The Role of Wind Speed and Wind Shear for Banner Cloud Formation

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ABSTRACT

Banner clouds are clouds that appear to be attached to the leeward face of a steep mountain. This paper investigates the role of wind speed and wind shear for the formation of banner clouds. Large-eddy simulations are performed to simulate the flow of dry air past an idealized pyramid-shaped mountain. The potential for cloud formation is diagnosed through the Lagrangian vertical parcel displacement, which in the case of a banner cloud shows a plume of large values in the lee of the mountain. In addition, vortical structures are visualized through streamlines and their curvature. A series of sensitivity experiments indicates that both the flow and the banner cloud occurrence are largely independent of the ambient wind speed $U$. On the other hand, the shear of the ambient wind has a profound impact on the location of the stagnation point on the windward face as well as on the flow geometry in the lee of the mountain. The relevant measure for shear is $H/H_s$, where $H$ denotes the height of the mountain and $H_s = U/\bar{U}$ is the scale height of the shear (with $\bar{U}$ denoting the scale of the shear). The simulations are also used to compute the line-of-sight velocity component seen by a hypothetical Doppler wind lidar positioned in the lee of the mountain; the analysis suggests that such sensitivities can potentially be detected using modern wind lidar technology.

1. Introduction

Banner clouds are clouds that appear to be attached to the leeward face of a steep mountain on otherwise cloud-free days (American Meteorological Society 2020). They can be observed, for instance, at Mount Zugspitze in the Bavarian Alps or at the Matterhorn in the Swiss Alps (Fig. 1). Banner clouds are interesting because their occurrence is somewhat counterintuitive: as an air parcel approaches the mountain, it is forced to rise and one would expect a cloud to occur on the windward side—as is often the case. In the event of a banner cloud exactly the opposite is true: the cloud occurs on the leeward side, while the windward side is cloud free.

During the last century, there have been only few investigations into this phenomenon, and these were mostly of descriptive nature (Küttner 2000). An overview of the early literature can be found in Schween et al. (2007). The first systematic observations were taken in the early 2000s in the form of time-lapse movies at Mount Zugspitze over the duration of 4 years (Schween et al. 2007). These observations resulted in a comprehensive definition of the phenomenon. The time-lapse movies were later combined with additional measurements, revealing numerous interesting properties of banner clouds at this mountain such as their diurnal and their seasonal cycle as well as the different wind regimes on the windward and leeward side of the mountain (Wirth et al. 2012).

At the same time there were attempts to elucidate various aspects of banner clouds using numerical simulations. Presumably the first successful simulation of a
banner cloud was carried through by Reinert and Wirth (2009). Later, Voigt and Wirth (2013) investigated various mechanisms that were previously hypothesized to lead to banner cloud formation; they found that the key mechanism is vertical uplift of air parcels, which turns out to be significantly larger in the lee of the mountain than on its windward side. Schappert and Wirth (2015) used trajectory calculations to better understand the Lagrangian aspects of the flow in the vicinity of the mountain. They showed that parcels that end up in the cloud must go around the mountain before they experience strong uplift on the leeward side. This implies that banner cloud formation requires a truly three-dimensional mountain, because in the case of two-dimensional orography there is no “flow-around option.” More recently, Prestel and Wirth (2016) systematically investigated the conditions that are conducive to banner cloud formation. They found that banner clouds occur preferentially for weakly stratified flow past a steep mountain.

All previous simulations were based on a model configuration with inflow–outflow geometry and a constant wind specified at the inflow boundary. The motivation for this choice was to keep the configuration as generic as possible. However, this leaves open the question how the banner clouds depend on the wind speed and wind shear. Such sensitivities are the topic of the present paper. More specifically, we investigate the occurrence and formation of banner clouds with the help of numerical simulations of flow past an idealized mountain. In our analysis we focus on the general flow properties, the occurrence of vortices in the lee of the mountain, as well as the strength of the leeside uplift as an indicator for the likelihood of banner cloud formation. We will show that the sensitivity with respect to wind speed is weak; on the other hand, the sensitivity with respect to wind shear turns out to be strong. The latter result motivated us to subsequently use our simulations in order to explore whether and to what extent the strong sensitivity to shear could be observed using modern Doppler wind lidar technology.

The paper is organized as follows. First, in section 2 we give an overview over our numerical model as well as the model configuration used. In section 3 we present our diagnostic tools to detect the occurrence of banner clouds as well as vortices. The results of the sensitivity studies are provided in section 4. In section 5 we interpret our results by visualizing vortical flow structures in the vicinity of the mountain, and in section 6 we explore the potential use of lidar observations. Finally, in section 7 we discuss our findings and draw our conclusions.

