A Two-Moment Bulk Parameterization of the Drop Collection Growth in Warm Clouds

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ABSTRACT

To improve the modeling of warm rain initiation, a two-moment bulk parameterization of the drop collection growth in warm clouds is developed by two steps: (i) its prototype is first derived based on the analytic solution of the stochastic collection equation (SCE) with the Golovin kernel, and (ii) the prototype is then revamped empirically to fit the numerical solution of SCE with the real hydrodynamic collection kernel, reaching the final version of the parameterization. Since the final version represents the self-collection of cloud drops explicitly, it replicates warm rain initiation well even when liquid water content (cloud-drop number concentration) is very low (high). It also replicates the autoconversion threshold and time delay of rain initiation via a small autoconversion rate.

1. Introduction

The climate models usually generate “too few and too bright” warm clouds in the tropical boundary layer (Nam et al. 2012). This modeling problem may originate in our insufficient knowledge of warm rain initiation (Rogers and Yau 1989; Blyth 1993). To be specific, the current cloud models cannot explain two phenomena: (i) warm clouds rain so quickly (in 30 min or less; Squires 1958), and (ii) precipitation can occur in warm clouds with low liquid water content (e.g., 0.5 g m^{-3} ; Rauber et al. 2007). This modeling problem can be mitigated by introducing plausible processes that broaden the cloud-drop spectrum, such as ultragiant aerosols (Johnson 1982; Blyth et al. 2003), cloud mixing (Latham and Reed 1977; Telford et al. 1984), and small-scale turbulence (Jonas 1996; Shaw 2003).

A new candidate of the processes is the radiative-cooling-induced broadening of cloud-drop spectrum that works just as dew drops form [Roach 1976; Austin

et al. 1995; Harrington et al. 2000; Zeng 2008; Brewster and McNichols 2018; Zeng 2018a; see Zeng (2018b) for review]. The spectrum broadening usually occurs near cloud top, just as observed (Small and Chuang 2008), because cloud drops there emit infrared radiation to space and thus undergo radiative cooling (Zeng 2008, 2018a). Since the spectrum broadening usually occurs near cloud top, it can accelerate precipitation initiation in all clouds, at least theoretically (Zeng 2018a). Hence its representation can be used to mitigate the too-few-and-too-bright problem in climate models.

However, the spectrum broadening cannot be incorporated into a climate model by bin representation because of its high computational expense, and therefore its parameterization is imperative. Hence, the parameterization needs to explicitly represent the basic spectrum broadening induced by gravitational drop collection (i.e., self-collection of cloud drops) first, which motivates this study.

The current parameterizations of warm rain initiation basically follow the general philosophy of Kessler (1969) in which total liquid water is divided into two parts: cloud water and rainwater (e.g., Liu and Daum 2004; Morrison et al. 2005; Liu et al. 2006; Thompson et al. 2008; Lee and Baik 2017). They represented the following four processes either explicitly or implicitly:

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- 1) accretion of cloud water by rainwater,
- 2) autoconversion of cloud water to rainwater as a result of cloud-drop self-collection (i.e., a cloud drop collecting another one becomes a raindrop),
- 3) self-collection of cloud drops (i.e., a cloud drop collecting another one remains in the same group of cloud drops), and
- 4) self-collection of raindrops (i.e., a raindrop collecting another one remains in the same group of raindrops).

Kessler (1969) proposed a one-moment parameterization to explicitly represent the accretion and autoconversion, but not the two self-collection processes, although the two self-collection processes broaden cloud-drop and raindrop spectra, respectively, and thus impact the autoconversion indirectly. In comparison to the Kessler parameterization (as well as other one-moment ones), a two-moment (or higher-order moment) parameterization can represent all of the four processes explicitly (Tzivion et al. 1987).

The current two-moment parameterizations emphasize the four processes differently for their own applications. Using the numerical results of the stochastic collection equation (SCE), Lee and Baik (2017) evaluated five parameterizations, which include their scheme and four other ones: Berry and Reinhardt (1974), Khairoutdinov and Kogan (2000), Liu and Daum (2004), and Seifert and Beheng (2006). In all the parameterizations except for Seifert and Beheng (2006), the cloud water content decreases almost linearly with time, indicating that the parameterizations represented a time-averaged effect of the autoconversion (or the self-collection of cloud drops) rather than an instantaneous one, which simplifies the autoconversion into a linear process (see Fig. 9 of Lee and Baik 2017). Moreover, all the parameterizations represented cloud-drop number with noticeable errors especially in cases of high cloud-drop number concentration (or small mean cloud-drop size) (see Fig. 8 of Lee and Baik 2017), which suggests that the autoconversion or the self-collection of cloud drops needs a better representation.

The self-collection of cloud drops was first represented as a time delay in rain initiation in an air parcel (Cotton 1972). Such technique of time delay is no longer in use, because it is difficult to trace an air parcel in a three-dimensional model (Tripoli and Cotton 1980). Another technique was proposed by Seifert and Beheng (2001, 2006) that tuned the parameterization to fit SCE results, but still needs to be improved especially when cloud-drop number concentration is high (Lee and Baik 2017). In this paper, a new technique is proposed to explicitly represent the self-collection of cloud drops in terms of mean cloud-drop size.

The paper consists of five further sections. Section 2 presents a numerical simulation of SCE, showing how the self-collection of cloud drops benefits the autoconversion. Section 3 proposes a new parameterization based on a dimensionless form of SCE with an analytic solution. Section 4 revamps the parameterization based on the SCE results with the real hydrodynamic collection kernel. Section 5 tests the parameterization with a bin model of an air parcel, and section 6 summarizes.

