A Budget Equation for the Amplitude of Rossby Wave Packets Based on Finite-Amplitude Local Wave Activity

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ABSTRACT

Recently, the authors proposed a novel diagnostic to quantify the amplitude of Rossby wave packets. This diagnostic extends the local finite-amplitude wave activity (LWA) of N. Nakamura and collaborators to the primitive-equations framework and combines it with a zonal filter to remove the phase dependence. In the present work, this diagnostic is used to investigate the dynamics of upper-tropospheric Rossby wave packets, with a particular focus on distinguishing between conservative dynamics and nonconservative processes. For this purpose, a budget equation for filtered LWA is derived and its utility is tested in a hierarchy of models. Idealized simulations with a barotropic and a dry primitive-equation model confirm the ability of the LWA diagnostic to identify nonconservative local sources or sinks of wave activity. In addition, the LWA budget is applied to forecast data for an episode in which the amplitude of an upper-tropospheric Rossby wave packet was poorly represented. The analysis attributes deficiencies in the Rossby wave packet amplitude to the misrepresentation of diabatic processes and illuminates the importance of the upper-level divergent outflow as a source for the error in the wave packet amplitude.

1. Introduction

The upper-tropospheric flow in the midlatitudes is characterized by the presence of planetary-scale waves, called Rossby waves (Rossby 1940; Rhines 2002). Rossby waves are important for understanding the atmospheric general circulation because they are able to transport energy and momentum across large distances (Charney and Drazin 1961; Hoskins and Karoly 1981). On the synoptic scale (i.e., zonal wavenumbers 6–8), Rossby waves often materialize in the form of Rossby wave packets [RWPs; see Wirth et al. (2018) for a recent review]. This means that their amplitude is large over a sub-planetary-scale region and decays to smaller values farther away. In the midlatitudes, such RWPs typically propagate in the zonal direction along sharp gradients of

potential vorticity (PV; Hoskins 1991; Martius et al. 2010). Embedded in such a RWP is a series of troughs and ridges, which can be seen as a manifestation of positive and negative PV anomalies, respectively, and which are usually associated with specific types of surface weather (Hoskins et al. 1985).

RWPs are interesting for several reasons. For instance, they sometimes act as precursors to extreme weather (see Wirth et al. 2018, and references therein). In addition, it was argued that RWPs play a role for predictability, although the precise mechanisms may vary from case to case and are currently under debate. In one study, long-lasting RWPs in the Northern Hemisphere were associated with above average forecast skill over Europe (Grazzini and Vitart 2015), while in another study a RWP actually transmitted errors committed over North America downstream in the zonal direction, leading to a complete forecast failure over Europe a few days later (Rodwell et al. 2013). Investigations about predictability often focus on the growth of small errors and the desire to pin down physical mechanisms that are responsible for the ensuing upscale error growth. Recently it has been shown that diabatic heating or numerical diffusion are able to significantly modify

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the structure of Rossby waves (Gray et al. 2014; Saffin et al. 2016; Harvey et al. 2018) and the misrepresentation of such processes in operational models may act as a source of forecast errors (Martínez-Alvarado et al. 2016; Baumgart et al. 2019).

Investigating the potential role of RWPs for predictability calls for 1) a diagnostic that appropriately quantifies local RWP amplitude, and 2) a budget equation for this diagnostic that allows one to associate the growth and decay of RWP amplitude to various physical mechanisms. Regarding the first item, we have recently presented a diagnostic for RWPs that is appropriate for the current purpose. Our diagnostic is based on local finite-amplitude wave activity (LWA; Huang and Nakamura 2016) extended to the framework of the primitive equations in isentropic coordinates, combined with a zonal filter to remove its phase dependence (Ghinassi et al. 2018). Due to its finite-amplitude nature, filtered LWA obeys an exact conservation relation even for highly nonlinear perturbations. Its application is, thus, not restricted to small amplitude waves. Furthermore, filtered LWA is local in longitude, and this allows one to track zonally propagating RWPs.

Recently, several PV-based diagnostics have been developed that allow one to quantitatively link the evolution of individual PV anomalies to physical processes (Chagnon et al. 2013; Teubler and Riemer 2016; Saffin et al. 2016). By contrast, our present diagnostics aims to describe the evolution of the amplitude of the whole wave packet rather than individual PV troughs and ridges, thereby providing information related to a larger spatial scale.

As we have shown in Ghinassi et al. (2018), our LWA diagnostic has a similar performance as more conventional methods based on the envelope of the meridional wind (Zimin et al. 2003, 2006; Glatt and Wirth 2014) when applied to small-amplitude plane-wave-like RWPs. At the same time, LWA was found more appropriate to follow individual RWPs into their fully nonlinear stage including cutoff formation and Rossby wave breaking. In addition, for our purpose the formulation of Ghinassi et al. (2018) is preferable to the quasigeostrophic formulation of Huang and Nakamura (2016) because we focus on the dynamics in the tropopause region and we aim to follow RWPs into the subtropics, where standard quasigeostrophic theory must be considered a poor approximation (Nakamura and Solomon 2011, hereafter NS11; Ghinassi et al. 2018).

It is the goal of the present paper to derive a budget equation for the filtered LWA of Ghinassi et al. (2018) and investigate its utility for quantifying processes associated with the growth and decay of RWP amplitude. For this purpose, we consider a hierarchy of models of varying complexity, allowing us to build confidence first

and subsequently show the value of our approach for real cases. We start by considering simulated RWPs in forced-dissipative configurations of two idealized models: a barotropic model on the sphere and a dry primitive-equation model. These simulations will show the ability of our diagnostic to distinguish between conservative RWP dynamics and nonconservative sources and sinks of LWA. In addition, for the three-dimensional flows we will partition the horizontal wind into several contributions following Teubler and Riemer (2016) and Baumgart et al. (2018). This will allow us to associate the impact on the wave amplitude evolution due to the contributions from the near tropopause, tropospheric deep and divergent wind. We will then apply the LWA budget to data from a weather forecast model for a case in which the misrepresentation of diabatic processes was suggested to be responsible for an error in an ensuing RWP (Rodwell et al. 2013). The comparison between the forecast and the respective analysis confirms the hypotheses of Rodwell et al. (2013), thus indicating the value of our LWA diagnostic.

The paper is organized as follows: in section 2 we present the different formulations of barotropic and isentropic LWA as well as their budget equations and our methodology. Sections 3, 4, and 5 then present our results for the barotropic model, the dry primitive-equation model, and the weather forecast model, respectively. Finally, section 6 provides a summary and our conclusions.

2. Theory and methodology

a. LWA in the barotropic model

In the barotropic model on a sphere, absolute vorticity q is given by

$$q(\lambda, \phi, t) = f + \zeta, \quad (1)$$

where λ is longitude, ϕ is latitude, t is time, $f = 2\Omega \sin\phi$ is the Coriolis parameter, Ω is the angular velocity of Earth's rotation,

$$\zeta(\lambda, \phi, t) = \frac{1}{a \cos\phi} \left[\frac{\partial v}{\partial \lambda} - \frac{\partial}{\partial \phi}(u \cos\phi) \right] \quad (2)$$

is relative vorticity, u and v are the zonal and meridional components of the horizontal wind $\mathbf{v} = (u, v)$, and a denotes Earth's radius. The dynamics of the flow is governed by

$$\frac{Dq}{Dt} = N_q, \quad (3)$$

where D/Dt denotes the material derivative and N_q represents material nonconservation of PV. In the barotropic model q plays the role of a potential vorticity, because

it is diagnostically related to the flow (u, v) and furthermore it is materially conserved in case of conservative flow.

Following [Huang and Nakamura \(2016\)](#), LWA is defined as

$$A(\lambda, \phi, t) = \frac{1}{\cos\phi} \left\{ \int_{l_S} [q(\lambda, \phi', t) - Q(\phi, t)] a \cos\phi' d\phi' + \int_{l_N} [Q(\phi, t) - q(\lambda, \phi', t)] a \cos\phi' d\phi' \right\}, \quad (4)$$

where ϕ' in an integration coordinate in the meridional direction; the integration extends over the sections l_N and l_S , which are the arcs along the meridian satisfying the conditions

$$l_N: q \leq Q, \phi' \geq \phi, \quad (5)$$

$$l_S: q \geq Q, \phi' \leq \phi. \quad (6)$$

The variable $Q(\phi, t)$ represents a specific value of PV that at any time is uniquely related to a given latitude ϕ through

$$\iint_{q>Q} dS = \iint_{\phi'>\phi} dS, \quad (7)$$

where $dS = a^2 \cos\phi' d\phi' d\lambda$ is the area element in spherical coordinates. This implies that A is Eulerian in longitude and partly Lagrangian in latitude. LWA is always nonnegative and it quantifies the vigor of Rossby waves in terms of their pseudomomentum (it has units of m s^{-1}). Denoting the meridional displacement of the wavy PV contour $Q(\phi, t)$ from its reference position ϕ by $\Delta\phi(\lambda, \phi, t)$, (4) can be rewritten in a compact form as

$$A(\lambda, \phi, t) = -\frac{1}{\cos\phi} \int_{\phi}^{\phi+\Delta\phi} (q - Q) a \cos\phi' d\phi'; \quad (8)$$

note that $\Delta\phi$ is defined positive northward and can be multivalued in case of multiple crossings of the PV contour along a meridian (a situation that happens when PV contours become highly distorted).

