A k - ε Turbulence Model for the Convective Atmosphere

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ABSTRACT: A k - ε turbulence model for the stable atmosphere is extended for the convective atmosphere. The new model represents the buoyancy-induced increase in the kinetic energy and scale of eddies, and is consistent with the Monin–Obukhov similarity theory for convective atmospheric boundary layers (ABLs). After being incorporated into an ABL model with the Coriolis force, the model is tested by comparing the ABL model results with the Businger–Dyer (BD) relationship. ABL model simulations are carried out to reveal the sensitivity of the vertical wind profile to model parameters (e.g., the Obukhov length, friction velocity, and geostrophic wind). When the friction velocity is consistent with geostrophic wind speed (or the turbulence in the inner regime is in equilibrium with that in the outer regime), the modeled wind profile is close to the BD relationship near the ground surface. Otherwise, the modeled wind profile deviates from the BD relationship, resembling the hockey stick transition model.

KEYWORDS: Boundary layer; Eddies; Parameterization; Subgrid-scale processes

1. Introduction

a. Motivation

Atmospheric turbulence is important not only in the atmospheric boundary layer (ABL) but also in clouds. Cumulus entrainment/detrainment, as a kind of moist turbulence, plays an important role in weather and climate models (Emanuel et al. 1994). Entrainment/detrainment usually occurs as a result of the inhomogeneous mixing between cloudy air parcels and their environment (Raymond and Blyth 1986). When the mixed air parcels move to the level of zero buoyancy [e.g., Raymond and Blyth 1986; Taylor and Baker 1991; see Emanuel (1994) for review], they induce downdrafts that in turn bring high ice crystal concentration (ICC), or activated ice nuclei, downward and therefore greatly increase ICC in the mixed-phase region (Zeng et al. 2009, 2011). Hence, the mixing of cloudy air parcels with their environment—especially near cloud boundary—is interesting to model.

Modeling the mixing near cloud boundary needs not only high spatial resolution with special numerical algorithms, but also proper turbulence parameterization, especially on the transition from the inside of clouds (or convective region) to the outside (or stably stratified region) (Grabowski 1989). The current cloud models generally used one-variable (i.e., turbulent kinetic energy) turbulence models to represent the subgrid effects, and assumed no difference in the mixing length between clouds and clear region (Klemp and Wilhelmson 1978).

Two-variable (e.g., k - ε) turbulence models (Launder and Spalding 1974) and higher-order turbulence closure (Mellor and Yamada 1982; Cheng et al. 2002; Umlauf and Burchard

2005) can be used to represent the difference in the mixing length between clouds and clear region [see Wilcox (2006) for a review of the turbulence models]. The former are preferred herein because they are simple yet effective. The standard k - ε model, one of the former, was widely used in many scientific and engineering simulations (Launder and Spalding 1974; Rodi 1987). It was also applied to the atmosphere after the parameters were tuned or new processes were introduced (e.g., Detering and Etling 1985; Duynkerke 1988; Apsley and Castro 1997; Sogachev 2009; Sogachev et al. 2012).

b. Status of the k - ε model

The standard k - ε model is expressed with two prognostic variables: the turbulent kinetic energy (TKE) k and its dissipation rate ε , which are defined as

$$k = \frac{1}{2} \overline{u_i u_i} \quad \text{and} \quad (1.1)$$

$$\varepsilon = 2\nu_0 \overline{s_{ij} s_{ij}}, \quad (1.2)$$

respectively, where u_i is the component of turbulent velocity in direction i , s_{ij} is the strain-rate fluctuation, ν_0 is the molecular viscosity, and the overbar indicates the Reynolds averaging.

In spite of its broad applications, the standard k - ε model does not work well for flow with large mean shear, jet spreading, or rotating turbulence. Its deficiencies originate in the equation for the dissipation rate ε , because the ε equation contains the terms of pressure and no effective way is available to handle the terms [see Zeng et al. (2020) for review]. To remove the deficiencies, Yakhot et al. (1992) and Shih et al. (1995) proposed two methods to improve the ε equation. Yakhot et al. (1992) improved the original ε equation via a renormalization group method, and Shih et al. (1995, hereafter S95) introduced a new ε equation based on the enstrophy equation. The latter approach is suitable for complicated ABLs because its framework is open to incorporate new processes such as atmospheric rotation and stratification (Zeng et al. 2020, hereafter Z20).

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S95 is quite different from the standard k - ε model. It redefines ε as the turbulent enstrophy δ times $2\nu_0$, or

$$\varepsilon = 2\nu_0\delta, \quad (1.3)$$

where

$$\delta = \frac{1}{2} \overline{\omega_i \omega_i}, \quad (1.4)$$

and ω_i is the component of turbulent vorticity in direction i .

The equation for enstrophy is different from the equation for the dissipation rate. It is obtained by multiplying the equation for vorticity ω_i with ω_i . Since the equation for vorticity ω_i involves no pressure, the equation for enstrophy contains no terms of pressure and thus all of its terms are clear in physics (Tennekes and Lumley 1972, p. 86). Hence, the equation for enstrophy provides a solid ground to obtain a closed equation for ε (S95).

On the other hand, the equation for TKE needs an expression of the dissipation rate for closure. Theoretical studies and numerical experiments show that the dissipation rate is approximately equal to $\varepsilon = 2\nu_0\delta$ at a high Reynolds number (Tennekes and Lumley 1972, p. 88; Grinstein et al. 2011; Yeung et al. 2012). Hence, ε in S95 has two names. It is called the dissipation rate so that S95 inherits the framework of the standard k - ε model (i.e., the TKE equation and the relationship of turbulent viscosity to k and ε). It is also called the turbulent enstrophy for its equation is derived from the enstrophy equation. With the new ε equation from the enstrophy equation, S95 removed the deficiencies in the standard k - ε model, including the well-known spreading rate anomaly of planar and round jets (thermals in convective ABLs can be treated as a kind of jet).

Following S95, Z20 extended the k - ε model for a stable atmosphere after incorporating the buoyancy damping of gravity waves (Bretherton and Smolarkiewicz 1989; Nilsson and Emanuel 1999; Raymond and Zeng 2000, 2005; Sobel and Bretherton 2000; Zeng 2001; Bretherton and Sobel 2002; Zeng et al. 2007, 2008), and showed the new model is consistent with Monin–Obukhov similarity theory (MOST) and field observations. In this paper, the new model is extended further for a convective atmosphere so that the model can work for both stable and unstable ABLs, providing a candidate for atmospheric turbulence modeling (Bretherton et al. 1999).

To our knowledge, few reports have been published on the use of the k - ε model for convective fluid. The rare use of the model may be attributed to the positive conversion of potential energy to TKE. Specifically, the positive conversion of potential energy to TKE in unstable fluid increases the kinetic energy and scale of eddies. However, the standard k - ε model and others (including S95 and Z20) represent only a decrease in the mean eddy scale (or the Taylor microscale) (see appendix B of Z20), which is opposite to the upscale tendency in unstable fluid. In this paper, the model is improved to represent the upscale tendency in unstable fluid.

This paper consists of five sections. Section 2 introduces the formulation of the new model, and section 3 shows its

consistency with MOST. Section 4 incorporates the model into an ABL model and then compares the ABL model results with the Businger–Dyer (BD) relationship as a test. Section 5 gives a summary and discussion.

