun.

Figures 3d–f show the transmittance errors associated with the optical depth scaling approach [Eqs. (3) and (4)]. In all cases considered, the errors are very small (below 0.002 almost without exception) for α between about 1° and 3° . This indicates that this approach would perform very well for simulating the radiation received by typical pyrheliometers. In the water cloud and aerosol cases, the errors are also very small for $\alpha < 1^\circ$. In the ice cloud case, however, positive errors appear for $\alpha < 1^\circ$, reaching up to 0.02 for $\alpha = 0.267^\circ$. (As discussed in connection to Fig. 5 below, these errors arise from the combination of the finite solar disk and the very sharp forward-scattering peak of ice crystals.) Finally, for $\alpha > 3^\circ$, negative errors dominate, with largest errors occurring for relatively large optical depths ($\tau \sim 5$ for the cloud cases; $\tau \sim 3$ for aerosols). For $\alpha = 5^\circ$, the errors are at most circa -0.004 , but for even larger half-FOV angles, the performance of the optical depth scaling approach deteriorates substantially, so that for $\alpha = 15^\circ$, the transmittance is underestimated by up to 0.031 in the water cloud case and by up to 0.021 in the ice cloud case.

An alternative view on the performance of the optical depth scaling approach is obtained by considering in Figs. 3g–i the relative errors in t_{sc} (i.e., the absolute errors in Figs. 3d–f divided by the t_{sc} shown in Figs. 3a–c). By this measure also, the performance is excellent in the water and ice cloud cases for α in the range of circa 1° – 3° , for relatively small values of τ . Especially in the ice cloud case, the relative error is below 1% in a rather large fraction of the (α, τ) parameter space, in fact up to $\tau = 5$ for $\alpha = 1.5^\circ$. Thus, the excellent performance in computing t_{sc} extends here to situations with very little direct radiation (for $\tau = 5$, $t_{dir} < 0.001$). However, much larger relative errors occur in the ice cloud case for very small half-FOV angles α (positive errors exceeding 20%). Furthermore, in all cases except for the ice cloud case for $\alpha \leq 1^\circ$, increasingly large negative relative errors in t_{sc} occur with increasing τ , up to $\sim -95\%$ for water clouds with $\tau = 10$. These large negative relative errors in t_{sc} at large values of τ are, however, probably unimportant for most applications, as they are associated with very small values of t_{sc} , and hence small absolute errors in t_{sc} , except at large half-FOV angles α . Finally, in the aerosol case, negative errors

actually prevail throughout the parameter space, but they are small when the aerosol layer is optically thin (e.g., almost always less than 3% for $\tau < 0.3$).

b. Error analysis for the optical depth scaling approach

It is not surprising that the performance of the optical depth scaling approach [Eqs. (3) and (4)] deteriorates with increasing τ . Indeed, this approximation is exact only for a pointwise sun, when only single scattering occurs. The latter condition is asymptotically true at the limit of $\tau \rightarrow 0$. With increasing τ , multiple scattering becomes increasingly important, thereby leading to increased errors in the computed transmittance. However, the amount of multiple scattering does not determine uniquely the magnitude of the errors.

To elucidate the role of first-order and multiple scattering, we consider in Fig. 4 the contributions to $t(2.5^\circ)$ stratified according to the number of scattering events experienced by the photons, for the ice cloud case (Fig. 4a) and the transported mineral aerosol case (Fig. 4b). The almost straight lines for both the direct transmittance t_{dir} and the apparent direct transmittance $t(2.5^\circ)$ in Figs. 4a and 4b demonstrate that in the range of optical depth considered ($\tau = 0$ – 5 in the midvisible), transmittance decreases quasi exponentially with optical depth, even when considering broadband solar radiation. The contribution from scattered radiation depends, however, nonmonotonically on optical depth. The maximum contribution to transmittance shifts toward higher τ with an increasing order of scattering (cf. Fig. 6 in Shiobara and Asano 1994). For the cases considered here, the contributions from first-, second-, and third-order scattering show broad maxima at $\tau \approx 0.7$, $\tau \approx 1.4$, and $\tau \approx 2.1$ (which correspond to slant optical depths of ~ 1 , 2 , and 3), respectively. Furthermore, consistent with Figs. 3b and 3c, the contribution of scattered radiation to $t(2.5^\circ)$ is much larger in the ice cloud case than in the aerosol case, and this is especially true for multiple (i.e., second or higher order) scattering. In the ice cloud case, the absolute contribution of multiple scattering to $t(2.5^\circ)$ peaks at 0.068, while in the aerosol case the maximum is only 0.0035. In the ice cloud case, multiple scattering makes a larger contribution to the transmittance than first-order scattering for $\tau \geq 1.8$, while in the aerosol case, this only occurs for $\tau \geq 3.8$. Remarkably, in spite of the large contribution of multiple scattering, the accuracy of the optical depth scaling approach is high in the ice cloud case, with absolute transmittance errors below 0.001 and relative errors below 5% up to $\tau = 5$. The corresponding errors are larger in the aerosol case, reaching nearly -0.002 and 31%, respectively.

FIG. 4. Contributions to broadband solar transmittance at a half-FOV angle $\alpha = 2.5^\circ$ in (a) the ice cloud case and (b) the transported mineral aerosol case. The black lines show the total transmittance and the gray, purple, blue, and cyan lines show the contributions from direct solar radiation and from radiation scattered once, twice, and three times, respectively. The total contribution of multiple scattering (photons scattered at least twice) is shown with the red lines. (c) For reference, the transmittance errors due to the optical depth scaling approach [Eqs. (3) and (4)] are plotted.

Another perspective on the errors of the optical depth scaling approach is obtained by noting that this approximation entails the following three assumptions. First, those scattering events in which photons experience scattering angles $\theta_s < \alpha$ have no impact on $t(\alpha)$; that is, they do not reduce the amount of radiation reaching the surface within an angular distance α from the center of the solar disk. Second, those photons that do experience scattering angles $\theta_s > \alpha$ never reach the surface within an angular distance of α from the center of the solar disk. Third, the sun is assumed to be a point source, so that the finite width of the solar disk does not influence $t(\alpha)$. Consequently, the transmittance errors for the optical depth scaling approach may be decomposed into three terms quantifying the effect on $t(\alpha)$ due to violations to each of these assumptions:

- Term 1: Because of multiple scattering, some photons that only experience scattering angles $\theta_s < \alpha$ reach the surface at angles larger than α from the center of the solar disk. Since the optical depth scaling approach does not account for this, this results in a positive error in $t(\alpha)$.
- Term 2: Because of multiple scattering, some photons that do experience scattering angles $\theta_s > \alpha$ reach the surface at angles smaller than α from the center

of the solar disk. This results in a negative error in the transmittance $t(\alpha)$ computed using the optical depth scaling approach.