2. Bin model simulation

In this section, a bin model simulation of SCE is carried out to show how the self-collection of cloud drops benefits the autoconversion (see Table 1 for other simulations). The simulation, in comparison to the previous ones (e.g., Berry and Reinhardt 1974; Tzivion et al. 1987; Zeng 2018a), focuses on the increase in mean cloud-drop size induced by drop collection during warm rain initiation.

The bin model of Zeng (2018a) is used herein to replicate drop collection growth. Its spectrum of liquid drops is described by $n(m, t)$, where t is time, m is drop mass, and $n(m, t)dm$ represents the number of drops with mass between m and $m + dm$ per unit volume of air. The spectrum is governed by SCE or

$$\frac{\partial n(m, t)}{\partial t} = \frac{1}{2} \int_0^m K(u, m-u) n(u, t) n(m-u, t) du - n(m, t) \int_0^\infty K(m, u) n(u, t) du, \quad (2.1)$$

where $K(m, u)$ is the collection kernel of two drops with mass m and u for hydrodynamic capture. The kernel is expressed as

$$K(m, u) = \pi(r_m + r_u)^2 E(m, u) |V_T(m) - V_T(u)|, \quad (2.2)$$

where r_m and r_u are the radii of drops with mass m and u , respectively; $V_T(m)$ is the terminal velocity of a drop with mass m ; and $E(m, u)$ is the collection efficiency of a drop of mass m with a drop of mass u .

The model is developed based on the method of Kovetz and Olund (1969). When its bin number is ~ 40 , it generates excessive large droplets that in turn bring about a large error of the autoconversion (Scott and Levin 1975). When its bin number is ~ 1000 , the error is reduced greatly and its result almost coincides with the analytic solution of SCE (Zeng 2018a). In this study, the bin model is used to simulate the autoconversion, self-collection of cloud drops, accretion, and self-collection of raindrops with 1024 bins.

The real hydrodynamic collection kernel is obtained using (2.2) with the drop terminal velocity V_T coming

TABLE 1. List of the numerical experiments.

Expt	Microphysics	LWC (g m^{-3})	p (hPa)	T ($^{\circ}\text{C}$)	z (km)	Notes
C075/P075	Bin/parameterization	0.75	900	20	1	Default experiments
C05/P05	Bin/parameterization	0.5	900	20	1	Sensitivity to LWC
C10/P10	Bin/parameterization	1.0	900	20	1	
C15/P15	Bin/parameterization	1.5	900	20	1	
C20/P20	Bin/parameterization	2.0	900	20	1	
C30/P30	Bin/parameterization	3.0	900	20	1	
C075P6	Bin	0.75	600	1.5	4.4	Sensitivity to height
C075P4	Bin	0.75	400	-20	7.5	
P075DT	Parameterization	0.75	900	20	1	Time step of 10 s
P30B	Parameterization	3.0	900	20	1	Sensitivity to raindrop breakup

from Beard (1976) and the collection efficiency E from de Almeida (1977) complemented by Mason (1971). The initial drop spectrum is expressed as

$$n(m, 0) = (N_0/m_0) \exp(-m/m_0), \quad (2.3)$$

where N_0 is the initial total number of drops per unit volume of air and m_0 the initial mean mass of drops. Suppose that $n(r, t)dr$ represents the number of drops with radius between r and $r + dr$ per unit volume of air. Since $n(r, t)dr = n(m, t)dm$, (2.3) is changed to

$$n(r, 0) = 4\pi(N_0/m_0)\rho_l r^2 \exp(-m/m_0),$$

where ρ_l is the density of liquid water, which is a Weibull distribution and is similar to the observed cloud-drop spectrum [Pruppacher and Klett 1997, (2-3) on p. 26].

Obviously, the liquid water content (LWC) per unit volume of air in (2.3),

$$L = N_0 m_0, \quad (2.4)$$

which does not change during drop collection. In addition, the model discretizes drop mass with 1024 bins. That is,

$$m_k = m_1 2^{(k-1)/20}, \quad k = 1, 2, \dots, 1024, \quad (2.5)$$

where $m_1 = 3.2 \times 10^{-17}$ kg, which corresponds to a radius of $0.2 \mu\text{m}$. Equation (2.1) is integrated with a time step of 0.25 s.

Experiment C075 is carried out to mimic an air parcel with $L = 0.75 \text{ g m}^{-3}$ at altitude $z = 1 \text{ km}$ where pressure $p = 900 \text{ hPa}$ and temperature $T = 20^{\circ}\text{C}$. Just like other experiments in Table 1, it neglects the drop sedimentation in an air parcel for two considerations. 1) Cloud drops have so small terminal velocity that their vertical advection is negligible. As a result, their sedimentation has little effect on the autoconversion rate. 2) Raindrops have large terminal velocity and thus their vertical advection

can change raindrop spectrum obviously (Tzivion et al. 1989). Since the real hydrodynamic collection kernel for raindrops is close to the Golovin form, the accretion is directly proportional to rainwater content but almost independent of raindrop spectrum [see (3.14) and (3.15) for further discussion]. Thus, raindrop sedimentation changes the accretion mainly via rainwater content and its effect on the autoconversion rate can be ignored.

Figure 1 displays the result of C075, showing an evolution of drop spectrum in 60 min. Generally speaking, rain forms in $\sim 40 \text{ min}$. To quantitatively analyze the rain formation, all drops are classified into two kinds: cloud drops with mass smaller than m^* and raindrops with mass larger than m^* , where the mass threshold m^* corresponds to a drop of radius $28 \mu\text{m}$ (see section 3b for the selection of m^*). Figure 2 displays the cloud and rainwater contents against time with a logarithmic scale, showing that the rainwater content increases from 20 to 40 min exponentially. In other words, little rain forms in the first 20 min.