2. Numerical model and model configuration

Regarding the numerical simulations we closely follow the work of Schappert and Wirth (2015), where the reader can find a detailed description. Here, only the key aspects will be provided. We use the nonhydrostatic anelastic version of the Eulerian–semi-Lagrangian fluid solver (EULAG) model in large-eddy simulation (LES) mode (Prusa et al. 2008), with the subgrid-scale fluxes of momentum and heat being parameterized through a turbulent kinetic energy (TKE) closure (Smolarkiewicz et al. 1997). The model is used to simulate three-dimensional turbulent flow of dry air past idealized orography in a nonrotating atmosphere. Orography is
represented through an immersed boundary approach (Mittal and Iacarino 2005; Smolarkiewicz et al. 2007). Surface stress in the flat portion of the lower boundary is parameterized through a bulk formula. Next to the obstacle the immersed boundary approach effectively represents a no-slip condition (Goldstein et al. 1993; Smolarkiewicz et al. 2007). There are no surface heat fluxes in our model configuration.

The model domain covers 8 km in the streamwise direction (coordinate $x$), 4.8 km in the spanwise direction (coordinate $y$), and approximately 3 km in the vertical direction (coordinate $z$). This domain turned out to be large enough for our purposes, which means that boundary effects do not affect our results in any substantial way. The grid spacing is equidistant in all three directions with $\delta x = \delta y = \delta z = 25$ m. Our orography is a square-based pyramid, being located 3 km downstream of the inflow boundary and centered in the spanwise direction. The center of its base marks the origin of the coordinate system. The pyramid has a height of $H = 1000$ m and a width of $L = 1000$ m. Note that the plots below only show the inner part of the model domain.

The inflow boundary is located at $x_{in} = -3000$ m. At this boundary we specify profiles of wind $\mathbf{u} = (u, v, w)$ and temperature $T$. The temperature is given in terms of potential temperature $\theta = T(p_0/p)^{\kappa}$, where $p$ denotes pressure, $p_0 = 1000$ hPa is a constant reference pressure, $\kappa = R/c_p$, $R$ is the gas constant for dry air, and $c_p$ is the specific heat at constant pressure. Potential temperature is specified such that the stratification is neutral ($\partial \theta/\partial z = 0$) for $z \leq H$ and stable ($\partial \theta/\partial z = 4$ K km$^{-1}$) for $z > H$. Neutral stratification below the mountain summit is motivated by the parameter sweep experiments of Prestel and Wirth (2016). These simulations indicated that weak stratification (in addition to a steep mountain) is a prerequisite for banner cloud formation, consistent with the observations of Wirth et al. (2012). Here, we consider neutral stratification to be a useful limit of a more or less unspecified “weak” stratification.

The wind at the inflow boundary has only a component in the streamwise direction $u_{in}(z)$. In this study we use various different profiles of $u_{in}(z)$ in order to explore the sensitivities. The profiles for the main simulations are lumped into three groups with three profiles in each group (Fig. 2). The first group of profiles (green lines in Fig. 2) is simply given by a constant wind of strength 5, 10, and 20 m s$^{-1}$. The second group of profiles (blue lines in Fig. 2) has zero wind at the bottom, linear shear up to an altitude of 1500 m, and a constant extension above. The shear is chosen such that it reaches 5, 10, and 20 m s$^{-1}$ at the summit level ($z = H$). The third group of profiles (red lines in Fig. 2) is specified such that $u_{in}(H) = 10$ m s$^{-1}$, but with different linear shear for $z \leq 1500$ m. We refer to these nine wind profiles as P-u1-u2, with u1 denoting the wind at the bottom and u2 denoting the wind at the summit level. For instance, the blue profiles are referred to as P-0-5, P-0-10, and P-0-20, respectively. In addition, for a scale analysis in section 4c we consider a few further inflow profiles as explained in that section. In all experiments we use the inflow wind speed at summit level in order to characterize the typical scale $U$ of the horizontal wind speed,

$$U = u_{in}(H).$$

(1)

Note that the flow profiles close to the mountain on its windward side may slightly deviate from the inflow profiles $u_{in}(z)$ owing to the parameterized surface friction. However, these deviations are restricted to a thin layer (about 100 m thick) close to the ground. Note also that in all cases the shear is in the sense of “forward shear,” meaning that the streamwise wind component increases with altitude.

We apply a rigid-lid condition at the upper boundary and simulated nonreflecting conditions at the spanwise and the outflow boundaries. The latter is effectively achieved by specifying the corresponding values of the inflow wind profile at these boundaries and putting these boundaries far enough away from the orography. Sponges are used in order to avoid wave reflections from the inflow boundary and from the upper boundary.
The model is run for 80 min altogether with a time step $\Delta t = 0.25$ s. The first 20 min are considered to be a spinup period, during which the flow and the advected tracer (see below) reach a statistically stationary state. Model data from the remaining 60 min are used in order to compute temporal averages. In the present paper we restrict our attention to analyzing the temporally averaged model variables and focus on their dependence on the wind profile specified at the inflow boundary. In addition, we eliminate the remaining (small) asymmetries in the spanwise direction by symmetrizing the time-averaged variables in the $y$ direction. This is achieved by calculating an appropriate arithmetic mean between the fields on either side of $y = 0$ (for details, see Schappert and Wirth 2015). The small modification resulting from this symmetrization makes it easier to focus on the generic properties of the flow.