- Term 3: Because of neglecting the finite width of the solar disk, the optical depth scaling approach misrepresents the transmitted solar radiation at small half-FOV angles.

We evaluated terms 1 and 2 by keeping track of the maximum θ_s experienced by each photon during its path to the surface. This analysis was conducted through Monte Carlo simulations in which a pointwise sun was assumed. Term 3 was evaluated as the difference in $t(\alpha)$ between simulations with a pointwise sun and a finite solar disk. The results are shown in Fig. 5.

There is some compensation between the multiple-scattering-related error terms 1 and 2 (Figs. 5a–c and Figs. 5d–f, respectively). However, at large values of α , the negative term 2 increases more rapidly than the positive term 1, which explains the increasingly negative errors in $t(\alpha)$ seen for $\alpha \gtrsim 5^\circ$ in Figs. 3d–f. These negative errors thus arise because, as a result of multiple scattering, some photons that have experienced scattering angles larger than α nevertheless reach the surface within an angular distance of α from the sun. For $\alpha < 5^\circ$, both terms 1 and 2 are generally below

FIG. 5. Decomposition of transmittance errors associated with the optical depth scaling approach for the (left) water cloud, (center) ice cloud, and (right) transported mineral aerosol. (a)–(c) Error term 1 and (d)–(f) error term 2 associated with multiple scattering and (g)–(i) error term 3 associated with the finite width of the solar disk. The contour interval is 0.005. See text for more details.

0.005. In particular, for $\alpha = 2.5^\circ$ in the ice cloud case, the absolute value of both terms 1 and 2 is less than 0.002 irrespective of τ . This indicates that the substantial contribution of multiple scattering to $t(2.5^\circ)$ in Fig. 4a

(up to 0.068) arises almost entirely from photons that have only experienced small scattering angles $\theta_s < 2.5^\circ$, and also that only few such photons “leak” to a larger angular distance from the sun. Finally, Fig. 5h shows that

FIG. 6. (a) Broadband optical depth scaling factor as a function of the half-FOV angle α for the water cloud (black), ice cloud (red), and transported mineral aerosol (ochre) cases discussed in section 3. The solid lines are estimates of k_0 based directly on the single-scattering albedo and phase function [Eq. (4)], while the closed circles represent effective values of k fitted to transmittances computed using the Monte Carlo model. (b) Spectral dependence of the scaling factor shown for half-FOV angles of $\alpha = 1.0^\circ$ (dashed lines for k_0 , crosses for k) and $\alpha = 2.5^\circ$ (solid lines for k_0 , closed circles for k). These values are shown for bands 3–18 in the Freidenreich and Ramaswamy (1999) solar radiation scheme. On the x axis, the band midpoint wavelength is indicated below the band number for selected bands.

the large positive errors in $t(\alpha)$ in the ice cloud case at very small values of α in Fig. 3e arise primarily from the neglect of the width of the solar disk. However, the associated error decreases rapidly with increasing α , being below 0.005 for $\alpha > 0.5^\circ$ and below 0.001 for $\alpha > 1^\circ$. In the water cloud and aerosol cases, in which the forward-scattering peak is not as sharp as in the ice cloud case, the corresponding error is very small. This analysis suggests that the width of the solar disk may have to be considered for CST applications (see also Reinhardt et al. 2014) or for narrow-FOV sun photometers, but it is a nonissue for modeling pyrheliometer measurements.

4. Optical depth scaling factor

Above, it was demonstrated that to a good approximation, the apparent direct solar radiation can be computed using Beer's law, if the actual optical depth is replaced by a scaled optical depth, where the scaling factor k_0 depends on the single-scattering albedo and the phase function [Eq. (4)]. Some further improvement can be obtained by fine-tuning the scaling factor based on the transmittances computed with the Monte Carlo model (Reinhardt et al. 2014). The symbol k is used for these effective scaling factors, to distinguish them from k_0 . Their derivation is detailed in appendix B. In this section, we discuss how the optical depth scaling factors depend on the

characteristics of the scattering particles and the half-FOV angle α . For illustrative purposes, we mainly focus on the broadband values (defined in appendix B).

To start with, we again consider the water cloud, ice cloud and mineral aerosol cases discussed in section 3. Figure 6a shows the broadband k_0 and k as a function of the half-FOV angle α , and Fig. 6b their spectral values computed for $\alpha = 1.0^\circ$ and $\alpha = 2.5^\circ$. The former angle is chosen as an example relevant for CST applications and sun photometers, while the latter represents typical pyrheliometers. The differences between k_0 and k are generally very small, both for the broadband values (Fig. 6a) and for the individual spectral bands (Fig. 6b). As an exception, at very small half-FOV angles, k is larger than k_0 in the ice cloud case. This is consistent with the positive transmittance errors seen when using the scaling factor k_0 (Figs. 3e,h), which are associated with the finite width of the solar disk (Fig. 5h). However, in agreement with the findings of Reinhardt et al. (2014), the difference between k and k_0 becomes appreciable only at half-FOV angles $\alpha \lesssim 0.5^\circ$. Hereafter, only the values of k are discussed.

Naturally, k decreases with an increasing half-FOV angle α (Fig. 6a). Also, k decreases with decreasing wavelength (Fig. 6b) because when the ratio of particle size to wavelength increases, the phase function diffraction peak grows stronger and sharper. Furthermore, k is smaller in the ice cloud case than in the water cloud

case and especially than in the aerosol case, mainly because of the differences in particle size. For example, for $\alpha = 2.5^\circ$, the broadband k equals 0.511 in the ice cloud case, 0.611 in the water cloud case and 0.916 in the mineral aerosol case. This suggests the following rule of thumb: when estimating the radiation received by pyrhemometers using Beer's law, the optical depth should be reduced by $\sim 50\%$ and $\sim 40\%$ for "typical" ice clouds and water clouds, but even for relatively large aerosol particles, the reduction is less than 10%. It is also of note that in the water and ice cloud cases, k is particularly sensitive to α at small half-FOV angles ($\alpha \lesssim 1^\circ$), which suggests that careful consideration of the CST system characteristics is needed for determining the value of k .

