Precipitation Enhancement in Squall Lines Moving over Mountainous Coastal Regions

FAN WUa AND KELLY LOMBARDOa

aDepartment of Meteorology and Atmospheric Science, The Pennsylvania State University, University Park, Pennsylvania

(Manuscript received 24 July 2020, in final form 21 June 2021)

ABSTRACT: A mechanism for precipitation enhancement in squall lines moving over mountainous coastal regions is quantified through idealized numerical simulations. Storm intensity and precipitation peak over the sloping terrain as storms descend from an elevated plateau toward the coastline and encounter the marine atmospheric boundary layer (MABL). Storms are most intense as they encounter the deepest MABLs. As the descending storm outflow collides with a moving MABL (sea breeze), surface and low-level air parcels initially accelerate upward, though their ultimate trajectory is governed by the magnitude of the negative nonhydrostatic inertial pressure perturbation behind the cold pool leading edge. For shallow MABLs, the baroclinic gradient across the gust front generates large horizontal vorticity, a low-level negative pressure perturbation, and thus a downward acceleration of air parcels following their initial ascent. A deep MABL reduces the baroclinically generated vorticity, leading to a weaker pressure perturbation and minimal downward acceleration, allowing air to accelerate into a storm’s updraft. Once storms move away from the terrain base and over the full depth of the MABLs, storms over the deepest MABLs decay most rapidly, while those over the shallowest MABLs initially intensify. Though elevated ascent exists above all MABLs, the deepest MABLs substantially reduce the depth of the high-θe layer above the MABLs and limit instability. This relationship is insensitive to MABL temperature, even though surface-based ascent is present for the less cold MABLs, the MABL thermal deficit is smaller, and convective available potential energy (CAPE) is higher.

KEYWORDS: Cold pools; Marine boundary layer; Precipitation; Squall lines; Coastal meteorology; Numerical analysis/modeling

1. Introduction

Precipitation enhancements are frequently observed over coastal regions (Ogino et al. 2016; Curtis 2019), with distinguishable spatial maxima centered on and surrounding coastlines. Tropical Rainfall Measuring Mission (TRMM) precipitation data show that precipitation amounts peak along coastlines and decrease rapidly within 300 km of the land–sea boundary (Ogino et al. 2016). Long-term (1931–2010) Global Precipitation Climatology Center (GPCC) V2018 reanalysis confirm these localized coastal maxima and reveal that their magnitudes have increased in recent decades, impacting coastal residents, offshore recreational and commercial activities, and coastal ecosystems (Curtis 2019). Physical mechanisms responsible for the precipitation enhancement are related to land–sea interactions (e.g., sea breeze), with 40%–60% of total rainfall over the Mediterranean and Maritime Continent associated with coastal circulations, for example (Bergemann et al. 2015). An important source of coastal precipitation is seaward-propagating mesoscale convection system (MCS), or squall lines, which are modified by these coastal circulations (Wu et al. 2009; Lombardo and Colle 2011, 2012, 2013; Li and Carbone 2015; Lombardo and Kading 2018, hereafter LK18). Therefore, advancing our knowledge of the mechanisms responsible for coastal precipitation enhancements necessitates an understanding of deep convective storm dynamics over coastal environments.

As deep convective storms move from inland to offshore, precipitation is dictated by characteristics of a storm’s cold pool and the ambient stable marine atmospheric boundary layer (MABL) air driving a local sea breeze (Moncrieff and Liu 1999; Zuidema et al. 2017; LK18). Given the importance of cold pools in the offshore movement of coastal storms (de Szoke et al. 2017), it is instructional to revisit the associated fundamental convective initiation (CI) mechanisms before considering modifications induced by the MABL. One of the leading theories on squall-line dynamics poses that squall-line strength and longevity are influenced by horizontal vorticity baroclinically generated by the cold pool and that of the ambient low-level wind shear, known as “RKW theory” (Rotunno et al. 1988; Moncrieff and Liu 1999; Tompkins 2001; Weisman and Rotunno 2004; Bryan and Rotunno 2014; Torri et al. 2015). Based on this theory, the vertical orientation of lifting at the cold pool leading edge (CPLE) can be determined by the ratio of cold pool intensity (C) to ambient low-level wind shear (Δu), or C/Δu [see Fig. 2 in Weisman and Rotunno (2004)]. The frontal lifting is downshear (upshear) tilted when C < Δu (C > Δu), or optimally upright when a balance is achieved (C = Δu).