2. Formulation for the convective atmosphere

In this section, the k - ε model of Z20 is extended for the convective atmosphere, beginning with its formulation. Its ε equation is then improved to incorporate thermals.

a. Equations of the k - ε model

Following the notation of Z20, the TKE equation is expressed as (Launder and Spalding 1974)

$$\frac{dk}{dt} - \nabla \cdot \left(\frac{\nu_t}{\sigma_k} \nabla k \right) = \nu_t (S^2 + G) - \varepsilon, \quad (2.1)$$

where t is time, ν_t is the turbulent viscosity, $\sigma_k = 1$ is the turbulent Prandtl number for k , and $\nu_t S^2$ and $\nu_t G$ represent the energy generation due to shear and buoyancy, respectively. The wind shear S in (2.1) is expressed in the Cartesian coordinate system $(x, y, z) = (x_1, x_2, x_3)$ as

$$S = \sqrt{2S_{ij}S_{ij}}, \quad (2.2)$$

where S_{ij} is the mean strain rate, the atmospheric stability G is expressed as

$$G = -\frac{g}{\sigma_\theta} \frac{\partial \theta}{\partial z}, \quad (2.3)$$

where θ is the potential temperature, σ_θ is the turbulent Prandtl number for θ , and g is the acceleration due to gravity.

The equation of ε is obtained from the enstrophy equation, yielding (S95)

$$\frac{d\varepsilon}{dt} - \nabla \cdot \left(\frac{\nu_t}{\sigma_\varepsilon} \nabla \varepsilon \right) = C_1 S \varepsilon - C_2 \frac{\varepsilon^2}{k + \sqrt{\nu_0 \varepsilon / C_\mu}} + B, \quad (2.4)$$

where $\sigma_\varepsilon = 1.2$ is the turbulent Prandtl number for ε , $C_2 = 1.9$, and

$$C_1 = \max[0.43, \eta/(\eta + 5)], \quad (2.5)$$

where the dimensionless variable η is defined as

$$\eta = Sk/\varepsilon. \quad (2.6)$$

Since k/ε represents the turbulence dissipation time scale, η is the ratio of the turbulent to mean strain time scale (Yakhot et al. 1992). On the other hand, since k/ε is proportional to the square of the mean eddy scale or Taylor microscale (Z20), η represents the dimensionless shear rate normalized with the mean eddy scale and molecular viscosity.

The turbulent viscosity ν_t in (2.1) and (2.4) is specified as

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}, \quad (2.7)$$

where

TABLE 1. Parameters in the turbulence model.

Parameter	Value or equation number
C_1	(2.5)
C_2	1.9
C_3	1.46
C_4	0.44
C_5	0.08
C_6	0.58
C_7	(3.19)
C_8	(3.20)
C_9	0.28
C_μ	(2.8)
σ_k	1.0
σ_ε	1.2
σ_θ	0.74

$$C_\mu = \left[4.0 + \frac{k}{\varepsilon} \sqrt{6(S_{ij}S_{ij} + \Omega_{ij}\Omega_{ij})} \cos\phi \right]^{-1}, \quad (2.8)$$

$$\phi = \frac{1}{3} \arccos \left[\sqrt{6} \frac{S_{ij}S_{jk}S_{ki}}{(S_{ij}S_{ij})^{3/2}} \right], \quad (2.9)$$

and Ω_{ij} is the mean-rotation-rate tensor (S95).

In addition, B in (2.4) represents the contribution of buoyancy to ε . In a stable region (Z20),

$$B = C_3 \frac{\varepsilon}{k} \nu_i G + C_4 \min \left(1, \sqrt{\frac{R_i}{C_5}} \right) N \varepsilon, \quad (2.10)$$

where C_3 , C_4 , and C_5 are constants, the gradient Richardson number is

$$R_i = -\frac{\sigma_\theta G}{S^2}, \quad (2.11)$$

and the Brunt–Väisälä frequency is

$$N = \left| \frac{g \partial \theta}{\theta \partial z} \right|^{1/2}. \quad (2.12)$$

All parameters of the model are listed in Table 1 for quick reference.

b. Representation of thermals

The expression of B in (2.10) works for a stable region (or $G < 0$). In this subsection, the expression for an unstable region (or $G > 0$) is discussed, beginning with the formation of thermals.

Observations of turbulent convection suggest that it could be represented as large eddies upon which small eddies are superimposed (Niemela et al. 2000). The relatively large eddies (or thermals) extract energy directly from potential energy, leading to an increase in mean eddy scale (see appendix B). Hence, following (2.10), the contribution of buoyancy to ε in an unstable region is represented by

$$B = C_3 \frac{\varepsilon}{k} \nu_i G + (C_6 - C_7 \eta - C_8 \eta^2) \min \left(1, \sqrt{\frac{|R_i|}{C_9}} \right) N \varepsilon, \quad (2.13)$$

where C_6 , C_7 , C_8 , and C_9 are coefficients to be determined.

The two terms on the right-hand side of (2.13) function similarly as their counterparts in (2.10). The first term represents the contribution of buoyancy to ε by assuming all eddies have only one scale: the mean eddy scale (or the Taylor microscale; Z20):

$$\lambda_m = \pi(\nu_0 k / \varepsilon)^{1/2}. \quad (2.14)$$

The second term on the right-hand side of (2.13) is introduced to amend the first one, because eddies, especially thermals, usually have scales different from λ_m . Its expression is chosen so that the turbulence model is consistent with MOST, which is discussed next.

3. Consistency with MOST

This section shows the proposed turbulence model is consistent with MOST and then determines its parameters C_6 – C_9 with MOST. To be specific, the section consists of three steps: 1) introducing the formulas of the similarity for unstable ABLs, 2) re-expressing them in terms of the Obukhov length L , and 3) expanding them in the Taylor's series so as to determine C_6 – C_9 .

a. MOST for unstable ABLs

Consider a steady horizontally homogeneous surface layer where MOST is applicable. The surface layer is hydrostatically unstable. Its momentum flux does not change with height, which is described as

$$u_*^2 = \nu_i S, \quad (3.1)$$

where u_* is the friction velocity. Its mean velocity and potential temperature profiles are expressed as (Businger et al. 1971)

$$\frac{k_0 z}{u_*} S = \left(1 - 15 \frac{z}{L} \right)^{-1/4}, \quad (3.2)$$

$$\frac{k_0 z}{\theta_*} \frac{d\theta}{dz} = \sigma_\theta \left(1 - 9 \frac{z}{L} \right)^{-1/2}, \quad (3.3)$$

where the von Kármán constant $k_0 = 0.41$ (Dyer 1974; S95; see footnote 1 for the discussion on k_0), θ_* is the surface-layer temperature scale, and L is the Obukhov length that is associated with θ_* as

$$L = \frac{u_*^2}{k_0 g \theta_* / \theta}. \quad (3.4)$$

Eliminating θ_* between (3.3) and (3.4) and then substituting (2.3) into the resulting equation yields

$$G = -\frac{u_*^2}{k_0^2 z L} \left(1 - 9 \frac{z}{L} \right)^{-1/2}. \quad (3.5)$$