3. Diagnostic tools

a. Diagnosing cloud occurrence

Obviously, cloud formation in the real atmosphere depends, among others, on the availability of moisture. In earlier work this was accounted for by introducing specific humidity as a model variable; a cloud can then be said to occur at those grid points where relative humidity exceeds 100% (Schappert and Wirth 2015). However, this approach introduces an element of arbitrariness, because the occurrence of a cloud then depends on the profiles of specific humidity prescribed at the inflow boundary.

Here, we adopt a different approach, which is well motivated by earlier studies. As shown, for example, by Voigt and Wirth (2013), the key mechanism for banner cloud formation is vertical uplift in the lee of the mountain. For this reason, we use as our key diagnostic the vertical parcel displacement $\Delta z$ that a parcel has experienced along its path since it has entered the model domain. A flow that produces a plume of large values of $\Delta z$ in the immediate lee of the mountain is considered to be conducive to banner cloud formation, because it implies an increased likelihood for cloud formation in the lee (Prestel and Wirth 2016). Given appropriate moisture conditions, the cloud will, thus, occur on the leeward side only.

Schappert and Wirth (2015) diagnosed vertical uplift through both a Eulerian technique (involving a quasi-conserved tracer) and a Lagrangian technique (involving trajectories). Despite some differences in the details, both techniques yielded qualitatively very similar results. We therefore choose in the current study the Eulerian technique, as it is more straightforward to implement. Essentially, we use a tracer $\chi$, which evolves according to

$$\frac{D\chi}{Dt} = M_{\chi} \approx 0,$$

where $D/Dt$ denotes the material rate of change following the flow and $M_{\chi}$ represents material nonconservation owing to the parameterized subgrid turbulence. As argued in Schappert and Wirth (2015), $M_{\chi}$ is close to zero almost everywhere in the domain, which means that the tracer is close to being materially conserved. The tracer is initialized as $\chi = z$, and we specify $\chi = z$ at the inflow boundary throughout the integration. Following earlier studies (Reinert and Wirth 2009; Voigt and Wirth 2013; Schappert and Wirth 2015; Prestel and Wirth 2016), this tracer $\chi$ can be used to obtain a Eulerian estimate of the vertical parcel displacement $\Delta z$ by simply computing

$$\Delta z = z - \chi.$$  

b. Diagnosing vortex geometry

In the present study we do not only aim to detect the sensitivities of diagnostics such as $\Delta z$, but we also try to understand them in terms of the geometry of the flow and associated vortical structures. Point of departure is the leeside bow-shaped vortex, which was sketched earlier in Voigt and Wirth (2013, their Fig. 4) and seems to be relevant for constant inflow profiles. However, this vortex turns out to be difficult to visualize using standard Eulerian metrics involving vorticity or variants of the Okubo–Weiss criterion (Haller 2005). This is consistent with previous experience in diagnosing vortices in complex flows (Jeong and Hussain 1995).

We therefore decided to resort to a Lagrangian technique. In the present context, the following approach proves to be useful. We consider a limited leeside volume defined by $500 \leq x \leq 2000$ m, $-950 \leq y \leq 950$ m, and $z \leq 1200$ m. At each grid point in this volume, we compute streamlines of the time mean wind both forward and backward (i.e., following the direction and against the direction of the wind, respectively) by the same distance such that one obtains a total arc length of 1000 m. The step size for this integration is chosen to be $\Delta s = 10$ m; this is significantly smaller than the grid spacing and turns out to be small enough such as to render our results independent of the precise value of $\Delta s$. We then compute the curvature $K$ along each streamline through

$$K = \left| \frac{d^2\gamma}{ds^2} \right|,$$
where $\gamma(s)$ is the arc-length parameterization of the streamline. For a grid point to belong to a vortex we require that the mean curvature $\langle K \rangle$ must exceed a threshold $K_0$, where $\langle \cdots \rangle$ denotes the average along the streamline.

4. Results

a. Sensitivity to wind speed

We start with the set of simulations $P-u-u$ (with $u = 5, 10, 20$) corresponding to the profiles with no shear at the inflow boundary. The results are summarized in Fig. 3. As expected, the flow is generally from left to right (blue streamlines in the plots), but the lee of the mountain features conspicuous vortical structures associated with recirculation in parts of the domain (cold colors in the top row). The leeside vortex in the vertical section and the corresponding two counterrotating vortices in the horizontal section have to be considered to be the “footprints” of a single three-dimensional bow-shaped vortex (cf. Voigt and Wirth 2013). There is upwelling both on the windward and on the leeward side of the mountain (warm colors in the middle row).

