Effects of Moisture in a Two-Layer Model of the Midlatitude Jet Stream

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ABSTRACT

The effects of moisture on the energetics of a statistically stationary, baroclinically unstable jet representing the midlatitude atmosphere are examined using a two-layer, β -plane shallow-water model. Flow is driven by a relaxation of the interface between the two layers to a baroclinically unstable profile. Moisture is input to the lower layer by evaporation. When supersaturation occurs, precipitation is triggered and the related latent heat release drives a mass transfer between the two layers. A comparison between dry and moist reference atmospheres shows that precipitation reduces eddy kinetic energy. This is related to the meridional distribution of precipitation, which occurs on the poleward side of the jet (where the interface field is raised). This latitudinal structure of precipitation is related to a correlation between poleward flow and ascent, which is analyzed using a shallow-water analog to the ω equation. The precipitation effect on the energy budget is predominately due to zonal- and time-averaged terms. Because of this, dry simulations in which the thermal forcing is modified to mimic the effect of zonally averaged precipitation are carried out and compared with their precipitating counterparts. These simulations show a similar reduction of baroclinic eddy kinetic energy; however, the barotropic eddy kinetic energy response shows a larger difference.

1. Introduction

The midlatitude atmosphere is characterized by an equator-to-pole temperature gradient and a corresponding zonal jet that increases with height in a manner consistent with thermal wind. Baroclinic instability of this jet extracts available potential energy and produces the cyclones and anticyclones that populate the midlatitude storm tracks. These storms produce a rich array of weather phenomena and, on average, a net poleward transport of heat and moisture. The poleward flux of heat can be understood through baroclinic instability, which produces eddies that act to reduce the temperature gradient responsible for the unstable jet. The poleward moisture flux can be viewed as a consequence of these same eddies: poleward-flowing warm, moist air and equatorward-flowing cold, dry air will have a net poleward moisture flux.

Moisture affects these dynamics. In particular, precipitation involves a phase change from vapor to liquid or ice, and the accompanying latent heat release can be viewed as a “forcing” of the dry dynamics. As such, precipitation produces localized heating events that

can be thought of as a direct forcing of synoptic- and mesoscale eddies (e.g., [Chang et al. 2002](#)). In the context of climate change, warming implies a moister atmosphere (e.g., [Held and Soden 2006](#)). Many initial-value problems (described below) suggest that increased moisture should lead to an increase in eddy kinetic energy and thus stronger storms. On the other hand, there may be competing mean-state changes, such as the surface temperature gradient decrease due to polar amplification of warming. In the context of dry dynamics, a reduced meridional temperature gradient implies weaker instability and weaker storms.

It is therefore interesting to examine the role of moisture in the energetics of synoptic-scale midlatitude storms and how the effects change with specific humidity. One approach to this problem has been to work with global climate models (GCMs). For example, in the context of global warming, [Ulbrich et al. \(2008\)](#) considered ensemble statistics of 23 coupled GCM runs and found an increase in storm activity, while [Lehmann et al. \(2014\)](#) found a decrease in Northern Hemisphere summer storm activity and an increase in Southern Hemisphere winter activity. In another study, [Frierson et al. \(2006\)](#) considered increases in moisture in a gray-radiation aquaplanet GCM (i.e., a GCM with no cloud or water vapor radiative feedbacks) and found a

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decrease in the eddy kinetic energy. [Laliberté et al. \(2015\)](#) showed in simulations of climate change that an atmospheric heat engine slowed down as a result of maintaining the hydrological cycle intensification, thereby producing weaker storms. In a review paper, [Schneider et al. \(2010\)](#) showed that eddy kinetic energy decreased with warming due to decreases in meridional potential temperature gradients and increases in static stability (despite increases in latent heat release).

A second approach to studying this problem is to consider baroclinic life cycles, a canonical initial-value problem calculation that simulates a baroclinic wave through its nonlinear evolution. For example, [Booth et al. \(2013\)](#) varied moisture content in the Weather Research and Forecasting (WRF) Model in a periodic channel and found that increased moisture led to an increase in storm strength. [Tierney et al. \(2018\)](#), in an effort to better represent how the midlatitude atmosphere is affected by global warming, ran simulations that varied the moisture and baroclinicity of the environment in tandem. They found nonmonotonic behavior: increases in moisture initially led to an increase in storm strength; however, storm strength decreased after a threshold value. [Waite and Snyder \(2013\)](#) also used WRF to study the effect of moisture on rotational and divergent energy spectra, and found that moisture excited more gravity waves compared to a similar dry simulation ([Waite and Snyder 2009](#)). [Pavan et al. \(1999\)](#) showed that the location of the initial moisture distribution was a factor in determining whether precipitation energized the flow. For example, when the subtropics were substantially subsaturated, eddy kinetic energy was diminished relative to a dry simulation. [Kirshbaum et al. \(2018\)](#) examined the sensitivity of moist baroclinic wave growth to environmental temperature for fixed relative humidity. They found that moist simulations showed an increase in eddy kinetic energy relative to dry simulations, yet as the atmosphere is warmed, competing effects of additional moisture led to little change in eddy kinetic energy. Other baroclinic life cycle studies include [Balasubramanian and Yau \(1996\)](#), who found an increase in the growth rate in a simulated moist cyclone compared to a dry simulation; [Balasubramanian and Garner \(1997\)](#), who found an increase in the growth rate of short baroclinic waves with moisture resulting in more eddy kinetic energy compared to dry simulations; and [Gutowski et al. \(1992\)](#), who similarly found an increase in eddy kinetic energy compared to dry simulations. Overall, these initial-value problem baroclinic life cycle results are suggestive of an energizing role for latent heat release, but the relevance of this to the statistical equilibria of the

climate is unclear. For example, these initial-value problems usually do not include the impact of the associated poleward energy transport by the baroclinic wave.

While GCMs and other models with comprehensive treatments of diabatic processes provide realism, their inherent complexity can make a thorough understanding of the fundamental dynamics difficult ([Held 2005](#)). A third approach has thus been to consider simpler, more idealized models. The earliest such study is [Gill \(1982\)](#), who considered a moist shallow-water system and found that moist gravity waves (i.e., gravity waves propagating in regions of precipitation) traveled slower than dry gravity waves. A similar model was used in [Lambaerts et al. \(2011a, 2012\)](#) to examine barotropic and baroclinic instability, respectively. Both studies considered initial-value problems where the background state was a Bickley jet (a commonly used wind profile that decays exponentially away from its central latitude). They found that precipitation enhanced the growth rate of the instability, as a result of the increase in available potential energy generated by precipitation. A caveat is that the initial moisture field had a relative humidity of greater than 99% across the entire domain, a situation unlikely to be found in the real atmosphere. [Lainé et al. \(2011\)](#) used a three-layer moist quasigeostrophic (QG) model and showed that the storm tracks weakened in moist simulations. [Lapeyre and Held \(2004\)](#) considered the statistical equilibrium of a two-layer QG model on a β plane with moisture in the lower layer. One of their key findings was that increasing the strength of latent heat release led to a transition from a jet-dominated regime toward a cyclone-dominated regime. Essentially, the direct forcing of eddies via latent heat release drove the system away from β -plane jet-dominated turbulence and closer to f -plane vortex-dominated turbulence. In their study, the base-state thermal wind shear was spatially uniform.