To analyze the processes in the first 20 min, Fig. 3 displays the number of cloud drops (or $\int_0^{m^*} n dm$) and their mean cloud-drop mass (or $\int_0^{m^*} mn dm / \int_0^{m^*} n dm$)

FIG. 1. Evolution of mass density $dM(\ln r)/d\ln r$ vs radius r in experiment C075, where $LWC = 0.75 \text{ g m}^{-3}$ and $M(\ln r)$ is the mass of drops with radius shorter than r . Thick line denotes the initial spectrum; time interval between lines is 10 min.

FIG. 2. Water contents of cloud drops (solid) and raindrops (dashed) vs time in experiment C075.

versus time. In the first 30 min, the number of cloud drops decreases with time due to the self-collection of cloud drops, and thus the mean cloud-drop mass increases correspondingly. Quantitatively speaking, the mean drop mass increases by 0.42% in 30 min, which corresponds to an increase in mean volume drop radius from 10.000 to 10.014 μm and an occurrence of the tail of cloud-drop spectrum at 30 min in Fig. 1. *Although the increase in mean drop mass looks small, it is a result of the self-collection of cloud drops.* Next, the increase is represented explicitly in a parameterization to embody the self-collection of cloud drops, which is different from its implicit representations (e.g., the threshold value of cloud water content) in Kessler (1969) and others.

3. Prototype of the proposed parameterization

In this section, a prototype of microphysics parameterization for warm clouds is developed based on a dimensionless form of SCE with an analytic solution. Since the dimensionless form of SCE is independent of cloud parameters, the prototype can be “universal” or independent of cloud parameters, providing a basic skeleton to develop the final form of the parameterization.

FIG. 3. Cloud-drop number (solid) and mean cloud-drop mass (dashed) vs time in experiment C075.

a. Analytic solution of SCE

An analytic solution of SCE is obtained after the collection kernel takes the Golovin form of

$$K(m, u) = b(m + u) \quad (3.1)$$

that is close to the real hydrodynamic collection kernel, where b is constant (Golovin 1963; Scott 1968). Using the dimensionless variables

$$T = bN_0m_0t, \quad (3.2)$$

$$x = m/m_0, \quad (3.3)$$

$$\phi(x, T) = \frac{m_0}{N_0}n(m, t), \quad (3.4)$$

Equation (2.1) is rewritten into a dimensionless form of

$$\begin{aligned} \frac{\partial \phi(x, T)}{\partial T} = & \frac{1}{2} \int_0^x x \phi(x, T) \phi(x - u, T) du \\ & - \phi(x, T) \int_0^\infty (x + u) \phi(x, T) du, \end{aligned} \quad (3.5)$$

which is solved by (Golovin 1963; Scott 1968)

$$\phi(x, T) = \frac{(1 - \tau) \exp[-x(\tau + 1)]}{x\sqrt{\tau}} I_1(2x\sqrt{\tau}), \quad (3.6)$$

with the total number of drops per unit volume of air at time t

$$N(t) = N_0(1 - \tau), \quad (3.7)$$

where $\tau = 1 - \exp(-T)$ and I_1 is an integral for Bessel functions or

$$I_1(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos \theta} \cos \theta d\theta.$$

Equation (3.5) is dimensionless. Suppose that all drops are classified into cloud drops and raindrops by a threshold of dimensionless drop mass $x^* = m^*/m_0$. Thus, the expressions of autoconversion and accretion depend only on x^* and thus become universal in warm rain initiation, exhibiting a similarity between N_0 , LWC , and b via (3.2)–(3.4) (see Figs. 4–7 of section 4 for the similarity of the final parameterization between different cases). Next, the expressions are derived in terms of x^* , using (3.5)–(3.7).

b. Autoconversion formulation

Since drops with radius larger than $28\ \mu\text{m}$ can initiate warm rain effectively, they are referred to as the collision–coalescence initiators (CCIs) (Mason 1971; Johnson 1993; Small and Chuang 2008). In response, total liquid water is divided into cloud water and rainwater, using radius $28\ \mu\text{m}$ as a separation size. In other words, CCIs are classified as a part of rainwater

because they accrete cloud drops (with radius smaller than $28\ \mu\text{m}$) effectively just as other raindrops.

The generation of CCIs due to cloud-drop collection is called the autoconversion of cloud water to rainwater (AUTO). Suppose that m^* represent the mass of a drop with radius $28\ \mu\text{m}$ and the initial drop spectrum of $n(m, 0)$ contain no raindrops (or no drops with mass larger than m^*). The autoconversion rate is thus expressed as

$$\text{AUTO} = \frac{d}{dt} \int_{m^*}^{\infty} mn(m, t) dm \quad (3.8)$$

at $t \rightarrow 0$.¹ Using the dimensionless variables in (3.2)–(3.4), the preceding expression is changed to

$$\text{AUTO} = bL^2 \frac{d}{dT} \int_{x^*}^{\infty} x\phi(x, T) dx \quad (3.9)$$

with the aid of (2.4). Substituting (3.6) into (3.9) gives

$$\text{AUTO} \approx bL^2 \left\{ \exp(-T) \frac{d}{dT} \int_{x^*}^{\infty} x \frac{(1-\tau) \exp[-x(\tau+1)]}{x\sqrt{\tau}} I_1(2x\sqrt{\tau}) dx \right\}, \quad (3.10)$$

where $\tau \rightarrow 0$. Since the term in the bracket is a function of only x^* (or m^*/m_0), (3.10) can be denoted as

$$\text{AUTO} \approx bL^2 f(x^*),$$

where the universal function

$$f(x^*) = \lim_{\tau \rightarrow 0} \left\{ \exp(-T) \frac{d}{dT} \int_{x^*}^{\infty} x \frac{(1-\tau) \exp[-x(\tau+1)]}{x\sqrt{\tau}} I_1(2x\sqrt{\tau}) dx \right\}.$$