The vertical sections in the top row as well as the horizontal sections in the middle row indicate that the flow geometry is very similar in all three runs, although the wind strength $U$ varies by a factor of 4. Note that the color bar was chosen to scale like $U$. It transpires that air

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**Fig. 3.** Sensitivity with respect to wind speed in the experiments with no shear at the inflow boundary: (left) P-5-5, (center) P-10-10, and (right) P-20-20. (a)–(c) Streamwise wind $u$ (color; m s$^{-1}$) and streamlines (blue lines) in a vertical section through the center of the mountain. (d)–(f) Vertical wind (color; m s$^{-1}$) and streamlines (blue lines) in a horizontal section at $z = H/2 = 500$ m. (g)–(i) Vertical displacement $D_z$ (color; m, zero contour green) in a vertical section through the center of the mountain. The mountain is indicated by dark gray shading in each panel. The green contour in the top row separates forward ($u > 0$) from backward ($u < 0$) streamwise flow. Note the different scaling of color bars in the top row and in the middle row.
parcels are advected along similar streamlines, but with a speed that roughly scales like $U$. A closer inspection indicates that the wind speed within the domain actually scales somewhat less than linearly with $U$. For instance, the magnitude of the return flow in the lee of the mountain is about $3.5 \text{ m s}^{-1}$ in experiment P-5-5 (Fig. 3a), increasing to $10 \text{ m s}^{-1}$ in experiment P-20-20 (Fig. 3c); strict linear scaling would predict $14 \text{ m s}^{-1}$ in P-20-20 (instead of $10 \text{ m s}^{-1}$).

How about vertical uplift $\Delta z$? The corresponding results are presented in the bottom row of Fig. 3. In all three cases there is a plume of large values in the lee of the mountain close to the summit. In contrast to the wind field (top and middle rows of the same figure), $\Delta z$ does not scale like $U$; instead, $\Delta z$ is quite similar in all three panels, with a maximum value around $650 \text{ m}$ in experiment P-5-5 (Fig. 3g), increasing to $680 \text{ m}$ in experiment P-20-20 (Fig. 3i). The approximate invariance of $\Delta z$ with $U$ is, again, consistent with the notion that the geometry of the streamlines is roughly independent of $U$ and that the air parcels travel along the streamlines with a speed that is proportional to $U$.

The only noticeable difference regarding $\Delta z$ between experiments P-5-5, P-10-10, and P-20-20 is the fact that the plume of large values is somewhat shorter in the experiments with weaker wind speeds. It turns out that this behavior is essentially due to internal gravity waves in the stably stratified part of the atmosphere above the mountain. These waves, which can be seen in the top row of Fig. 3, are excited by the presence of the mountain, and the first occurrence of downwelling behind the mountain resulting from the gravity waves is likely to limit the streamwise extent of the plume cloud. Consider a parcel that oscillates in the vertical with fixed frequency $f_s$. As this parcel is advected by the ambient horizontal wind, its horizontal wavelength becomes $\lambda \propto U/f_s$. In the case of an internal gravity wave, the parcel’s oscillation frequency is given by $f_s = N \cos \alpha$, where $\alpha$ is the angle between its phase line and the vertical. To the extent that the angle $\alpha$ does not sensitively depend on $U$, the above heuristic argument carries over to the current situation and allows one to understand why the horizontal wavelength $\lambda$ and, hence, the streamwise extent of the $\Delta z$ plume should increase for increasing wind speed.

We tested the above hypothesis by repeating the previous set of simulations with neutral stratification throughout the domain. This was achieved by prescribing $\partial u / \partial z = 0$ at the inflow boundary throughout the depth of the domain. In these new runs the differences between the different inflow profiles are even smaller than those shown in Fig. 3. For illustration we show the corresponding results for $\Delta z$ in Fig. 4. Now, the leeside plume of large values of uplift is approximately independent of the wind speed. This corroborates that the weak dependence of the plume size on wind speed in our original set of simulations (bottom row of Fig. 3) must be essentially due to the internal gravity waves above the mountain.

b. Sensitivity to wind shear

We now investigate the sensitivity to increasing the shear while leaving $U = 10 \text{ m s}^{-1}$ constant. More specifically, we consider the experiments P-10-10, P-5-10, and P-0-10 (red lines in Fig. 2). The results are presented in Fig. 5. Note the change of some of the color bars between Figs. 3 and 5.