A budget equation for LWA is obtained by differentiating (8) with respect to time and applying the Leibniz rule (a full derivation is given in [appendix A](#)). This yields

$$\frac{\partial A}{\partial t} \cos\phi = T_C + T_Q + T_N, \quad (9)$$

where

$$T_C = \int_{\phi}^{\phi+\Delta\phi} \nabla' \cdot [(q - Q)\mathbf{v}] a \cos\phi' d\phi', \quad (10a)$$

$$T_Q = \int_{\phi}^{\phi+\Delta\phi} \frac{\partial Q}{\partial t} a \cos\phi' d\phi', \quad (10b)$$

$$T_N = - \int_{\phi}^{\phi+\Delta\phi} N_q a \cos\phi' d\phi', \quad (10c)$$

where ∇' is the standard nabla operator in spherical coordinates, with the spatial derivatives computed with respect to λ and ϕ' . The term T_C represents the local rate of change of LWA through conservative dynamics, which includes advective processes and momentum exchange with the mean flow. Under conservative dynamics, T_C can be expressed as the divergence of a flux (e.g., appendix B in [Huang and Nakamura 2016](#)). This implies that LWA is globally conserved on the sphere. The term T_Q arises from the diabatic modification of $Q(\phi, t)$ and represents the nonlocal, nonconservative impact that remote processes may have on A . The term T_N represents nonconservative processes affecting the LWA evolution more locally. Unless specified otherwise, this term will be calculated as a residual from (9). Note that T_C and T_N are truly local only in longitude, since their values on a given point involve an integration in the meridional direction.

b. The primitive-equation version of LWA

According to [Ghinassi et al. \(2018\)](#), LWA in the framework of the primitive equations with potential temperature θ as a vertical coordinate is defined as

$$A(\lambda, \phi, \theta, t) = \frac{1}{\cos\phi} \left\{ \int_{l_S} [q(\lambda, \phi', \theta, t) - Q(\phi, \theta, t)] \sigma a \cos\phi' d\phi' + \int_{l_N} [Q(\phi, \theta, t) - q(\lambda, \phi', \theta, t)] \sigma a \cos\phi' d\phi' \right\}, \quad (11)$$

where now

$$q = \frac{f + \zeta_\theta}{\sigma} \quad (12)$$

is Ertel PV, with $\sigma = -g^{-1}(\partial p / \partial \theta)$ denoting the isentropic layer density and ζ_θ the vertical component of relative vorticity. The latter is defined as in (2), and the subscript θ means that the partial derivatives are performed along isentropic surfaces. The relations (5) and (6) remain valid except that q is now defined as in (12). The relation between a given latitude ϕ and the corresponding PV value $Q(\theta, \phi, t)$ on each isentrope is defined through

$$\iint_{q>Q} dM = \iint_{\phi'>\phi} dM, \quad (13)$$

where $dM = \sigma dS$ is the isentropic layer mass in differential form. In contrast with the barotropic framework,

the horizontal wind along isentropes can now be horizontally divergent, which implies that for a given ϕ the associated value $Q(\theta, \phi, t)$ may be time dependent even for adiabatic flow.

There is an important difference between our formulation of LWA in the primitive equations [(11) and (14)] and the canonical formulation of finite-amplitude wave activity (FAWA) in the earlier work of NS11: we prefer to use latitude ϕ as a coordinate rather than mass-equivalent latitude (as they do). We do this in order to facilitate the budget equation to be derived below, which is the primary focus in our paper. To be sure, for all aspects concerning wave–mean flow interaction, the canonical form of FAWA (NS11) is more appropriate and should be used. The latter in fact is formulated in terms of a set of Q contours that are invariant with time (when the flow is conservative) and involves equivalent latitude as meridional coordinate, resulting in a tighter connection with the underlying physics contained in Kelvin’s circulation theorem.

Using compact notation, A from (11) can be rewritten as

$$A(\lambda, \phi, \theta, t) = -\frac{1}{\cos\phi} \int_{\phi}^{\phi+\Delta\phi} (q - Q) \sigma a \cos\phi' d\phi'. \quad (14)$$

To derive the budget equation, we differentiate (14) with respect to time and obtain

$$\frac{\partial A}{\partial t} \cos\phi = T_C + T_Q + T_N, \quad (15)$$

where

$$T_C = \int_{\phi}^{\phi+\Delta\phi} \nabla' \cdot [(q - Q) \sigma \mathbf{v}] a \cos\phi' d\phi', \quad (16a)$$

$$T_Q = \int_{\phi}^{\phi+\Delta\phi} \frac{\partial Q}{\partial t} \sigma a \cos\phi' d\phi', \quad (16b)$$

and $\mathbf{v} = (u, v)$ now denotes the isentropic horizontal wind. Appendix B provides a full derivation of (15) as well as the explicit form of T_N , which contains the nonconservative terms from the momentum and heat equation. As before, the term T_C represents the local conservative dynamics; note that this term may now include vertical RWP propagation. It contains contributions from both Rossby waves and gravity waves, even though the gravity wave contribution is generally smaller for baroclinic eddies than the Rossby wave contribution (Tung 1986; Methven 2013). The term related to the pseudomomentum exchange through gravity waves involves a time derivative,

preventing T_C in the primitive equations from being expressed in terms of the divergence of a flux. This also implies that LWA in the primitive equation is not exactly globally conserved on the sphere. The term T_Q represents the nonlocal effect due to the rate of change of $Q(\theta, \phi, t)$, which may be due to either conservative or nonconservative processes. Note that this is a main difference compared to the FAWA formulation of NS11, in which Q is invariant with time in the conservative limit. In the framework of the primitive equations in isentropic coordinates, the fact that we evaluate LWA at constant latitude means that its conservation relation (in the zonally averaged sense) is different from the canonical form of the Eliassen–Palm (E-P) relation for the FAWA of NS11. Such differences are explored in appendix C. Finally, the term T_N describes the effect of nonconservative processes, including diabatic heating, radiation, friction and PV mixing (NS11; Huang and Nakamura 2017). In our isentropic analysis we always compute this term as a residual from (15).

c. Zonal filtering

LWA as defined above still contains the full phase information of the underlying waves, with nodes of zero LWA that may arise when a PV contour intersects the associated latitude circle. However, in this work we aim to focus on the role and the budget of entire RWPs; that is, we want to discount the phase information within individual RWPs. For this reason, we apply a zonal filter to the original LWA through convolution with a Hann window (Harris 1978). The full width at half maximum λ_d of the Hann window is allowed to depend on latitude ϕ and is defined through

$$\lambda_d(\phi) = \frac{2\pi a \cos\phi}{s_d(\phi)}, \quad (17)$$

where $s_d(\phi)$ is the so-called dominant wavenumber. The latter is defined as the wavenumber that corresponds to the maximum of the zonal Fourier spectrum of the meridional wind v . In addition we apply a moving average along latitude (with a latitudinal window of 10°) to obtain a smooth transition between the values of s_d at different latitudes. In the following we only consider filtered LWA, which henceforth will be denoted by A for simplicity.

d. Wind partitioning in the LWA budget

In the framework of the primitive equations, we further consider a partitioning of the conservative term into different contributions following Teubler and Riemer (2016)

and Baumgart et al. (2018). This is achieved by partitioning the wind $\mathbf{v}$ in the term T_C in (16a). Owing to the fact that partial differentiation, integration, and filtering are linear operations, this partitioning of $\mathbf{v}$ translates to a corresponding partitioning of T_C . We first apply Helmholtz partitioning to the horizontal wind $\mathbf{v}$ to separate the nondivergent rotational component from the divergent component $\mathbf{v}_{\text{div}}$ (following Lynch 1989). Thereafter, the rotational component is further partitioned through piecewise PV inversion (PPVI; Davis 1992) a contribution from the lower troposphere (called *tropospheric deep*, $\mathbf{v}_{\text{TSD}}$) and a contribution from the upper levels (called *near tropopause*, $\mathbf{v}_{\text{nTP}}$). The separation between the upper and lower troposphere is defined through the 600-hPa pressure level. The spatial domain used for the inversion extends between 11° and 80°N, and vertical boundary conditions for PV inversion are specified in terms of potential temperature anomalies at 875 and 125 hPa. PV anomalies are determined with respect to a time-mean background state $\mathbf{v}_{\text{bg}}$, which is specified as an appropriate time average. Further details can be found in Teubler and Riemer (2016). The full wind $\mathbf{v}$ can thus be written as

$$\mathbf{v} = \mathbf{v}_{\text{bg}} + \mathbf{v}_{\text{nTP}} + \mathbf{v}_{\text{TSD}} + \mathbf{v}_{\text{div}} + \mathbf{v}_{\text{res}}, \quad (18)$$

where $\mathbf{v}_{\text{res}}$ includes the residual that arises from nonlinear effects in the PV inversion operation and from uncertainties related to the boundary conditions. The magnitude of $\mathbf{v}_{\text{res}}$ is generally smaller compared to the magnitude of the other terms in (18) when PPVI is applied to real atmospheric data. In the Portable University Model of the Atmosphere (PUMA) experiment, we found that locally the magnitude of $\mathbf{v}_{\text{res}}$ may be similar to the magnitude of $\mathbf{v}_{\text{TSD}}$ and $\mathbf{v}_{\text{div}}$. However, we did not find any evidence that this residual wind compromises the physical interpretation of our results (see section 4 for further discussion). Therefore, the contribution from $\mathbf{v}_{\text{res}}$ will always be neglected.

We then combine the background term and the near-tropopause term into a modified near-tropopause term $\tilde{\mathbf{v}}_{\text{nTP}} = \mathbf{v}_{\text{nTP}} + \mathbf{v}_{\text{bg}}$ to finally obtain

$$\mathbf{v} = \tilde{\mathbf{v}}_{\text{nTP}} + \mathbf{v}_{\text{TSD}} + \mathbf{v}_{\text{div}}. \quad (19)$$

Substituting this partitioning into the definition of T_C yields

$$T_C = T_{\text{nTP}} + T_{\text{TSD}} + T_{\text{div}}. \quad (20)$$

In the following, the three terms on the right-hand side will be referred to as the near-tropopause, the tropospheric-deep, and the divergent terms, respectively.

e. Computation of LWA and its budget from model data

First, LWA is calculated using the algorithm of Ghinassi et al. (2018). Thereafter, the terms in the budget equations, (9) and (15), are computed as follows. For a given time step n we consider data at time $n - 1$, n , and $n + 1$. The time derivatives are computed using centered differences between time steps $n + 1$ and $n - 1$. We then evaluate the integrals for T_C and T_Q at all three time steps $n - 1$, n , and $n + 1$ using the algorithms described in section 2 of Ghinassi et al. (2018). Subsequently, we apply some temporal filtering to the term T_C as follows:

$$\tilde{T}_C^{(n)} = \frac{1}{4} [T_C^{(n-1)} + 2T_C^{(n)} + T_C^{(n+1)}], \quad (21)$$

where $T_C^{(n)}$ denotes the term at the discrete time step n . This is done to make the instantaneous quantity consistent with the effective averaging due to the finite-difference time derivatives in $\partial A / \partial t$ and T_Q . Finally, the term T_N is computed as a residual unless stated otherwise. Once all terms have been obtained, the zonal filter is applied to all of them to remove their phase information.