Our study expands on [Lapeyre and Held \(2004\)](#) by allowing for a meridionally varying forcing profile taken to represent a more Earthlike thermal forcing. Further, we use a two-layer β -plane shallow-water model. This allows for order-one variations in the Coriolis parameter and for ageostrophic motion to be generated by the dynamics. Nonetheless, where appropriate, approximations similar to those assumed by QG in formulating diagnostics are used.

A principal result is that moisture has a strong dissipative effect on the dynamics of the statistical equilibrium of the moist two-layer model. For example, as moisture is increased, a progressively weaker eddy field results. Much like the midlatitude atmosphere, there is more evaporation equatorward of the jet in all of our simulations. While in the atmosphere this is

often attributed to large saturation specific humidities (due to the high sea surface temperatures), in our simulations no meridional structure on the thermodynamics is imposed. That is, there is no meridional prescription in the boundary conditions or governing equations that would preferentially lead to evaporation being equatorward of the jet. Instead, this latitudinal dependence of specific humidity is related to a correlation between poleward transport and vertical ascent. This is examined in more detail using the shallow water analog of the ω equation to relate ascent to other dynamical terms.

In [section 2](#), the governing equations and numerical model used along with parameter values are described. The energy equation for this system and its associated energy budget are introduced. [Section 3](#) compares dry and moist atmosphere reference simulations. In [section 4](#), the sensitivity of the system is examined as the humidity is varied significantly using three physically meaningful methods. [Section 5](#) attempts to recover the results of our moist simulations by adjusting the thermal forcing in dry simulations to mimic moisture effects. In [section 6](#), we present a summary and conclusions.

2. Methodology

a. Model equations and forcing description

We use a two-layer rotating shallow-water model with a tracer representing the moisture in the lower layer. The model equations were derived in [Lambaerts et al. \(2011b\)](#) and is an extension of the model described in [Bouchut et al. \(2009\)](#). The lower-layer moisture variable is passively advected until it reaches supersaturation. When moisture reaches supersaturation, precipitation occurs, and the associated latent heat release is modeled as a transfer of mass from the lower layer into the upper layer, thereby affecting the dynamics of the flow. Physically, one can think of this as a conversion of the lower-layer fluid with a relatively small potential temperature into the upper-layer fluid with a larger potential temperature. Since precipitation removes moisture in our model, evaporation in the moisture equation is required in order to achieve statistical stationarity. Evaporation is governed by the bulk aerodynamic formula. [Figure 1](#) shows a schematic representation of the two-layer model with precipitation and evaporation.

To force the model, the interface field separating the two layers is relaxed to a baroclinically unstable profile. This represents a thermal (or radiative) forcing that supplies available potential energy necessary for baroclinic instability. Rayleigh drag in the lower layer

FIG. 1. Schematic of the two-layer model with precipitation and evaporation. Precipitation induces a lowering of the interface field η_2 . Adapted from [Lambaerts et al. \(2012\)](#).

represents surface friction. The equations of motion are given by

$$\frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 + \mathbf{f} \times \mathbf{u}_1 = -g \nabla (h_1 + h_2) - r \mathbf{u}_1, \quad (1)$$

$$\frac{\partial \mathbf{u}_2}{\partial t} + \mathbf{u}_2 \cdot \nabla \mathbf{u}_2 + \mathbf{f} \times \mathbf{u}_2 = -g \nabla (h_1 + s h_2) - \frac{\mathbf{u}_2 - \mathbf{u}_1}{h_2} \alpha P, \quad (2)$$

$$\frac{\partial h_1}{\partial t} + \nabla \cdot (h_1 \mathbf{u}_1) = -\alpha P + \frac{h_1^f(y) - h_1}{\tau_h}, \quad (3)$$

$$\frac{\partial h_2}{\partial t} + \nabla \cdot (h_2 \mathbf{u}_2) = \alpha P + \frac{h_2^f(y) - h_2}{\tau_h}, \quad (4)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (q \mathbf{u}_1) = E - P, \quad (5)$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the lower- and upper-layer horizontal velocities, and h_1 and h_2 are the thicknesses of the lower and upper layers, respectively. The Coriolis parameter is given by $f = f_0 + \beta y$, g is gravitational acceleration, s is the stratification parameter that is related to reduced gravity via $g' = g(s - 1)$, r is the Rayleigh drag inverse time scale, τ_h is the thermal forcing time scale, $h_1^f(y)$ and $h_2^f(y)$ are the lower- and upper-layer thermal forcing profiles. These equations are identical to [Lambaerts et al. \(2012\)](#) except for the rightmost terms in (3) and (4), which represent thermal forcing.

We can also write $h_1 = H_1 - \eta_1 + \eta_2$ and $h_2 = H_2 - \eta_2$ where η_1 is the free surface and η_2 is the interface field. We use equal mean-layer depths so that $H_1 = H_2 \equiv H$. Note that η_1 represents a material surface of potential temperature at the *bottom* of the atmosphere while the

FIG. 2. (left) The η_2 target forcing profile. (center) Zonal thermal wind shear $u_2 - u_1$. (right) Meridional gradient of lower-layer potential vorticity associated with the forcing profile (PV_1^f), with the shaded area showing the region of baroclinic instability.

tropopause is a rigid lid, see Fig. 1. Since $\eta_1 \ll \eta_2$, we will discuss our results primarily in terms of the interface field between the two layers, η_2 . This interface field can be thought of as a surface of constant potential temperature in the midtroposphere.

Moisture is represented by q , which is the specific humidity normalized by the mean lower-layer depth. That is,

$$q(x, y, t) = \frac{1}{H} \int_{\eta_1}^{H+\eta_2} Q(x, y, z, t) dz,$$

where $Q(x, y, z, t)$ is the three-dimensional specific humidity (g kg^{-1}). We note that a similar moisture evolution equation to (5) is obtained by a Galerkin truncation in the vertical of the continuous primitive equations (Frierson et al. 2004). Evaporation follows the bulk aerodynamic formula and is given by

$$E = e_0 |\mathbf{u}_1| (q_s^o - q), \quad (6)$$

where q_s^o is the saturation specific humidity over the surface representing the ocean, and e_0 is a constant of proportionality (related to the product of the drag coefficient and air density). Precipitation occurs when moisture reaches supersaturation and is modeled as a relaxation back toward saturation. That is,

$$P = \frac{q - q_s^a}{\tau} H(q - q_s^a), \quad (7)$$

where $H(\cdot)$ is the Heaviside function, τ is the relaxation time scale of precipitation, and q_s^a is the lower-layer saturation specific humidity.

For most of our simulations, the surface and lower-layer saturation humidities are fixed to the same value. That is, we fix q_s^o in (6) to be equal to q_s^a in (7): $q_s^o = q_s^a \equiv q_s$. This definition neglects the surface to lower-layer temperature difference (i.e., the same value of saturation humidity implies the same temperature

by the Clausius–Clapeyron relation). Further, q_s^o and q_s^a are constant throughout the domain. Because of this and because the prognostic humidity variable, q , [governed by (5)–(7)] does not depend on η_2 , this implies that there are no thermodynamic constraints on where the evaporation and precipitation should occur in our simulations.

The thermal forcing (or target) profiles for the surface and interface fields are given by

$$\eta_1^f(y) = 0 \quad \text{and} \quad \eta_2^f(y) = \Delta\eta \tanh(y/L_j),$$

where $\Delta\eta = f_0 U_0 L_j / g'$ and U_0 and L_j are the speed and width of the jet, respectively. These profiles induce a thermal wind of the form, $u_2 - u_1 = U_0 \text{sech}^2(y/L_j)$. In the literature, this velocity profile is known as a Bickley jet and has been commonly used in both oceanic and atmospheric contexts (Poulin and Flierl 2003; Irwin and Poulin 2014; Lambaerts et al. 2011b, 2012).