Correspondingly, (3.2) is rewritten in terms of L as

$$S = \frac{u_*}{k_0 z} \left(1 - 15 \frac{z}{L} \right)^{-1/4}. \quad (3.6)$$

b. Reformulation in terms of the Obukhov length L

To compare the similarity with the turbulence model, (3.1)–(3.6) are re-expressed in terms of L or the dimensionless height:

$$\tilde{z} = -15z/L. \quad (3.7)$$

Thus, (3.5) and (3.6) give

$$\frac{G}{S^2} = \left(\frac{1 + \tilde{z}}{1 + 0.6\tilde{z}} \right)^{1/2} \frac{\tilde{z}}{15}. \quad (3.8)$$

Substituting (3.6) into (3.1) yields the expression of ν_t , or

$$\nu_t = k_0 u_* z (1 + \tilde{z})^{1/4}. \quad (3.9)$$

After multiplying (2.7) with S and then substituting (2.6) and (3.1) into the resulting equation, we obtain

$$k = \frac{u_*^2}{C_\mu \eta}. \quad (3.10)$$

Using (3.10), (2.6) is changed to

$$\varepsilon = \frac{Su_*^2}{C_\mu \eta^2}. \quad (3.11)$$

The preceding expressions, (3.5)–(3.11), are used to rewrite the energy and enstrophy Eqs. (2.1) and (2.4). For the steady surface layer, the energy equation, (2.1), is simplified to

$$\varepsilon = \nu_t (S^2 + G) + \frac{d}{dz} \left(\frac{\nu_t}{\sigma_k} \frac{dk}{dz} \right),$$

which is further changed to

$$\frac{\eta^{-2}}{C_\mu} = 1 + \frac{G}{S^2} + \frac{k_0^2 \tilde{z} (1 + \tilde{z})^{1/4}}{\sigma_k C_\mu} \frac{d}{d\tilde{z}} \left[\frac{\tilde{z} (1 + \tilde{z})^{1/4} d\eta^{-1}}{d\tilde{z}} \right] \quad (3.12)$$

with the aid of (2.6), (2.7), (3.1), (3.6), and (3.11). Similarly, the ε equation, (2.4), is simplified to

$$\begin{aligned} & -C_1 S \varepsilon + C_2 \frac{\varepsilon^2}{k} - C_3 \frac{\varepsilon}{k} \nu_t G \\ & + (C_7 \eta + C_8 \eta^2 - C_6) \min \left(1, \sqrt{\frac{|R_t|}{C_9}} \right) N \varepsilon = \frac{d}{dz} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{d\varepsilon}{dz} \right) \end{aligned}$$

for the steady surface layer, which is further changed to

$$\begin{aligned} & -C_1 + C_2 \eta^{-1} - C_3 C_\mu \eta \frac{G}{S^2} \\ & + (C_7 \eta + C_8 \eta^2 - C_6) \min \left(1, \sqrt{\frac{|R_t|}{C_9}} \right) \frac{N}{S} \\ & = \frac{k_0^2 \eta^2 \tilde{z}^2 (1 + \tilde{z})^{1/2}}{\sigma_\varepsilon} \frac{d}{d\tilde{z}} \left\{ \tilde{z} (1 + \tilde{z})^{1/4} \frac{d[\tilde{z} (1 + \tilde{z})^{1/4} \eta^2]^{-1}}{d\tilde{z}} \right\} \quad (3.13) \end{aligned}$$

with the aid of (2.6), (3.1), (3.6), and (3.11).

Equations (3.12) and (3.13) come from the energy and the ε equations, respectively, and thus have their own solutions of η . If the two solutions of η from (3.12) and (3.13) are close to each other, the turbulence model is consistent with MOST (see Z20 for more discussions). Otherwise, the turbulence model is inconsistent with MOST and thus is ill-posed. In the next

subsections, the solutions of η from the two equations are compared, showing that the proposed model is consistent with MOST.

c. Consistency near $\tilde{z} = 0$

The two solutions of η from (3.12) and (3.13) near $\tilde{z} = 0$ are compared to obtain two constraints on the model parameters. Expanding η in the Taylor series of $\tilde{z}$ first and then substituting it into (3.12) gives

$$\eta = \frac{1}{\sqrt{C_\mu}} - \frac{1}{2\sqrt{C_\mu - k_0^2/\sigma_k}} \frac{\tilde{z}}{15} + \dots \quad (3.14)$$

with the aid of (3.8). Equation (3.14) is then substituted into (3.13), giving

$$\begin{aligned} & C_2 \sqrt{C_\mu} - C_1 + \left\{ \frac{C_\mu C_2}{2\sqrt{C_\mu - k_0^2/\sigma_k}} - C_3 \sqrt{C_\mu} \right. \\ & \left. + \left(\frac{C_7}{\sqrt{C_\mu}} + \frac{C_8}{C_\mu} - C_6 \right) \frac{\sigma_\theta}{\sqrt{C_9}} \right\} \frac{\tilde{z}}{15} \\ & = \frac{k_0^2}{\sigma_\varepsilon} + \frac{2k_0^2}{\sigma_\varepsilon} \left\{ \frac{15}{4} - \frac{\sqrt{C_\mu}}{2\sqrt{C_\mu - k_0^2/\sigma_k}} \right\} \frac{\tilde{z}}{15} \end{aligned}$$

after retaining the first two order terms. The preceding equation requires

$$C_2 \sqrt{C_\mu} - C_1 = \frac{k_0^2}{\sigma_\varepsilon}, \quad (3.15a)$$

$$\begin{aligned} & C_3 \sqrt{C_\mu} - \frac{C_\mu C_2}{2\sqrt{C_\mu - k_0^2/\sigma_k}} - \left(\frac{C_7}{\sqrt{C_\mu}} + \frac{C_8}{C_\mu} - C_6 \right) \frac{\sigma_\theta}{\sqrt{C_9}} \\ & = \frac{2k_0^2}{\sigma_\varepsilon} \left\{ \frac{\sqrt{C_\mu}}{2\sqrt{C_\mu - k_0^2/\sigma_k}} - \frac{15}{4} \right\}, \quad (3.15b) \end{aligned}$$

where (3.15a) is a constraint of σ_ε for neutral and stable ABLs (S95; Z20) and (3.15b) is a constraint of C_6 – C_9 for unstable ABLs. In other words, when C_6 – C_9 satisfies (3.15), the two solutions of η from (3.12) and (3.13) are close to each other near $\tilde{z} = 0$.