3. LWA budget in the barotropic model simulation

This section aims to test our diagnostic framework in a barotropic model simulation to identify local sources and sinks of LWA. The interpretation of these results is rather straightforward and, therefore, a helpful step toward the situation in more complex three-dimensional flows, which will be the subject of the two subsequent sections.

a. Model setup

We consider a forced dissipative model configuration by specifying the nonconservative term in (3) as follows:

$$N_q = F - \alpha(q - q_0) - \nu \nabla^4 (q - q_0). \quad (22)$$

Here, q_0 denotes the zonally symmetric initial PV and F represents pseudo-orographic forcing. The latter is defined such as to provide a source of LWA that is both local and transient,

$$F(\lambda, \phi, t) = -\nabla \cdot [\mathbf{v}_0 f \Gamma(\lambda, \phi) \sin^2 \Omega_f t], \quad (23)$$

where $\mathbf{v}_0$ is the horizontal wind at initial time and $\Gamma(\lambda, \phi)$ is a nondimensional Gaussian mountain. The mountain has a maximum height of 0.15, is centered at 30°N, 30°E, and has an elliptic base with a standard deviation of

7.5° and 30° in the zonal and meridional directions, respectively. The period of the forcing is set to be 24 days by choosing $\Omega_f = \pi/24 \text{ days}^{-1}$. The interaction of the flow with the orography gives rise to Rossby waves that subsequently propagate eastward in the sense of the group velocity. Therefore, in this setup orography has to manifest as a source of LWA. The second term on the right-hand side of (22) represents linear relaxation toward the zonally symmetric initial state. In our simulations we chose $\alpha = (15 \text{ days})^{-1}$. The third term on the right-hand side of (22) represents hyperdiffusion, which acts to dissipate enstrophy near the grid scale. The coefficient of hyperviscosity is set to $\nu = 10^{15} \text{ m}^4 \text{ s}^{-1}$. We expect both the linear relaxation and the hyperdiffusion to act as a sink of LWA.

As an initial condition, we specify a purely zonal background flow $u_0(\phi)$ following Held and Phillipps (1987),

$$u_0(\phi) = A \cos \phi - B \cos^3 \phi + C \sin^2 \phi \cos^6 \phi, \quad (24)$$

with $A = 25$, $B = 30$, and $C = 300 \text{ m s}^{-1}$. This profile u_0 consists of a barotropically stable westerly jet at $\phi = \pm 30^\circ$ and easterly winds at the equator and in the deep tropics. The wind profile is associated with substantial meridional gradients of q_0 around $\phi = \pm 30^\circ$, which allows RWP to propagate downstream of the forcing.

For the numerical simulations we use a standard pseudospectral method with triangular truncation at T89 and a leapfrog time-stepping scheme in combination with a Robert–Asselin–Williams filter (Williams 2011) to damp the growth of the computational mode. The corresponding physical grid has a resolution of $2^\circ \times 2^\circ$ in longitude and latitude. The model is integrated with a time step $\Delta t = 900 \text{ s}$ for 24 days, which corresponds to one period of the pulsation of our forcing.

LWA and the corresponding terms in the LWA budget equation are computed globally using 6-hourly model output of q and (u, v) . This is done in order to simulate the availability of model data from more complex models or reanalyses, which usually do not provide output every model time-step, but typically only every few hours.

b. Results

At first, we consider day 10, at which the orographic forcing is close to its maximum and, therefore, likely to be stronger than the dissipation. The potential vorticity field at this time (Fig. 1a) shows an area of waviness located above the forcing region and extending downstream, and it is in these regions that we find elevated values of A (Fig. 1b). Over these regions, $(\partial A / \partial t) \cos \phi$ is positive, which means that LWA increases (Fig. 1c). The

conservative term T_C at day 10 (Fig. 1d) is negative over the forcing region and positive more downstream, suggesting that LWA is radiated away from the orographic source. This happens especially in the southeast direction, although a small positive region is also found northeast of the forcing. We believe that one of the factors related to such asymmetry in the meridional component of RWP radiation is the spherical geometry, which profoundly affects the horizontal propagation of the waves (Hoskins and Karoly 1981; Held 1983; Orlanski 2003). There is a large region above the orographic forcing with positive values of the nonconservative term T_N , which confirms its role as a source of LWA, as expected. A region in which the local nonconservative term is negative is found farther downstream and equatorward of the orographic forcing, and it is presumably associated with damping of LWA due to friction and diffusion. The contribution from T_Q is not shown in these maps, since its magnitude is much smaller compared to the other budget terms (see Fig. 2 below).

As indicated in the previous section, we generally computed the nonconservative term T_N as a residual. In the barotropic simulation, we alternatively computed T_N using the explicit formula (10c), with N_q defined according to (22). Comparing the result (Figs. 1f,i) with the corresponding computation from the residual (Figs. 1e,k) shows broad agreement both in terms of the overall shape and in the amplitude of the respective fields. This indicates that the numerical error incurred due to the finite differencing in computing (9) together with numerical diffusion introduced by the time filter are noticeable, but small.

At day 20, the orographic forcing is close to zero, and at this time we expect nonconservative processes to dominate and act as a sink of LWA. Figure 1k shows that this is, indeed, the case: T_N is negative everywhere. LWA itself (Fig. 1h) has continued an overall eastward shift, and we can identify three wave packets: The first lies poleward of the orography (north of 65°N) and is well extended in longitude, and we will refer to it as the *poleward RWP*. The other two wave packets are more longitudinally confined and located on the equatorward flank of the jet (the location of jet maximum is at around 30°N); we will refer to them as the *first equatorward RWP* (at around 20°N , 60° – 120°E) and the *second equatorward RWP* (at 15°N , 120°E – 160°W). The values of $(\partial A / \partial t) \cos \phi$ at this time are negative in a large portion of the domain (Fig. 1i), implying an overall decrease of LWA in the absence of forcing. The poleward wave packet is almost stationary, consistent with the fact that only a small (compared to the wave packet extension) region of negative LWA tendency is found on the rear of the RWP. The first equatorward wave packet is decaying

FIG. 1. Analysis of the barotropic model simulation after (left) 10 and (right) 20 days of integration. (a),(g) Potential vorticity, (b),(h) filtered LWA A , (c),(i) $(\partial A/\partial t)\cos\phi$, (d),(j) conservative term T_C , (e),(k) local non-conservative term T_N computed as a residual, and (f),(l) T_N computed by meridionally integrating N_q from (22). Color bar and units for (c)–(e) and (i)–(l) are the same. The gray shading in (a) represents the region in which the amplitude of the pseudo-orographic forcing exceeds 0.075. The rectangles in (b) and (h) mark the area used later for spatial averaging.

FIG. 2. Time evolution of the LWA budget terms in the barotropic simulation averaged over a finite region that includes the orography: $(\partial A/\partial t)\cos\phi$ (red line), conservative term T_C (blue line), local nonconservative term T_N estimated a residual (green line) and computed by meridionally integrating N_q from (22) (dashed green line), and nonlocal nonconservative term T_Q (dashed black line).

$[(\partial A/\partial t)\cos\phi]$ is negative here], whereas the dipole of positive/negative $(\partial A/\partial t)\cos\phi$ associated with the second equatorward wave packet suggests that it is propagating eastward along the zonal direction. The conservative term T_C (Fig. 1j) shows, at low latitudes, areas of positive and negative values in close proximity to each other, indicating RWP propagation in a southerly or southwesterly direction.

Figure 1 clearly indicates that the nonconservative term T_N may be either predominantly positive (day 10) or negative (day 20), and in our experiment this depends on whether the weak dissipation is overwhelmed by strong orographic forcing (day 10) or not (day 20). By contrast, the conservative term T_C shows approximately as much positive as negative values. In fact, this is to be expected since finite-amplitude wave activity is globally conserved for purely conservative conditions in the barotropic model (Nakamura and Zhu 2010; Solomon and Nakamura 2012). This means that conservative processes essentially act to redistribute LWA across the surface of the sphere without affecting globally integrated LWA.

To further analyze the LWA budget, Fig. 2 shows the evolution of spatially averaged terms from (9). Here, spatial averaging is performed over a finite region delimited by 60° – 10° N, 20° – 120° E (marked in Figs. 1b and h), which contains the orographic forcing. During the first 16 days the nonconservative term T_N is positive implying that orographic LWA generation dominates over LWA dissipation. The averaged conservative term is zero until approximately day 7, suggesting that the

LWA created by the forcing is still confined within the region used for averaging. Thereafter, this term becomes negative, which means that LWA is radiated away leading to a net export from the considered region. After day 16 the nonconservative term T_N becomes negative, which means that dissipation of LWA starts to dominate over forcing. The term T_Q is very small and always negative (dashed line in Fig. 2). The two methods to compute the nonconservative term T_N in Fig. 2 indicates, again, small differences; in fact, one can interpret the difference between the solid and the dashed green line as a broad estimate for the uncertainty related with the computation of the budget terms and numerical diffusion. This suggests that the main features discussed above are significant and not a result of numerical errors and demonstrate the possibility of quantifying nonconservative sources and sinks of LWA using (9).