Figure 2 shows the thermal forcing profile η_2^f , the thermal wind zonal velocity, and the meridional gradient of lower-layer QG potential vorticity associated with the forcing profile, $PV_1^f = \beta y - f_0 \eta_2^f / H$. The forcing profile η_2^f is lowered equatorward and raised poleward of the jet. This forcing profile induces the thermal wind zonal velocity shown in the center panel, which shows a maximum of 20 m s^{-1} at $y = 0 \text{ km}$ and decays rapidly outside this central latitude.¹ The region of baroclinicity is roughly between $y = -1000$ and $+1000 \text{ km}$ (right panel, shaded region).

The moisture parameter α [see (3) and (4)] is related to the latent heat of vaporization and q_s is the vertically average saturation humidity in the lower layer. However, in order to specify α and q_s , we follow Lambaerts et al. (2012) and use the ratio of the speeds of dry and moist gravity waves as an approximation of the moist to

¹ We will use latitude and y interchangeably.

TABLE 1. Parameter values used in reference simulations.

Parameter	Value
L_x	14 000 km
L_y	7000 km
$H_1 = H_2$	5 km
g	10 m s^{-2}
g'	1 m s^{-2}
f_0	10^{-4} s^{-1}
β	$2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$
U_0	20 m s^{-1}
L_j	1000 km
r	$(10 \text{ days})^{-1}$
τ_h	20 days
α	1000 m kg g^{-1}
q_s	0.6 g kg^{-1}
τ	100 s
e_0	$3.31 \times 10^{-8} \text{ m}^{-1}$

dry static stability ratio. In their study, the ratio of moist to dry static stability is 0.5, implying that moist gravity waves travel at roughly half the speed of dry gravity waves. This speed is controlled by the product αq_s (Lambaerts et al. 2011b). We choose $\alpha q_s = 600 \text{ m s}^{-1}$, so that moist gravity waves travel at approximately 60% of the speed of dry gravity waves. In particular, we take $\alpha = 1000 \text{ m kg g}^{-1}$ and $q_s = 0.6 \text{ g kg}^{-1}$ to be reference values. Note that typical values of q_s near Earth's surface are about 10 g kg^{-1} ; q_s , however, corresponds to a depth average over the lower half of the troposphere and, as such, our choice of 0.6 g kg^{-1} is not unreasonable. Following Lambaerts et al. (2011a, 2012), the relaxation time scale of precipitation is $\tau = 5\Delta t \approx 100 \text{ s}$. To choose a reference value of e_0 , we consider the domain- and time-averaged values of relative humidity and precipitation rates. We find that a value of $e_0 = 3.31 \times 10^{-8} \text{ m}^{-1}$ (which corresponds to an evaporation time scale of 70 days) gives a relative humidity of approximately 80% and a precipitation rate of approximately 3 mm day^{-1} , comparable to Earth's observed values. To systematically examine the effect of changing moisture in the flow, we vary the parameters q_s^o , q_s^a , and e_0 in physically meaningful ways (section 4). Table 1 shows parameter values used in our reference simulations.

b. Numerical methods

To solve the system given in (1)–(5), we developed a finite difference code on an Arakawa C grid (Arakawa and Lamb 1977) with leapfrog time stepping. The system is solved in a zonally periodic channel with walls at the north and south boundaries, where free-slip boundary conditions ($\partial_y u = \partial_y h = \partial_y q = v = 0$, in which ∂_y denotes partial differentiation with respect to y) are applied. All simulations use 512×256 grid points in the

horizontal, giving a resolution of $\Delta x = \Delta y \approx 27 \text{ km}$. For ease of numerical integration, we resolve the fast barotropic gravity waves, thus limiting the size of the time step to $\Delta t = 0.25(2gH)^{1/2} \approx 20 \text{ s}$. However, since these waves are small in amplitude [$\eta_1/(2H) \ll 1$], we will make the rigid lid approximation ($\eta_1 \approx 0$) in our diagnostic calculations. To ensure numerical stability, small-scale dissipation is implemented via a bi-Laplacian hyperviscosity (with a diffusivity coefficient of $A_h = f_0 \Delta x^4 / \pi^4 \approx 5.7 \times 10^{11} \text{ m}^4 \text{ s}^{-1}$) in the momentum equation and Laplacian viscosity in the specific humidity equation (with a diffusivity coefficient of $A_h = 2f_0 \Delta x^2 / \pi^2 \approx 1.5 \times 10^4 \text{ m}^2 \text{ s}^{-1}$).

The specific humidity variable, q , is initialized at 50% of the lower-layer saturation specific humidity q_s^a . All other fields are initialized with small random noise. Simulations are run for 1000 days with time averaging computed over the last 500 days. To test the robustness of our averaging choice, some simulations were run up to 2000 days with different averaging windows (not shown). For example, we averaged over 1000–1500 and 1500–2000 days and found similar values for means and standard deviations, indicating approximate statistical stationarity.

c. Energy and energy budget

In our analysis, we separate the flow into zonal and eddy components using the following notation: $f(x, y, t) = \bar{f}(y, t) + f'(x, y, t)$. We use the dry energy norm for the two-layer shallow-water system as a metric to diagnose the energy of the flow. This is given by

$$E = \left\langle h_1 \frac{|\mathbf{u}_1|^2}{2} + h_2 \frac{|\mathbf{u}_2|^2}{2} + g \frac{(h_1 + h_2)^2}{2} + g' \frac{h_2^2}{2} \right\rangle, \quad (8)$$

where $\langle \dots \rangle = A^{-1} \iint_A \dots dA$ is the domain average.

The energy budget of the system is found by taking the time derivative of (8). One finds

$$\frac{dE}{dt} = \left\langle \text{Radiation} + \text{Drag} - g' \alpha \bar{\eta}_2 \bar{P} - g' \alpha \overline{\eta'_2 P'} - \frac{\alpha P}{2} |\mathbf{u}_2 - \mathbf{u}_1|^2 \right\rangle, \quad (9)$$

with definitions of radiation (or thermal forcing) and drag terms in the appendix [(A1) and (A2), respectively]. The precipitation-related energy tendencies result from a zonal forcing (P_{zonal}), an eddy forcing (P_{eddy}), and the “drag” arising from the momentum transfer associated with the transfer of mass between the layers due to precipitation (P_{drag}):

$$P_{\text{zonal}} \equiv -g' \alpha \bar{\eta}_2 \bar{P}, \quad (10)$$

$$P_{\text{eddy}} \equiv -g' \overline{\alpha \eta_2' P'}, \quad (11)$$

$$P_{\text{drag}} \equiv -\frac{\alpha P}{2} |\mathbf{u}_2 - \mathbf{u}_1|^2.$$

The P_{zonal} and P_{eddy} terms are analogous to the diabatic generation of zonal and eddy available potential energy in the Lorenz energy cycle (Lorenz 1955) [(A3) and (A4), respectively].