The effect of cloud droplet and ice crystal size is further illustrated in Fig. 7a, which shows broadband k (for $\alpha = 1.0^\circ$ and $\alpha = 2.5^\circ$) for water clouds with $r_e = 3\text{--}30\ \mu\text{m}$ and for ice clouds with $r_e = 5\text{--}60\ \mu\text{m}$, again assuming the GHM model for the latter. As expected, in all cases k shows a monotonic decrease with increasing r_e . However, for ice clouds with large crystals $r_e > 30\ \mu\text{m}$, the values of k for $\alpha = 2.5^\circ$ saturate to ~ 0.5 , as virtually all of the diffraction pattern then falls at scattering angles smaller than α . Nevertheless, $k = 0.5$ is not an absolute lower limit, as even much lower values can occur, depending on ice crystal morphology (see below). Furthermore, even for the same r_e , k is somewhat smaller for ice clouds than water clouds, mainly because for a given value of r_e , the projected-area mean radius r_p is larger for ice clouds than water clouds [see the discussion related to Eq. (7)]. Also, k may depend on other characteristics of the size distribution beyond r_e . For water clouds with $\nu_e = 0.2$, k is generally slightly larger than for $\nu_e = 0.1$, by circa 0–0.015, but the effect of ν_e is modest compared to the much larger effect of r_e .

Figure 7b displays the values of broadband k computed for 11 aerosol types in the OPAC database, for eight values of RH. There is some scatter in k related to differences in refractive index and the shape of the size distribution, but overall, a roughly linear decrease of k with aerosol r_e is seen for both $\alpha = 1.0^\circ$ and $\alpha = 2.5^\circ$. Also, Fig. 7b reiterates the point that for aerosols, the optical depth scaling factor is much closer to 1 than for water and ice clouds. In fact, in all cases with $r_e < 2\ \mu\text{m}$, $k > 0.90$ for $\alpha = 2.5^\circ$ and $k > 0.97$ for $\alpha = 1.0^\circ$.

The impact of ice crystal morphology on k is addressed in Fig. 8. In addition to the three Baum et al. (2014) ice cloud bulk scattering models [which all assume severely roughened (SR) ice crystals], values of k are shown for nine individual habits in the Yang et al. (2013) database, for three assumptions about ice crystal

FIG. 7. (a) Effective broadband optical depth scaling factor k as a function of cloud particle effective radius r_e , for water clouds with a droplet size effective variance of $\nu_e = 0.10$ (black) and $\nu_e = 0.20$ (gray) and for ice clouds with a size–shape distribution following the GHM model (red) and for half-FOV angles of $\alpha = 1.0^\circ$ (crosses) and $\alpha = 2.5^\circ$ (closed circles). (b) Values of k for 11 aerosol types in the OPAC database and for 8 values of RH (0%, 50%, 70%, 80%, 90%, 95%, 98%, and 99%). In most but not all cases, r_e increases with increasing RH. The transported mineral aerosol type (which is nonhygroscopic) is highlighted with an open square. In both (a) and (b), the dark blue curves show numerical fits for the broadband scaling factor discussed in section 5. For clarity, only the curves for $\nu_e = 0.10$ are displayed for water clouds.

roughness: SR, moderately roughened (MR), or completely smooth (CS). An ice crystal effective radius of $r_e = 30\ \mu\text{m}$ is assumed. For SR ice crystals, k depends only moderately on the habit, especially for $\alpha = 2.5^\circ$, for which the values lie between 0.49 and 0.55. For MR ice crystals, the values are slightly lower (0.40–0.54 for $\alpha = 2.5^\circ$), and for CS crystals, they are even lower and

FIG. 8. Effective broadband optical depth scaling factor k for ice clouds with $r_e = 30 \mu\text{m}$ (crosses for $\alpha = 1.0^\circ$ and closed circles for $\alpha = 2.5^\circ$). The scaling factors are given for different assumptions about ice crystal morphology: for the general habit mixture (GHM), solid columns (SC), and aggregates of solid columns (AggSC) ice cloud scattering models provided by Baum et al. (2014) and for nine individual habits in the Yang et al. (2013) database. The Baum et al. (2014) scattering models assume severely roughened (SR) ice crystals, while three roughness options are considered for the Yang et al. (2013) habits: SR (red), moderately roughened (MR; cyan), and completely smooth (CS; blue).

depend very strongly on the habit (0.17–0.51 for $\alpha = 2.5^\circ$ and 0.21–0.61 for $\alpha = 1.0^\circ$). In particular, plates and plate aggregates stand out for their very low values of k . The low values of k for CS crystals are related to rays that are transmitted through the crystals, entering and exiting through exactly parallel crystal faces (Haapanala et al. 2017). Such ray paths [which contribute to P_{ray} in Eq. (6)] are especially common for CS platelike crystals. For MR and SR crystals, however, the crystal surface slopes are tilted randomly for each incident ray (Yang et al. 2013), which spreads out the phase function contribution by transmitted rays to a larger range of scattering angles, resulting in a higher k . There is observational evidence that the scattering by natural ice crystals most often differs from their idealized counterparts (such as the CS crystals) (Cole et al. 2014; Ulanowski et al. 2014), also in the near-forward directions (DeVore et al. 2012; Haapanala et al. 2017), which could be due to surface roughness or other non-ideal features like irregularities and inhomogeneity. At any rate, Fig. 8 shows that the optical depth scaling factor k is highly sensitive to assumptions about ice crystal morphology. This also applies to the circumsolar

ratio (Reinhardt et al. 2014) and the radiance distribution around the sun (Haapanala et al. 2017).