d. Consistency at $\tilde{z} \gg 1$

The two solutions of η from (3.12) and (3.13) at $\tilde{z} \gg 1$ are compared to obtain two other constraints on the model parameters. When $\tilde{z}$ is very large, $S \rightarrow 0$ and thus $\eta \rightarrow 0$, because the mean eddy scale λ_m is limited by the depth of the unstable layer. Meanwhile, (3.8) shows $G/S^2 \approx \tilde{z}/(15\sqrt{0.6})$, which suggests $\eta^{-1} \propto \sqrt{\tilde{z}}$ based on (3.12). Thus, when $\tilde{z} \rightarrow \infty$ (or the altitude approaches to the top of the surface layer), $1/\tilde{z} \rightarrow 0$; subsequently, $\eta^{-1}/\sqrt{\tilde{z}}$ is expanded in the Taylor series of $1/\tilde{z}$ as

$$\frac{\eta^{-1}}{\sqrt{z}} = C_{\infty 1} + \frac{C_{\infty 2}}{\tilde{z}} + \dots, \quad (3.16)$$

where $C_{\infty 1}$ and $C_{\infty 2}$ are constants. Substituting (3.16) into (3.12) yields

$$C_{\infty 1} = \frac{3k_0^2}{16\sigma_k} + \sqrt{\frac{9k_0^4}{256\sigma_k^2} + \frac{C_\mu}{15\sqrt{0.6}}}, \quad (3.17a)$$

$$C_{\infty 2} = \frac{16\sigma_k C_\mu [1 - (45\sqrt{0.6})^{-1}] + k_0^2 C_{\infty 1}}{32\sigma_k C_{\infty 1} - 2k_0^2}. \quad (3.17b)$$

Then, substituting (3.16) into (3.13) yields one constraint on the constants or

$$C_6 = \left\{ C_2 C_{\infty 1} - \frac{C_3 C_\mu}{15\sqrt{0.6} C_{\infty 1}} \right\} \sqrt{\frac{15\sqrt{0.6}}{\sigma_\theta}}. \quad (3.18a)$$

In addition, when $\tilde{z}$ is moderately large, substituting (3.16) into (3.13) yields one more constraint:

$$C_7 \approx C_1 C_{\infty 1} \sqrt{\frac{15\sqrt{0.6}}{\sigma_\theta}} = 0.213, \quad (3.18b)$$

with the aid of (3.18a).

e. Consistency in the vertical profile of η

The two vertical profiles of η from (3.12) and (3.13) are compared through $0 \leq \tilde{z} \leq 15$, yielding the last constraints on the model parameters. Equation (3.12) is solved numerically for η , using the boundary conditions $\eta = C_\mu^{-1/2}$ at $\tilde{z} = 0$ and $\eta = (C_{\infty 1}\sqrt{15} + C_{\infty 2}/\sqrt{15})^{-1}$ at $\tilde{z} = 15$ (or $z = -L$). Specifically, (3.12) is solved numerically using 1024 grid points with a second-order finite-difference scheme. Its result is displayed by the black line in Fig. 1. Since (3.12) is independent of C_6 – C_9 , this η – $\tilde{z}$ profile from (3.12) is used to represent MOST and thus is used as the standard to test other profiles from (3.13).

Equation (3.13), just like (3.12), is solved numerically once C_6 – C_9 are given. Its η – $\tilde{z}$ profile is then compared with that of (3.12) to show whether the turbulence model is consistent with MOST. In the proposed model, C_6 – C_9 is determined as follows. Given $C_3 = 1.46$, $C_6 = 0.58$ is obtained from (3.18a).

With the estimate of C_7 in (3.18b), an η – $\tilde{z}$ profile can be obtained from (3.13). However, the profile deviates a little from the standard from (3.12). To better match the standard from (3.12), C_7 is adjusted to

$$C_7 = 0.213/\max(16|R_i| - 5.5, 1) \quad (3.19)$$

with

$$C_8 = 0.21 C_7 \quad (3.20)$$

and $C_9 = 0.28$, where C_9 is obtained from (3.15b). Now, all the model parameters are determined and listed in Table 1.

Once the proposed model uses (3.19) and the other parameters in Table 1, an η – $\tilde{z}$ profile is obtained from (3.13) and then compared with that from (3.12) in Fig. 1. Generally speaking,

FIG. 1. Consistency in the η – $\tilde{z}$ profile between the present turbulence model (red) and the result from MOST (black line). For comparison, the blue line represents the result from the standard k – ϵ model with no thermals (or $C_6 = C_7 = C_8 = 0$); the green dot indicates the model of Shih et al. (1995) for neutral ABLs.

the two η – $\tilde{z}$ profiles from (3.13) and (3.12) coincide quite well, indicating that the proposed turbulence model is consistent with MOST.

For comparison, the η – $\tilde{z}$ profile of the standard k – ϵ model (i.e., $C_3 = 1.46$, $C_6 = C_7 = C_8 = 0$) is obtained from (3.13) similarly and is displayed in Fig. 1. Obviously, the η – $\tilde{z}$ profile of the standard k – ϵ model is different from that of (3.12), suggesting that the standard k – ϵ model is not consistent with MOST. Such inconsistency between the standard k – ϵ model and MOST is understandable, because the standard k – ϵ model does not incorporate thermals or mistreat them as regular eddies (see appendix B).

4. ABL model and its test

In this section, the proposed turbulence model is introduced into an ABL model to simulate the convective ABL, beginning with the ABL model setup. ABL model results are then compared with the Businger–Dyer (BD) relationship to test the proposed turbulence model.

a. Setup of the ABL model

The ABL model is similar to its counterpart in Z20. It considers a steady horizontally homogeneous ABL with the Coriolis force. Its mean horizontal wind components U and V in the (x, y) direction are governed by

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z} \left(\nu_t \frac{\partial U}{\partial z} \right) + f(V - V_g), \quad (4.1a)$$

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial z} \left(\nu_t \frac{\partial V}{\partial z} \right) - f(U - U_g), \quad (4.1b)$$

where f is the Coriolis parameter, and (U_g, V_g) are the two components of geostrophic wind in the (x, y) direction. Its prognostic equations of k and ε are expressed as

$$\frac{\partial \ln k}{\partial t} = \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_k} \frac{\partial \ln k}{\partial z} \right) + \frac{\nu_t}{\sigma_k} \left(\frac{\partial \ln k}{\partial z} \right)^2 + \frac{\nu_t}{k} (S^2 + G) - \frac{\varepsilon}{k}, \quad (4.1c)$$

$$\begin{aligned} \frac{\partial \ln \varepsilon}{\partial t} = & \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \ln \varepsilon}{\partial z} \right) + \frac{\nu_t}{\sigma_\varepsilon} \left(\frac{\partial \ln \varepsilon}{\partial z} \right)^2 + C_1 S - \frac{C_2 \varepsilon}{k + \sqrt{\nu_0 \varepsilon / C_\mu}} \\ & + C_3 \frac{\nu_t}{k} G + (C_6 - C_7 \eta - C_8 \eta^2) \min \left(1, \sqrt{\frac{|R_t|}{C_9}} \right) N. \end{aligned} \quad (4.1d)$$

At the bottom boundary of the model (or $z = 0$), $U = V = 0$, and $\eta = C_\mu^{-1/2}$; thus

$$k = \frac{u_*^2}{\sqrt{C_\mu}} \quad \text{and} \quad (4.2a)$$

$$\varepsilon = \frac{u_*^3}{k_0 z_0} \left(1 - \frac{15z_0}{L} \right)^{-1/4}, \quad (4.2b)$$

which are obtained from (3.10) and (3.11), respectively, with the aid of (3.6). At the top boundary of the model (or $z = 3$ km), the boundary conditions are the same as those in Z20, including $U = U_g$ and $V = V_g$.