Theoretically, an accurate yet complicated expression of $f(x^*)$ can be obtained from the preceding equation first and then be revamped to incorporate the difference between the real collection kernel and the Golovin one. Since the expression needs to take an empirical revamp eventually, a simple yet approximate expression of $f(x^*)$ (or the prototype of the autoconversion rate) is estimated/assumed first based on its original expression. Once the simple expression of $f(x^*)$ is obtained, its accuracy is tested with the bin model simulations in section 4 eventually.

For a practical application, it is assumed that the autoconversion rate takes the following simple expression of x^{*-1} (or m_0/m^* the mean drop mass normalized by m^*):

$$\text{AUTO} = bL^2 \exp\left(-\frac{a}{x^{*-1} - c}\right), \quad (3.11)$$

where a , b , and c are constants to be determined by comparing (3.11) to the SCE results with the real

collection kernel (see section 4a). Expression (3.11) is further modified to take account of the difference between the real collection kernel and the Golovin one. The difference is discussed first via the collection kernel of Long (1974) that is closer to the real collection kernel than the Golovin form, especially for small drops. Specifically, the Long form is close to the Golovin form or $K(m, u) \propto m + u$ when the larger drop has a radius larger than $50\ \mu\text{m}$, and otherwise $K(m, u) \propto m^2 + u^2$.

To take account of the difference in collection kernel between $m + u$ and $m^2 + u^2$ at cloud-drop size, $bm_0/m^* = b/x^*$ is introduced to replace b in (3.11), yielding

¹ The autoconversion represents the generation of CCIs in bulk while a cloud-drop spectrum is given. Since its rate depends only on the spectrum rather than spectrum history, the time of the given spectrum is defined as $t = 0$ in (3.8) for the sake of convenience.

$$\text{AUTO} = \frac{b}{x^*} L^2 \exp\left(-\frac{a}{x^{*-1} - c}\right), \quad (3.12)$$

where b as well as a and c will be determined by comparing (3.12) with bin model results.

Similarly, the decrease in cloud-drop number concentration due to the self-collection of cloud drops is obtained based on (3.7), giving

$$\frac{b'}{x^*} N_0 L \left[1 + \exp\left(\frac{x^{*-1} - c}{a'}\right) \right] \quad (3.13)$$

with the aid of (2.4) and (3.2), where a' and b' are constants to be determined; the term in the bracket is introduced to represent the effect of mean drop size on the decrease in cloud-drop number during the self-collection.

c. Accretion formulation

Since the real collection kernel of raindrops is close to the Golovin form (3.1) (Long 1974), the accretion of cloud water by rainwater (ACCR) is expressed in terms of $n(m, t)$ and m^* as

$$\text{ACCR} = \int_0^{m^*} mn(m, t) \int_{m^*}^{\infty} b(m+u)n(u, t) du dm. \quad (3.14)$$

Since raindrop mass is much larger than cloud-drop mass (or $u \gg m$), (3.14) is simplified to

$$\text{ACCR} \approx b \int_0^{m^*} mn(m, t) dm \int_{m^*}^{\infty} un(u, t) du,$$

which is denoted as

$$\text{ACCR} \approx b L_c L_r, \quad (3.15)$$

where L_c and L_r are the water contents of cloud drops and raindrops, respectively. A similar expression of (3.15) was obtained by Ziegler (1985) and Seifert and Beheng (2001). Please note that (3.15) is almost independent of cloud-drop and raindrop spectra. Once raindrop mass is close to cloud-drop mass (or $u \sim m$), (3.14) is not accurate and thus needs an amendment (see section 4c for further discussion).

Similarly, the decrease in cloud-drop number during the accretion is expressed as

$$\begin{aligned} & \int_0^{m^*} n(m, t) \int_{m^*}^{\infty} b(m+u)n(u, t) du dm \\ & \approx b \int_0^{m^*} n(m, t) dm \int_{m^*}^{\infty} un(u, t) du, \end{aligned} \quad (3.16)$$

which is directly proportional to the cloud-drop number concentration and the rainwater content L_r and is almost independent of cloud-drop and raindrop spectra (Ziegler 1985; Seifert and Beheng 2001).

In summary, (3.15) and (3.16) work well for large raindrops (with mass $u \gg m^*$), because $u \gg m^* > m$ brings about the degeneration of the second integral of (3.14) into the corresponding part of (3.16). However, the expressions are not accurate for small raindrops (with mass $u \sim m^*$), especially right after the auto-conversion occurs, and thus need to be adjusted (see section 4c for their adjustment).

In addition, although the real hydrodynamic collection kernel is close to the Golovin form for raindrops, they are still different. Theoretically, b in (3.15) and (3.16) vary as functions of the difference between the real collection kernel and the Golovin form. Hence, the magnitudes of b in (3.15) and (3.16) need to be adjusted based on the bin model simulations with the real collection kernel.

4. Revamp of the parameterization

The preceding prototype of the parameterization was proposed based on the analytic solution of SCE with the Golovin collection kernel. Since the real hydrodynamic collection kernel is different from the Golovin form especially at small drop size (Long 1974), it needs to be revamped to match the real collection kernel. In this section, the prototype is revamped against the bin model simulations with the real collection kernel.