A final issue worth discussing is the relationship between the optical depth scaling factor k (or k_0) used for the computation of the apparent direct solar radiation and the optical depth scaling employed in delta-two-stream approximations (e.g., Joseph et al. 1976; Zdunkowski et al. 1980; Räisänen 2002). In the latter, the optical depth is scaled as $\tau^* = k_{2\text{str}}\tau$, where

$$k_{2\text{str}} = 1 - \omega f \quad (8)$$

and f is the forward-peak scattering fraction. While other choices are possible (e.g., Räisänen 2002), it is usually assumed

$$f = g^2, \quad (9)$$

where the asymmetry parameter g is the average value of the cosine of scattering angle:

$$g = \int_0^\pi \cos\theta_s P_{11}(\theta_s) \sin\theta_s d\theta_s. \quad (10)$$

The purpose for scaling τ (and ω and g) in delta-two-stream approximations is to improve the accuracy of the computed total (direct plus diffuse) radiative fluxes. Nevertheless, in some radiation schemes the delta-scaled optical depth τ^* is also used for diagnosing the direct radiative flux (see the discussion in Sun et al. 2016). Therefore, it is of interest to study if $k_{2\text{str}}$ can be applied to estimating the radiation measured by pyrheliometers.

In Fig. 9, k computed for $\alpha = 2.5^\circ$ and the spectral band $\lambda = 0.685\text{--}0.870 \mu\text{m}$ is compared with $k_{2\text{str}}$, for a large number of water cloud, ice cloud, and aerosol cases. Two general comments can be made. First, in almost all cases, $k_{2\text{str}}$ is smaller than k , and often by a very wide margin, especially for aerosols and water clouds. Only for a few cases with CS ice crystals, $k_{2\text{str}}$ is close to or even slightly above k . Thus, in general, the use of $k_{2\text{str}}$ would overestimate the radiation measured by pyrheliometers. This was previously demonstrated by Sun et al. (2016) for dust aerosols. In fact, in agreement with their study, we find that for aerosols, the use of unscaled optical depth would be better, or at least less wrong, than the use of two-stream delta scaling (i.e., the optimal value of k is much closer to 1 than $k_{2\text{str}}$). For ice clouds, however, the opposite is generally true. Second, there is no one-to-one relationship between $k_{2\text{str}}$ and k . Rather, for any given value of $k_{2\text{str}}$, k varies widely. In fact, this is not surprising. For $\alpha = 2.5^\circ$, the effective scaling factor k is generally within 0.01 of the scaling factor k_0 defined by Eq. (4). As k_0 depends on the phase function values up

FIG. 9. Effective optical depth scaling factor for the spectral band $\lambda = 0.685\text{--}0.870\ \mu\text{m}$ for a half-FOV angle of $\alpha = 2.5^\circ$ compared with the optical depth scaling factor $1 - \omega g^2$ typically applied in δ -two-stream approximations. Black (gray) dots represent water clouds with an effective droplet radius of $r_e = 3\text{--}30\ \mu\text{m}$ and a droplet size effective variance of $\nu_e = 0.10$ ($\nu_e = 0.20$). Red, cyan, and blue dots represent ice clouds consisting of severely roughened (SR), moderately roughened (MR) and completely smooth (CS) crystals with $r_e = 5\text{--}60\ \mu\text{m}$, including the three Baum et al. (2014) ice cloud scattering models and nine individual habits in the Yang et al. (2013) database. The ochre dots represent aerosols (11 aerosol types for 8 values of RH).

to $\theta_s = \alpha = 2.5^\circ$ only, while $k_{2\text{str}}$ depends on the phase function for the entire range $0^\circ\text{--}180^\circ$, there is no physical reason for k_0 or k to be closely related to $k_{2\text{str}}$. Therefore, separate parameterization(s) independent of $k_{2\text{str}}$ should be developed for k . This is the topic of the next section.

5. Parameterization

The effective optical depth scaling factors k for water clouds (subscript “liq”), ice clouds (ice), and aerosols (aer) were parameterized as a function of the effective radius r_e of the scattering particles:

$$k_{\text{liq}} = a_{\text{liq},1} r_e^{-1} + a_{\text{liq},2} r_e^{-1/2} + a_{\text{liq},3} + a_{\text{liq},4} r_e^{1/2}, \quad (11)$$

$$k_{\text{ice}} = a_{\text{ice},1} r_e^{-1} + a_{\text{ice},2} r_e^{-1/2} + a_{\text{ice},3} + a_{\text{ice},4} r_e^{1/2}, \quad (12)$$

$$k_{\text{aer}} = a_{\text{aer},1} + a_{\text{aer},2} r_e. \quad (13)$$

The range of r_e considered in numerical fitting was $3\text{--}30\ \mu\text{m}$ for water clouds, $5\text{--}60\ \mu\text{m}$ for ice clouds, and circa

$0.1\text{--}3.1\ \mu\text{m}$ for aerosols. The water cloud parameterization coefficients were computed for two values of droplet size effective variance ($\nu_e = 0.1$ and $\nu_e = 0.2$). For ice clouds, the coefficients were evaluated separately for the three Baum et al. (2014) ice cloud bulk scattering models and for the nine individual habits in the Yang et al. (2013) database, the latter for three roughness assumptions. For the half-FOV angle α , 18 values between 0.267° and 5° were considered. Furthermore, the parameterization coefficients were computed for three choices of spectral resolution: for broadband solar radiation ($0.174\text{--}10\ \mu\text{m}$), for 4 separate bands ($0.174\text{--}0.685$, $0.685\text{--}1.220$, $1.220\text{--}2.381$, and $2.381\text{--}10\ \mu\text{m}$) and for 17 spectral bands [i.e., all Freidenreich and Ramaswamy (1999) bands with a nonnegligible contribution to surface insolation]. All parameterization coefficients are provided in the online supplemental material. A subset of them, including the broadband and four-band coefficients for $\alpha = 1.0^\circ$ and $\alpha = 2.5^\circ$ for water clouds with $\nu_e = 0.1$, for ice clouds following the GHM size–shape distribution, and for aerosols, is reproduced in Tables 1–3.