Moving density currents, such as cold pool outflows and sea breezes, are associated with increased low-level vertical wind shear and can alter this balance. The associated increase in low-level mass convergence can also promote CI (e.g., Droegemeier and Wilhelmson 1985; Wilson and Schreiber 1986; Carbone et al. 1990; Kingsmill 1995; Moncrieff and Liu 1999; Banacos and Schultz 2005), though not always. A deep convective storm over coastal Queensland, Australia, during the Coastal Convective Interactions Experiment (CCIE) was anticipated to intensify as it moved into the marine environment through deeper cold-pool-lifting by the increased low-level vertical wind shear, though the storms dissipated (Soderholm et al. 2016).
Dissipation was hypothesized to result from the elevated level of free convection (LFC) in the marine environment, though differences in density between the outflow and MABL may have played a role as well. The vertical slope at the interface of the two moving fluids is sensitive to their buoyancies, tilting toward the denser fluid (van der Wiel et al. 2017; Cafaro and Rooney 2018), thus impacting an air parcel’s vertical trajectory, CI, and precipitation.

Storm dynamics are also influenced by the marine environment following the initial engagement with the MABL. Perhaps counterintuitively, sea surface temperature (SST) is not a good predictor for MCS longevity over the coastal waters (Lombardo and Colle 2012). Rather, the presence of strong environmental 0–3-km wind shear is critical, and can help sustain storms >100 km offshore (Lombardo and Colle 2012). Contrasting case studies of northeastern U.S. quasi-linear convective systems (QLCSs) showed that bores can form following the collision between a QLCS’s cold pool and a MABL, which can successfully move storms over the stable layer (Lombardo and Colle 2013). Here, enhanced low-level vertical wind shear helps trap the wave energy within the boundary layer maintaining the bore (LK18), rather than provide a source of horizontal vorticity. Further, the reduced buoyancy gradient across the cold pool–MABL interface limits the baroclinic development of vorticity. Rather, pre-collision cold pool–MABL buoyancy differences determine bore development, with bores forming when the MABL is denser than the cold pool (LK18). Though bores provided a mechanism to successfully move storms over the coastal waters, conditions in the free troposphere determine storm intensity and precipitation (LK18; Lombardo 2020).

To add further complexity, squall lines often occur over coastal regions with orography along or immediately upstream of the coastline (Kömişçü et al. 1998; Teng et al. 2000; Mapes et al. 2003; Warner et al. 2003; Cohuet et al. 2011; Pucillo et al. 2020), influencing CI and preexisting convection. Prior studies have primarily focused on the impact of inland orography on squall-line evolution, specifically a storm’s ability to successfully traverse a mountain range (Frame and Markowski 2006; Reeves and Lin 2007; Letkewicz and Parker 2010, 2011; Smith et al. 2015). Terrain-crossing squall lines often propagate discretely, leading to heterogeneous precipitation patterns across the ridge (Teng et al. 2000; Frame and Markowski 2006). Storms initially intensify through orographic lift as they ascend a mountain, but weaken in the lee slope as their cold pools become partially blocked by the terrain. Gravitational descent down the lee slope further thins and warms the cold pools, limiting their ability to lift parcels to their LFCs, resulting in a precipitation shadow. Storms can ultimately reintensify at the terrain base if the flow transitions from supercritical to subcritical, supporting intense vertical motion (Frame and Markowski 2006). The environment in the lee also influences ridge-crossing, with success favored in the presence of comparatively high leeside instability, though mean wind plays an increasingly important role when convective available potential energy (CAPE) is substantially reduced (Letkewicz and Parker 2010, 2011), such as in the marine environment.

To the authors’ knowledge, only a limited number of studies have examined deep convective storm evolution over mountainous coastal regions. Daytime onshore sea breezes over coastal Columbia were shown to modify coastal precipitation development and propagation as they ascend the coastal terrain as upslope flows (López and Howell 1967; Warner et al. 2003). Gravity waves were also shown to support the offshore propagation of organized convection over western South America, though the waves formed from diurnal variations in elevated heating over the coastal mountain peak rather than through cold pool–MABL interactions (Mapes et al. 2003). Systematic quantification of coastal squall-line dynamics, specifically the impact of elevated topography and onshore sea-breeze flows, is absent in the literature. Given the globally ubiquitous nature of squall lines over mountainous coastal regions, and the impact of these environmental features on heavy, convective precipitation, it is prudent to identify the physical processes controlling storm evolution. Advancements in our understanding of the fundamental dynamics may improve prediction of these hazardous storms. Toward this end, idealized numerical experiments are designed to quantify the modification to storm dynamics by a mountain and moving MABLs. Section 2 describes these experiments, as well as the base-state, terrain, and MABL characteristics. Methods to identify the CPLE and to diagnose components of the vertical acceleration are introduced as well. Section 3 describes storm evolution in the constructed coastal environment, and the dynamics of enhanced precipitation over the terrain slope is presented in section 4. Storm evolution once over the MABL is presented in section 5, with discussions and conclusions in sections 6 and 7, respectively.