4. LWA budget in a dry general circulation model

In this section, we apply the primitive-equation version of the LWA budget to simulations of idealized RWP life cycles in a dry general circulation model. These simulations allow us to study the full three-dimensional flow in a quasi-realistic midlatitude setup, including the partitioning of the conservative term into the different contributions as described in (20).

a. Model setup and simulation

For our simulations, we use the so-called PUMA (Fraedrich et al. 2005). PUMA is a spectral model that solves the primitive equations on the sphere. Our experiments include neither moisture nor orography. All tendencies from physical processes are parameterized as linear relaxation like in Held and Suarez (1994). This corresponds to linear friction in the momentum equation and Newtonian cooling in the heat equation, respectively. The reference temperature for the Newtonian cooling term includes a local dipole in the Northern Hemisphere, superimposed on an otherwise zonally symmetric profile similar as in Frisius et al. (1998) and Fraedrich et al. (2005). This dipole locally increases cooling at higher latitudes and warming at lower latitudes. The strength of the dipole is largest at lower levels and decreases to zero in the upper troposphere and above. The precise formulation of the reference temperature field for the Newtonian cooling is provided in appendix D. This device creates a local region of enhanced low-level baroclinicity, resulting in downstream eddy development like in an idealized storm-track experiment (Chang and Orlanski 1993; Frisius et al. 1998). What we expect to see is the growth and the upward and eastward propagation of baroclinic

eddies originating from the low-level dipole region, and this should manifest itself as a source of LWA in the upper troposphere through the conservative term T_C . Newtonian cooling, which implies a direct diabatic modification of PV, may act either as a source or as a sink of LWA on the instantaneous flow field; however, we expect such linear relaxation to overall damp LWA. Similarly, friction and hyperdiffusion are expected to always damp LWA as for the barotropic case.

PUMA is run for 2 years under perpetual equinox conditions (no seasonal or diurnal cycles) with spectral resolution T42, with 20 vertical levels and with a time step of 30 min. Friction is applied only to the four lowest levels. The relaxation time scale for Newtonian cooling is set to 4 days near the surface increasing to 40 days in the upper troposphere and above. Every six hours the model output is interpolated onto a regular $2^\circ \times 2^\circ$ (longitude $\times$ latitude) grid on equispaced pressure levels ranging from 850 to 100 hPa. The first year of the simulation is discarded to eliminate the model spin up, and our analysis is performed only on the second year. LWA is computed only for the Northern Hemisphere.

b. Results

An animated sequence of snapshots of PV in the upper troposphere (not shown) reveals that our simulation, indeed, produces RWPs that originate over the dipole region, then grow while propagating downstream, and eventually decay, which is often associated with wave breaking. In our PUMA simulation we have less control over the generation of RWPs than in the previous barotropic model simulation. In the latter, in fact, RWPs were directly generated by the pseudo-orographic forcing term. By contrast, in the PUMA simulation the opposing tendencies from baroclinic instability and general dissipation result in a chaotic transient flow, and it is not a priori clear when exactly this leads to LWA generation in the upper troposphere. This fact motivates us to consider the time-averaged behavior first and thereafter to analyze episodes of transient RWPs.

The 1-yr climatologies of relevant quantities on the 315-K isentropic surface are shown in Fig. 3. This particular isentropic surface is chosen since it intersects the tropopause in the midlatitudes, where a strong isentropic PV gradient is found. Time-averaged PV (Fig. 3a) shows that the isentropic PV gradient is larger above the heating dipole (associated with stronger westerlies in the upper troposphere, not shown), whereas downstream the PV gradient appears weaker. Time-averaged LWA (Fig. 3b) features a longitudinally extended maximum in the midlatitudes downstream of the heating dipole. This maximum arises from the transient Rossby wave activity

FIG. 3. Time-averaged variables at the 315-K isentropic surface from the PUMA model simulation: (a) PV (potential vorticity units, $1 \text{ PVU} = 10^{-6} \text{ K kg}^{-1} \text{ m}^2 \text{ s}^{-1}$), (b) filtered LWA, (c) conservative term T_C , and (d) local nonconservative term T_N . The letters C and W in (a) mark the centers of the cold and warm anomalies in the relaxation temperature field. The rectangular box in (b) marks the area for the spatial averaging used in Fig. 4. Units and color bar are the same for (c) and (d).

associated with RWPs in this region. Another secondary maximum of LWA is found to the south of the warm anomaly; this confined pocket of LWA appears to be associated with very low values of PV and is arguably a result of the diabatic heating happening below.

Regarding the time-mean LWA budget, we first note that the tendency term $(\partial A / \partial t) \cos \phi$ as well as T_Q are very small (not shown) as expected for a climatology. The conservative term T_C (Fig. 3c) is positive throughout the domain shown, most notably over the dipole region and immediately downstream. This is in stark contrast with the barotropic model simulation above, where we saw positive and negative areas of similar size

and magnitude. The key difference between these two model configurations is their dimensionality. The barotropic model describes two-dimensional horizontal flow, and the conservative term only redistributes wave activity with zero net source or sink upon horizontal averaging. Similarly, the conservative term can be expected to merely redistribute wave activity in the PUMA simulation; however, this process now includes also the vertical dimension. This implies that the conservative term at one particular isentropic level spanning the upper troposphere may well be positive everywhere throughout the domain, as it is the case in our simulation. The conservative LWA generation is balanced by nonconservative term T_N (Fig. 3c). This is plausible because the term T_N includes linear relaxation of temperature toward a zonally symmetric state, which corresponds to a tendency to suppress waviness.

We now shift our attention to the transient RWPs that underlies the climatological mean picture. Similar as for the barotropic model, we produced time series by averaging all terms in the LWA budget over a finite box (shown in Fig. 3b) that contains the region of enhanced transient Rossby wave activity. A first look at the time series (not shown) reveals that the increase of LWA $[(\partial A/\partial t)\cos\phi]$ tends to occur in burst-like events, followed by a rapid decay of the growth rate. This behavior is likely to be an upper-tropospheric signature of periodic bursts of baroclinic eddy activity, which is observed both in idealized simulations and in real data of storm tracks (Novak et al. 2017; Ambaum and Novak 2014). To obtain an average picture of such events, we now consider a composite average by selecting those maxima of $(\partial A/\partial t)\cos\phi$ that exceeded $10 \text{ m s}^{-1} \text{ day}^{-1}$ and then taking a time window of ± 5 days centered over the respective peaks. With this criterion, a total of 46 episodes of LWA growth and decay that occurred during our one year integration are examined. The resulting composite (Fig. 4a) shows that the growth rate of LWA during these transient events indeed occurs with peaks of positive values of $(\partial A/\partial t)\cos\phi$ followed by a rapid decrease; $(\partial A/\partial t)\cos\phi$ then becomes negative about 1 day after the maximum in the composite. These episodes of LWA growth are accompanied by positive values of the conservative term T_C , consistent with our earlier interpretation that the growth on the chosen upper-tropospheric isentrope is essentially due to conservative upward propagation of LWA. The nonconservative term T_N is always negative and indicates a sharp decrease immediately after the peaks in $(\partial A/\partial t)\cos\phi$ and T_C . This suggests that the LWA damping through linear relaxation and dissipation increases to counteract the conservative growth of LWA shortly after the burst, eventually leading to a negative tendency of $(\partial A/\partial t)\cos\phi$.

FIG. 4. Composite mean of the time series of the terms from the LWA budget equation [(15)], averaged over the storm-track region (indicated by the box in Fig. 3b) on the 315-K isentrope in the PUMA simulation. (a) $(\partial A/\partial t)\cos\phi$ (red line), conservative term T_C (blue line), local nonconservative term T_N (green line), and nonlocal nonconservative term T_Q (dashed black line). (b) As in (a), but for T_C and its partitioning into near-tropopause (T_{NTP} ; blue dots), tropospheric-deep (T_{TSd} ; triangles) and divergent (T_{div} ; plus signs) contributions. Both composites are centered on the peaks of $(\partial A/\partial t)\cos\phi$; see text for discussion.

The term T_Q , which is related to the nonconservative modification of $Q(\phi, t)$, is one order of magnitude smaller compared to the other terms; it decreases during the peaks of $(\partial A/\partial t)\cos\phi$ and it recovers immediately afterward.

Figure 4b disentangles the different contributions to T_C for the composite event as detailed in (20). The background PV state for the PPVI for the PUMA experiment is defined as the annual mean. Figure 4b shows that in the time mean the conservative dynamics LWA tendency is largely associated with the near-tropopause wind contribution T_{NTP} , which also includes the contribution from $\mathbf{v}_{bg}$. The tropospheric-deep wind contribution T_{TSd} is always positive, but has a lower magnitude compared to T_{NTP} . This is consistent with the fact that

the influence of PV and potential temperature perturbations near the surface is lower compared to the PV anomalies located into the mid- and upper troposphere. The magnitude of T_{Tsd} slightly increases about 1 day after the peaks of T_C . This growth input from the lower levels is likely to be associated with the positive feedback of upper-level PV anomalies interacting with the lower PV and temperature anomalies during baroclinic instability (Hoskins et al. 1985; Methven et al. 2005). The T_{div} contribution is always negative and acts to decrease the wave packets amplitude, in particular after the T_C peak. We also investigated the contribution to the conservative dynamics of LWA that would arise from $\mathbf{v}_{\text{res}}$ and found that it has a magnitude comparable to T_{Tsd} and T_{div} only in the vicinity of the dipole and decreases to smaller values farther away (not shown). Therefore, such neglected contribution would only marginally affect the LWA dynamics over the region where the averaging is performed, which is located mostly downstream of the dipole.