Consider an air parcel with unit mass (instead of unit volume in section 3) of air that contains clouds drops with number N_c and water content q_c , and raindrops with number N_r and water content q_r . Thus, its variables of cloud drops and raindrops change with time due to drop collection and breakup, which is described by

$$\frac{\partial q_c}{\partial t} = -A_q - C_q, \quad (4.1)$$

$$\frac{\partial N_c}{\partial t} = -S_c - C_n, \quad (4.2)$$

$$\frac{\partial q_r}{\partial t} = A_q + C_q, \quad (4.3)$$

$$\frac{\partial N_r}{\partial t} = \frac{1}{m^*} A_q - S_r + B_r, \quad (4.4)$$

using the following parameterization rates per unit mass of air: A_q is the autoconversion of cloud water to rainwater, C_q is the accretion of cloud water to rainwater, C_n

is the decrease of cloud-drop number due to the accretion of cloud drops by raindrops, S_c is the decrease of cloud-drop number due to the self-collection of cloud drops, S_r is the decrease of raindrop number due to the self-collection of raindrops, and B_r is the increase of raindrop number due to raindrop breakup. These rates depend on two dimensionless mean drop masses of cloud and rainwater or

$$\hat{m}_c = \frac{q_c}{m^* N_c}, \quad (4.5)$$

$$\hat{m}_r = \frac{q_r}{m^* N_r}. \quad (4.6)$$

Obviously, $\hat{m}_c < 1$ and $\hat{m}_r > 1$.

a. Autoconversion rate

The expression of A_q is obtained from (3.12). Since (3.12) is expressed per unit volume of air, it is changed to A_q per unit mass of air. Let ρ_a denote air density. Substituting L in (3.12) with $\rho_a q_c$ gives

$$A_q = \rho_a b \hat{m}_c q_c^2 \exp\left(-\frac{a}{\hat{m}_c - c}\right), \quad (4.7)$$

where $b_1 = 8 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$ and $c = 0.045$. When $q_r \rightarrow 0$ (or no rainwater), (4.10) degenerates into (4.7) except that the exponent 0.75 is introduced for a better fit of the bin model results.

The adjustment of (4.7) to (4.10) is introduced empirically so that the parameterization fits the bin model results. Figure 4 displays the modeled autoconversion A_q in C075. It also displays A_q in the parameterization that is computed with (4.10) and the values of $\hat{m}_c$, q_c , and q_r from C075. Generally speaking, the autoconversion in the parameterization agrees well with that in C075, showing that the empirical adjustment of (4.7) to (4.10) is good.

To test whether (4.10) is workable with other cloud-drop numbers and water contents, two experiments C10 and C05 are carried out that use the same setup as C075 except for $\text{LWC} = 1.0$ and 0.5 g m^{-3} , respectively. Their modeled autoconversion rates are also displayed in Fig. 4 against their corresponding ones in the parameterization, showing that (4.10) works well with other cloud water contents. The figure also exhibits a similarity of the parameterization between different cases just as that of the bin model results.

where $\hat{m}_c$ replaces x^{*-1} in (3.12) to represent the mean cloud-drop mass in the parameterization model.

CCIs collect large cloud droplets more efficiently than small ones, because the collection efficiency E of two droplets increases significantly with increasing the radius of the smaller droplet (Mason 1971; de Almeida 1977). As a result, once CCIs form, they decrease the mean cloud-drop mass (see Fig. 3), although the cloud-drop spectrum is still becoming broader and the autoconversion rate is becoming larger. Since a cloud-drop spectrum cannot be described completely by two parameters of N_c and q_c (Liu et al. 2007), other parameters are needed to describe the broadness of cloud-drop spectrum after CCIs form. Seifert and Beheng (2001) proposed a new parameter: the ratio of rainwater to cloud water,

$$\chi = q_r / q_c. \quad (4.8)$$

With the aid of χ , an effective mean cloud-drop mass

$$\hat{m}_{ca} = \frac{\hat{m}_c + 0.047\chi^{0.25}}{1 + \chi^{0.25}} \quad (4.9)$$

is introduced to approximately represent the cloud-drop spectrum broadness first. It is then used to adjust (4.7) to

$$A_q = \rho_a b_1 \hat{m}_c q_c^2 \frac{1}{0.032 + \chi} \left(\exp\left\{ - \left[\frac{0.016}{\max(\hat{m}_{ca} - c, 0.0001)} \right]^{0.75} \right\} + 0.024\chi \right), \quad (4.10)$$

b. Self-collection of cloud drops

The self-collection of cloud drops impacts the autoconversion indirectly via cloud-drop spectrum. To be specific, it decreases cloud-drop number by cloud-drop coalescence, which is described by S_c . Next, S_c is expressed in terms of q_c , N_c , and $\hat{m}_c$.

The expression of S_c is obtained just as that of A_q . First, substituting L in (3.13) with $\rho_a q_c$ and then adjusting the resulting expression just as (4.7) gives

$$S_c = \rho_a b' \hat{m}_c N_c q_c \left[1 + \exp\left(\frac{\hat{m}_c - c}{a'}\right) \right], \quad (4.11)$$

where the term in the brackets is introduced to represent the effect of the real collection kernel on cloud-drop spectrum.