In our discussion we focus on the differences between the three runs. The first feature to note is that shear has a strong impact on the location of the stagnation point separating upwelling from downwelling on the windward face of the mountain. The location of the stagnation point can be inferred from the plots of $u$ (top row in Fig. 5), because downwelling on the windward face implies backward flow ($u < 0$) and upwelling implies forward flow ($u > 0$); as a consequence, the intersection of the zero contour of $u$ (green contour) with the windward face of the mountain indicates the location of the stagnation point. The same information can be obtained more straightforwardly from vertical
sections of the vertical wind shown in Fig. 6. Apparently, the stagnation point is much higher for the experiment with strong shear. Adding wind shear is effectively associated with an additional component of the pressure field that has a upward gradient on the windward face; this produces a tendency for enhanced downwelling, which makes the stagnation point move upward.

Presumably the most striking result is that shear has a strong impact on the vertical wind field in the lee of the mountain (middle row in Fig. 5). Without shear (left column) there is upwelling close to the leeward face of the mountain, changing to downwelling at about 1000 m downstream. This is consistent with a vortex whose axis is oriented in the spanwise direction (y direction). However, this behavior changes qualitatively with the addition of shear (right column): now there is an extended region of upwelling on the plane \( y = 0 \) extending far into the lee, with no sign of downwelling at intermediate distances from the mountain. Instead, there are streamwise streamers of downwelling on both sides adjacent to the plane \( y = 0 \), suggesting two counterrotating vortices whose axes are oriented in the streamwise direction (x direction).

The strong sensitivity of the vertical wind field to shear becomes even more striking in Fig. 6, showing plots of \( w \) in a streamwise vertical section. In this plot both the streamlines and the vertical wind field are qualitatively very different depending on whether or not the inflow profile has shear.

To what extent does the strong sensitivity of the flow field translate to changes in the vertical parcel displacement \( \Delta z \)? The corresponding plots are presented in the bottom row of Fig. 5. Interestingly, all three experiments show a distinct plume of large values in the lee of
Nevertheless, there are some differences in the size and strength of this plume depending on whether or not the inflow profile has shear. For instance, the maximum amplitude of the P-0-10 experiment is about $D_z \approx 450$ m (Fig. 5i), while maximum amplitude of the P-10-10 experiment (i.e., constant inflow) is $D_z \approx 650$ m (Fig. 5g). Intuitively this is consistent with the fact that the volume of air that impinges on the mountain per unit time is smaller in the shear case. Another significant sensitivity concerns the values of $D_z$ close to the windward face of the mountain, indicating much smaller values for stronger shear. This finding is consistent with the (abovementioned) fact that the stagnation point on the windward face is located higher for the stronger shear, which reduces the potential for upwelling.

c. Sensitivity to the wind profile revisited

The strong sensitivity to wind shear reported in the previous subsection motivated us to revisit the sensitivity to wind speed for sheared profiles. More specifically we consider the experiments P-0-5, P-0-10, and P-0-20. The results (Fig. 7) indicate that, again, both the flow geometry and the leeside plume of large values of $\Delta z$ is approximately independent of the wind speed $U$. The situation is analogous to the runs for the constant wind profiles in Fig. 3.

This prompts the question. What is the appropriate metric for vertical shear that determines the flow geometry in the lee of the mountain? The vertical shear at the inflow boundary scales like

$$U_z = \frac{u_{\text{in}}(H) - u_{\text{in}}(0)}{H}. \quad (5)$$

However, Fig. 7 indicates that $U_z$ cannot be the appropriate metric, because $U_z$ varies by a factor of 4 in these three experiments, while the resulting flow geometry is nearly invariant. Rather, we suggest that the relevant metric involves the scale height $H_s$ of the shear, defined as

$$H_s = \frac{U}{U_z}. \quad (6)$$

Substituting (5) into (6) and using (1) yields

$$\frac{H}{H_s} = \frac{U - u_{\text{in}}(0)}{U}. \quad (7)$$

We hypothesize that this ratio $H/H_s$ is the relevant measure for the dependence of the flow geometry on the wind and its shear in the present context. The value of $H/H_s$ is zero in all three experiments without shear in Fig. 3; by contrast, the value of $H/H_s$ is 1 for all three experiments from Fig. 7. Both results are consistent with the near invariance of the flow geometry within each set of experiments. On the other hand, the three experiments from the previous section (i.e., P-10-10, P-5-10, and P-0-10) are associated with $H/H_s = 0.5$ and 1, respectively, and these different values are consistent with the substantial qualitative differences that we found in the latter set of experiments.