The model is evaluated with the BD relationship. To be specific, the BD relationship is expressed in terms of the dimensionless wind shear $\phi_M(z)$. That is (Businger et al. 1971; Dyer 1974; Baas et al. 2006; Salesky and Chamecki 2012),¹

$$\phi_M = \left(1 - 15 \frac{z}{L} \right)^{-1/4} \quad \text{when } L < 0, \quad (4.3a)$$

$$\phi_M = 1 + 4.7 \frac{z}{L}, \quad \text{when } L > 0. \quad (4.3b)$$

In a numerical experiment, the modeled dimensionless wind shear ϕ_M is computed with

$$\phi_M = \frac{k_0 z}{u_*} S. \quad (4.4)$$

¹ The slope of ϕ_M , 4.7, is used in the present model. The slope of 4.7 was first obtained by Businger et al. (1971) with the von Kármán constant $k_0 = 0.35$. Almost the same slope (i.e., 4.64) was obtained by Baas et al. (2006) with observational data and $k_0 = 0.4$ (see their Fig. 6), where the slope is treated as an averaged value of many cases and an in situ slope usually deviates from the averaged value. On the other hand, other slopes (e.g., 1.6, 6) with $k_0 = 0.4$ have been proposed (e.g., Höglström 1988; Nakanish 2001; Zilitinkevich et al. 2013). Theoretically, the model can match the other slopes by adjusting its coefficients with the procedure in section 3.

Since MOST requires (4.4) is equal to (4.3) (Monin and Obukhov 1954), the difference between the modeled ϕ_M and the BD relationship can be treated as an index to measure the error of the proposed turbulence model.

b. Control experiments for stable and unstable ABLs

The control experiment for stable ABLs, STABLE, is the same as that in Z20. It simulates the Leipzig wind profile (Mildner 1932; Lettau 1950) with the following parameters: the Obukhov length $L = 0.58$ km, roughness length $z_0 = 0.3$ m, friction velocity $u_* = 0.65$ m s⁻¹, $U_g = 17.5$ m s⁻¹, $V_g = 0$, $f = 1.13 \times 10^{-4}$ s⁻¹, and air density $= 1.25$ kg m⁻³. In addition, it uses 1000 grid points with a model top of 3 km and a grid size of 3 m to discretize (4.1). It integrates the model equations for 30 h so that it reaches a steady state. Its time series of maximum turbulent viscosity is displayed in Fig. 2, showing that the model becomes steady eventually. Its time-averaged variables over the last 15 h are used to represent its steady state. Figure 3 displays its dimensionless wind shear ϕ_M at the steady state, showing the modeled ϕ_M is close to the BD relationship up to ~400 m in altitude.

The control experiment for unstable ABLs, UNSTABLE, takes the same setup as STABLE except for the stability in (3.5), $L = -0.58$ km, $z_0 = 0.2$ m, and $u_* = 0.34$ m s⁻¹, where the friction velocity is smaller than that in STABLE (Beare 2007; Fang et al. 2020). In addition, a weakly stable layer [i.e., its vertical gradient of potential temperature is about 0.1°C (100 m)⁻¹] is overlaid above the ABL to mimic the transition from ABL to the free atmosphere.

UNSTABLE sets the stable-layer bottom at 1 km altitude. Two similar experiments are carried out that use the same setup as UNSTABLE except the stable-layer bottom altitudes are set at 1.5 and 2 km. The three experiments output almost the same variables near the surface but different turbulent viscosity in the upper model domain. Their turbulent viscosity at the steady state is displayed in Fig. 4, showing that the stable-layer location is important to determine the turbulent viscosity in the upper portion of the ABL.

c. Modeling the first kind of deviation of the BD relationship in a stable ABL

A total of 10 experiments are carried out to reveal the sensitivity of ϕ_M to model parameters in a stable ABL (see Table 2 for experiment summary). They are designed to simulate two phenomena: the BD relationship and its temporary deviation (Baas et al. 2006; Salesky and Chamecki 2012; Sun et al. 2012, 2016).

Of the ten experiments, S2–S6 are designed to simulate the first kind of deviation of the BD relationship caused by turbulence intermittency. Experiments S2–S6 use the same setup as STABLE except for $L = 0.3, 0.2, 0.1, 0.05$, and 0.02 km, respectively. Figure 2 displays their time series of maximum turbulent viscosity, showing that turbulence intermittency occurs when the Obukhov length L is below 0.2 km. To further exhibit the turbulence intermittency, the time series of k , ε , ν_t , and S in S6 with $L = 0.02$ km are displayed in Fig. 5 as an example, where a shaded area represents a quiescent period.

FIG. 2. Maximum turbulent viscosity vs time in the six experiments for stable ABLs with different Obukhov length L (see Table 2 for the model parameters).

To reveal the processes of turbulence intermittency, S4 with $L = 0.1$ km is chosen for analysis herein for its “simple” oscillation of turbulence intermittency (Fig. 2). As shown by its time series of k , ε , ν_t , and S in Fig. 6, the wind shears at 49.5 and 79.5 m are out of phase, maintaining a constant bulk wind shear (or U_g divided by the ABL depth). Specifically, in the shaded area of Fig. 6, the strong wind shear at 49.5 m intensifies the turbulence and vertical mixing that in turn lead to weak wind shear at the original altitude but strong wind shear at 79.5 m; after the wind shear and turbulence at 79.5 m reach their extreme and then decay, the wind shear and turbulence at 49.5 m are intensified again. Such out-of-phase turbulent variables between different altitudes can be used to explain the complicated turbulence intermittency in the other experiments such as U6.

Returning to the discussion on the vertical profile of ϕ_M in S2–S6, when L is short, the modeled wind shear varies with time (Figs. 5 and 6), which leads to a temporary deviation of ϕ_M from the BD relationship. Even so, the time-averaged ϕ_M in S2–S6 is still close to the BD relationship (Fig. 3).

d. Modeling the second kind of deviation of the BD relationship in a stable ABL

Experiments S2TS and S2TW are designed to simulate the second kind of deviation of the BD relationship. The deviation is caused by asynchronous changes in the friction velocity and geostrophic wind. To be specific, when the friction velocity and geostrophic wind speed satisfy a one-to-one relationship, the ABL stays at MOST and thus the BD relationship is maintained. Otherwise, the BD relationship is violated and the ABL deviates from MOST, exhibiting as the hockey stick transition (HOST; Sun et al. 2012, 2016) where the inner and outer regimes of ABL turbulence are separable and interact with each other (Hutchins and Marusic 2007a,b; Mathis et al. 2009; Salesky and Anderson 2018; see appendix A for details).

The one-to-one relationship between the friction velocity and geostrophic wind is hidden in the Ekman layer model,

which is shown by Eq. (A.2) in appendix A. Since the Ekman layer model employs a constant turbulent viscosity, the one-to-one relationship is replaced by a dimensionless number that is derived with a two-layer model in appendix A.

The dimensionless number, representing the approximate ratio between the wind speeds at the bottoms of the inner and outer regimes, is expressed as (see appendix A)

$$N_s = [fk_0 z_0 (U_g^2 + V_g^2)/u_*^3]^{1/2} \phi_M. \quad (4.5)$$

FIG. 3. Dimensionless wind shear ϕ_M vs dimensionless height z/L from the BD relationship (black solid line) and the 14 experiments with $N_s = 0.085$ – 0.087 in Table 2.