We now apply our diagnostic to a snapshot of the flow containing a large RWP (see Fig. 5), in order to obtain insight into its dynamics at a particular time, which is the main scope of the present work. The map of filtered LWA (Fig. 5b) shows a major RWPs in the vicinity of the dipole (approximately at 40°N, 50°–100°E) associated with a PV trough and a ridge located immediately to the east (Fig. 5a). This RWP will be referred to as the first RWP in the following. Another region of waviness is located farther downstream (approximately between 120° and 190°E), where the flow looks more nonlinear and resembles wave breaking (see PV map in Fig. 5a). Compared to the first RWP, this second RWP is associated with smaller values of LWA, while they are distributed over a larger region. The tendency $(\partial A/\partial t)\cos\phi$ associated with the first RWP is negative at the trailing edge and positive at the leading edge, suggesting eastward propagation. Another region of positive $(\partial A/\partial t)\cos\phi$ is found in proximity of the large amplitude ridge belonging to the second RWP, implying an amplitude increase of this structure. The nonconservative term T_N is mainly negative over the considered domain, especially where the first RWPs is located. This is consistent with Newtonian cooling acting as a sink of wave amplitude. In the second RWP the contribution from T_N is very small.

The partitioning of T_C into the different contributions (Fig. 6) emphasizes again the large differences between the two RWPs considered. In the region of the first RWP T_C is almost entirely related to its tropospheric-deep contribution T_{Tsd} (Fig. 6c), and this is consistent with the RWP growth due to baroclinic instability. On the other hand, the near-tropopause contribution T_{nTP} is

FIG. 5. Snapshots of relevant variables on the 315-K isentropic in the PUMA simulation after 28 days of integration. (a) Potential vorticity (PVU), (b) filtered LWA A , (c) observed tendency term $(\partial A/\partial t)\cos\phi$, (d) conservative term T_C , and (e) local non-conservative term T_N . The letters C and W in (a) mark the centers of the cold and warm anomalies, respectively, in the relaxation temperature field.

slightly negative in the vicinity of the first RWP and positive farther downstream (Fig. 6b). This suggests that part of the LWA from the first RWP is transferred to the second region of waviness through near-tropopause dynamics, as one would expect from the downstream development of LWA associated with the first RWP. Last, the divergent contribution T_{div} (Fig. 6d) is very small, except in the neighborhood of the dipole of the

FIG. 6. Partitioning of the conservative term in the LWA budget at day 28 on the 315-K isentropic in the PUMA simulation. (a) Full conservative term T_C , (b) contribution T_{nTP} , (c) contribution T_{Tsd} , and (d) contribution T_{div} . Here only the area between 25° and 80°N is shown to exclude boundary effects related to the PPVI, which usually are larger close to the southern boundary (located at 11°N). Units and color bar are the same for all plots.

diabatic forcing, where its contribution is negative. We consider this to be consistent with the fact that vertical motion associated with diabatic heating and cooling maximizes close to the heating dipole. These vertical motions translate into horizontal divergence of the isentropic wind. In the present case, the effect of the divergence contribution is to reduce the overall amplitude of the first RWP. As we will see in the next section, the divergent contribution in our PUMA simulation is much weaker than in the real atmosphere, and we relate this to the lack of moist processes in our dry primitive-equation model.

5. LWA budget in the operational weather forecast model

We now move on and apply our diagnostic to real meteorological data analyzing the propagation of a

particular RWP that occurred in April 2011. The RWP propagated from North America toward Europe, where it produced wave breaking and eventually evolved into blocking. This episode was associated with below average forecast skill in the medium range over Europe in most numerical weather prediction models (Rodwell et al. 2013). Our analysis uses data from the integrated forecasting system of the European Centre for Medium-Range Weather Forecasts (ECMWF). We compare the ECMWF forecasts issued at 0000 UTC 10 April 2011 with the respective verifying analyses to investigate the difference between the two in terms of LWA and its evolution. We retrieved 6-hourly data for both the forecasts and the analyses on pressure levels between 100 and 850 hPa with a grid resolution of 1° for the Northern Hemisphere. LWA and the related budget terms (including the partitioning of the conservative term) were computed as described in section 2. Here, we focus only on the origin of the error during the first 48 h of lead time. Later, the RWP produced wave breaking and this led to a complete forecast failure at around 7 days of lead time; however, this large amplitude stage of the RWP will not be analyzed in the present work. During the first 48 h a cyclone was found over North America, associated with heavy precipitation over a region spanning from the Midwest toward the U.S. East Coast. We focus on this time window to see how the diabatic modification of PV associated with moist processes manifests itself in the nonconservation of LWA.

An overview over the selected episode is given in Fig. 7 showing a sequence of maps of upper-tropospheric PV on the 325-K and mean sea level pressure. At the initial time (Fig. 7a) the upper-level flow consisted of a trough located over the Rocky Mountains and a ridge downstream over the U.S. Great Plains. This trough–ridge couplet was part of a more extended RWP. At the surface, there was a cyclone developing immediately to the east of the upper-level trough. Subsequently, the upper-tropospheric trough and the ridge moved eastward and became more tilted, and the surface cyclone deepened and moved in a northeastward direction (Figs. 7b,c). During the first 24 h some appreciable differences are already visible when comparing the forecast and the analysis, especially regarding the magnitude of PV in the upper-tropospheric wave packet. These differences in the upper-level PV field become more pronounced after 48-h lead time. In particular, the PV trough located at 90°W appears broader and less filamented in the forecast than in the verifying analysis. Another region of interest is the one located around 55°N, 75°W: here the longitudinal extent of the ridge is broader in the forecast, and higher values of PV compared to the analysis are found in the trough immediately above the

FIG. 7. Evolution of the flow in the upper troposphere and at the surface during an episode in April 2011 (a)–(c) in the ECMWF model forecast issued at 0000 UTC 10 Apr 2011 and (d)–(f) in the corresponding analyses. The color shows PV on the 325-K isentrope (PVU) and the black contours show the 990-, 995-, and 1000-hPa isobars of surface pressure.

dynamical tropopause (cf. Figs. 7c,f). The fact that these upper-level differences are found right above the surface cyclone suggests that an erroneous diabatic modification of the upper-level PV (associated with the physical processes within the cyclone) may have occurred in the forecast.

The same episode is shown from an LWA perspective in Fig. 8, which provides the forecast fields (left column), the verifying analyses (center column), as well as the difference between the two (right column). It can be seen how LWA identifies two separate RWPs, the one from the previous figure located over the North American continent, as well as an additional one over Europe. While the latter seems to be well represented in the forecast during the first 24 h, the RWP over North America shows an appreciable error in terms of LWA as early as 24 h into the forecast. In particular, the RWP amplitude is too strong in the forecast immediately to the south of the Great Lakes region (Fig. 8h). After 48 h, this amplitude error persists and shifts toward the North Atlantic, and this eastward shift seems to be associated with the eastward propagation of the RWP. At the same time, the forecast LWA is too weak on the northern flank of the RWP (see the band of negative LWA

difference between the United States and Canada in Figs. 8h and 8i).

The terms from the LWA budget, averaged over the first 48-h lead time, are shown in Fig. 9 both for the forecast (left column) and the verifying analysis (right column). The corresponding forecast errors, computed as the difference “forecast minus analysis,” are shown in Fig. 10. What emerges from this comparison is that the conservative term T_C is more negative in the forecast, especially at the trailing edge of the wave packet (cf. Figs. 9b,e and Fig. 10b). The nonconservative term T_N is positive in both the forecast and the analysis, which means that the overall contribution from nonconservative processes in this case was to increase the RWP amplitude. However, the magnitude of T_N is larger in the forecast. The net effect of the conservative and nonconservative terms corresponds to an overestimation of the forecast LWA tendency over the U.S. East Coast (positive values in Fig. 10a).

Finally, we turn to the partitioning of T_C into the different contributions (Fig. 11). As background state for the PPVI we chose a 30-day mean centered on 13 April from the analysis for both the forecast and the analysis. The near-tropopause contribution T_{nTP}

FIG. 8. Evolution of filtered LWA on the 325-K isentropes for the same episode as shown in Fig. 7: (a)–(c) ECMWF model forecast, (d)–(f) verifying analysis, and (g)–(i) the difference (forecast minus analysis), for the first 2 days into the forecast.

recovers a large part of the full term (cf. Fig. 11a with Fig. 9b). This result is quite similar as in our earlier PUMA simulation (see the respective panels in Fig. 9). The deep-troposphere contribution T_{TSd} (Fig. 11b) is

positive both in the forecast and the analysis. This is consistent with the growth of the RWP through baroclinic instability, which in turn is related to a strong horizontal temperature gradient over North America

FIG. 9. Terms of the LWA budget on the 325-K isentropes, averaged between 10 and 12 Apr 2011, for (a)–(c) the ECMWF model forecast and (d)–(f) the corresponding verifying analysis. (a), (d) Tendency term $(\partial A/\partial t) \cos \phi$, (b), (e) conservative term T_C , and (c), (f) local nonconservative term T_N .

FIG. 10. Error (forecast minus analysis) in the LWA budget terms from Fig. 9: (a) tendency term $(\partial A/\partial t)\cos\phi$, (b) conservative term T_C , and (c) local nonconservative term T_N . Contours for all plots are between -10^{-4} and 10^{-4} ms^{-2} at an interval of $5 \times 10^{-5} \text{ ms}^{-2}$. The zero contour is not shown, and dashed contours represent negative values.

during this period (not shown here). This process seems to be well represented in the forecast (Fig. 12b). Interestingly, the largest error arises from the divergent contribution T_{div} (Fig. 12c): in the forecast this term is associated with a considerable decrease of LWA (Fig. 11c), much stronger than suggested by the verifying analysis (Fig. 11f). The negative values of T_{div} in the forecast are found in the middle of the ridge, ahead of the approaching PV trough (Fig. 7), similar as for the RWP examined in the PUMA simulation (Fig. 6d). In the present real case, however, the magnitude of the divergent wind contribution T_{div} is much larger than in the PUMA simulation due to a stronger divergent wind in the upper levels, and this is likely to be associated with moist processes below.