2. Methods

a. Model configuration

Large-eddy simulations are performed with the Cloud Model 1 (CM1) version 19, a nonhydrostatic, cloud-resolving numerical model (Bryan and Fritsch 2002). The 800 km (horizontal) × 20 km (vertical) two-dimension domain is sufficiently large to limit the influence of the lateral boundaries on storm evolution. The horizontal domain has 200-m grid spacing, and the vertical resolution is stretched from 50 m below 3 km to 250 m above 10 km. A terrain-following coordinate system is used, following Gal-Chen and Somerville (1975). The horizontal boundaries are open-radiative, with free-slip conditions at the top and bottom boundaries. Numerical parameterizations used to estimate physical processes include the Morrison double-moment microphysical scheme (Morrison et al. 2009), which includes predictions of cloud droplets, cloud ice, rain, snow, and graupel, and a TKE subgrid-scale turbulence scheme (Deardorff 1980). No radiation and surface fluxes are included to eliminate the influence of time-varying parameters (i.e., diurnal cycle) and isolate the influence of the MABL and orography on storms. The model is integrated for 480 min using a Runge–Kutta scheme, with a large time step of 0.75 s. Other model configurations follow LK18.
b. Orography

Plateau-shaped terrain is included as a simplified representation of the coastal terrain structure in regions that experience coastal squall lines, such as the eastern United States, China, the Mediterranean, and South America. A 1.5-km plateau is included from 0 to 300 km, with a slope extending from the plateau top to the base (300–360 km), with values based on commonly observed coastal orography in squall-line-active regions. A storm moving over this terrain in the absence of a MABL is used as a reference ("CTRL") for comparison to additional terrain experiments, and to simulations which include a MABL at the terrain base (see section 3a). Initial experiments quantify the plateau’s influence on storm evolution, by removing the feature ("NoTER") and extending the elevated surface across the full domain ("OnlyTER"). The bulk of the experiments quantify storm response as they descend the plateau and collide with a moving MABL, with depths of 500, 1000, and 1500 m and potential temperature perturbations ($\theta_0$) of 2, 3, 4, and 5 K relative to the base state. MABL values are informed by observations and prior work (Lombardo and Colle 2013; LK18; Lombardo 2020).

c. Base-state environment

Simulations are initialized with a horizontally homogeneous base-state environment by using an analytic thermodynamic profile, informed by observed soundings during coastal squall-line events (Fig. 1a; Lombardo 2020). Vertical wind shear is 15 m s$^{-1}$ in the lowest 3.5 km. For regions with terrain, the profile below the surface is truncated by the model to maintain the horizontally homogeneous initial conditions, which is commonly used in idealized simulations of squall lines over mountainous regions (Frame and Markowski 2006; Letkewicz and Parker 2011; Reeves and Lin 2007). In this way, the initial conditions inhibit artificial convection caused by a horizontally heterogeneous base-state environment. With the lower part of the profile removed below the terrain surface, base-state environmental variables over the mountainous region vary and deviate from those at "sea level." For example, surface-based convective available potential energy (SBCAPE) and most-unstable convective available potential energy (MUCAPE) are 738.69 and 917.50 J kg$^{-1}$ over the plateau top, respectively, but 1596.95 and 1630.53 J kg$^{-1}$ at the bottom. Additional base-state thermodynamic parameters are provided in Table 1. Base-state wind shear also decreases from 15 m s$^{-1}$ at the plateau base to 9 m s$^{-1}$ at the top. Although the instability is diminished on the plateau top, it is able to support a mature squall line.

d. Storm and MABL initiation

Storms are initiated at $t = 0$ on the plateau top through momentum forcing (Morrison et al. 2015), with 20-km-wide convergence centered 200 km from the left boundary, which decreases from the ground to 10 km above ground level (AGL).\footnote{Sensitivity to storm initiation location was tested (centered at 100 km, 150 km) with similar results.} A 0.2 m s$^{-1}$ magnitude forcing is applied from 0 to 55 min, which ramps down from 55 to 60 min. Storms reach maturity before encountering the plateau edge. The MABL is inserted at $t = 0$ as a block of relatively dense air (LK18; Lombardo 2020), from the base of the plateau to the domain edge (360–800 km). This allows the MABL to move toward the sloping terrain as a sea breeze. Water vapor mixing ratio within the MABL is held constant, resulting in a cloud layer for cooler, deeper MABLs. The MABL is not continuously forced.
which leads to a sloped leading edge as it moves toward and ascends the slope. The inclusion of a MABL reduces the instability at the base of terrain by providing a stably stratified surface-based layer, with properties governed by the MABL depth and \( \theta_e \) (Fig. 1b, Table 2).