It is demonstrated in Fig. 7a that the parameterized broadband k_{liq} agrees very closely with the actual values derived from Monte Carlo calculations (for visual clarity, only the curves for $\nu_e = 0.1$ are shown). The parameterized broadband k_{ice} for the GHM model also agrees well with the Monte Carlo values. For aerosols, the parameterization captures the general decrease of k with increasing aerosol size but there is substantial scatter around the linear relationship (Fig. 7b). This scatter arises from the differences in refractive index and the assumed shape of the size distribution between the different aerosol types. Therefore, Eq. (13) is best considered as a first-guess parameterization. It is suggested that the best solution would be to (i) employ the refractive index and size distribution information available in an aerosol model to parameterize k_{aer} based on Eq. (4), and to (ii) possibly adjust k_{aer} slightly downward, since the use of Eqs. (3) and (4) tends to slightly underestimate the contribution of scattered radiation to the transmittance for aerosols (Figs. 3f,i). Furthermore, spherical particles have been assumed here, but in reality, solid aerosols are generally nonspherical. For dust consisting of angular, sharp-edged particles, forward scattering can be as much as 50% stronger than for volume-equivalent spheres (Kalashnikova and Sokolik 2002). For spheroids, which may be more appropriate for modeling the scattering by dust far from its source regions (Russell et al. 2004), the differences from spheres are smaller. For near-forward scattering integrated over typical instrument FOVs, Russell et al. (2004) report spheroid

TABLE 1. Parameterization coefficients for water clouds ($\nu_e = 0.1$) for use in Eq. (11), where α is the half-opening angle and $\Delta\lambda$ the spectral band.

$\alpha(^{\circ})$	$\Delta\lambda$ (μm)	$a_{\text{liq},1}$	$a_{\text{liq},2}$	$a_{\text{liq},3}$	$a_{\text{liq},4}$
1.0	0.174–10.00	$-2.996\,21 \times 10^0$	$3.594\,97 \times 10^0$	$-1.937\,66 \times 10^{-1}$	$4.284\,95 \times 10^{-2}$
	0.174–0.685	$-3.476\,03 \times 10^0$	$4.550\,43 \times 10^0$	$-7.067\,45 \times 10^{-1}$	$1.012\,21 \times 10^{-1}$
	0.685–1.220	$-3.260\,34 \times 10^0$	$3.584\,31 \times 10^0$	$-4.406\,26 \times 10^{-2}$	$1.746\,82 \times 10^{-2}$
	1.220–2.381	$1.061\,73 \times 10^{-1}$	$-5.272\,54 \times 10^{-1}$	$1.473\,27 \times 10^0$	$-1.241\,24 \times 10^{-1}$
	2.381–10.00	$9.181\,43 \times 10^{-1}$	$-1.235\,17 \times 10^0$	$1.553\,97 \times 10^0$	$-8.575\,00 \times 10^{-2}$
2.5	0.174–10.00	$-3.630\,27 \times 10^{-1}$	$1.340\,77 \times 10^0$	$1.199\,31 \times 10^{-1}$	$3.308\,54 \times 10^{-2}$
	0.174–0.685	$9.411\,70 \times 10^{-1}$	$-9.288\,75 \times 10^{-2}$	$4.967\,69 \times 10^{-1}$	$1.115\,45 \times 10^{-3}$
	0.685–1.220	$-1.060\,72 \times 10^0$	$2.270\,16 \times 10^0$	$-1.915\,38 \times 10^{-1}$	$6.402\,54 \times 10^{-2}$
	1.220–2.381	$-3.894\,09 \times 10^0$	$4.789\,26 \times 10^0$	$-6.790\,49 \times 10^{-1}$	$9.152\,30 \times 10^{-2}$
	2.381–10.00	$-6.186\,66 \times 10^{-1}$	$3.238\,05 \times 10^{-1}$	$1.161\,95 \times 10^0$	$-9.409\,71 \times 10^{-2}$

versus sphere differences of at most 7%, while Ge et al. (2011) find differences of $\sim 10\%$. Note that differences in single-scattering properties between nonspherical and spherical particles depend not only on the shapes but also on the metric (e.g., volume, projected area, or r_e) used to define the radius of equivalent spheres.

For completeness, we also represent a parameterization for scaling the optical depth due to Rayleigh scattering:

$$k_{\text{Ray}} = 1 - a_{\text{Ray}}\alpha^2, \quad (14)$$

where α is in degrees and the coefficient $a_{\text{Ray}} = 1.67 \times 10^{-4}$ includes the contributions of both single and multiple scattering, estimated for a cloud-free and aerosol-free midlatitude summer atmosphere with a surface albedo of 0.2 and a solar zenith angle of 45° . In fact, the effect of multiple scattering depends strongly on the spectral band and the surface albedo, but a more sophisticated parameterization is hardly warranted as Rayleigh scattering contributes very little to the scattered radiation near the sun (e.g., for $\alpha = 2.5^{\circ}$, the broadband t_{sc} is usually below 10^{-4}).

Finally, the transmittance $t(\alpha)$ can be computed as

$$t(\alpha) = t_{\text{gas}} \exp\left(-\frac{k_{\text{liq}}\tau_{\text{liq}} + k_{\text{ice}}\tau_{\text{ice}} + k_{\text{aer}}\tau_{\text{aer}} + k_{\text{Ray}}\tau_{\text{Ray}}}{\cos\theta_0}\right). \quad (15)$$

Here, k_{liq} , k_{ice} , k_{aer} , and k_{Ray} are the optical depth scaling factors for the value of α considered; t_{gas} is the direct-beam transmittance for gaseous absorption; and τ_{liq} , τ_{ice} , τ_{aer} , and τ_{Ray} are the total column optical depths for cloud liquid water, cloud ice, aerosols, and Rayleigh scattering. The gaseous transmittance and the optical depths depend on the spectral band considered. Here, they are computed using the solar radiation scheme of Freidenreich and Ramaswamy (1999).

Figure 10 illustrates the performance of the parameterization presented above. In each figure panel, the contribution of scattered radiation to the broadband solar transmittance t_{sc} is indicated on the x axis, while the y axis gives the difference between the parameterized and Monte Carlo transmittances. Similar to Fig. 9, a large number of cases are considered: water clouds with $r_e = 3\text{--}30\,\mu\text{m}$ (for $\nu_e = 0.1$ and $\nu_e = 0.2$), ice clouds with $r_e = 5\text{--}60\,\mu\text{m}$ [for the three Baum et al. (2014) models and the nine single-habit cases with three roughness

TABLE 2. Parameterization coefficients for ice clouds for use in Eq. (12) for the general habit mixture model (Baum et al. 2014), where α is the half-opening angle and $\Delta\lambda$ is the spectral band.