In summary, both the local nonconservative term T_N and the divergent contribution T_{div} to the conservative term T_C are associated with substantial errors in the forecast. Both terms are directly or indirectly related to diabatic heating happening in the mid- and upper troposphere (Hoskins et al. 1985; Teubler and Riemer 2016). Our analysis thus suggests that the origin of the RWP amplitude error was a poor representation of diabatic processes in the forecast model. This result is

consistent with the original interpretation of Rodwell et al. (2013).

6. Summary and conclusions

The current work proposes a novel diagnostic to quantify the processes associated with the generation and dissipation of upper-tropospheric RWPs in terms of filtered LWA, which is a measure of the pseudomomentum associated to wave packets. The diagnostic is based on a budget equation for filtered LWA and allows one to distinguish between conservative dynamics and nonconservative processes. We applied this technique to a hierarchy of models, namely, a barotropic model, a dry primitive-equation model, and a state-of-the-art weather forecast model. For the latter two, we drew on recent work by Ghinassi et al. (2018), who extended LWA to the framework of the primitive equations in isentropic coordinates. The budget term representing conservative dynamics was further partitioned into contributions from the near-tropopause wind, the tropospheric-deep wind, and the divergent wind close to the tropopause, following previous work by Teubler and Riemer (2016) and Baumgart et al. (2018).

Our diagnostic was first applied to an idealized simulation using the barotropic model on the sphere with local pseudo-orographic forcing as a source for RWPs. The simplicity of the model and its configuration allowed us to anticipate the behavior of the individual terms of the LWA budget, and we found a good correspondence between these expectations and our model results. In particular, as expected the orographic forcing and the dissipation were reflected in the nonconservative term T_N of the LWA budget. Importantly, these sources and sinks showed a pronounced zonal asymmetry, which demonstrates the added value of LWA compared to a zonally averaged diagnostic.

We then applied our diagnostic to simulations of a dry primitive-equation model. A forced-dissipative model configuration was used with locally enhanced baroclinicity in order to generate an idealized storm track, in which RWPs grow due to baroclinic instability. A key difference with respect to the foregoing barotropic simulation is the fact that in a three-dimensional model RWPs and the associated LWA may also propagate in the vertical. It follows that on a given upper-tropospheric isentrope the vertical propagation of an RWP may appear as net increase or decrease of LWA on that isentrope, although this process is clearly associated with conservative dynamics. The dynamics of these simulated baroclinic life cycles clearly appeared in our LWA budget: in the upper troposphere we found that Rossby wave activity is continuously excited via baroclinic growth, which at

FIG. 11. Partitioning of the conservative term T_C into the different wind contributions on the 325-K isentropic averaged between 10 and 12 April (with 6-hourly data) for (a)–(c) the forecast and (d)–(f) the analysis for (a),(d) T_{nTP} , (b),(e) T_{TSd} , and (c),(f) T_{div} .

the same time is balanced by the dissipation from non-conservative processes (linear relaxation and hyperdiffusion). It was shown that the new diagnostic is, indeed, able to identify these local conservative and nonconservative sources and sinks of LWA and their impact on the RWPs amplitude evolution.

Finally, the diagnostic framework was applied to an observed episode simulated by the operational model of the ECMWF. We chose an episode of reduced predictive skill, which was associated with a propagating RWP that eventually evolved into a blocking flow regime. We applied our LWA budget diagnostic to both the forecast and the analysis and obtained insight into error sources regarding the RWP dynamics by comparing the two. Differences in RWP amplitude during early lead times were found to be related to non-conservative processes at the tropopause and with upper-tropospheric divergent flow, the latter likely to be associated with diabatic heating in the midtroposphere. These findings are consistent with the study of [Rodwell et al. \(2013\)](#), who linked the forecast error in this particular RWP to a misrepresentation of diabatic processes over the U.S. Midwest associated with a mesoscale convective system, which was present at the time of the forecast initialization. At the same time, we demonstrated that the origin of the error in the RWP occurred not only during the forecast initialization but

persisted also at some time later into the forecast. Our results are also consistent with the analysis performed by [Baumgart et al. \(2018, 2019\)](#), who linked the early stage of error growth to the diabatic modification of PV. A similar scenario was also found by [Martínez-Alvarado et al. \(2016\)](#), who linked an error in a ridge embedded in an RWP to the misrepresentation of both the diabatic modification of PV and the divergent outflow in the upper troposphere.

A key advantage of LWA in comparison with its linear variant is that all terms can be computed from any PV field without the danger of singularities. On the other hand, obviously there are some caveats with our proposed method. It is important to remark that the partly Lagrangian nature of LWA can sometimes prevent a straightforward interpretation in connection with identifying RWPs. For instance, in case of a pronounced and longitudinally extended reversal of the meridional PV gradient, high (low) PV values at a given latitude may become associated with high LWA values at a more poleward (equatorward) latitude in the Northern Hemisphere. If this is the case, large values of LWA or its budget may be associated with meteorological phenomena at a somewhat different latitude. The choice of a suitable isentropic that avoids such longitudinally extended PV gradient reversals is thus desirable and helps to mitigate this problem.

FIG. 12. Error (forecast minus analysis) in the different contributions to the conservative term in the LWA budget corresponding to the fields displayed in Fig. 11: (a) near-tropopause term, (b) tropospheric-deep term, and (c) divergent term. Contours for all plots are between -2×10^{-4} and $2 \times 10^{-4} \text{ m s}^{-2}$ at an interval of 10^{-4} m s^{-2} . The zero contour is not shown, and dashed contours represent negative values.

In summary, our work suggests that the LWA budget analysis adds a valuable tool for diagnosing the dynamics of RWPs at a regional scale. In particular, this tool can be used in order to attribute errors associated with the zonal propagation of RWPs to conservative and nonconservative processes, respectively. Future work has to show to what extent the results from our case study are systematic and can be generalized to the majority of the cases of interest.

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APPENDIX A

Derivation of (9)

In our compact notation, barotropic LWA on the sphere can be written as

$$A(\lambda, \phi, t) = -\frac{1}{\cos\phi} \int_{\phi}^{\phi+\Delta\phi} (q - Q)a \cos\phi' d\phi'. \quad (\text{A1})$$

Differentiating (A1) with respect to time and applying the Leibniz rule one obtains

$$\begin{aligned} \frac{\partial A}{\partial t} \cos\phi = & - \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial t} [(q - Q)a \cos\phi' d\phi' \\ & - \frac{\partial \Delta\phi}{\partial t} [(q - Q)a \cos\phi']_{\phi'=\phi+\Delta\phi} \\ & + \frac{\partial \phi}{\partial t} [(q - Q)a \cos\phi']_{\phi'=\phi}. \end{aligned} \quad (\text{A2})$$

The second term on the right-hand side vanishes because $q(\lambda, \phi + \Delta\phi, t) = Q(\phi, t)$ at any time [from the definition of $Q(\phi, t)$], and the third term on the right-hand side vanishes because ϕ is a coordinate, which does not explicitly depend on time. By contrast, the reference PV distribution $Q(\phi, t)$ may vary with time in the presence of nonconservative processes. We thus obtain

$$\frac{\partial A}{\partial t} \cos\phi = - \int_{\phi}^{\phi+\Delta\phi} \frac{\partial q}{\partial t} a \cos\phi' d\phi' + \int_{\phi}^{\phi+\Delta\phi} \frac{\partial Q}{\partial t} a \cos\phi' d\phi'. \quad (\text{A3})$$

From (3) we have

$$\frac{\partial q}{\partial t} = -\nabla \cdot (\mathbf{v}q) + N_q. \quad (\text{A4})$$

Substituting this into (A3) and carefully distinguishing between ϕ and ϕ' yields (9).

APPENDIX B

Derivation of (15)

Using the same compact notation as before, the primitive-equations version of LWA on a given isentrope reads

$$A(\lambda, \phi, \theta, t) = -\frac{1}{\cos\phi} \int_{\phi}^{\phi+\Delta\phi} (q - Q)\sigma a \cos\phi' d\phi'. \quad (\text{B1})$$

Differentiating (B1) with respect to time yields

$$\begin{aligned} \frac{\partial A}{\partial t} \cos \phi &= - \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial t} [(q-Q)\sigma] a \cos \phi' d\phi' \\ &= - \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial t} (q\sigma) a \cos \phi' d\phi' \\ &\quad + \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial t} (Q\sigma) a \cos \phi' d\phi', \end{aligned} \quad (\text{B2})$$

because the boundary terms vanish for the same reasons as in [appendix A](#). The equation for the mass-weighted PV in isentropic coordinates [see, e.g., (3.8.8) in [Andrews et al. \(1987\)](#)] reads

$$\frac{\partial}{\partial t} (q\sigma) = -\nabla \cdot (q\sigma\mathbf{v}) + N_1, \quad (\text{B3})$$

where

$$N_1 = -\nabla \cdot \begin{pmatrix} -Y + \dot{\theta} \partial v / \partial \theta \\ X - \dot{\theta} \partial u / \partial \theta \end{pmatrix}, \quad (\text{B4})$$

(X , Y) denote the nonconservative terms in the isentropic horizontal momentum equation, and $\dot{\theta}$ is the material rate of change of potential temperature due to diabatic processes. In addition, we have

$$\frac{\partial}{\partial t} (Q\sigma) = \frac{\partial Q}{\partial t} \sigma + Q \frac{\partial \sigma}{\partial t} \quad (\text{B5})$$

and

$$\frac{\partial \sigma}{\partial t} = -\nabla \cdot (\sigma\mathbf{v}) + N_2 \quad (\text{B6})$$

with $N_2 = \partial(\sigma\dot{\theta})/\partial\theta$ [see, e.g., (3.8.1c) in [Andrews et al. \(1987\)](#)]. Substituting (B3) and (B5) into (B2) and carefully distinguishing between ϕ and ϕ' yields (15), with the expression for T_N reading

$$T_N = \int_{\phi}^{\phi+\Delta\phi} (-N_1 + N_2) a \cos \phi' d\phi'. \quad (\text{B7})$$