Figure 4 displays the bin-modeled rate of the decrease of cloud-drop number due to the self-collection of cloud drops S_c in C10, C075, and C05. To fit the bin model results, (4.11) is adjusted to

$$S_c = \rho_a b_2 \hat{m}_c N_c q_c \left[1 + \exp\left(\frac{\hat{m}_{ca} - c}{0.0007} + \frac{6.2\chi}{1 + \chi}\right) \right], \quad (4.12)$$

FIG. 4. Comparison in A_q (autoconversion rate; red and black) and S_c (decrease rate of cloud-drop number due to cloud-drop self-collection; blue and green lines) between a bin model (solid) and its corresponding parameterization (dashed). Data for LWC = (left) 1.0, (middle) 0.75, and (right) 0.5 g m⁻³ come from C10, C075, and C05, respectively.

where $b_2 = 0.025 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$. Using (4.12) and the values of $\hat{m}_c$, q_c , and q_r from C075, S_c in the parameterization is computed and displayed in Fig. 4 for comparison. The good agreement in S_c between the bin model and the parameterization shows that the adjustment to (4.12) is good even while LWC takes different values.

c. C_q and C_n of the accretion

Raindrops accrete cloud drops and thus decrease the number and water content of cloud drops. Their accretion rate C_q is expressed in (3.15). Following the procedure from (3.12) to (4.7), $C_q = \rho_a b q_c q_r$ is obtained after substituting L_c and L_r in (3.15) with $\rho_a q_c$ and $\rho_a q_r$, respectively. To fit the bin-modeled C_q in C10, C075, and C05, the expression is adjusted to

$$C_q = \rho_a b_3 q_c q_r \left[1 + 3.2 \hat{m}_r^{-0.07} \left(\frac{\rho_{a0}}{\rho_a} \right)^{0.25} \exp(0.62 \sqrt{\hat{m}_r}) \right], \quad (4.13)$$

where $b_3 = 0.224 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$, ρ_{a0} is the air density near the surface with air pressure 1013.25 hPa and temperature 300 K, and the term in the brackets is introduced to represent (i) the difference between the real collection kernel and the Golovin one and (ii) the effect of air density on the real collection kernel. In (4.13) as well as (4.14) and (4.15), $\hat{m}_r$ is substituted with $\min(\hat{m}_r, 12)$ to project the difference between the real hydrodynamic collection kernel and the Golovin one when the larger drop has a radius around 50 μm (i.e., between 28 and 64 μm in the parameterization) (Long 1974).

Figure 5 displays the modeled C_q in C10, C075, and C05 against their corresponding parameterization values computed with (4.13), showing that (4.13) is good. Since the terminal speed of large drops is sensitive to air density (Beard 1976), two experiments C075P4 and C075P6 are carried out to test the sensitivity of (4.13) to air density (or pressure). C075P4 mimics an air parcels with LWC = 0.75 g m⁻³, $p = 400$ hPa, and $T = -20^\circ\text{C}$ at $z = 7.5$ km, whereas C075P6 with LWC = 0.75 g m⁻³, $p = 600$ hPa, and $T = 1.5^\circ\text{C}$ at $z = 4.4$ km. Their modeled C_q agree well with the corresponding values in the parameterization, one of which is displayed in Fig. 6 as an example.

Since accretion brings about a decrease in cloud-drop number concentration, its expression of the decrease is obtained by adjusting (3.16), following the procedure to (4.13). That is,

$$C_n = \rho_a b_4 N_c q_r \left[1 + 3.2 \hat{m}_r^{-0.12} \left(\frac{\rho_{a0}}{\rho_a} \right)^{0.25} \exp(0.96 \sqrt{\hat{m}_r}) \right], \quad (4.14)$$

where $b_4 = 0.078 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$. Figures 5 and 6 display the modeled C_n in C10, C075, C05, and C075P4 against their corresponding parameterization values, showing (4.14) is good.

d. Self-collection of raindrops

The self-collection of raindrops brings about a decrease in raindrop number. Its expression S_r is obtained based on (3.7) just as (3.13), yielding $S_r = \rho_a b N_r q_r$. To fit the bin model results, the expression is further adjusted to

FIG. 5. As in Fig. 4, but for C_q (accretion rate of cloud water to rainwater; red and black) and C_n (decrease rate of cloud-drop number due to the collection of cloud drops by raindrops; blue and green).

$$S_r = \rho_a b_5 N_r q_r \left[1 + \hat{m}_r^{-0.1} \left(\frac{\rho_{a0}}{\rho_a} \right)^{0.25} \exp(0.55 \sqrt{\hat{m}_r}) \right], \quad (4.15)$$

where $b_5 = 1.1 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-1}$, and the term in the bracket is introduced to represent the effects of the real collection kernel and air density. Figure 7 displays the bin-modeled S_r in C10, C075, and C05 against the corresponding parameterization values, showing (4.15) is good. C075P4 and C075P6 are further compared to their counterpart parameterization, showing the sensitivity of (4.15) to air density or pressure is good too (figure omitted).

e. Breakup of raindrops

If there was no raindrop breakup (or $B_r = 0$), the raindrop number concentration N_r would continue to decrease due to raindrop self-collection, resulting in excessive mean raindrop mass $\hat{m}_r$. Figure 8 displays $\hat{m}_r$ (or its corresponding radius) versus time in an air parcel model with the present parameterization or experiment P30 with no raindrop breakup (see section 5a for experiment details), showing that $\hat{m}_r$ becomes too large in 20 min.

The excessive mean raindrop mass in Fig. 8 can be mitigated by introducing B_r . Consider an air parcel with only large raindrops, where there are two processes: raindrop collection and breakup. The parcel approaches an equilibrium between raindrop collection and breakup given sufficient time and eventually stays at a steady state of raindrop distribution (Srivastava 1971; Feingold et al. 1988; Tzivion et al. 1989). Feingold et al. (1988) obtained an analytical

solution of the SCE with raindrop breakup when the collection and breakup kernels are constants. Since (4.6) shows

$$d \ln \hat{m}_r / dt = d \ln q_r / dt - d \ln N_r / dt,$$

and the raindrop collection and breakup change no rainwater content q_r (or $d \ln q_r / dt = 0$), the analytic solution can be reexpressed in a differential form as

FIG. 6. As in Fig. 5, but for Exp. C075P4 and its corresponding parameterization, where the air parcel stays at altitude $z = 7.5 \text{ km}$ or $p = 400 \text{ hPa}$.