While the kinetic energy (KE) in the shallow-water system is cubic, for diagnostic purposes we approximate it as quadratic:

$$\text{KE} = \left\langle h_1 \frac{|\mathbf{u}_1|^2}{2} + h_2 \frac{|\mathbf{u}_2|^2}{2} \right\rangle \approx \left\langle H \frac{|\mathbf{u}_1|^2}{2} + H \frac{|\mathbf{u}_2|^2}{2} \right\rangle.$$

For convenience, we also decompose the velocities into their linear barotropic and baroclinic representations:

$$\mathbf{u}_{\text{bt}} = \frac{\mathbf{u}_1 + \mathbf{u}_2}{\sqrt{2}} \quad \text{and} \quad \mathbf{u}_{\text{bc}} = \frac{\mathbf{u}_2 - \mathbf{u}_1}{\sqrt{2}}.$$

The KE of the flow is then decomposed into the zonal barotropic kinetic energy and barotropic eddy kinetic energy as

$$\overline{\text{BT}} = \left\langle \frac{|\overline{\mathbf{u}}_{\text{bt}}|^2}{2} \right\rangle \quad \text{and} \quad \text{BT}' = \left\langle \frac{|\mathbf{u}'_{\text{bt}}|^2}{2} \right\rangle,$$

where we have normalized by the layer thickness. Similar definitions are also used for zonal and eddy baroclinic kinetic energies (denoted by BC and BC'). These definitions are consistent, for example, with barotropic and baroclinic energy in a quasigeostrophic model and it should be borne in mind that they are different from what one would get had we instead defined $\mathbf{u}_{\text{bt}}$ using the variable layer thicknesses. Zonal and eddy available potential energies ($\overline{\text{APE}}$ and APE' , respectively) are defined as

$$\overline{\text{APE}} = \left\langle \frac{g'}{H} \frac{\overline{\eta_2}^2}{2} \right\rangle \quad \text{and} \quad \text{APE}' = \left\langle \frac{g'}{H} \frac{\eta_2'^2}{2} \right\rangle.$$

3. Results

We first compare a passive simulation (for which $\alpha = 0$, so that precipitation does not feed back on the dynamics) to an active simulation ($\alpha = 1000 \text{ m kg g}^{-1}$) with the reference parameter values stated in Table 1 and described at the end of section 2a.

a. Characteristics of the flow

Figure 3 shows snapshots of upper-layer QG potential vorticity, $\text{PV}_2 = \zeta_2 + \beta y + f_0 \eta_2 / H$ and lower-layer

relative humidity in the passive (left) and active (right) simulations. The snapshots were taken at $t = 1000$ days and are representative of the 500-day averaging period used. In both cases, PV_2 is dominated by a meandering jet centered about $y = 0 \text{ km}$, with the jet structure appearing more regular in the active simulation. In general, lower-layer fields appear more regular with less small-scale features than their upper-layer counterparts. For example, lower-layer QG potential vorticity shows a strong wavenumber-3 pattern (not shown) similar to that evident in the relative humidity and precipitation fields presented in the bottom row of Fig. 3. Additionally, lower-layer eddies are confined mainly to the center latitudes. This is related to the limited latitudinal extent of baroclinicity imposed by the forcing, which drives geostrophic turbulence only between about $y = -1000$ and $+1000 \text{ km}$ (Fig. 2). As eddies move outside of this range, they are dissipated by the bottom drag, so that a confinement of eddies to the center latitudes results. David et al. (2017) find a similar restriction of eddies to the forcing region in the context of a barotropic QG model. Our simulations show this to be a robust behavior, for example, varying the bottom drag or the relaxation time scale of the forcing does not qualitatively change this behavior (not shown).

Figure 4 shows the zonal- and time-averaged meridional profiles of η_2 , P , q , and u in the passive and active simulations (blue and orange lines). The top-left panel shows that, in both simulations, η_2 is qualitatively similar to the target forcing profile with deviations occurring mainly between $y = 0$ and $y = 2000 \text{ km}$. The top-right panel shows that precipitation is confined to a relatively narrow latitudinal band poleward of the jet. The limited latitudinal extent of precipitation and the magnitude of precipitation rates differ from Earth's atmosphere. Nonetheless, a similar local maximum in P is found in Earth's midlatitudes. Comparing the meridional profiles of η_2 and P shows that precipitation occurs where the interface field is raised. This has significant implications for the energetics [see (10)] as discussed in the next section.

The bottom-left panel of Fig. 4 shows relative humidity, q/q_s . There are two local maxima of relative humidity, one at the wall at $y = -L_y/2$ and one that coincides with the latitude of maximum precipitation. Between these maxima, there is a region of subsaturation with relative humidity reaching $\approx 55\%$ equatorward of the jet. This subsaturation gives rise to strong evaporation at these latitudes. The bottom-right panel of Fig. 4 shows the upper- and lower-layer zonal winds. While both simulations show a similar distribution of winds, the active simulation shows a slight poleward

FIG. 3. Snapshots of (top) upper-layer potential vorticity (PV_2) and (bottom) lower-layer relative humidity (q/q_s) at $t = 1000$ days for the (left) passive and (right) active reference simulations. Black contours denote regions of precipitation $P > 0$.

displacement of upper-layer zonal winds u_2 . In addition, the winds on the flanks of the upper-layer jet accelerate relative to the passive simulation. A poleward displacement of the jet stream in response to increased moisture was also found in Frierson et al. (2006) and in response to warming in Yin (2005).

b. Energetics

Table 2 compares time- and domain-averaged KE and APE between our passive and active simulations. Precipitation drastically reduces KE' and APE' . Specifically, APE' is reduced by 37% and the barotropic and baroclinic KE' (BT' and BC') are reduced by 47% and 37%, respectively. Meridional distributions of KE' show a reduction in magnitude at each latitude band, approximately matching the domain-averaged reduction (not shown). This is qualitatively consistent with a similar reduction in KE' observed by Frierson et al. (2006) in GCMs. Zonal energy

components are also reduced, but by a much smaller amount ($<7\%$).

The time-averaged components of the energy budget from (9) are computed in Table 3. The terms related to precipitation forcing in the passive simulation (for which precipitation does not affect the dynamics) are italicized. Precipitation acts as an energy sink, the overwhelming majority of which comes from P_{zonal} [see (10)]. By comparison, P_{drag} and P_{eddy} are an order of magnitude smaller. That P_{zonal} constitutes a sink is consistent with numerical simulations in Pavan et al. (1999) and observational estimates in Romanski and Rossow (2013). In our simulation, the magnitude is surprisingly large, comparable to bottom drag.

Counterintuitively, P_{zonal} , which acts on the $\overline{APE}$ budget, leaves the zonal energy approximately unchanged but drastically reduces eddy energy. How then does the action of P_{zonal} lead to a decrease in eddy energy? To explain this, consider the Lorenz energy cycle

FIG. 4. Zonal- and time-averaged meridional profiles of (top left) η_2 , (top right) P , (bottom left) q/q_s , and (bottom right) upper- and lower-layer zonal winds u for the passive and active reference simulations.

(Lorenz 1955), which describes the conversion pathways between the zonal and eddy components of KE and APE. The conversion pathway of interest is $\overline{\text{APE}} \rightarrow \text{APE}' \rightarrow \text{KE}'$:

$$\overline{\text{APE}} \rightarrow \text{APE}' = -g' \overline{\eta_2 v_2'} \frac{\partial \overline{\eta_2}}{\partial y} \quad \text{and}$$

$$\text{APE}' \rightarrow \text{KE}' = g' \overline{h_2} \eta_2' \nabla \cdot \mathbf{u}_2'.$$

Table A1 shows the domain- and time-averaged values of these conversion terms. Comparing active to passive

TABLE 2. Domain- and time-averaged kinetic and available potential energy ($\text{m}^2 \text{s}^{-2}$) of active and passive simulations. Values in parenthesis indicate standard deviation defined via time variations. Boldface font is used to emphasize changes $>10\%$.