To further test the hypothesis, we consider another key feature of the flow, namely, the location of the stagnation point on the windward face of the pyramid. More specifically, we determined the height $H_{\text{stag}}$ of the stagnation point as the altitude of the point (on the plane $y = 0$) where the vertical wind close to the pyramid changes its sign. We then carried through a number of simulations where we independently varied $U$ (between 5 and 20 m s$^{-1}$) and $U_z$ (between 0 and 15 m s$^{-1}$ km$^{-1}$). Table 1 provides the values of $H/H_s$ as well as for $H_{\text{stag}}$ for these simulations. To the extent that the flow geometry is

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**Fig. 6.** Vertical wind (color; m s$^{-1}$) and streamlines (blue lines) in a vertical section through the center of the mountain for experiment (a) P-10-10 and (b) P-0-10. The same color bar applies to both panels.
determined by $H/H_s$, the value $H_{stag}$ should only depend on $H/H_s$. Correspondingly, we plot in Fig. 8 the normalized height $H_{stag}/H$ of the stagnation point against $H/H_s$. To a good approximation, most of the points in this plot collapse onto a one-dimensional submanifold (instead of filling the two-dimensional plane). This can be interpreted as a confirmation of our hypothesis, namely, that the value of $H/H_s$ determines to a large extent the flow geometry on the windward side. Only the simulations with constant inflow profile slightly deviate from this behavior (corresponding to the leftmost points in the plot). The underlying reason for this behavior is the parameterized surface friction in the upstream part of the domain, which slows down the streamwise flow in a thin layer above the ground, and this effectively lifts the stagnation point. Indeed, we were able to prove this connection through an additional set of simulations with constant inflow profile and zero surface friction in the flat part of the domain: in these latter simulations (not shown) the stagnation point was at $z = 0$.

We conclude that in the current context the impact of vertical shear on the flow geometry can be estimated through the value of $H/H_s$: When $H/H_s \ll 1$, the shear is negligible and the situation corresponds to a constant inflow profile; when $H/H_s \approx 1$, the flow is strongly affected by the shear; and intermediate values of $H/H_s$ between 0 and 1 indicate intermediate shear between the former two extremes.

5. Vortex visualization

The strong sensitivities with respect to shear beg for an explanation in terms of flow structures and how these
change with the addition of shear. In the previous section we already hinted at different vortex alignments in the runs with and without shear. We now address this issue more systematically by visualizing vortex-like flow structures in the lee of the mountain.

First, we attempt to visualize the flow with the help of streamlines for which the uplift $D_z$ exceeds a given threshold (Fig. 9). Quite apparently, the geometry of the streamlines is very different depending on whether or not the ambient wind has vertical shear. In the case of shear (Fig. 9b) the streamlines have a noticeable straight vertical section in the lee, corresponding to the upwelling that was mentioned in connection with Fig. 6. On the other hand, for the constant ambient wind profile (Fig. 9a), the streamlines are more curved indicating a more vortex-like behavior, consistent with the streamlines in the vertical and horizontal sections from Figs. 5a and 5d. Animated versions of the two panels of Fig. 9 can be found in “Supplement 1” in the online supplemental material, showing the different leeside vortices even more clearly than the snapshots provided here.

We now aim to reduce the amount of information contained in Fig. 9 by computing the curvature of streamlines as detailed in section 3. Figure 10a shows an isosurface of $h_K$ for the experiment P-10-10 without shear. Clearly, high values of curvature are concentrated in a bow-shaped tube corresponding to the so-called box vortex known from the engineering literature (Martinuzzi and AbuOmar 2003). Apparently, this bow-shaped structure is consistent with the geometry of the streamlines in the horizontal and vertical sections shown in Figs. 3b and 3e as well as Fig. 9a. As was shown in Prestel and Wirth (2016), this vortex is primarily generated by surface friction: the two spanwise faces of the mountain generate vorticity with vertical orientation (visible in the horizontal section of Fig. 3e), while the windward face of the mountain generates vorticity with horizontal orientation (visible in the vertical section of Fig. 3b). Figure 10a shows that these features in the respective sections combine to form a single three-dimensional vortex, which is located in the immediate lee of the mountain.

The same type of visualization for the experiment P-0-10 with shear is shown in Fig. 10b. The shape of the $h_K$ isosurface is somewhat less coherent than previously, but nevertheless one can identify a three-dimensional structure in the lee of the mountain; its two vertically oriented lower sections resemble those in Fig. 10a, whereas the horizontally oriented top part of the arch is shifted into the lee and partly missing.

The differences in the leeside vortical structures between the two experiments can be understood qualitatively in terms of the Helmholtz vortex theorem. The theorem says that vortex lines in conservative barotropic flow are material lines moving with the flow. Obviously, our flow is neither conservative nor barotropic, but away from the surface the impact of friction is small and baroclinicity is possibly of minor importance in the immediate lee of the mountain. With these assumptions,

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**TABLE 1. Analysis of the stagnation point on the windward face of the mountain in 13 different numerical experiments.** The different entries in the table represent simulations that differ in their combination of $U$ and $U_z$, where the velocity scale $U$ increases in the row direction and the shear scale $U_z$ increases in the column direction of the table. There are three pieces of information given for each table cell: its name (e.g., P-5-10), the value of $H/H_s$ as obtained from (7), and the height of the stagnation point (m) as inferred from the respective numerical simulation. The asterisks in the lower-left cells of the table indicate the part of the phase space that cannot be realized by our model configuration.