FIG. 7. As in Fig. 4, but for S_r (decrease rate of raindrop number due to raindrop self-collection).

$$d\hat{m}_r/dt = -(\hat{m}_r - \hat{m}_{re})/\tau, \quad (4.16)$$

$$B_r = 0 \quad (4.19)$$

where τ is the time scale in approach of the steady state and $\hat{m}_{re}$ the mean raindrop mass at the steady state divided by m^* , based on (24)–(26) of Feingold et al. (1988). The analytic solution also shows that τ is inversely proportional to the rainwater content $\rho_a q_r$. That is,

$$\tau = 5.46 \rho_{a0} / (\rho_a q_r), \quad (4.17)$$

where τ is in second and q_r in kg/kg; the coefficient is chosen, based on the simulations of SCE with the real collection and breakup kernels, resulting in $\tau = 20$ min while $q_r = 5 \times 10^{-3}$ kg kg $^{-1}$ and $p = 900$ hPa (Feingold et al. 1988; Tzivion et al. 1989; Pruppacher and Klett 1997, 651–652). In addition, $\hat{m}_{re} = 4.5 \times 10^4$, corresponding to a mean volume raindrop radius of 1 mm, is estimated based on the tropical rain observations of Willis (1984).

Since the rainwater content q_r is constant during raindrop collection and breakup, (4.16) is rewritten as

$$B_r = S_r + N_r(\hat{m}_r - \hat{m}_{re})/(\tau \hat{m}_r), \quad (4.18)$$

with the aid of (4.6) and (4.4). Please note that both B_r and S_r change the raindrop number concentration in (4.4) via $B_r - S_r$.

The laboratory observations of drop collision show that most collisions involving large drops (radius > 0.3 mm) result in breakup and other collisions of smaller drops result in drop coalescence (Low and List 1982). Hence, it is assumed that (4.18) works for large drops (radius $\gg 0.3$ mm) and

for small drops (radius $\ll 0.3$ mm). For the drops with radius close to 0.3 mm, B_r is obtained by averaging (4.18) and (4.19) with a weight of $\hat{m}_r^2$ against a drop with radius 0.3 mm, reaching

FIG. 8. Mean volume raindrop radius vs time in experiment P30 with no raindrop breakup (blue) and P30B with raindrop breakup (red).

$$B_r = [S_r + N_r(\hat{m}_r - \hat{m}_{re})/(\tau\hat{m}_r)]/[1 + (4151/\hat{m}_r)^2], \quad (4.20)$$

where $\hat{m}_r = 4151$ corresponds to a mean volume raindrop radius of $1.5 \times 0.3 = 0.45$ mm.

To exhibit the effect of raindrop breakup on mean raindrop mass $\hat{m}_r$, experiment P30B is carried out that uses the same setup as P30 except for (4.20). The experiment is compared with P30 in Fig. 8, showing that the excessive raindrop mass is removed after the raindrop breakup term B_r in (4.20) is used.

5. Comparison to the numerical simulations of SCE

The final form of the parameterization consists of the expressions of A_q , C_q , C_m , S_c , S_r , and B_r in terms of N_c , q_c , N_r , and q_r . In the preceding section, the expressions are computed using N_c , q_c , N_r , and q_r from the bin model, and their values are then compared with their counterparts in the bin model as a test of the expressions. However, a further question is whether their errors are accumulated to distort the modeling of warm rain initiation, which is addressed in this section. That is, the parameterization model in (4.1)–(4.4) is used to simulate warm rain initiation against the bin model, addressing whether the parameterization can work well with no error accumulation and, more importantly, can replicate the autoconversion threshold in Kessler (1969) and the time delay of rain initiation in Cotton (1972).

a. Parameterization experiments

The parameterization model of (4.1)–(4.4) have four prognostic variables: N_c , q_c , N_r , and q_r . If the parameterization is accurate, the model will output the same N_c , q_c , N_r , and q_r as the bin model. Hence, the difference in N_c , q_c , N_r , and q_r between the parameterization model and the bin model can be used to measure the accuracy of the parameterization in bulk. In this section, six experiments of the parameterization model, named with a starting letter of P, are carried out against their counterparts that are named with a starting letter of C (see Table 1 for experiment summary).

Experiment P075 uses (4.1)–(4.4) to simulate N_c , q_c , N_r , and q_r versus time. It takes the same variables (e.g., T and p) as C075 except that its initial values of N_c , q_c , N_r , and q_r are computed using the initial cloud-drop spectrum in C075. Since drop collection does not change LWC, both P075 and C075 have a constant LWC of 0.75 g m^{-3} .

P075 integrates (4.1)–(4.4) with a time step of 0.25 s just as in C075. Its values of q_c , N_c , and q_r are displayed against time in Fig. 9. The parameterization model outputs almost the same results as the bin model, showing that the parameterization represents the warm

FIG. 9. Comparison of (top) cloud-drop number concentration, (middle) cloud water content, and (bottom) rainwater content between experiment C075 of the bin model (red) and experiment P075 of the parameterization model (blue).

rain initiation well (see Figs. 10 and 12 for more comparisons).