	Passive	Active	% change
BT'	22.1 (0.9)	11.7 (2.0)	-47.1%
BC'	6.8 (0.4)	4.2 (0.6)	-37.3%
APE'	6.8 (0.5)	4.2 (0.7)	-37.3%
BT	28.3 (0.7)	26.5 (1.1)	-6.4%
BC	17.7 (0.2)	17.0 (0.5)	-4.0%
APE	253.8 (0.8)	253.0 (2.3)	-0.3%

reference simulations, there is a decrease of approximately 23% in $\overline{\text{APE}} \rightarrow \text{APE}'$ due to a reduction of $\overline{\eta_2 v_2'}$. This, in turn, leads to a decrease ($\approx 18\%$) in $\text{APE}' \rightarrow \text{KE}'$. A further description of the generation and dissipation terms can be found in the appendix.

A time series of P_{eddy} shows a signal that oscillates between positive and negative values (not shown); however, the time average is a small negative value (i.e., P_{eddy} is a weak energy sink). That P_{eddy} is, overall, an energy sink in our reference simulation is opposite to that in Pavan et al. (1999). However, P_{eddy} does become a clear and significant energy source when the

TABLE 3. Domain and time average of terms in the energy budget ($10^{-2} \text{m}^3 \text{s}^{-3}$). Values in parenthesis indicate standard deviation. Italicized values do not contribute to the energy budget.

	Passive	Active
Radiation	4.7 (0.2)	6.2 (0.4)
Drag	-4.7 (0.2)	-2.9 (0.3)
P_{zonal}	-4.0 (0.5)	-3.0 (0.9)
P_{eddy}	-0.2 (0.2)	-0.2 (0.2)
P_{drag}	-0.3 (0.1)	-0.1 (0.1)

precipitation forcing is strong (section 4). That P_{eddy} is small is related to a poor correlation between P' and η'_2 . Specifically, η'_2 is largest in the central latitudes while P' is largest at higher latitudes. Moreover, in the latitude bands over which the two do both show significant variance, precipitation tends to fall between extrema of η'_2 . That is, P' and η'_2 are out of phase, and this gives a weak correlation and a small P_{eddy} . This out-of-phase nature of P' and η'_2 is further described in section 3c.

c. Why does precipitation occur north of the jet?

To understand why precipitation occurs north of jet, we consider the relationship between divergence and precipitation. Lower-layer convergence implies upward motion. In a more complete model, this upward motion would imply a reduction of q_s and a corresponding increase in relative humidity. In our model, convergence implies an increase in q and, since q_s is constant, a corresponding increase in relative humidity

also results. We also find that lower-layer convergence is related to meridional velocity, with northward velocities implying negative horizontal divergence. Relative humidity, then, increases as fluid parcels move toward the north. It follows that saturation values are reached in the northern portion of the domain (and north of the jet, in particular). This, then, is where the bulk of the precipitation occurs. These dynamics are analogous to those of the “warm conveyor belt,” in which warm, moist air is advected northward and upward. In the Northern Hemisphere, this causes precipitation on the northeastern part of the storm (Chang and Song 2006; Field and Wood 2007).

To examine the link between northward flow and lower-layer convergence, we turn to the ω equation (Hoskins et al. 1978), which computes the vertical velocity (or, equivalently, horizontal divergence) induced by the QG component of the flow. Neglecting thermal forcing and bottom drag, the ω equation for the two-layer shallow-water system is given by

$$\nabla^2 \delta_1^\omega - 2k_d^2 \delta_1^\omega = \frac{f_0}{g'H} \left[\underbrace{\nabla^2 J(\psi_{\text{bt}}, \psi_{\text{bc}})}_{F^{\text{th}}} - \underbrace{J(\psi_{\text{bt}}, \nabla^2 \psi_{\text{bc}}) + J(\psi_{\text{bc}}, \nabla^2 \psi_{\text{bt}})}_{F^{\text{vort}}} - \underbrace{\sqrt{2}\beta \frac{\partial \psi_{\text{bc}}}{\partial x}}_{F^\beta} \right] - \underbrace{\frac{\alpha}{H} \nabla^2 P}_{F^P}, \quad (12)$$

where $k_d = f_0(g'H)^{-1/2}$ is the baroclinic deformation wavenumber, ψ_{bt} is the barotropic streamfunction, ψ_{bc} is the baroclinic streamfunction, and $J(\cdot, \cdot)$ is the Jacobian operator. The streamfunctions are determined using the pressure fields normalized by the Coriolis parameter at the central latitude f_0 . Solving (12) for δ_1^ω and comparing with the δ_1 output by the model shows good agreement, with correlation coefficients of 0.98 and 0.82 for the passive and active simulations, respectively. This indicates that most of the divergence is induced by the geostrophically balanced part of the flow.

We identify the vorticity advection and thermal advection forcing terms as $F^{\text{vort}} = -[J(\psi_{\text{bt}}, \nabla^2 \psi_{\text{bc}}) + J(\psi_{\text{bc}}, \nabla^2 \psi_{\text{bt}})]$ and $F^{\text{th}} = \nabla^2 J(\psi_{\text{bt}}, \psi_{\text{bc}})$, respectively. Using a generalized version of the ω equation, Räisänen (1995) and Stepanyuk et al. (2017) showed that these terms contribute roughly equally in the midtroposphere of Earth's atmosphere.

Since (12) is linear, we can decompose $\delta_1^\omega = \delta_1^{\text{th}} + \delta_1^{\text{vort}} + \delta_1^\beta + \delta_1^P$, to isolate the divergence induced by each forcing term on the right-hand side. For example, the divergence due to thermal advection forcing δ_1^{th} can be calculated by solving

$$\nabla^2 \delta_1^{\text{th}} - 2k_d^2 \delta_1^{\text{th}} = \frac{f_0}{g'H} F^{\text{th}}.$$

Figure 5 shows scatterplots of v_1 against each component of δ_1^ω . To generate this figure, 20 points along the latitude of maximum precipitation are randomly sampled at 15 random outputs over the last 500 days; these patterns are robust. Similar to Räisänen (1995) and Stepanyuk et al. (2017), we find that δ_1^{vort} and δ_1^{th} dominate δ_1^ω . We also find that these terms show the strongest anticorrelation with v_1 . The precipitation component δ_1^P is also anticorrelated with v_1 . By contrast, δ_1^β shows a weak positive correlation with v_1 . However, the sum $\delta_1^{\text{th}} + \delta_1^{\text{vort}}$ dominates δ_1^β , so that, overall, northward flow implies convergence.

This is consistent with classic synoptic meteorology where lower-layer convergence is associated with positive vorticity advection occurring east of troughs where the flow is northward (Martin 2006). This lower-layer convergence implies increases in relative humidity. For a parcel to reach supersaturation, a sufficiently large time-integrated convergence is necessary. This implies precipitation occurs following

FIG. 5. Scatterplot between lower-layer meridional velocity v_1 and the components of lower-layer divergence δ_1^ω computed using the ω equation, (12).

a significant poleward displacement. As such precipitation occurs predominantly on the poleward side of the jet, giving rise to an asymmetric distribution of P about the jet. This conclusion does not depend on the inhomogeneous evaporation distribution that results from the flow dependence of the bulk formula (6), as we have found similar results in simulations with prescribed spatially uniform evaporation rates.