APPENDIX C

Link between the LWA Budget and the E-P Relation in the Primitive Equations

To link the LWA budget in the primitive equations with the canonical E-P relation for finite-amplitude wave activity [see (30) of [NS11](#)] we start performing the zonal average (denoted with an overbar) of our (15); that is,

$$\frac{\partial \bar{A}}{\partial t} \cos \phi = \bar{T}_C + \bar{T}_Q + \bar{T}_N. \quad (\text{C1})$$

The zonal mean of T_C yields

$$\begin{aligned} \bar{T}_C &= \frac{1}{2\pi} \int_0^{2\pi} \int_{\phi}^{\phi+\Delta\phi} \nabla' \cdot [(q-Q)\sigma\mathbf{v}] a \cos \phi' d\phi' d\lambda \\ &= \frac{1}{2\pi} \int_0^{2\pi} \int_{\phi}^{\phi+\Delta\phi} \frac{1}{a \cos \phi'} \frac{\partial}{\partial \lambda} [(q-Q)\sigma u] a \cos \phi' d\phi' d\lambda \\ &\quad + \frac{1}{2\pi} \int_0^{2\pi} \int_{\phi}^{\phi+\Delta\phi} \frac{1}{a \cos \phi'} \frac{\partial}{\partial \phi'} [(q-Q)\sigma v \cos \phi'] \\ &\quad \times a \cos \phi' d\phi' d\lambda \\ &= 0 + \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial \phi'} [\overline{(q-Q)\sigma v} \cos \phi'] d\phi', \end{aligned} \quad (\text{C2})$$

since the integral in $d\lambda$ is zero due to periodic boundaries in longitude. The integral in $d\phi'$ is

$$\begin{aligned} \int_{\phi}^{\phi+\Delta\phi} \frac{\partial}{\partial \phi'} [\overline{(q-Q)\sigma v} \cos \phi'] d\phi' &= -\overline{(q-Q)\sigma v} \cos \phi \\ &= -\cos \phi \overline{\sigma v q_e^*} \Big|_{\phi'=\phi}, \end{aligned} \quad (\text{C3})$$

since $q(\lambda, \phi + \Delta\phi, \theta, t) = Q(\phi, \theta, t)$ at latitude $\phi + \Delta\phi$. In the above expression $(\cdot)^* \equiv \overline{\sigma(\cdot)}/\bar{\sigma}$ denotes the mass-weighted zonal average and $q_e(\lambda, \phi, \phi', \theta, t) \equiv q(\lambda, \phi', \theta, t) - Q(\phi, \theta, t)$. The zonal mean of T_Q yields

$$\bar{T}_Q = \frac{1}{2\pi} \int_0^{2\pi} \int_{\phi}^{\phi+\Delta\phi} \frac{\partial Q}{\partial t} \sigma a \cos \phi' d\phi' d\lambda = \frac{1}{2\pi a} \int_D \frac{\partial Q}{\partial t} dM, \quad (\text{C4})$$

where D is the domain defined by

$$D = [(q \geq Q) \cap (\phi' \leq \phi)] \cup [(q < Q) \cap (\phi' > \phi)].$$

Finally, the whole expression for the zonal average of (15) is

$$\frac{\partial \bar{A}}{\partial t} \cos \phi = -\cos \phi \overline{\sigma v q_e^*} \Big|_{\phi'=\phi} + \frac{1}{2\pi a} \int_D \frac{\partial Q}{\partial t} dM + \bar{T}_N. \quad (\text{C5})$$

Note that the above expression is different from the canonical form of the E-P relation of [NS11](#), since in our formulation we evaluate A at fixed latitude ϕ . The second term on the rhs of (C5) in fact contains both the adiabatic change of Q due to the residual mean circulation plus all nonconservative processes [$Q(\phi, \theta, t)$ is not conserved even for conservative flow]. Another difference is that the first term on the right-hand side in our expression involves v and q_e at latitude ϕ , while the

TABLE D1. Parameters in (D2).

Parameter	Value
λ_1	60°E
λ_2	60°E
ϕ_1	32.1°N
ϕ_2	51.6°N
d_{λ_1} for $\lambda \geq \lambda_1$	20 rad ⁻²
d_{λ_1} for $\lambda < \lambda_1$	15 rad ⁻²
d_{λ_2} for $\lambda \geq \lambda_2$	12 rad ⁻²
d_{λ_2} for $\lambda < \lambda_2$	5 rad ⁻²
d_{ϕ_1} for $\phi \geq \phi_1$	20 rad ⁻²
d_{ϕ_1} for $\phi < \phi_1$	10 rad ⁻²
d_{ϕ_2} for $\phi \geq \phi_2$	9 rad ⁻²
d_{ϕ_2} for $\phi < \phi_2$	5 rad ⁻²

corresponding term in (30) of NS11 involves $\hat{v}$ and $\hat{q}$ (where the hat denotes the deviation from the mass-weighted zonal average) evaluated at the mass-equivalent latitude Φ_M associated with the constant $Q(\phi, \theta)$.

APPENDIX D

Relaxation Temperature Profile in PUMA

The zonally symmetric part $T_R(\phi, \sigma_{\text{lev}})$ of the relaxation temperature field is equal to the one of Held and Suarez (1994); namely,

$$T_R(\sigma_{\text{lev}}, \phi) = \max\{T_{\text{tp}}, [T_{\text{srf}} - (\Delta T_R)_{\text{EP}} \sin^2 \phi - (\Delta \theta)_z \ln \sigma_{\text{lev}} \cos^2 \phi] \sigma_{\text{lev}}^\kappa\}, \quad (\text{D1})$$

where $\sigma_{\text{lev}} = p/p_s$ is the model vertical coordinate (with $p_s = 1000$ hPa), $\kappa = R/c_p = 2/7$, $R = 287$ J kg⁻¹ K⁻¹ is the gas constant for dry air, $T_{\text{tp}} = 200$ K is the temperature at the tropopause, $T_{\text{srf}} = 315$ K, and $(\Delta T_R)_{\text{EP}} = 50$ K is the temperature difference between the equator and the poles and $(\Delta \theta)_z = 10$ K.

A cold and a warm anomaly are added to $T_R(\phi, \sigma_{\text{lev}})$, such that the new field $\tilde{T}_R(\phi, \sigma_{\text{lev}})$ becomes

$$\begin{aligned} \tilde{T}_R(\sigma_{\text{lev}}, \phi, \lambda) = & T_R(\sigma_{\text{lev}}, \phi) + f(\sigma_{\text{lev}})\{T_R(\sigma_{\text{lev}}, \phi) \\ & - (\Delta T)_{\text{dip}} \sum_{j=1}^2 (-1)^j \exp[-d_{\lambda_j} \cos^2 \phi (\lambda - \lambda_j)^2 \\ & - d_{\phi_j} (\phi - \phi_j)^2]\}, \end{aligned} \quad (\text{D2})$$

where $(\Delta T)_{\text{dip}}$ is the difference between the warm ($j = 1$) and cold ($j = 2$) anomaly, which is set to $(\Delta T)_{\text{dip}} = 40$ K. The function $f(\sigma_{\text{lev}})$ provides weighting of the temperature anomalies along the vertical such that they decrease from their surface value to zero in the upper troposphere and above:

FIG. D1. Relaxation temperature $\tilde{T}_R$ on the lowest model level for PUMA.

$$f(\sigma_{\text{lev}}) = \begin{cases} \sin\left[\frac{\pi}{2}\left(\frac{\sigma_{\text{lev}} - 0.4}{1 - 0.4}\right)\right], & \text{if } \sigma_{\text{lev}} \geq 0.4, \\ 0, & \text{if } \sigma_{\text{lev}} < 0.4. \end{cases} \quad (\text{D3})$$

Values for parameters of (D2) are given in Table D1, and the modified relaxation temperature in the lowest model level is shown in Fig. D1.

REFERENCES

- Ambaum, M. H. P., and L. Novak, 2014: A nonlinear oscillator describing storm track variability. *Quart. J. Roy. Meteor. Soc.*, **140**, 2680–2684, <https://doi.org/10.1002/qj.2352>.
- Andrews, D. G., J. R. Holton, and C. B. Leovy, 1987: *Middle Atmosphere Dynamics*. Academic Press, 489 pp.
- Baumgart, M., M. Riemer, V. Wirth, F. Teubler, and S. T. K. Lang, 2018: Potential vorticity dynamics of forecast errors: A quantitative case study. *Mon. Wea. Rev.*, **146**, 1405–1425, <https://doi.org/10.1175/MWR-D-17-0196.1>.
- , P. Ghinassi, V. Wirth, T. Selz, G. C. Craig, and M. Riemer, 2019: Quantitative view on the processes governing the up-scale error growth up to the planetary scale using a stochastic convection scheme. *Mon. Wea. Rev.*, **147**, 1713–1731, <https://doi.org/10.1175/MWR-D-18-0292.1>.
- Chagnon, J. M., S. L. Gray, and J. Methven, 2013: Diabatic processes modifying potential vorticity in a North Atlantic cyclone. *Quart. J. Roy. Meteor. Soc.*, **139**, 1270–1282, <https://doi.org/10.1002/qj.2037>.
- Chang, E. K. M., and I. Orlanski, 1993: On the dynamics of a storm track. *J. Atmos. Sci.*, **50**, 999–1015, [https://doi.org/10.1175/1520-0469\(1993\)050<0999:OTDOAS>2.0.CO;2](https://doi.org/10.1175/1520-0469(1993)050<0999:OTDOAS>2.0.CO;2).
- Charney, J. G., and P. G. Drazin, 1961: Propagation of planetary-scale disturbances from the lower into the upper atmosphere. *J. Geophys. Res.*, **66**, 83–109, <https://doi.org/10.1029/JZ066i001p00083>.
- Davis, C. A., 1992: Piecewise potential vorticity inversion. *J. Atmos. Sci.*, **49**, 1397–1411, [https://doi.org/10.1175/1520-0469\(1992\)049<1397:PPVI>2.0.CO;2](https://doi.org/10.1175/1520-0469(1992)049<1397:PPVI>2.0.CO;2).
- Fraedrich, K., E. Kirk, U. Luksch, and F. Lunkeit, 2005: The Portable University Model of the Atmosphere (PUMA): Storm track dynamics and low-frequency variability. *Meteor. Z.*, **14**, 735–745, <https://doi.org/10.1127/0941-2948/2005/0074>.