$\alpha(^{\circ})$	$\Delta\lambda$ (μm)	$a_{\text{ice},1}$	$a_{\text{ice},2}$	$a_{\text{ice},3}$	$a_{\text{ice},4}$
1.0	0.174–10.00	$-3.836\,34 \times 10^0$	$4.297\,48 \times 10^0$	$-4.332\,66 \times 10^{-1}$	$6.083\,29 \times 10^{-2}$
	0.174–0.685	$-1.366\,72 \times 10^0$	$2.315\,11 \times 10^0$	$-4.853\,85 \times 10^{-2}$	$3.747\,68 \times 10^{-2}$
	0.685–1.220	$-5.198\,52 \times 10^0$	$5.524\,05 \times 10^0$	$-7.149\,35 \times 10^{-1}$	$8.007\,01 \times 10^{-2}$
	1.220–2.381	$-9.539\,14 \times 10^0$	$8.563\,89 \times 10^0$	$-1.200\,04 \times 10^0$	$1.038\,75 \times 10^{-1}$
	2.381–10.00	$-6.111\,58 \times 10^0$	$4.681\,33 \times 10^0$	$1.115\,59 \times 10^{-1}$	$-1.194\,05 \times 10^{-3}$
2.5	0.174–10.00	$-3.558\,01 \times 10^{-1}$	$1.065\,27 \times 10^0$	$2.368\,99 \times 10^{-1}$	$1.713\,47 \times 10^{-2}$
	0.174–0.685	$1.565\,38 \times 10^{-1}$	$4.170\,55 \times 10^{-1}$	$3.830\,66 \times 10^{-1}$	$7.353\,66 \times 10^{-3}$
	0.685–1.220	$-2.863\,42 \times 10^{-2}$	$9.373\,11 \times 10^{-1}$	$2.547\,23 \times 10^{-1}$	$1.627\,49 \times 10^{-2}$
	1.220–2.381	$-2.452\,98 \times 10^0$	$3.281\,48 \times 10^0$	$-2.632\,11 \times 10^{-1}$	$5.085\,47 \times 10^{-2}$
	2.381–10.00	$-8.609\,54 \times 10^0$	$7.839\,76 \times 10^0$	$-1.057\,65 \times 10^0$	$9.487\,21 \times 10^{-2}$

TABLE 3. Parameterization coefficients for aerosols for use in Eq. (13), where α is the half-opening angle and $\Delta\lambda$ the spectral band.

$\alpha(^{\circ})$	$\Delta\lambda (\mu\text{m})$	$a_{\text{aer},1}$	$a_{\text{aer},2}$
1.0	0.174–10.00	9.99583×10^{-1}	-1.30225×10^{-2}
	0.174–0.685	1.00102×10^0	-1.99908×10^{-2}
	0.685–1.220	9.98745×10^{-1}	-8.75551×10^{-3}
	1.220–2.381	9.97739×10^{-1}	-2.92232×10^{-3}
	2.381–10.00	9.98437×10^{-1}	-6.69538×10^{-4}
2.5	0.174–10.00	9.93417×10^{-1}	-4.60182×10^{-2}
	0.174–0.685	9.96553×10^{-1}	-6.54197×10^{-2}
	0.685–1.220	9.92263×10^{-1}	-3.53396×10^{-2}
	1.220–2.381	9.87706×10^{-1}	-1.35234×10^{-2}
	2.381–10.00	9.90714×10^{-1}	-3.21955×10^{-3}

assumptions], and 11 aerosol types for eight values of RH.

Figures 10a–c show results for a midvisible optical depth $\tau = 1$, for which t_{sc} is close to its maximum, for half-FOV angles $\alpha = 1.0^{\circ}$, $\alpha = 2.5^{\circ}$, and $\alpha = 5^{\circ}$. In these cases, the accuracy of parameterized t_{sc} is generally excellent for water and ice clouds. In all water cloud cases and most ice cloud cases, the errors in t_{sc} are below 0.002, and the relative errors in t_{sc} are consistently less than 2%, except for a few cases for $\alpha = 1.0^{\circ}$. While slightly larger errors in t_{sc} appear in some ice clouds cases (up to 0.005 for $\alpha = 1.0^{\circ}$), these errors are very small compared to the large spread of t_{sc} associated with differences in ice crystal size, habit, and roughness (cf. Fig. 8). For aerosols, the parameterization is less accurate than for water and ice clouds, which is expected based on the substantial scatter of points around the regression lines in Fig. 7b. The absolute errors in t_{sc} reach up to 0.006 for $\alpha = 2.5^{\circ}$ and 0.011 for $\alpha = 5^{\circ}$, and the relative errors very often exceed 10%.

Figure 10d illustrates a limitation of the current parameterization. At large half-FOV angles and optical depths, t_{sc} is generally underestimated, for the parameter values considered here ($\alpha = 5^{\circ}$, $\tau = 5$) typically by 0.001–0.004. This is also seen when the optical depth scaling factor is computed directly from Eq. (4) (Figs. 3d–f), and it results from an increased contribution of multiple scattering to t_{sc} (Fig. 5). Optimizing the value of k based on Monte Carlo results [Eq. (B3) in appendix B] alleviates the negative bias only slightly. It also gives rise to a slight positive bias at lower optical depths, which can be discerned from Fig. 10c. In an absolute sense, the negative errors in t_{sc} are smaller at the half-FOV angles most relevant for pyrhemeter measurements ($\alpha \approx 2.5^{\circ}$) or sun photometers and CST applications ($\alpha \sim 1^{\circ}$).

Figure 11 addresses cases in which either two cloud layers or cloud and aerosol coexist in the same atmospheric

column. Here, the total (cloud plus aerosol) midvisible optical depth τ is fixed to either 1 or 3, and the same cloud and aerosol microphysical properties are assumed as in Figs. 1–6 (for other details, see the figure caption). For example, Fig. 11b indicates that, for $\tau = 1$ and $\alpha = 2.5^{\circ}$, the broadband t_{sc} decreases gradually from 0.173 to 0.022 when the ice cloud contribution to τ decreases incrementally from 1 to 0 and the mineral aerosol contribution increases incrementally from 0 to 1. Concurrently, the parameterization error in t_{sc} increases from -0.0011 to 0.0015 . More generally, Fig. 11 suggests that the presence of different scatterer types in the same atmospheric column does not compromise the accuracy of the parameterization.