Another similar experiment P075DT is carried out to test the sensitivity of the parameterization to time step. The experiment takes the same setup as P075 except for a time step of 10 s that is usually used in cloud modeling. Its result, as shown in Fig. 10, is close to those of the bin model and P075 with a slight difference after rain forms. Since the experiment uses a time integration with the first-order accuracy of time step, the difference can be reduced further by a finite-difference scheme with the second-order accuracy of time step.

FIG. 10. As in Fig. 9, but for experiment P075DT that uses a time step of 10 s.

b. Autoconversion threshold

The present parameterization can replicate the characteristics of the Kessler parameterization, which is shown using the results from the bin model, present parameterization, and Kessler parameterization. In the parameterization of Kessler (1969), the autoconversion rate

$$A_q = a_k(q_c - q_{c0}) \quad (5.1)$$

when q_c is larger than q_{c0} the threshold value of cloud water content; otherwise, the autoconversion rate equals zero, where a_k is a constant. Obviously, the Kessler parameterization has two characteristics: (i) there is no autoconversion when cloud water content is low and (ii) the autoconversion rate is directly proportional to cloud water content.

To be comparable with the Kessler parameterization, the average autoconversion rate is computed from the bin and parameterization models using

$$A_q = q_{cBR}/T_{BR}, \quad (5.2)$$

where T_{BR} is the time scale of the autoconversion and q_{cBR} is the rainwater content at $t = T_{BR}$ (Berry and Reinhardt 1974). After $q_{cBR} = 0.1\text{LWC}$ is set, the average autoconversion rates in all the experiments of the bin and parameterization models are obtained, and

then displayed against LWC in Fig. 11. Generally speaking, the autoconversion rates in both the bin model and the present parameterization are directly proportional to LWC.

To fit the autoconversion rate from the bin model, the Kessler parameterization adjusts its two constants, a_k and q_{c0} , to model the rain initiation for a given cloud. If a cloud is convective with a life span of 45 min, for example, the Kessler parameterization is adjusted to match the bin-modeled result at LWC of $\sim 1.5 \text{ g m}^{-3}$, which is shown in Fig. 11. The autoconversion threshold q_{c0} corresponds to the small autoconversion rate at a low LWC (e.g., 0.1 g m^{-3}), because a small autoconversion rate cannot generate sufficient CCIs to initiate rain within the life span of a given cloud.

In summary, the present parameterization replicates the autoconversion rate versus LWC just as the bin model does. It also replicates the two characteristics of the Kessler parameterization: the autoconversion threshold of q_{c0} and the positive correlation to LWC.

c. Time delay of rain initiation

Cotton (1972) pointed out the time delay of rain initiation in an air parcel. Figure 12 displays the time series of rainwater content in the six bin model experiments with different values of LWC, showing the time delay becomes longer with decreasing LWC. Such time delay, just like the autoconversion threshold, is associated with the self-collection of cloud droplets.

Since the present parameterization properly incorporates the self-collection of cloud droplets, it replicates the time delay of rain initiation well, which is clear in Fig. 12. To be specific, the present parameterization replicates the autoconversion threshold of Kessler (1969) and the time delay of Cotton (1972) via a small autoconversion rate at low LWC. The long time delay at a LWC less than 1 g m^{-3} is reasonable because it corresponds to the threshold of $\sim 1 \text{ g m}^{-3}$ in Kessler (1969) or no autoconversion when $\text{LWC} < 1 \text{ g m}^{-3}$. Figure 12 also shows that the present parameterization replicates the cloud-drop number concentration of the bin model well even while LWC varies significantly.

In summary, the present parameterization incorporates the concepts of the autoconversion threshold and the time delay of rain initiation in the one-moment schemes of Kessler (1969) and Cotton (1972). It properly replicates the nonlinear development of warm rain initiation in the two-moment scheme of Seifert and Beheng (2006). It can be extended into a multimoment scheme to consider other processes such as raindrop sedimentation and the relative dispersion of cloud-drop spectrum (Tzivion et al. 1987; Liu et al. 2007).

FIG. 11. Average autoconversion rate vs LWC in the bin model (red circles), present parameterization (blue line), and Kessler parameterization (black line).

6. Conclusions

A simple two-moment microphysics parameterization for warm clouds is developed with two steps: (i) its prototype is derived based on a dimensionless form of SCE with an analytic solution first, and then (ii) the prototype is revamped empirically against the numerical solution of SCE with the real hydrodynamic collection kernel. Once its final form is obtained, it is tested by comparing two air parcel models: one with the parameterization and the other with the bin representation. The two models agree quite well in warm rain initiation, exhibiting the good performance of the parameterization.

The parameterization represents the self-collection of cloud drops explicitly. As a result, it can replicate warm rain initiation well even when LWC is very low or drop number concentration is very high. It can also replicate the autoconversion threshold of Kessler (1969) and the time delay of Cotton (1972) via a small autoconversion rate.

The parameterization uses an intermediate variable of mean cloud-drop mass $\bar{m}_c$ to explicitly represent the self-collection of cloud drops. Since $\bar{m}_c$ is sensitive to other factors such as radiation, aerosols, and turbulent mixing, it can be modified to incorporate other processes in warm rain initiation, such as the radiative effect, aerosol concentration, and cloud entrainment (e.g., Blyth 1993; Gerber

FIG. 12. Time series of (top) rainwater content and (bottom) cloud-drop number concentration from the bin model (red) and parameterization model (blue). Numbers beside lines indicate LWC (g m^{-3}) used in each experiment at 900 hPa in Table 1.

et al. 2008; Lu et al. 2012; Jensen and Nugent 2017; Zeng 2018a). Hence, the parameterization provides an opportunity to mitigate the “too-white-and-too-dense” problem in weather and climate models in the future.

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