<table>
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<th>$U_z$ (m s$^{-1}$ km$^{-1}$)</th>
<th>$U$ (m s$^{-1}$)</th>
<th>$H/H_s$</th>
<th>Stagnation point (m)</th>
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<td>15</td>
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<td>15</td>
<td>*</td>
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**FIG. 8.** Normalized height $H_{stag}/H$ of the windward stagnation point plotted against $H/H_s$ for the 13 different simulations from Table 1. Each of the lightly colored circles represents one experiment, and the more heavily colored circles at $H/H_s = 0, 0.5$, and 1 represent experiments that happen to lie on top of each other in this graph.
any vortex that has been generated by surface friction at the lateral faces of the mountain is simply advected into the lee. For a constant inflow wind profile, the vortex keeps its orientation with respect to the mountain, which means that it leans slightly backward with altitude like the leeward face of the mountain (Fig. 10a). This situation yields the familiar arch-shaped vortex described in Voigt and Wirth (2013). On the other hand, adding vertical shear makes this vortex tilt away from the mountain at upper levels (Fig. 10b). This means that air parcels are swirling around a vortex core that is tilted into the lee—rather than slightly against the shear as before. It follows that parcels rise in a corkscrew fashion along a forwardly tilted vortical structure (cf. Fig. 9b).

While the air parcels ascend along the leeside vortical structures in the case with shear, their overall motion is increasingly affected by the streamwise advection. This reduces the curvature of their streamlines, suggesting that our diagnostic $\mathcal{K}$ turns less appropriate to detect the vortical structure. To test this hypothesis we repeated the computation of $\mathcal{K}$ in this experiment, except that for the computation of the streamlines we used a modified wind field $\vec{u} = (0, v, w)$ where we set the streamwise component to zero. The idea is to obtain hypothetical streamlines that indicate the sense of rotation in the $y-z$ plane. The corresponding visualization (Fig. 11) indicates that the upward extensions of the two legs of the bow vortex extend far into the lee at higher levels. Additional visualization (not shown here) indicate that these two legs are composed of two counter-rotating vortices with their axis oriented along the streamwise direction. The region of strong upwelling on the plane of symmetry $y = 0$ from Fig. 6b is consistent
with the orientation of these two vortices and their opposite sense of rotation.

Care needs to be exercised when interpreting the visualizations in Figs. 10 and 11. Remember that our diagnostic is based on the time-averaged wind, which prevents one from seeing any transient features. We produced animations of the time evolution of various variables in our simulations; these animations indicate that the flow is transient on multiple time scales, with vortex-like features in the lee separating off the mountain in a quasi-periodic fashion. An example is given in the “Supplement 2,” showing the spanwise wind (color; m s\(^{-1}\)) as well as streamlines in a horizontal section at \(z = 600\) m. Apparently, some of these transient vortical features survive the time averaging, and this is what appears in Figs. 10 and 11. On the other hand, transient aspects like the advection of the vortex with the flow cannot possibly be captured with our present vortex visualization, because it is based on the time-mean flow field.

6. Potential use of wind lidar observations

We have found that the flow geometry changes in some fundamental way as the scale height of the shear of the ambient wind changes. This prompts the question whether or not one would be able to detect related sensitivities in a field campaign using a Doppler wind lidar similar as in Gerber et al. (2017). Such a lidar is able to probe an entire volume through scans in various planes, but it only retrieves the line-of-sight component of the three-dimensional wind. We, therefore, used our simulations in order to estimate the potential of wind Doppler lidar observations to verify the sensitivities that we found.

For this purpose, we computed the line-of-sight component of the simulated wind fields seen by a hypothetical wind lidar located at \((x, y, z) = (1000, 0, 0)\) m, that is, on the axis of symmetry in the lee of the mountain. We focus on the sensitivity to shear with \(U\) kept fixed, corresponding to our set of simulations P-10-10, P-5-10, and P-0-10. The result is given in Fig. 12. The top row shows the line-of-sight velocity for a scan in the streamwise direction (i.e., in the plane \(y = 0\)), and the bottom row shows the corresponding plots for a scan in the spanwise direction (i.e., in the plane \(x = 1000\) m). For both scan orientations there are very significant qualitative differences in the retrieved velocity field. This suggests that, given enough return signal, a wind lidar should be able to distinguish such differences. In particular, the retrieved signal in the streamwise scan reflects the fundamental differences pointed out in connection with the vertical wind field (Fig. 6). Similarly, there is strong sensitivity of the retrieved signal in the spanwise scan, with regions of strong flow away from the lidar in the shear case (Fig. 6f), but almost exclusively flow toward the lidar in the case without shear (Fig. 6d). These results suggest that a carefully designed measurement campaign should be able to verify whether and to what extent the strong sensitivities found in this paper are relevant in the real world.