6. Summary

The contribution of scattered radiation to the radiation received from directions near the sun is a relevant issue for the interpretation of direct solar radiation measurements by pyrhemeters and sun photometers and for concentrating solar technology applications. Here, this topic was addressed using a Monte Carlo radiative transfer model. The radiation received from near-solar directions was studied as a function of the cloud and aerosol properties and the half-FOV angle α around the sun.

It was shown that the contribution of scattered radiation t_{sc} to the transmittance at a given half-FOV angle α depends strongly on the type of the particles considered, especially their size, because the phase function forward-scattering peak grows sharper with an increasing ratio of particle size to wavelength. Thus, for a given optical depth, t_{sc} is typically much larger for water clouds than aerosols, and larger still for ice clouds. Furthermore, for ice clouds, t_{sc} depends strongly on ice crystal morphology (i.e., assumptions about ice crystal habit and roughness).

The Monte Carlo calculations were compared with a simple approach, in which the apparent direct transmittance is estimated through Beer's law by replacing the optical depth with a scaled optical depth, which excludes the part of the phase function corresponding to scattering angles smaller than α . Overall, this approach performs very well, especially for cases with relatively small optical depth. Two limitations were, however, identified. First, at very small half-FOV angles ($\alpha < 0.5^{\circ}$ – 1°), the transmittance is overestimated for ice clouds. This issue arises from the combination of a very sharp forward-scattering peak and the finite width of the solar disk. Second, the transmittance is generally underestimated at large optical depths, as multiple scattering causes the actual transmittance to

FIG. 10. The contribution of scattered radiation to the transmittance computed with the Monte Carlo model [$t_{sc}(\alpha)$ in Eq. (2)] and the associated parameterization errors for (a) a half-FOV angle of $\alpha = 1.0^\circ$ and a midvisible optical depth of $\tau = 1$, (b) $\alpha = 2.5^\circ$ and $\tau = 1$, (c) $\alpha = 5^\circ$ and $\tau = 1$, and (d) $\alpha = 5^\circ$ and $\tau = 5$. The black and gray dots represent water clouds, while the red, cyan, and blue dots represent ice clouds and the ochre dots represent aerosols, as explained in the caption of Fig. 9. The dashed gray lines indicate fractional errors of $\pm 2\%$ and $\pm 10\%$ in the contribution of scattered radiation.

decrease slower than the optical depth scaling approach suggests. In practice, this is only relevant at half-FOV angles of at least several degrees.

It was also demonstrated that the optical depth scaling factor used in delta-two stream approximations ($f = 1 - \omega g^2$) is generally substantially smaller than the scaling factor derived from Monte Carlo calculations [or directly from the single-scattering properties through Eq. (4)] for $\alpha = 2.5^\circ$. Therefore, direct solar radiation computed using the two-stream optical depth

scaling would seriously overestimate that measured by pyrheliometers.

Parameterizations were derived for computing the optical depth scaling factor for water clouds, ice clouds, and aerosols as a function of the effective radius of the scattering particles. Parameterization coefficients are provided for several values of α , for different spectral bands and in the case of ice clouds, for many assumptions about ice crystal morphology. Overall, the accuracy of the parameterization is quite high for water and

FIG. 11. The contribution of scattered radiation to the transmittance computed with the Monte Carlo model [$t_{sc}(\alpha)$ in Eq. (2)] and the associated parameterization errors for half-FOV angles of (a) $\alpha = 1.0^\circ$, (b) $\alpha = 2.5^\circ$, and (c) $\alpha = 5.0^\circ$ for cases in which the sum of cloud and aerosol midvisible optical depth is fixed to either $\tau = 1$ or $\tau = 3$ [the two groups of points being well separated, see (a)]. The large red symbols represent cases in which there is only a water cloud with $r_e = 10 \mu\text{m}$ and $\nu_e = 0.10$ (closed circle), a GHM ice cloud with $r_e = 30 \mu\text{m}$ (closed triangle), or transported mineral aerosol (asterisk) in the atmospheric column. The gray dots represent cases with coexisting water and ice cloud in the same column, with the fractional contribution of ice to τ increasing incrementally from 0.1 to 0.9. Similarly, the blue dots represent coexisting water cloud and aerosol, and the cyan dots represent coexisting ice cloud and aerosol. The dashed gray lines indicate fractional errors of $\pm 2\%$ and $\pm 10\%$ in the contribution of scattered radiation.

ice clouds (assuming the ice crystal morphology is known), although at large values of α and τ , a tendency to underestimated transmittance prevails. For aerosols, the accuracy is more modest. The accuracy could be improved by developing model-specific parameterizations based on the size distribution and refractive index information available in aerosol models, and eventually by considering the effects of aerosol nonsphericity. In principle, the parameterizations developed here could be employed operationally, for example, for computing time-varying optical depth scaling factors in an NWP model based on the simulated cloud and aerosol properties. Similarly, adjustable scaling factors could be used in other applications such as optical depth retrievals with sun photometers, if external information on the atmospheric particle types, sizes, and ideally, morphology, is available.

A final point worth stressing is that a prerequisite for an accurate computation of the apparent direct transmittance is the knowledge of the optical properties of the scattering particles present in the atmosphere. An especially critical yet rather poorly known factor is the ice crystal phase function at near-forward-scattering angles.

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APPENDIX A

Treatment of Finite Solar Disk and Limb Darkening

The procedure for finding out the initial position of each photon in the Monte Carlo simulations with a finite solar disk consists of three steps. In step 1, the angular distance δ from the center of the solar disk is determined assuming uniform brightness of the solar disk:

$$\delta = \delta_{\max} \sqrt{RN_1}, \quad (\text{A1})$$

where $\delta_{\max} = 0.267^\circ$ is the average half width of the solar disk as seen from Earth, and RN_1 is a random number distributed uniformly between 0 and 1. The square root is needed because the solid angle covered by an annulus of width $d\delta$ increases proportionally to δ .