FIG. 4. Vertical profiles of the turbulent viscosity from experiments STABLE (blue) and UNSTABLE (red line) and from the Leipzig observations (black circles). Golden and green lines represent the two experiments that take the same setup as UNSTABLE except for the bottom altitude of the stable layer overlaid (or number beside each dashed line).

Obviously, the number N_s depends on three kinds of variables: 1) macrovariables of geostrophic wind and the Coriolis parameter, 2) microvariable of friction velocity, and 3) ground variable of roughness length. The number can be used to express the consistency between the macro and microvariables

over a given ground surface. When $N_s = 0.085$ (where 0.085 is obtained with the Leipzig dataset), the macro and microvariables are consistent (i.e., the turbulence in the inner regime is in equilibrium with that in the outer regime) and thus the ABL stays at MOST. Otherwise, the ABL deviates from MOST, exhibiting as HOST where the turbulence in the inner regime is not in equilibrium with that in the outer regime.

To simulate the second kind of deviation of the BD relationship, S2TS and S2TW are carried out that use the same setup as S2 except for $u_* = 1.3 \text{ m s}^{-1}$ (strong) and 0.35 m s^{-1} (weak surface turbulence), respectively. Thus, S2TS, S2, and S2TW hold $N_s = 0.03$, 0.085 , and 0.214 , respectively. Their wind speed and wind shear are displayed in Fig. 7. When $N_s = 0.085$, the modeled ϕ_M is close to the BD relationship. When $N_s \neq 0.085$, the modeled ϕ_M deviates from the BD relationship, which resembles the two-layer model of HOST.

To further test the second kind of deviation, an additional two experiments are carried out that use other model parameters but maintain $N_s = 0.085$. To be specific, S7 uses $U_g = 7 \text{ m s}^{-1}$ and $u_* = 0.35 \text{ m s}^{-1}$, whereas S8 uses $z_0 = 0.2 \text{ m}$ and $u_* = 0.57 \text{ m s}^{-1}$. Although their parameters are different from those of the Leipzig dataset, their modeled ϕ_M is still close to the BD relationship (Fig. 3), which contrasts the experiments with $N_s \neq 0.085$, too.

In summary, when the model parameters satisfy $N_s = 0.085$, the modeled ϕ_M is close to the BD relationship. Once a sudden change of geostrophic wind occurs, N_s deviates from 0.085 and thus the turbulence in the inner regime deviates from its equilibrium with the wind shear caused by the geostrophic wind, bringing about a deviation of ϕ_M from the BD relationship. On the other hand, since the friction velocity u_* is a microscale variable and responds to the change in geostrophic wind, the equilibrium in turbulence between the inner and outer regimes (or $N_s = 0.085$) is then reinstated gradually so that ϕ_M becomes close to the relationship again (e.g., Cheng et al. 2002), which is consistent with the statistical correlation

TABLE 2. List of the ABL experiments.

Expt	L (m)	U_g (m s^{-1})	z_0 (m)	u_* (m s^{-1})	N_s	Max height of display in Fig. 3 (m)
STABLE	580	17.5	0.3	0.65	0.085	600
S2	300	17.5	0.3	0.65	0.085	465
S3	200	17.5	0.3	0.65	0.085	225
S4	100	17.5	0.3	0.65	0.085	87
S5	50	17.5	0.3	0.65	0.085	39
S6	20	17.5	0.3	0.65	0.085	15
S2TS	300	17.5	0.3	1.30	0.030	—
S2TW	300	17.5	0.3	0.35	0.214	—
S7	300	7.0	0.3	0.35	0.085	291
S8	300	17.5	0.2	0.57	0.085	498
UNSTABLE	−580	4.5	0.2	0.34	0.087	180
U2	−300	4.5	0.2	0.34	0.087	120
U3	−200	4.5	0.2	0.34	0.087	93
U4	−100	4.5	0.2	0.34	0.087	48
U5	−50	4.5	0.2	0.34	0.087	27
U6	−20	4.5	0.2	0.34	0.087	12
U2TS	−300	4.5	0.2	0.50	0.049	—
U2TW	−300	4.5	0.2	0.17	0.246	—

FIG. 5. Times series of (top to bottom) TKE, ε , turbulent viscosity, and wind shear, respectively, at altitude $z = 4.5$ (black), 16.5 (red), 19.5 (green), and 28.5 m (blue) in experiment S6 with the Obukhov length $L = 20$ m. The shaded area marks a quiescent period for analysis.

between the observed ϕ_M and the BD relationship (Baas et al. 2006; Salesky and Chamecki 2012).

e. Modeling the deviation of the BD relationship in convective ABLs

Eight experiments are designed to reveal the sensitivity of ϕ_M to model parameters in unstable ABLs (see Table 2 for experiment

parameters). Of the eight experiments, U2–U6 take the same setup as UNSTABLE (or $N_s = 0.087$) except for the Obukhov length $L = -0.3, -0.2, -0.1, -0.05$, and -0.02 km, respectively. Their vertical profiles of ϕ_M are displayed in Fig. 3, showing that the profiles are close to the BD relationship near the surface.

To examine the deviation of ϕ_M from the BD relationship at $N_s \neq 0.085$, Experiments U2TS and U2TW

FIG. 6. As in Fig. 5, but in experiment S4 with the Obukhov length $L = 100$ m. The shaded area marks a period from very weak to significant turbulence.

are carried out that use the same setup as U2 except for $u_* = 0.5$ (strong) and 0.17 m s^{-1} (weak surface turbulence), respectively. Hence, U2TS, U2, and U2TW hold $N_s = 0.049$, 0.087 , and 0.246 , respectively. Their wind shears are displayed in Fig. 8, showing that the modeled ϕ_M is close to the BD relationship when $N_s \approx 0.085$ but not when N_s is away from 0.085 .

In addition, the modeled deviation of ϕ_M from the BD relationship varies with atmospheric stability. The comparison between Figs. 8 and 7 shows that, given a deviation of N_s from 0.085 , the modeled deviation of ϕ_M from the BD relationship in an unstable ABL is much stronger than in a stable ABL. Such sensitivity of the deviation is consistent with the observations of Salesky and Chamecki (2012)

FIG. 7. (left) Dimensionless wind shear ϕ_M vs z/L and (right) wind speed vs height in experiments S2 ($u_* = 0.65 \text{ m s}^{-1}$; blue), S2TS ($u_* = 1.3 \text{ m s}^{-1}$; red), and S2TW ($u_* = 0.35 \text{ m s}^{-1}$; green). Black line represents the BD relationship.

that the deviation is stronger during daytime than during nighttime.

f. Comparison to the standard $k-\epsilon$ model

In contrast to the proposed one, the standard $k-\epsilon$ model outputs ϕ_M with bias, which is shown with a new experiment. Since the conventional buoyancy representation in the standard $k-\epsilon$ model is equivalent to (2.13) with $C_6 = C_7 = C_8 = 0$ (Duynkerke 1988; Apsley and Castro 1997), the new experiment takes the same setup as U2 except for $C_6 = C_7 = C_8 = 0$. Its wind shear ϕ_M is compared with the BD relationship in Fig. 9. Generally speaking, the experiment generates too large of a wind shear, whereas the proposed turbulence model (or U2) generates a reasonable wind shear near the surface.