The correlation between lower-layer convergence and meridional velocity also has important implications for P_{eddy} [see (11)]. This correlation, along with the correlation between v_1 and v_2 (>0.9) and the assumption of geostrophy ($v_2 \approx g'/f_0 \partial_x \eta'_2$), implies that $P' \sim \partial_x \eta'_2$. Taken together, these correlations and assumption of geostrophy imply that $\overline{\eta'_2 P'} \sim \overline{\eta'_2 \partial_x \eta'_2} \approx 0$, so that a weak P_{eddy} results.

4. Sensitivity to changes in moisture

In this section, we examine the sensitivity of the energetics with respect to changes in humidity, varying equilibrium levels of q using three methods:

- M1: by fixing e_0 and varying $q_s \equiv q_s^a = q_s^o$. That is, we vary the saturation specific humidity of the ocean surface and lower layer in tandem. Physically, increased q_s represents a global warming scenario in that higher temperatures lead to a higher saturation specific humidity.
- M2: by fixing e_0 and q_s^a and varying q_s^o . Qualitatively, varying q_s^o mimics moving from a desert-like surface toward an ocean-like one.
- M3: by fixing $q_s = q_s^a = q_s^o$ and varying e_0 . These simulations vary the rate of evaporation by varying the drag coefficient in the bulk aerodynamic

formula and are classified by the time scale of evaporation:

$$T_e \equiv \frac{1}{e_0 \times 5 \text{ m s}^{-1}} \approx \frac{1}{e_0 |\mathbf{u}_1|},$$

where $|\mathbf{u}_1| \approx 5 \text{ m s}^{-1}$ is the approximate root-mean-square speed found in the dry simulation.

Larger q_s and q_s^o but shorter T_e correspond to larger equilibrium values of specific humidity, q . In the M2 and M3 suites, larger q also implies larger relative humidity. In the M1 suite, however, this is not the case. Instead, as q increases, relative humidity remains approximately constant, as is typically the case in climate change simulations (e.g., Held and Soden 2006).

Two distinct behaviors emerge. In one, P_{eddy} is negligible and in the other, it becomes a significant energy source. In the regime where P_{eddy} is negligible, increased moisture corresponds to decreased KE' . In the other regime, KE' increases with moisture and P_{eddy} becomes a significant energy source. Also, in this second regime, both the amount and latitudinal extent of precipitation increase. Because of this, we will refer to these two behaviors as the “weak precipitation regime” and “strong precipitation regime.”

The top row of Fig. 6 shows the domain and time average of KE' components for M1 (left), M2 (center), and M3 (right). All plots are organized so that moisture increases from left to right. In the M1 and M2 suites, increased moisture implies decreased KE' , with both BT' and BC' decreasing from left to right; $\overline{\text{KE}}$, by contrast is essentially independent of moisture (not shown but consistent with Table 2). Available potential energy behaves similarly. This same behavior is also evident for a portion of the M3 suite. That is, drier simulations

FIG. 6. (top) Domain- and time-averaged eddy energy components normalized by passive reference simulation. (middle) Zonal- and time-averaged profiles (solid and dotted lines indicate weak and strong regimes, respectively). (bottom) Averaged energy budget terms (P_{eddy} and P_{drag} are omitted in the M1 and M2 panels because they are near zero). The (left) M1, (center) M2, and (right) M3 suites are defined in section 4. In the top and bottom rows, circles and triangles denote weak and strong precipitation regimes, and black markers indicate the active reference simulation. In the bottom row, error bars denote the temporal standard deviation.

(longer T_e) show this behavior, however, the two simulations for which $T_e < 50$ days do not. These two simulations are what we refer to as the strong precipitation regime and all others fall into the weak regime. The two regimes are denoted in the figure using triangles and circles, respectively.

The middle row of Fig. 6 shows the zonal- and time-averaged meridional precipitation profiles with solid and dotted lines corresponding to weak and strong precipitation, respectively. In the weak regime, precipitation is confined between $y = 0$ and 2000 km and has rates increasing from about 10 to 30 mm day^{-1} as q_s and q_s^o are increased and as T_e is decreased. This amplification of the climatological structure of precipitation is consistent with GCM projections of future

climate (Held and Soden 2006). In the strong regime, precipitation becomes wider at $T_e = 25$ days and spans the whole domain at $T_e = 12.5$ days. The latitude at which maximal precipitation occurs also shifts northward, with this shift largest in the $T_e = 12.5$ days simulation. In addition to the poleward shift of precipitation, there is also a corresponding poleward shift of the zonal jet (not shown). A similar poleward shift in the jet stream has also been found in GCM simulations of climate change (e.g., Frierson et al. 2006; Yin 2005).

The bottom row of Fig. 6 shows terms in the energy budget for the three suites with weak and strong regimes denoted by circles and triangles, respectively. The amount of energy input by the thermal forcing

FIG. 7. Lower-layer vorticity for (left) the active reference simulation ($T_e = 70$ days) and (center) the simulation with $T_e = 12.5$ days. Black contours denote regions of precipitation. (right) Lower-layer eddy kinetic energy spectra for the M3 suite of simulations. Solid and dashed lines denote weak and strong precipitation regimes, respectively.

is independent of moisture in the weak precipitation regime. The dissipation mechanism, however, varies. At low q , bottom drag exceeds P_{zonal} . As q is increased, however, P_{zonal} increases and bottom drag decreases. At the highest levels of q considered (within the weak regime), P_{zonal} dominates over bottom drag as the dominant dissipation mechanism. In the strong regime, thermal forcing increases markedly with moisture. The bottom drag sink also increases with moisture, consistent with a stronger flow in the lower layer. Finally, P_{eddy} also shows distinctly different behavior in the two regimes. It is essentially negligible in the weak regime. In the strong regime, however, it increases significantly with moisture. Pavan et al. (1999) found P_{eddy} to be a significant energy source in their moist baroclinic life cycle simulations. This behavior is consistent with our strong precipitation regime but inconsistent with our weak one. However, much of previous literature (described in the introduction) have different sensitivities between initial-value problems, like baroclinic life cycles of Pavan et al. (1999), and boundary value problems, as we perform here. We next examine a case from the strong regime in more detail.

A closer look at a strong precipitation case

Here, we take a closer look at the simulation with $T_e = 12.5$ days to demonstrate some of the interesting behavior of the strong precipitation regime. Figure 7 compares snapshots of the lower-layer vorticity for $T_e = 12.5$ days and our reference simulation ($T_e = 70$ days). The former shows a much more turbulent lower layer compared to the latter (cf. center and left panels). While eddy activity in the reference case is confined between $y = -2000$ and $+2000$ km, for the $T_e = 12.5$ days case, it extends throughout the domain. Moreover, the reference case shows a more regular, large-scale pattern whereas the $T_e = 12.5$ days run shows many more small-

scale features. In particular, there is a significant intensification of lower-layer cyclones, with the strongest of these cyclones intensifying by a factor of 4 relative to the reference simulation.

This intensification is a result of a convergence-precipitation feedback. Specifically, precipitation induces convergence, which can, in turn, lead to further precipitation. In this simulation, the precipitation component of divergence δ_1^P increases by a factor of about 4 relative to the active reference simulation (not shown). This process is suggestive of a transition to diabatic Rossby waves, where diabatically generated PV anomalies are comparable to adiabatic PV anomalies (Snyder and Lindzen 1991; Moore and Montgomery 2004). Diabatic Rossby waves have also been found to be the dominant form of modal instability for sufficiently warm and moist climate states in GCM simulations (O’Gorman et al. 2018).