e. Identification of CPLE

The CPLE is identified, tracked, and used as a point of reference for evaluating the evolving storm physical processes. Previous studies have decomposed vertical accelerations (e.g., Frame and Markowski 2006), with a substantial decrease in precipitation over the sloping terrain emphasized in the analysis of total column precipitation mass from the simulations with and without a MABL. Here, similar analyses are performed on the plateau, with precipitation mixing ratio plumes exceeding 12 km in altitude prior to reaching the plateau top eastern edge (300 km), with minimal differences.

f. Decomposition of vertical accelerations and pressure perturbations

A number of studies have decomposed vertical accelerations and pressure perturbations to diagnose the physical processes dominating storm evolution (Jeekan and Romps 2015, hereafter JR15; Dawson et al. 2016; Peters 2016; Schenkman et al. 2016; LK18; Huang et al. 2019). Here, similar analyses are performed on the plateau, with precipitation mixing ratio plumes exceeding 12 km in altitude prior to reaching the plateau top eastern edge (300 km), with minimal differences.

### Table 1. Base-state stability indexes of most unstable convective available potential energy (MUCAPE, J kg\(^{-1}\)), most unstable convective inhibition (MUCIN, J kg\(^{-1}\)), surface-based convective available potential energy (SBCAPE, J kg\(^{-1}\)), surface-based convective inhibition (SBCIN, J kg\(^{-1}\)), lifting condensation level (LCL, m AGL), and level of free convection (LFC, m AGL) on the plateau, on the slope, and at sea level. All values are from single-point locations.

<table>
<thead>
<tr>
<th>Index</th>
<th>Plateau (0–300 km)</th>
<th>Slope (330 km)</th>
<th>Bottom (360–800 km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUCAPE (J kg(^{-1}))</td>
<td>917.50</td>
<td>1491.70</td>
<td>1630.53</td>
</tr>
<tr>
<td>MUCIN (J kg(^{-1}))</td>
<td>1.61</td>
<td>3.18</td>
<td>17.04</td>
</tr>
<tr>
<td>SBCAPE (J kg(^{-1}))</td>
<td>738.69</td>
<td>1382.14</td>
<td>1596.95</td>
</tr>
<tr>
<td>SBCIN (J kg(^{-1}))</td>
<td>0.10</td>
<td>0.10</td>
<td>13.01</td>
</tr>
<tr>
<td>LCL (m)</td>
<td>363.29</td>
<td>398.28</td>
<td>799.74</td>
</tr>
<tr>
<td>LFC (m)</td>
<td>553.18</td>
<td>574.87</td>
<td>1142.85</td>
</tr>
</tbody>
</table>

### Table 2. Base-state SBCAPE (J kg\(^{-1}\)) and MUCAPE (J kg\(^{-1}\)) at the plateau base (360–800 km) for the experiments with a MABL, with depths of 500, 1000, and 1500 m and potential temperature perturbations (\( \theta' \)) of \([-2, -3, -4, -5] \) K relative to the base state. All values are from single-point locations.

<table>
<thead>
<tr>
<th>( \theta' ) (K)</th>
<th>500 m SBCAPE (J kg(^{-1}))</th>
<th>500 m MUCAPE (J kg(^{-1}))</th>
<th>1000 m SBCAPE (J kg(^{-1}))</th>
<th>1000 m MUCAPE (J kg(^{-1}))</th>
<th>1500 m SBCAPE (J kg(^{-1}))</th>
<th>1500 m MUCAPE (J kg(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>1350.95</td>
<td>1544.97</td>
<td>1182.24</td>
<td>1414.97</td>
<td>1196.59</td>
<td>1227.97</td>
</tr>
<tr>
<td>(-3)</td>
<td>1255.76</td>
<td>1544.97</td>
<td>1093.13</td>
<td>1414.97</td>
<td>1017.14</td>
<td>1046.87</td>
</tr>
<tr>
<td>(-4)</td>
<td>1163.53</td>
<td>1544.97</td>
<td>1006.98</td>
<td>1414.97</td>
<td>613.74</td>
<td>916.85</td>
</tr>
<tr>
<td>(-5)</td>
<td>1074.28</td>
<td>1544.97</td>
<td>923.83</td>
<td>1414.97</td>
<td>545.06</td>
<td>916.85</td>
</tr>
</tbody>
</table>
Without terrain ("NoTER"), the squall line is more intense during the initial two hours [Figs. 2b(1), 2b(2), 3b], in part due to the larger environmental instability and low-level vertical wind shear than over the plateau top (Table 1). The storm weakens over the following several hours, though the smaller precipitation mass between 300 and 360 km is associated with natural storm evolution following initiation rather than terrain [Figs. 2b(3), 2b(4), 3b]. By 420 min, the squall line is almost fully decayed (Fig. 3b), showing that the presence of a plateau extends the lifetime of the storm.