The excessive wind shear in the experiment, as shown by the right panel of Fig. 9, is caused by the insufficient turbulent viscosity (or vertical mixing), which is understandable from its thermal representation. That is, the standard $k-\epsilon$ model overlooks thermals (or mistreats them as regular eddies), leading to the insufficient vertical mixing and subsequently excessive wind shear near the surface (see appendix B for details).

5. Conclusions and discussion

a. Remarks

The $k-\epsilon$ turbulence model for the stable atmosphere (Z20) is extended for the unstable (or convective) ABL. The new model represents the buoyancy-induced increase in the kinetic energy and scale of eddies. Its theoretical analysis shows that it is consistent with MOST (Fig. 1).

The turbulence model is further introduced into an ABL model with the Coriolis force, addressing the sensitivity of vertical wind shear to model parameters. The ABL model results are then compared with the BD relationship, which is summarized as follows:

- The modeled vertical wind shear ϕ_M is close to the BD relationship near the surface, when $N_s = 0.085$ or the turbulence in the inner regime is in equilibrium with that in the outer regime.
- The modeled ϕ_M deviates from the BD relationship when $N_s \neq 0.085$ (or the turbulence in the inner regime is not in

FIG. 8. Dimensionless wind shear ϕ_M vs z/L in experiments U2 ($u_* = 0.34 \text{ m s}^{-1}$; red), U2TS ($u_* = 0.5 \text{ m s}^{-1}$; blue), and U2TW ($u_* = 0.17 \text{ m s}^{-1}$; green). Black line represents the BD relationship.

FIG. 9. (left) Dimensionless wind shear ϕ_M vs z/L and (right) turbulent viscosity vs height in U2 (or present turbulence model; red) and an experiment (blue) that takes the same setup as U2 except for the conventional buoyancy representation with no thermals (or $C_6 = C_7 = C_8 = 0$). Black line represents the BD relationship.

equilibrium with that in the outer regime), which resembles the HOST model of Sun et al. (2012, 2016).

- When the Obukhov length is short, the ABL model replicates turbulence intermittency that in turn brings about a temporary deviation of ϕ_M from the BD relationship. Even so, the time-averaged ϕ_M is still close to the BD relationship.

In the present experiments, the friction velocity u_* is treated as an independent input variable. In fact, u_* varies in response to the changes of other model parameters such as geostrophic wind and surface parameters. Such response is not discussed in this paper and needs to be explored in the future.

b. Turbulence parameterization versus convection representation

Since turbulence and convection are different processes in the ABL, their parameterizations are different too, though they may work together as subgrid processes in an atmospheric model. Consider, for example, a fluid confined between two horizontal surfaces maintained at constant temperature (see Emanuel 1994, p. 47). Once its Rayleigh number exceeds a critical value, convective cells arise even without turbulence. The convective cells, as a result, transport momentum and heat vertically. If the cells are treated as subgrid players in an atmospheric model, they should be represented via a convection parameterization. In this case, the parameterization is independent of turbulence and related to the linear equations of convection, to some extent.

In contrast, the turbulence parameterization is introduced to represent the vertical transport of momentum caused by turbulent eddies. Since the parameterization is related to wind shear via the nonlinear terms of momentum advection, it can exist even without convection (e.g., in a neutral atmosphere).

In fact, convective cells and turbulent eddies usually coexist as subgrid players in an atmospheric model. Their effects thus should be represented via their parameterizations, respectively. On the other hand, convective cells interact with turbulent eddies, making the parameterizations complicated. Hence, some subgrid parameterizations are proposed that combine the two parameterizations together (Soares et al. 2004; Siebesma et al. 2007; Suselj et al. 2019).

With the advance in computational power, high-resolution models with explicit representation of convective cells become possible. To be specific, in a three-dimensional high-resolution model (usually referred to as an eddy-resolving model), convective cells are resolved whereas turbulent (or subgrid) eddies are represented via a turbulence parameterization (e.g., Deardorff 1974; Nakanishi 2001). Since the model results are sensitive to the turbulence (or subgrid) parameterization, it is interesting to develop a proper turbulence parameterization (Deardorff 1974; Cuijpers and Duynkerke 1993; van Zanten et al. 2011). In this study, a new candidate of the turbulence parameterization is proposed for high-resolution models, and it does not include the effects of convective cells on the vertical transport of momentum and heat.

The proposed turbulence parameterization is calibrated with MOST. Since MOST works in a very thin layer near the ground surface and convective cells are rare there (because of the restraint of ground surface), the ABL model in section 4 is thus used to test the proposed turbulence parameterization against MOST. On the other hand, the ABL model cannot be compared with observations above the thin layer, because it overlooks the effects of convective cells and other synoptic factors (see Fig. 4 for an example). In the future, the proposed turbulence parameterization will be implemented into a three-dimensional high-resolution ABL model to simulate convective

FIG. A1. Schematic for the independent effects of geostrophic wind and surface parameters on the momentum flux u_*^2 at $z = H$.

cells and then replicate the phenomena in the upper portion of the ABL, such as the overshooting of thermals or counter-gradient diffusion.

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APPENDIX A

Consistency between the Micro- and Macroparameters

The surface layer, as suggested by the observations (Acevedo and Fitzjarrald 2003; Sun et al. 2012, 2016), is often decoupled with the upper layer of the atmospheric boundary layer (ABL), because the former is influenced strongly by the cooling/heating ground surface. The dramatic transition from the former to the latter resembles a hockey stick and thus is referred to as the hockey stick transition (HOST; Sun et al. 2012, 2016). Such separation of the two regimes exists even in neutral and unstable ABL (Hutchins and Marusic 2007a,b; Mathis et al. 2009; Salesky and Anderson 2018). In this appendix, a simple two-layer model is proposed to discuss the consistency among the model parameters of u_* , z_0 , and U_g .

The ABL, as shown in Fig. A1, is divided into two layers: the surface layer with depth H and the layer above (referred to as the Ekman layer). The upper layer is dominated by large eddies (Hutchins and Marusic 2007b; Sun et al. 2016). If it has a constant turbulent viscosity ν_t , its analytic solution of the Ekman spiral is obtained (Stull 1988, p.211), yielding the wind shear

$$S_+ = [f(U_g^2 + V_g^2)/\nu_t]^{1/2} \quad (\text{A.1})$$

and the constraint

$$u_*^2 = [f(U_g^2 + V_g^2)/\nu_t]^{1/2} \quad (\text{A.2})$$

at $z = H$. As the two equations show, S_+ is directly proportional to the bulk wind shear in the ABL and the microscale variable of u_* is constrained by the macro variable of geostrophic wind.

On the other hand, the surface layer is dominated by small eddies due to the restriction of ground surface and is influenced by the cooling/heating ground surface that works independently of geostrophic wind (Acevedo and Fitzjarrald 2003; Sun et al. 2016). Since u_* varies with ground surface parameters (e.g., roughness length z_0 , and ground surface temperature T), its change is not often synchronized with the change in geostrophic wind, violating (A.2) and thus bringing about a two-layer model just as HOST.