The right panel of Fig. 7 shows the lower-layer eddy kinetic energy spectra for simulations in the M3 suite. Simulations belonging to the weak and strong precipitation regimes are indicated in solid and dotted lines, respectively. Simulations with weak precipitation have a steep spectrum close to k_h^{-4} (where k_h is the horizontal wavenumber). Conversely, the experiments with strong precipitation show a spectrum that shallows significantly toward k_h^{-3} . That the spectrum shallows implies that smaller scales are significantly energized in the strong precipitation regime.

This case approaches the “strong latent heat release” regime discussed in Lapeyre and Held (2004). In their study of homogeneous turbulence on a β plane, increasing the strength of latent heat release resulted in a transition of the flow from a jet-dominated regime toward a cyclone-dominated one, typical of f -plane turbulence. While the jet is still present in this case, the flow does exhibit a marked increase in cyclone activity.

FIG. 8. (top) Plot of η_2^{eff} for simulations with (left) varying q_s and (right) varying T_e , defined by (13). Note that these profiles are staggered to better illustrate the differences between cases. (bottom) Meridional gradient of lower-layer potential vorticity associated with η_2^{eff} forcing (PV_1) for (left) varying q_s and (right) varying T_e .

5. Effective forcing

For most of parameter space, the energy budget analysis shows that P_{zonal} dominates other moist terms; P_{zonal} is proportional to the product of zonally averaged P and η_2 fields (10). In our simulations, further averaging in time before taking the product did not significantly affect the results (not shown). As a result, it is tempting to interpret the dominant precipitation effect as equivalent to a change in the target profile to which η_2 is relaxed. This begs the question of whether the KE' reduction is primarily due to this change in the target thermal profile.

To investigate this, we define the “effective thermal forcing” by combining the dry thermal forcing profile with $\bar{P}(y)$:

$$\eta_2^{\text{eff}}(y) \equiv \eta_2^f(y) - \alpha \tau_h \bar{P}(y), \quad (13)$$

where $\bar{P}(y)$ is the zonal and time average of P . Since M1 and M2 simulations have similar energetics, effective forcing runs from only the M1 and M3 suites are presented.

The top row of Fig. 8 shows the resulting η_2^{eff} for the cases in the M1 (left) and M3 (right). Note that in each case, η_2^{eff} varies between -2 and $+2$ km, but the profiles are staggered to better visualize the changes between cases. The bottom row shows the meridional gradient of PV_1 associated with the η_2^{eff} target forcing profile. Since the PV_2 gradient of the forcing is positive definite (not shown), negative values in dPV_1/dy indicate where the forcing is baroclinically unstable. Compared to the passive simulation (black dotted), a new region of baroclinicity is created between about $y = 1500$ and 2000 km at the expense of the baroclinicity slightly north of the central latitude. In other words, some portion of the baroclinicity shifts poleward.

FIG. 9. Percentage change in BT' (blue) and BC' (orange) between effective forcing simulations and their active simulation counterparts for the (left) M1 and (right) M3 suites. Weak and strong precipitation simulations are denoted by circles and triangles, respectively.

Figure 9 shows the percentage change in BT' and BC' between η_2^{eff} and their active simulation counterparts. Weak and strong precipitation simulations are denoted by circles and triangles, respectively. In both suites and for most of parameter space, BC' is similar between simulations using η_2^{eff} and active simulations: differences do not exceed 20% and are less than 5% in the drier cases. (More substantial differences, however, are seen in the simulations with $q_s = 0.8 \text{ g kg}^{-1}$ and $T_e \leq 50$ days.) BT' and BC' also show good agreement: $<10\%$ difference for all weak precipitation simulations and $<30\%$ difference in the strong precipitation cases (not shown). BT' , on the other hand, shows larger discrepancies: even in the driest simulations, differences reach values $>10\%$ and generally increase as humidity increases. Strong precipitation simulations show large differences, but this is expected because the definition of η_2^{eff} neglects P_{eddy} .

These results indicate that, for relatively dry simulations in the weak precipitation regime, a dry model can mostly reproduce moist simulation energetics by accounting for baroclinicity changes. Nonetheless that there are discrepancies in the energetics between simulations indicates that there must be other ways in which precipitation interacts with the dynamics. These interactions then indirectly impact the energy budget. For example, active simulations show a skewness toward cyclones that is absent in the effective forcing simulations (Lapeyre and Held 2004; Lambaerts et al. 2011b).

6. Conclusions

In this study, the effect of moisture on the energetics of forced-dissipative equilibria of a baroclinic jet on a

β plane was examined using a two-layer shallow-water model. Moisture was confined to the lower layer, where it was input by evaporation and removed by precipitation, which occurred upon supersaturation. The associated latent heat release was represented by a transfer of mass between the two layers, thus providing a feedback between moisture and the dry dynamics. No latitudinal dependence of evaporation or precipitation was imposed by the model setup. A strong latitudinal dependence did, however, emerge. Unlike in more realistic simulations, where for example sea surface temperature influences evaporation, here the meridional structure was determined solely by the dynamics of the unstable jet.

The main result shows precipitation to act as an energy sink. Specifically, the zonally averaged component of the precipitation effect on the energetics P_{zonal} indirectly acts to reduce eddy kinetic energy. That P_{zonal} is an energy sink is related to the latitudinal structure of precipitation. In particular, we found precipitation to be largely confined to higher latitudes where the interface thickness field is raised. Physically, this corresponds to heating in relatively cool areas. In isolation, this would imply a reduction of available potential energy. However, this reduction is balanced by an increase in thermal forcing. Instead, P_{zonal} reduces the conversion of APE to APE' . This, in turn, reduces the conversion of APE' to KE' , leading to weaker eddies. That moist simulations produce weaker eddies is similar to the results found using more comprehensive models (Frierson et al. 2006). That precipitation occurs preferentially poleward of the jet in our model is related to a correlation between the lower-layer convergence and poleward meridional velocity. This correlation is

primarily a result of positive vorticity advection, which was dominant in an ω -equation analysis of lower-layer divergence. Evaporation occurred preferentially at lower latitudes for similar reasons.

To examine the sensitivity of the system, we varied atmospheric moisture using three methods (M1, M2, and M3, corresponding, respectively, to varying q_s , q_s^o , and T_e). As moisture is increased in the model, the eddy kinetic energy decreased monotonically in the majority of the simulations. A notable exception occurred when the evaporation time scale was short: $T_e < 50$ days. In this regime, the eddy component of the precipitation forcing (latent heating of the warm phase of eddies generates eddy available potential energy) is nonnegligible and energizes the flow. However, we also note that for these simulations, the domain- and time-averaged values of relative humidity and precipitation rates are both quite large. In the majority of experiments performed, direct forcing of eddies by precipitation was found to be negligible. Values of q_s , q_s^o , and T_e (or e_0) exceeding those presented in the sensitivity experiments resulted in a rapid draining of the lower-layer thickness. Using different forms of bottom drag (including quadratic drag and a drag normalized by different powers of lower-layer thickness) did not arrest the draining of the lower-layer thickness for larger values of the variational parameters. This suggests other physical processes missing from the two-layer model, such as a representation of vertical moist convection, are likely important for this regime.