In step 2, limb darkening is taken into account by using the formula (Böhm-Vitense 1989)

$$L_{\text{norm}}(\beta) = a + b \cos \beta + c \cos^2 \beta. \quad (\text{A2})$$

Here, $L_{\text{norm}}(\beta)$ is the radiance normalized by its peak value, the coefficients a , b , and c depend on the spectral band, and β is the angular distance from the center of the

solar disk toward the limb (0° – 90°), which can be approximated (with a maximum error of $\delta_{\max}$) as

$$\beta \approx \sin^{-1}(\delta/\delta_{\max}). \quad (\text{A3})$$

The values of $L_{\text{norm}}(\beta)$ vary from 1 at the center of the solar disk to ~ 0.14 – 0.73 at the limb, with strongest limb darkening in the UV region. To simulate limb darkening, another random number RN_2 is drawn. If it happens that

$$\text{RN}_2 > L_{\text{norm}}(\beta), \quad (\text{A4})$$

the value of δ is rejected, and the algorithm returns to step 1. Otherwise, it proceeds to step 3.

In step 3, a third random number RN_3 is used to select a directional angle γ within the solar disk, so that $\gamma = 0^\circ, 90^\circ, 180^\circ$, and 270° refer to directions north, west, south, and east of the center of the disk, respectively:

$$\gamma = 360^\circ \times \text{RN}_3. \quad (\text{A5})$$

The initial zenith angle θ and azimuth angle ϕ of the photon are then computed as follows:

$$\theta = \theta_0 - \delta \cos \gamma, \quad (\text{A6})$$

$$\phi = \phi_0 + \frac{\delta \sin \gamma}{0.5(\sin \theta_0 + \sin \theta)}, \quad (\text{A7})$$

where θ_0 and ϕ_0 are the zenith and azimuth angles of the center of the solar disk. The denominator in Eq. (A7) is needed because at a zenith angle θ' , an angular distance of $\Delta\delta$ in the zonal direction corresponds to a change of $\Delta\phi = \Delta\delta/\sin\theta'$ in the azimuth angle.

APPENDIX B

Derivation of Effective Optical Depth Scaling Factors

For each water cloud, ice cloud, and aerosol case considered, Monte Carlo calculations were performed for a set of 23 midvisible ($0.55\ \mu\text{m}$) optical depths between 0 and 10. A midlatitude summer atmosphere, a solar zenith angle of $\theta_0 = 45^\circ$, and a surface albedo of 0.2 were assumed. For each spectral band i , this provided a set of cloud (or aerosol) transmittances:²

$$t_i^*(\tau_i) = t_i(\tau_i)/t_i(0), \quad (\text{B1})$$

where $t_i(\tau_i)$ is the transmittance for a cloud (or aerosol) spectral optical depth of τ_i and $t_i(0)$ is the transmittance for the cloud-free and aerosol-free case. The corresponding cloud (or aerosol) transmittance for the optical depth scaling approach is

$$t_{\text{scaled},i}^*(k_i, \tau_i) = \exp\left(\frac{-k_i \tau_i}{\cos \theta_0}\right). \quad (\text{B2})$$

The transmittances defined by Eqs. (B1) and (B2) were used to evaluate numerically the mean-square error (MSE) in the transmittance curve:

$$\text{MSE}(k_i) = \int_0^1 [t_{\text{scaled},i}^*(k_i, \tau_i) - t_i^*(\tau_i)]^2 dt_i^*, \quad (\text{B3})$$

and the value of k_i minimizing the MSE was defined as the effective optical depth scaling factor for spectral interval i . This procedure is illustrated in Fig. B1 for a GHM ice cloud with $r_e = 30\ \mu\text{m}$, for the spectral interval 0.685 – $0.870\ \mu\text{m}$. At a small half-FOV angle of $\alpha = 0.5^\circ$, the use of the scaling factor $k_{0,i}$ computed from Eq. (4) overestimates the cloud transmittance (Fig. B1a; see also Fig. 3e), mainly because of neglecting the finite width of the solar disk (Fig. 5h). Optimizing k_i through Eq. (B3) successfully eliminates this bias. At $\alpha = 5^\circ$ (Fig. B1b), the use of $k_{0,i}$ underestimates the transmittance, especially at relatively large optical depths ($\tau \sim 5$), which can be ascribed to the increased role of multiple scattering (Figs. 5b,e). The use of optimized k_i alleviates but does not eliminate this bias, and introduces a small positive bias at smaller optical depths. These features are ubiquitous at large values of α and can be seen in the behavior of the parameterization errors in Figs. 10c and 10d. Overall, Eq. (B3) puts most weight to relatively small values of optical depth ($\tau \sim 0$ – 2), as it is at small values of τ that the transmittance changes rapidly.

The procedure for computing the broadband values of the effective optical depth scaling factor k was similar, but the MSE in broadband transmittance corresponding to a spectrally uniform scaling factor k was evaluated and minimized:

$$\text{MSE}(k) = \int_0^1 \left\{ \sum_i w_i [t_{\text{scaled},i}^*(k, \tau_i) - t_i^*(\tau_i)] \right\}^2 dt^*, \quad (\text{B4})$$

where w_i is the fractional contribution of spectral interval i to the radiation transmitted to the surface in the cloud-free and aerosol-free case and t^* refers to the broadband cloud or aerosol transmittance. Finally, the broadband value of optical depth scaling factor k_0

² Unlike in the main text, the spectral interval i is marked here explicitly, while the dependence on the half-FOV angle α is not shown.

FIG. B1. Illustration of the fitting procedure for the effective optical depth scaling factor k . The errors in cloud transmittance for the optical depth scaling approach ($t_{\text{scaled}}^* - t^*$) are plotted against the cloud transmittance from the Monte Carlo calculations t^* for a GHM ice cloud with $r_e = 30 \mu\text{m}$. The spectral interval is $\lambda = 0.685\text{--}0.870 \mu\text{m}$, and two half-FOV angles are considered: (a) $\alpha = 0.5^\circ$ and (b) $\alpha = 5.0^\circ$. The open circles represent the case in which the scaling factor k_0 is computed directly from the single-scattering properties [Eq. (4)] and the closed circles represent the case in which k is optimized through Eq. (B3). The numerical values of k_0 and k are given in each panel. The extra x-axis scale on the top of the panels indicates cloud optical depth.

corresponding to Eq. (4) (shown in Fig. 6) was computed by minimizing the mean-square difference in broadband transmittance to the case in which $k_{0,i}$ was evaluated separately for each interval i :

$$\text{MSE}(k_0) = \int_0^1 \left\{ \sum_i w_i [t_{\text{scaled},i}^*(k_0, \tau_i) - t_{\text{scaled},i}^*(k_{0,i}, \tau_i)] \right\}^2 dt^*. \quad (\text{B5})$$

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