For a continuously elevated 1.5-km surface ("OnlyTER"), the storm’s initial evolution is very similar to the CTRL storm [Figs. 2c(1), 2c(2), 3c]. After 120 min, it is weaker and shallower than in the other experiments [Figs. 2c(3), 2c(4), 3c], with a similar decay time to the NoTER storm (Figs. 3b,c). Therefore, the reintensification of the CTRL storm is in response to the sloping terrain, which will be discussed in sections 4 and 5.

b. Precipitation evolution with a MABL

A squall line’s precipitation is modified as it descends the plateau slope and encounters the MABL, though the precipitation trends differ while a storm is over the slope versus at the plateau base (Fig. 4). Therefore, analyses are presented in three phases: when the storm is 1) over the plateau top (0–90 min), 2) over the slope (90–150 min), and 3) at the base (150–480 min). Over the plateau top, the magnitude and structure of the precipitation mass are essentially the same among the experiments. Over the slope, precipitation mass diverges as a function of MABL depth and $\theta'$. Generally, precipitation is largest for the storm encountering the deepest MABL and declines with decreasing MABL depth. Such precipitation differences are associated with

---

**Fig. 2.** Precipitation mixing ratio (g kg$^{-1}$, including rain, snow, and graupel, shaded), cloud mixing ratio (black contours of 0.5 g kg$^{-1}$, including cloud water and cloud ice), and $u$–$w$ wind (reference vector: 10 m s$^{-1}$) at (top to bottom) 60, 120, 180, and 240 min for simulations including [a(1)]–[a(4)] a 1.5-km plateau (CTRL), [b(1)]–[b(4)] no terrain (NoTER), and [c(1)]–[c(4)] a 1.5-km elevated surface across the full domain (OnlyTER). Terrain is shaded gray.
differences in storm intensity, highlighted by temporally zoomed-in time series of maximum vertical velocity ($w_{\text{max}}$), total precipitation mass, and rain rate (Fig. 5; Cecil and Zipser 1999; Xu and Wang 2015; Grant and van den Heever 2016). As storms initially collide with the upslope moving MABL, $w_{\text{max}}$ increases from 3–4 to 13–18 m s$^{-1}$, with the largest and earliest $w_{\text{max}}$ increases for storms encountering the deepest MABLs (Fig. 5a). For the CTRL storm, the collision with the ambient upslope flow in the absence of a MABL generates the weakest $w_{\text{max}}$. Total precipitation mass and rain rate (magnitude and timing) respond to this change in vertical motion, with the most intense and earliest precipitation enhancements during storm interactions with the deepest MABLs (Figs. 5b,c). The relationship between precipitation mass and MABL $u_0$ is less clear, though precipitation is initially largest in the experiment with deepest and coldest (1500-m-deep, $-2^\circ$K-$u_0$) MABL. Details of and dynamics associated with this precipitation peak are presented in section 4.

Once the squall lines reach the base and move away from the slope (after 150 min), the trend of precipitation mass with MABL depth reverses, with values generally continuing to increase until 240 min for storms over the shallowest MABLs, and declining for storms over the deepest MABLs (Fig. 4). While no robust relationship exists between MABL $\theta'$ and storm precipitation, more precipitation mass is produced for storms over the least cold ($-2^\circ$K-$\theta'$) 500-m-deep and 1500-m-deep MABLs. Further, the impacts of MABL $\theta'$ are more obvious for storms over the deepest MABL; Precipitation mass decreases more quickly while over the $-5^\circ$K-$\theta'$ MABL than the $-2^\circ$K-$\theta'$ MABL.

**c. Storm-scale evolution over terrain with no MABL**

Transitions in precipitation trends are presented in detail for the CTRL and a subset of representative MABL experiments, $-5^\circ$K-$\theta'$ for all depths, with consistent processes and trends for storms over the less cold ($-2^\circ$K, $-3^\circ$K, $-4^\circ$K-$u_0$) MABL. Details of and dynamics associated with this precipitation peak are presented in section 4.