7. Discussion and conclusions

We carried through idealized large-eddy simulations in order to explore the sensitivity of banner cloud formation to the strength and the shear of the ambient wind field. Cloud formation was diagnosed through the upward vertical displacement \(\Delta z\) of air parcels. A high potential for banner cloud formation is then associated with those model experiments that show a plume of large values of \(\Delta z\) in the lee of the mountain. In addition, we tried to visualize the vortices in the lee of the mountain.

One key result is that the impact of shear should be measured by \(H/H_s\), where \(H_s = U/U_z\) is the scale height of the background wind profile, \(H\) is the height of the mountain, \(U\) denotes the scale of the wind speed, and \(U_z\) is the scale of the wind shear. The value of \(H/H_s\) has a profound impact on the flow geometry both on the windward and on the leeward side of the mountain. In particular, for \(H/H_s \approx 1\) there is a narrow region of strong upwelling extending into the lee on the plane of symmetry \(y = 0\), which does not exist in the absence of shear \((H/H_s = 0)\). In addition, larger values of \(H/H_s\) are...
associated with a higher location of the stagnation point on the windward side of the mountain, which implies less uplift on the windward side in comparison with a constant wind profile. This reduces the likelihood for cloudy air on the windward side of the mountain, which makes the situation more consistent with the definition of a banner cloud (Schween et al. 2007). By contrast, the model configuration with a constant wind profile, which was used in practically all previous simulation studies, is less appropriate to study banner clouds, because the positive values of $D_z$ on the windward face of the mountain reduce the likelihood to observe cloud-free conditions there. For this reason, we suggest that future simulations should use a sheared instead of a constant background wind profile. Such a profile is also more consistent with the observed general tendency for the wind speed to increase with altitude.

Given a fixed value of $H/H_s$, the likelihood of banner cloud formation does not sensitively depend on the background wind speed $U$. A slight dependence on wind speed is introduced by the internal gravity waves above the mountain, because their wavelength depends on $U$. Overall, however, the impact of gravity waves is weak, because we chose stratification to be neutral below the summit of the mountain; the latter choice was motivated by earlier results showing that banner clouds require (among others) weak stratification for their existence (Prestel and Wirth 2016). Our finding that banner cloud formation does not sensitively depend on $U$ is consistent with the observations of Wirth et al. (2012), who showed that (within statistical uncertainty) banner cloud occurrence at Mount Zugspitze is independent of the wind speed.

We visualized the vortical structures in the lee of the mountain by computing streamlines from the time mean wind field as well as their curvature. For the experiments without wind shear we obtained the classical bow-shaped vortex (see Fig. 4 in Voigt and Wirth 2013), which is generated by surface friction as the air parcels pass the mountain (Prestel and Wirth 2016). Increasing the wind shear by increasing $H/H_s$ effectively tilts this vortex into the lee such that one is left with two separate branches of the vortex, both of which extend into the lee in the streamwise direction. This change in the leeside vortical structure is consistent with the fundamental changes in the wind field that we found. Our argument with the vortex tilting due to the increased shear is based on Helmholtz’s vortex theorem. Strictly speaking, this theorem does not apply, because the time mean flow is not inviscid.

**FIG. 12.** Sensitivity of the line-of-sight component of the wind (color; m s$^{-1}$) as seen by a hypothetical wind lidar located at $(x, y, z) = (1000, 0, 0)$ m. Positive values denote flow away from the lidar, negative values denote flow toward the lidar. Results are shown for experiment (left) P-10-10, (center) P-5-10, and (right) P-0-10. (a)–(c) Vertical section in the streamwise direction through the center of the mountain. The dark gray triangle depicts the mountain, and the light gray shading indicates the area that is inaccessible to the lidar observation. (d)–(f) Vertical section in the spanwise direction at $x = 1000$ m, i.e., in the lee of the mountain.
even away from the frictional boundary layer owing to the effects of the transient eddies. Nevertheless, the argument still seems to keep some heuristic value. A more detailed investigation of the transient flow would be interesting but is beyond the scope of the present paper.

We also used our simulations in order to compute the line-of-sight component of the wind field as seen by a hypothetical wind lidar positioned in the lee of the mountain. It turns out that the lidar instrument should see very significant qualitative differences between the shear and the no-shear case. Of course, in such a measurement campaign it would be necessary to estimate the wind shear on the windward side simultaneously with the lidar observations on the leeward side. However, a broad estimate of the wind shear (like from a radiosonde or through unmanned aircraft) should be sufficient, because we do not anticipate complex flow geometry on the windward side. Hopefully, some time in the future such an observational campaign will be able to show to what extent the results from our idealized simulations carry over to the real world.

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