The violation of (A.2) is discussed next, beginning with the formulas in section 3 for the surface layer. In the surface layer, the turbulent viscosity

$$\nu_t = u_* k_0 H / \phi_M(H) \quad (\text{A.3})$$

and the wind shear

$$S_- = u_* \phi_M(H) / (k_0 H) \quad (\text{A.4})$$

at $z = H$, which are obtained from (3.1) and (4.4), respectively.

As shown by (A.4) and (A.1), the surface layer requires that the wind shear at $z = H$ is determined by the microparameter of u_* whereas the Ekman layer requires that the wind shear is determined by the macrovariable of geostrophic wind U_g (Fig. A1). Hence, there is a difference between S_+ and S_- that can be treated as an inconsistency between u_* and U_g .

The inconsistency between u_* and U_g , once formed, is diminished gradually via large eddies. Large eddies transport momentum and heat efficiently between the surface layer and the upper layer, and thus mix them sufficiently. As a result, the ABL reinstates its steady state so that $S_+ = S_-$. The steady state of $S_+ = S_-$, with the aid of (A.2) and (A.3), is expressed with

$$S_+/S_- = N_s (u_* k_0 z_0 / \nu_t)^{1/2} H / z_0 = 1, \quad (\text{A.5})$$

where the dimensionless number

$$N_s = [fk_0 z_0 (U_g^2 + V_g^2)/u_*^3]^{1/2} / \phi_M(H). \quad (\text{A.6})$$

The number N_s can be explained using physics. Constraint (A.5) is rewritten as

$$N_s = z_0 (u_* k_0 z_0)^{-1/2} / (H \nu_t^{-1/2}) \quad (\text{A.7})$$

where $u_* k_0 z_0$ represents the turbulent viscosity at $z = z_0$. In (A.7), the numerator $z_0 (u_* k_0 z_0)^{-1/2}$ and the denominator $H \nu_t^{-1/2}$ are proportional to the wind speed at $z = z_0$ and H , respectively, if (A.1) was extended downward to cover the surface layer. In other words, N_s represents the approximate ratio between the wind speeds at the bottoms of the two layers.

Since ν_t varies vertically, N_s cannot be determined by (A.7). Instead, N_s is determined with (A.6) and the Leipzig dataset if the dataset is treated as an ideal case of Monin–Obukhov similarity theory (MOST) (Mildner 1932; Lettau 1950; Sundararajan 1979). To be specific, after taking $H = 0.1|L|^{A1}$ (A.6) becomes

$$N_s = 0.68 [fk_0 z_0 (U_g^2 + V_g^2)/u_*^3]^{1/2} \quad (\text{A.8a})$$

for stable ABL and

$$N_s = 1.26 [fk_0 z_0 (U_g^2 + V_g^2)/u_*^3]^{1/2} \quad (\text{A.8b})$$

for unstable ABLs, with the aid of (4.3). Substituting the parameters of the Leipzig dataset into (A.8a) yields

$$N_s = 0.085. \quad (\text{A.9})$$

In summary, when the model parameters of z_0 , U_g , and u_* satisfy (A.9), the turbulence in the surface layer is in equilibrium with that in the upper one, yielding MOST and the Businger–Dyer (BD) relationship; otherwise, the surface layer is decoupled with the upper one, yielding HOST.

The proposed connection between (A.9) and the BD relationship is tested in section 4b. When the experiments (e.g., S2) take $N_s = 0.085$, their modeled dimensionless wind shear is close to the BD relationship near the ground surface (Fig. 3). When the experiments (e.g., S2TS and S2TW) take N_s away from 0.085, their dimensionless wind shear deviates from the BD relationship, exhibiting the characteristics of HOST (Figs. 7 and 8).

APPENDIX B

Buoyancy-Induced Increase in the Taylor Microscale

The standard k – ε model takes only the first term on the right-hand side of (2.13) (Duynkerke 1988; Apsley and Castro

1997). This appendix shows why the standard k – ε model is not suitable for an unstable region and why the second term in (2.13) is imperative.

Turbulence in the convective ABL is different from that in the neutral ABL. It is not isotropic. It is usually organized into thermals and plumes (Kaimal et al. 1976; Lenschow and Stephens 1980; Young 1988; Niemela et al. 2000; Salesky and Anderson 2018). Updraft curtains in the surface layer usually merge to form large updrafts, and large eddies can reach a scale of the mixing-layer depth (Stull 1988). As a result, the mean eddy scale has a tendency to become large in convective ABLs. This upscale tendency (or buoyancy-induced increase in mean eddy scale) is different from the downscale energy cascade in the inertial subrange, and is represented by the second term on the right-hand side of (2.13), which is discussed mathematically in the following paragraphs.

Taking the logarithm of (2.14) first, then differentiating the resulting equation with respect to time, and finally substituting (2.1) and (2.4) into the resulting equation yields an equation on the increase of the Taylor microscale or mean eddy scale, or

$$\begin{aligned} \frac{2d\lambda_m}{\lambda_m dt} - \frac{1}{k} \nabla \cdot \left(\frac{\nu_t}{\sigma_k} \nabla k \right) + \frac{1}{\varepsilon} \nabla \cdot \left(\frac{\nu_t}{\sigma_\varepsilon} \nabla \varepsilon \right) \\ = \left(C_\mu \eta - C_1 + \frac{C_2 - 1}{\eta} \right) S \\ + \left[(C_7 \eta + C_8 \eta^2 - C_6) \min \left(1, \sqrt{\frac{|R_t|}{C_9}} \right) - (C_3 - 1) \frac{C_\mu N}{\sigma_\theta S} \right] N, \end{aligned} \quad (\text{B.1})$$

with the aid of (2.6) and

$$G = \sigma_\theta^{-1} N^2. \quad (\text{B.2})$$

The second term on the right-hand side of (B.1) represents the contribution of buoyancy to λ_m , showing that λ_m increases with time due to buoyancy if

$$(C_7 \eta + C_8 \eta^2 - C_6) \min \left(1, \sqrt{\frac{|R_t|}{C_9}} \right) > (C_3 - 1) \frac{C_\mu N}{\sigma_\theta S} \eta. \quad (\text{B.3})$$

However, the standard k – ε model uses $C_3 = 1.46$ and $C_6 = C_7 = C_8 = 0$ (Duynkerke 1988; Apsley and Castro 1997) and thus does not satisfy (B.3), indicating that the mean eddy scale always decreases with time due to buoyancy. In other words, the standard k – ε model overlooks the buoyancy-induced increase in eddy scale and mistreats thermals as regular eddies. In contrast, the present model with the parameters in Table 1 satisfies (B.3) when η is large, suggesting that the model can represent the buoyancy-induced increase in the mean eddy scale.

Umlauf and Burchard (2005) proposed that C_3 could be negative for unstable stratification in the standard k – ε model. Such conjecture is consistent with the proposed ε equation, because (B.3) requires $C_6 - C_7 \eta - C_8 \eta^2 < 0$ when η is large.

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^{A1} In strict terms, H is defined as an altitude to separate the inner and outer regimes in the boundary layer (Hutchins and Marusic 2007a,b). Observations and modeling suggest that H corresponds to the midpoint of the log layer in neutral ABLs (Mathis et al. 2009) and is proportional to $|L|$ in unstable ABL (Salesky and Anderson 2018). Based on the studies, $H \approx 0.1|L|$ is proposed in the two-layer model.

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