Once the squall lines reach the base and move away from the slope (after 150 min), the trend of precipitation mass with MABL depth reverses, with values generally continuing to increase until 240 min for storms over the shallowest MABLs, and declining for storms over the deepest MABLs (Fig. 4). While no robust relationship exists between MABL $\theta'$ and storm precipitation, more precipitation mass is produced for storms over the least cold ($-2^\circ$K-$\theta'$) 500-m-deep and 1500-m-deep MABLs. Further, the impacts of MABL $\theta'$ are more obvious for storms over the deepest MABL; Precipitation mass decreases more quickly while over the $-5^\circ$K-$\theta'$ MABL than the $-2^\circ$K-$\theta'$ MABL.
MABLs. Cross sections of mixing ratio ($q$), equivalent potential temperature ($\theta_e$), and buoyancy ($B$) provide insight into structural changes for individual storms. For the CTRL storm interacting with the upslope ambient flow (Fig. 6a), the leading line convective plumes become shallower as the squall line descends down the slope [120 min; Fig. 7a(1)]. As the cold pool moves downslope, its depth decreases (< 1 km) and the associated $B$ increases [Fig. 7c(1); e.g., Frame and Markowski (2006)], resulting in a decline of cold pool speed from 13.03 m s$^{-1}$ on the plateau top (90–100 min) to 5.43 m s$^{-1}$ on the slope (120–130 min).$^2$

This thinning and warming reduce the cold-pool-driven lifting, contributing to relatively shallow (ground to 4 km) vertical motion [Fig. 7c(1)]. Though air parcels rise to their lifting condensation levels [LCLs; Fig. 7b(1)] and LFCs (0.99 km AGL), the updraft core, defined loosely as a plume of relatively high $\theta_e$, is comparatively thin while over the slope [Fig. 7b(1)], with lower $\theta_e$ values than the boundary layer source region, resulting in the smallest precipitation [Fig. 7a(1)].

At the plateau base (180 min), the storm intensifies [Fig. 7a(2)] as the cold pool deepens to > 2 km [Figs. 7b(2),c(2)], more effectively lifting surface parcels to their LFC in the presence of 1597 J kg$^{-1}$ of SBCAPE (Table 1). The associated vertical motion is tropospheric-deep, drawing high-$\theta_e$ boundary layer air into the storm [Fig. 7b(2)], leading to greater precipitation [Fig. 7a(2)]. The squall line remains well-organized and continues across the domain through 300 min [Figs. 7a(3)–c(3)].

d. Storm-scale evolution over terrain including a MABL

Squall-line vertical motion depth and magnitude are enhanced over the slope as storms collide with the advancing MABLs (Figs. 6b–d), with CI favored for progressively deeper MABLs (Figs. 8–10). The collision between a storm’s cold pool and the deepest MABL (Fig. 6d) initiates an intense ($q > 7$ g kg$^{-1}$), deep (>7 km) convective plume [Fig. 8a(1)] and deep ascent [Figs. 8b(1),c(1)], contributing to a substantial increase in precipitation (Fig. 4). Despite the overall reduction in low-level potential instability in the downstream marine environment, relatively undiluted, high-$\theta_e$ air from above the MABL is transported into the updraft [Fig. 8b(1)]. Low-CAPE (455.22 J kg$^{-1}$), negatively buoyant ($-5.96 \times 10^{-2}$ m s$^{-2}$) MABL air is mechanically lifted into the storm as well, with 10-min-average mean vertical displacement ($z_{10}$) of 3.7 km (Table 3). The more buoyant air ($> -0.07$ m s$^{-2}$) near the MABL top rises unhindered (> 6 km; Fig. 11c), while the least buoyant air ($< -0.10$ m s$^{-2}$) achieves smaller vertical excursions and descends after the initial lift (Fig. 11c).

Once at the plateau base, the squall begins to decay [Figs. 8a(2)–c(2),a(3)–c(3)]. Compared with the CTRL, the cold pool experiences a similar deepening and strengthening at the plateau bottom, but the MABL reduces the boundary layer $\theta_e$, decreasing the low-level horizontal buoyancy gradient (Table 3). Consequently, updrafts become elevated above the surface [Fig. 8c(2)]. Negatively buoyant air from the upper marine layer rises into the storm along sloped trajectories (Table 4), while the least buoyant parcels descend into the westward MABL flow (Fig. 11), illustrating that the cold pool outflow and surface-base lifting has weakened. Relatively rapidly, the updraft slows, contracts, and begins to tilt downshear [Figs. 8b(2)–c(2),b(3)–c(3)], contributing to a loss of precipitation [Figs. 8a(2),a(3)].