Since P_{zonal} was the dominant moisture forcing in the energy budget, we tested whether including the zonal- and time-averaged component of precipitation $\bar{P}(y)$ into the definition of the thermal forcing would result in dry simulations that showed similar energetics to their moist counterparts. Including $\bar{P}(y)$ into the target profile affects the baroclinicity of the forcing. This served to reduce baroclinic instability near the center latitudes and to add a new unstable region slightly poleward. Simulation with this “effective forcing” showed $\overline{BC}$, $\overline{BT}$, and $\overline{BC}$ to very closely match those in the corresponding active simulations, less than 10% difference for most of parameter space; $\overline{BT}$, by contrast, showed larger ($>10\%$) differences even in the driest simulations. The reasons for this difference did not appear related to direct effects of precipitation on the energetics, but to a more subtle modification of the dynamics. For example, we found eddy barotropization terms and vorticity skewness to be affected by moisture (not shown). A similar skewness effect was also reported by Lapeyre and Held (2004) and Lambaerts et al. (2011a). The partial success of the effective forcing

dry simulations is suggestive of the utility of constructing dry GCM analogs to moist GCMs (Butler et al. 2010; Voigt and Shaw 2016; Li et al. 2019).

In most of our simulations, the lower-layer flow appeared more regular than is typical of the atmosphere. This result was robust to a wide range of parameters such as the drag coefficient, jet forcing width, and the thermal relaxation time scale. We did not, however, consider other factors such as rough terrain or differential heating between land and ocean, which might have served to produce a more turbulent lower-layer flow. Additionally, our truncation to only two layers in the vertical was severe and it may be that increased vertical resolution, such as a continuously stratified model, would lead to a more turbulent flow near the surface. An increased vertical resolution would also better represent the vertical structure of moisture and allow for a more realistic parameterization of precipitation that permits higher baroclinic modes to be excited. Furthermore, the runaway convergence–precipitation feedback would be suppressed since the ascent of diabatically heated parcels would be halted at altitudes with similar environmental temperatures. Therefore, future work includes testing the robustness of our main results in a continuously stratified model.

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APPENDIX

Energy Budget

The full energy budget in (9) includes radiation and drag terms. Here, we describe these terms in detail. First, the radiation term is composed of terms associated with the thermal relaxation:

$$\begin{aligned} \text{Radiation} = & \frac{g'}{\tau_h} \eta_2 (\eta_2^f - \eta_2) - \frac{g}{\tau_h} \eta_1^2 + \frac{1}{2\tau_h} [|\mathbf{u}_1|^2 (h_1^f - h_1) \\ & + |\mathbf{u}_2|^2 (h_2^f - h_2)]. \end{aligned} \quad (\text{A1})$$

The first two terms come directly from the thickness equations, while the third term is associated with the cubic definition of kinetic energy in the shallow-water system. The bottom drag term is given by

$$\text{Drag} = -rh_1 |\mathbf{u}_1|^2. \quad (\text{A2})$$

The precipitation forcing on the energy budget affects both the zonal and eddy available potential energy budgets. The main forcing component P_{zonal} appears in the zonal available potential energy (APE) budget, given by

$$\begin{aligned} \frac{d}{dt} \overline{\text{APE}} = & \left(\underbrace{g' \overline{\eta_2} \partial_y (\overline{h_2} \overline{v_2})}_{\overline{\text{KE} \rightarrow \text{APE}}} - \underbrace{g' \overline{\eta_2} \partial_y (\overline{\eta_2' v_2'})}_{\overline{\text{APE} \rightarrow \text{APE}'}} - \underbrace{g' \overline{\alpha} \overline{\eta_2} \overline{P}}_{P_{\text{zonal}}} \right. \\ & \left. + \underbrace{g' \tau_h^{-1} \overline{\eta_2} (\overline{\eta_2^f} - \overline{\eta_2})}_{Z_{\text{rad}}} \right). \quad (\text{A3}) \end{aligned}$$

The direct forcing of eddies by precipitation P_{eddy} is found in the eddy available potential energy (APE') budget, given by

$$\begin{aligned} \frac{d}{dt} \text{APE}' = & \left(\underbrace{g' \overline{h_2} \overline{\eta_2' \nabla \cdot \mathbf{u}_2'}}_{\text{APE}' \rightarrow \text{KE}'} - \underbrace{g' \overline{\eta_2' v_2'} \partial_y \overline{\eta_2}}_{\overline{\text{APE} \rightarrow \text{APE}'}} - \underbrace{g' \overline{\alpha} \overline{\eta_2' P'}}_{P_{\text{eddy}}} \right. \\ & \left. - \underbrace{g' \tau_h^{-1} \overline{\eta_2' \eta_2}}_{E_{\text{rad}}} \right). \quad (\text{A4}) \end{aligned}$$

To better understand how precipitation affects eddy kinetic energy, Table A1 shows the time and domain average of each component in the above APE budgets for the passive and active reference simulations.

In the APE budget, Z_{rad} and $\overline{\text{KE}} \rightarrow \overline{\text{APE}}$ are energy sources for all simulations. The sum, $Z_{\text{rad}} + \overline{\text{KE}} \rightarrow \overline{\text{APE}}$, is approximately constant between passive and active simulations, though the relative magnitudes of each term change somewhat. On the other hand, $\overline{\text{APE}} \rightarrow \text{APE}'$ is an energy sink. In the active simulations, P_{zonal} provides an additional energy sink. As a result, the magnitude of $\overline{\text{APE}} \rightarrow \text{APE}'$ decreases between the passive to the active simulation by $\approx 30\%$. This implies less APE' production.

In the APE' budget, E_{rad} and $\text{APE}' \rightarrow \text{KE}'$ are energy sinks while $\overline{\text{APE}} \rightarrow \text{APE}'$ is an energy source. The latter term is equal and opposite to the corresponding term in the APE budget; P_{eddy} is negligible in our control simulations but can be a significant energy source in the strong precipitation regime described in section 4. Since APE' is reduced between passive to active simulations (see Table 2), this implies a weaker E_{rad} . As

TABLE A1. Time- and domain-averaged terms of the zonal (upper half of table) and eddy (lower half of table) available potential energy budgets ($10^{-2} \text{ m}^2 \text{ s}^{-3}$). Italicized values do not contribute to the energy budget.

	Passive	Active
$\overline{\text{KE}} \rightarrow \overline{\text{APE}}$	4.5	4.2
$\overline{\text{APE}} \rightarrow \text{APE}'$	-12.8	-9.9
P_{zonal}	-4.0	-3.0
Z_{rad}	8.3	8.7
$\text{APE}' \rightarrow \text{KE}'$	-8.7	-7.1
$\overline{\text{APE}} \rightarrow \text{APE}'$	12.8	9.9
P_{eddy}	-0.2	-0.2
E_{rad}	-4.0	-2.4

stated above, $\overline{\text{APE}} \rightarrow \text{APE}'$ is reduced between the passive to active simulation. Perhaps unsurprisingly, this leads to a similar reduction in $\text{APE}' \rightarrow \text{KE}'$ from the passive to the active simulation leading to weaker KE' .

Hence, the energy pathway from P_{zonal} to a reduction in KE' is that precipitation causes a reduction in $\overline{\text{APE}} \rightarrow \text{APE}'$, which, in turn, causes a reduction in $\text{APE}' \rightarrow \text{KE}'$. This is a “dry” pathway by which the action of P_{zonal} on $\overline{\text{APE}}$ (in active simulations) can decrease KE' , consistent with our finding that dry simulations with effective forcing can emulate the active simulations (section 5).

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