- Frisius, T., F. Lunkeit, K. Fraedrich, and I. N. James, 1998: Storm-track organization and variability in a simplified atmospheric global circulation model. *Quart. J. Roy. Meteor. Soc.*, **124**, 1019–1043, <https://doi.org/10.1002/qj.49712454802>.
- Ghinassi, P., G. Fragkoulidis, and V. Wirth, 2018: Local finite-amplitude wave activity as a diagnostic for Rossby wave packets. *Mon. Wea. Rev.*, **146**, 4099–4114, <https://doi.org/10.1175/MWR-D-18-0068.1>.
- Glatt, I., and V. Wirth, 2014: Identifying Rossby wave trains and quantifying their properties. *Quart. J. Roy. Meteor. Soc.*, **140**, 384–396, <https://doi.org/10.1002/qj.2139>.
- Gray, S. L., C. M. Dunning, J. Methven, G. Masato, and J. M. Chagnon, 2014: Systematic model forecast error in Rossby wave structure. *Geophys. Res. Lett.*, **41**, 2979–2987, <https://doi.org/10.1002/2014GL059282>.
- Grazzini, F., and F. Vitart, 2015: Atmospheric predictability and Rossby wave packets. *Quart. J. Roy. Meteor. Soc.*, **141**, 2793–2802, <https://doi.org/10.1002/qj.2564>.
- Harris, F. J., 1978: On the use of windows for harmonic analysis with the discrete Fourier transform. *Proc. IEEE*, **66**, 51–83, <https://doi.org/10.1109/PROC.1978.10837>.
- Harvey, B., J. Methven, and M. H. P. Ambaum, 2018: An adiabatic mechanism for the reduction of jet meander amplitude by potential vorticity filamentation. *J. Atmos. Sci.*, **75**, 4091–4106, <https://doi.org/10.1175/JAS-D-18-0136.1>.
- Held, I. M., 1983: Stationary and quasi-stationary eddies in the extratropical troposphere: Theory. *Large-Scale Dynamical Processes in the Atmosphere*, B. J. Hoskins and R. P. Pearce, Eds., Academic Press, 127–168.
- , and P. J. Philipps, 1987: Linear and nonlinear barotropic decay on the sphere. *J. Atmos. Sci.*, **44**, 200–207, [https://doi.org/10.1175/1520-0469\(1987\)044<0200:LANBDO>2.0.CO;2](https://doi.org/10.1175/1520-0469(1987)044<0200:LANBDO>2.0.CO;2).
- , and M. J. Suarez, 1994: A proposal for the intercomparison of the dynamical cores of atmospheric general circulation models. *Bull. Amer. Meteor. Soc.*, **75**, 1825–1830, [https://doi.org/10.1175/1520-0477\(1994\)075<1825:APFTIO>2.0.CO;2](https://doi.org/10.1175/1520-0477(1994)075<1825:APFTIO>2.0.CO;2).
- Hoskins, B. J., 1991: Towards a PV- θ view of the general circulation. *Tellus*, **43A**, 27–35, <https://doi.org/10.3402/tellusa.v43i4.11936>.
- , and D. J. Karoly, 1981: The steady linear response of a spherical atmosphere to thermal and orographic forcing. *J. Atmos. Sci.*, **38**, 1179–1196, [https://doi.org/10.1175/1520-0469\(1981\)038<1179:TSLROA>2.0.CO;2](https://doi.org/10.1175/1520-0469(1981)038<1179:TSLROA>2.0.CO;2).
- , M. E. McIntyre, and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.*, **111**, 877–946, <https://doi.org/10.1002/qj.49711147002>.
- Huang, C. S. Y., and N. Nakamura, 2016: Local finite-amplitude wave activity as a diagnostic of anomalous weather events. *J. Atmos. Sci.*, **73**, 211–229, <https://doi.org/10.1175/JAS-D-15-0194.1>.
- , and —, 2017: Local wave activity budgets of the wintertime Northern Hemisphere: Implications for the Pacific and Atlantic storm tracks. *Geophys. Res. Lett.*, **44**, 5673–5682, <https://doi.org/10.1002/2017GL073760>.
- Lynch, P., 1989: Partitioning the wind in a limited domain. *Mon. Wea. Rev.*, **117**, 1492–1500, [https://doi.org/10.1175/1520-0493\(1989\)117<1492:PTWIAL>2.0.CO;2](https://doi.org/10.1175/1520-0493(1989)117<1492:PTWIAL>2.0.CO;2).
- Martínez-Alvarado, O., E. Madonna, S. L. Gray, and H. Joos, 2016: A route to systematic error in forecasts of Rossby waves. *Quart. J. Roy. Meteor. Soc.*, **142**, 196–210, <https://doi.org/10.1002/qj.2645>.
- Martius, O., C. Schwierz, and H. C. Davies, 2010: Tropopause-level waveguides. *J. Atmos. Sci.*, **67**, 866–879, <https://doi.org/10.1175/2009JAS2995.1>.
- Methven, J., 2013: Wave activity for large-amplitude disturbances described by the primitive equations on the sphere. *J. Atmos. Sci.*, **70**, 1616–1630, <https://doi.org/10.1175/JAS-D-12-0228.1>.
- , B. J. Hoskins, E. Heifetz, and C. H. Bishop, 2005: The counter-propagating Rossby-wave perspective on baroclinic instability. Part IV: Nonlinear life cycles. *Quart. J. Roy. Meteor. Soc.*, **131**, 1425–1440, <https://doi.org/10.1256/qj.04.23>.
- Nakamura, N., and D. Zhu, 2010: Finite-amplitude wave activity and diffusive flux of potential vorticity in eddy-mean flow interaction. *J. Atmos. Sci.*, **67**, 2701–2716, <https://doi.org/10.1175/2010JAS3432.1>.
- , and A. Solomon, 2011: Finite-amplitude wave activity and mean flow adjustments in the atmospheric general circulation. Part II: Analysis in the isentropic coordinate. *J. Atmos. Sci.*, **68**, 2783–2799, <https://doi.org/10.1175/2011JAS3685.1>.
- Novak, L., M. H. P. Ambaum, and R. Tailleux, 2017: Marginal stability and predator–prey behaviour within storm tracks. *Quart. J. Roy. Meteor. Soc.*, **143**, 1421–1433, <https://doi.org/10.1002/qj.3014>.
- Orlanski, I., 2003: Bifurcation in eddy life cycles: Implications for storm track variability. *J. Atmos. Sci.*, **60**, 993–1023, [https://doi.org/10.1175/1520-0469\(2003\)60<993:BIELCI>2.0.CO;2](https://doi.org/10.1175/1520-0469(2003)60<993:BIELCI>2.0.CO;2).
- Rhines, P. B., 2002: Rossby waves. *Encyclopedia of Atmospheric Sciences*, Elsevier, 1923–1939.
- Rodwell, M. J., and Coauthors, 2013: Characteristics of occasional poor medium-range weather forecasts for Europe. *Bull. Amer. Meteor. Soc.*, **94**, 1393–1405, <https://doi.org/10.1175/BAMS-D-12-00099.1>.
- Rossby, C.-G., 1940: Planetary flow patterns in the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **66** (Suppl.), 68–87.
- Saffin, L., J. Methven, and S. L. Gray, 2016: The non-conservation of potential vorticity by a dynamical core compared with the effects of parametrized physical processes. *Quart. J. Roy. Meteor. Soc.*, **142**, 1265–1275, <https://doi.org/10.1002/qj.2729>.
- Solomon, A., and N. Nakamura, 2012: An exact Lagrangian-mean wave activity for finite-amplitude disturbances to barotropic flow on a sphere. *J. Fluid Mech.*, **693**, 69–92, <https://doi.org/10.1017/jfm.2011.460>.
- Teubler, F., and M. Riemer, 2016: Dynamics of Rossby wave packets in a quantitative potential vorticity–potential temperature framework. *J. Atmos. Sci.*, **73**, 1063–1081, <https://doi.org/10.1175/JAS-D-15-0162.1>.
- Tung, K. K., 1986: Nongeostrophic theory of zonally averaged circulation. Part I: Formulation. *J. Atmos. Sci.*, **43**, 2600–2618, [https://doi.org/10.1175/1520-0469\(1986\)043<2600:NTOZAC>2.0.CO;2](https://doi.org/10.1175/1520-0469(1986)043<2600:NTOZAC>2.0.CO;2).
- Williams, P. D., 2011: The raw filter: An improvement to the Robert–Asselin filter in semi-implicit integrations. *Mon. Wea. Rev.*, **139**, 1996–2007, <https://doi.org/10.1175/2010MWR3601.1>.
- Wirth, V., M. Riemer, E. K. M. Chang, and O. Martius, 2018: Rossby wave packets on the midlatitude waveguide—A review. *Mon. Wea. Rev.*, **146**, 1965–2001, <https://doi.org/10.1175/MWR-D-16-0483.1>.
- Zimin, A. V., I. Szunyogh, D. Patil, B. R. Hunt, and E. Ott, 2003: Extracting envelopes of Rossby wave packets. *Mon. Wea. Rev.*, **131**, 1011–1017, [https://doi.org/10.1175/1520-0493\(2003\)131<1011:EEORWP>2.0.CO;2](https://doi.org/10.1175/1520-0493(2003)131<1011:EEORWP>2.0.CO;2).
- , —, B. R. Hunt, and E. Ott, 2006: Extracting envelopes of nonzonally propagating Rossby wave packets. *Mon. Wea. Rev.*, **134**, 1329–1333, <https://doi.org/10.1175/MWR3122.1>.