Collision with a moderately deep (1000-m) MABL (Fig. 6c) yields smaller, shallower mixing ratios [Fig. 9a(1)], lower $\theta_e$ [Fig. 9b(1)], and smaller vertical motion [Fig. 9c(1)] within the convective plumes. Average vertical velocity of MABL air is lower (Table 3), with updrafts $\geq 4$ m s$^{-1}$ ($\geq 12$ m s$^{-2}$) restricted to below 5 km [3 km; Fig. 9c(1)]. Despite their larger CAPE (705.38 J kg$^{-1}$), MABL parcels descend after rising only 2–3.5 km, with the exception of the most buoyant MABL parcels

$^2$The calculations of cold pool speed follow the method in Grant and van den Heever (2016).
which ascend to 5 km (Fig. 11b; Table 3). Once at sea level, even the least buoyant marine air parcels ascend into the updrafts, with larger vertical displacements, velocities, and CAPE than in the deepest MABL experiment (cf. Figs. 11e,f; Table 4). The MABL reduces the available potential instability, however, and by 300 min, only weak elevated, weak convection remains [Figs. 9a(3)–c(3)].

Collision with the shallowest (500-m) MABL (Fig. 6b) produces even less convective precipitation [Fig. 10a(1)], though low-level $u_e$ [Fig. 10b(1)] and vertical motion [Fig. 10c(1)] are larger than for the prior experiment. Parcels ascend more slowly along sloped trajectories with smaller average vertical displacements (Fig. 11a; Table 3), regardless of their larger CAPE (1052.54 J kg$^{-2}$); After rising 2–3 km, they all descend toward the surface (Fig. 11a). Once away from the terrain, intensity of the now elevated storm is similar to the CTRL [Figs. 10a(2)–c(2)], though the least buoyant MABL air remains below 1 km (Fig. 11d). Unlike the CTRL, there are signs of weakening as the storm traverses the domain. The high-$u_e$ conduit is no longer tropospheric-deep [Fig. 10b(3)], with evidence of a downshear tilt.

4. Mechanism for the precipitation enhancement over the slope

Pressure perturbation and vertical acceleration analyses provide additional insight into the observed precipitation enhancement for storms over mountainous coastal regions, and the associated ascent characteristics after the initial collision between cold pool and upslope flow.

a. Storm-scale physical processes over terrain with no MABL

Collision of the cold pool and upslope flow (no MABL) yields positive total vertical acceleration ($dw/dt$) around the CPLE from the surface to 2 km, dominated by inertial processes ($a_i$; Fig. 12a), suggesting that cold-pool-driven lifting is the major process responsible for the low-level ascent. The smaller buoyant acceleration ($a_b < 2 \times 10^{-2}$ m s$^{-2}$) also contributes positively to the lifting of high-$\theta_e$ ambient layer ahead of (i.e., east of) the CPLE, with negative acceleration behind (i.e., west of) the CPLE (Fig. 12a). Following the initial lift, parcel ascent above 2 km is inhibited by a negative $dw/dt$ center (primarily $a_i$, < $-6 \times 10^{-2}$ m s$^{-2}$) displaced above the cold pool behind the CPLE (Fig. 12a). The force driving these accelerations is primarily the nonhydrostatic component of the perturbation pressure gradient force (NPPGF; Fig. 12b). Positive inertial acceleration at the CPLE is due to an upward inertial NPPGF associated with a near-surface $p_{iab}$ maxima. Negative inertial acceleration west of the CPLE is caused by a downward NPPGF associated with a low-level $p_{iab}$ minima (Figs. 12b,c). This negative NPPGF limits the vertical displacement of the
CTRL air parcels after their initial lift, limiting the generation of precipitation.

More in-depth physical understanding of these pressure perturbations, which are critical in determining the fate of the air parcels, can be revealed by decomposing the $p_{0i}(nh)$ field into its individual contributing processes, which include the convergence/deformation of the wind field ("splat" term, positive contribution) and the rotation of the wind field ("spin" term, negative contribution; see the appendix for details). Physically, positive $p_{0i}(nh)$ is associated with convergence (splat) around the CPLE (Fig. 12d) as the cold pool encounters the upslope flow. Negative $p_{0i}(nh)$ behind the CPLE is associated with horizontal vorticity (spin; Fig. 12d), generated baroclinically due to horizontal gradients of buoyancy ($-\partial B/\partial x$; Figs. 12e,f; Trapp and Weisman 2003; Xu et al. 2015; Markowski 2016). The associated vorticity minimum is displaced slightly behind the baroclinic generation ($-\partial B/\partial x$) minimum due to rearward advection following its development (Fig. 12d). These physical processes explain the shallow updraft and the small precipitation enhancement over the slope. The near-surface positive $p_{0i}(nh)$ associated with the cold pool–upslope flow convergence accelerates parcels upward. The elevated negative $p_{0i}(nh)$ associated with baroclinically generated horizontal vorticity due to the sharp $B$ gradient across the cold pool interface then accelerates parcels downward, suppressing updraft development.

b. Storm-scale physical processes over terrain including a MABL

The magnitudes and locations of these vertical accelerations and nonhydrostatic pressure perturbations are altered by the upslope moving MABL. Generally, the near-surface positive total vertical accelerations at the CPLE ($dw/dt$) and the associated positive $p_{0i}(nh)$ are similar to those for the CTRL storm, with minimal differences in their locations and magnitudes (Table 5; Fig. 13). The most notable differences are for the storm colliding with the deepest MABL, with 15%–20% larger total acceleration ($4.09 \times 10^{-2} \text{ m s}^{-2}$), owing to 15%–20% greater convergence at the CPLE (Table 5; Fig. 13g), due to the deeper, stronger upslope flow (Fig. 6d). Considering parcel motion above this initial cold pool lift, the most critical differences lay behind the CPLE, in the negative $dw/dt$ above the cold pool ($dw/dt$). The presence of the MABL decreases the horizontal buoyancy gradient between cold pool and adjacent ambient atmosphere, which reduces the baroclinically generated horizontal vorticity (Figs. 13c,f,i), the associated negative $p_{0i}(nh)$ (Figs. 13b,e,h), and the downward inertial acceleration (Figs. 13a,d,g; Table 5). As a result, the magnitude of the
downward total vertical acceleration is reduced and its center is displaced upward, allowing for deeper and stronger ascending motion, and subsequently more precipitation. MABL depth determines the reduction in magnitude and increase in height of these minima, with weaker and more elevated minima for deeper MABLs (Figs. 13a,d,g).

5. Processes governing storm evolution at the plateau base

Arrival at the plateau base allows the CTRL storm and those encountering the shallowest MABLs to reintensify, while those moving over the deeper MABLs begin to decay. Analyses range from 150 to 170 min, beginning once the storms reach the plateau base (Figs. 14–17). With no MABL, the storm’s cold pool deepens and yields larger ascent at a new CPLE (denoted by “A” in Fig. 14b(1)) that forms ~4 km downstream of the original boundary [denoted by “B” in Fig. 14b(1)], that now exhibits vertically stacked positive and negative centers limiting ascent [Figs. 14a(1)–c(1)]. As the storm moves across the bottom, positive acceleration around the new CPLE deepens and intensifies, with buoyant processes initially playing a more dominant role [Figs. 14b(2),c(3)]. Consequently, surface-based high-\(\theta_e\) air is lifted to the LFC [~1 km; Figs. 14a(2),a(3)] and moisture is fluxed vertically, promoting CI and precipitation [Figs. 14c(2),c(3)]. However, elevated negative acceleration is still present behind the CPLE, contributing to sloped and weaker ascent [Figs. 14c(2)].

In contrast, advancement over the deepest MABL results in essentially the elimination of the horizontal buoyancy gradient across the CPLE, and the lowest levels are ultimately dominated by the westward moving MABL [Fig. 15a(1)]. Positive vertical acceleration at the CPLE weakens as the near-surface convergence dissipates, and is replaced by downward buoyant acceleration (\(a_b\)) from the negatively buoyant MABL [Figs. 15b(1)], limiting low-level ascent and vertical moisture flux [Fig. 15c(1)]. Ascent becomes elevated above 2 km and displaced 4–6 km ahead of the diffuse CPLE after 160 min [denoted by “A” in Figs. 15b(2),b(3)], likely forced by an elevated bore propagating faster than the decaying CPLE, which is addressed in section 6a. Lack of lower-level ascent prevents low-level air parcels from being lifted (to their LFC), including the elevated high-\(\theta_e\) air, which is now below the region of ascent, contributing to the decay of storm [Figs. 15c(2),c(3)].

When the depth of the coldest MABL is reduced, the cold pool is initially more buoyant than the MABL following its descent down the slope [Fig. 16b(1)]. The shallowest MABLs have the smallest inland penetration distance (Fig. 6b), allowing the cold pool to adiabatically warm during its descent down the full length of the slope, without modification by the MABL. A new elevated region of (bore-induced) buoyant acceleration (\(a_b\)) forms 1–2 km ahead of the CPLE and atop the stable MABL [centered near 2 km in altitude, indicated by “A” in Figs. 16b(1)–b(3)], while the original \(\text{d}w/\text{d}t\) couplet becomes vertically stacked west of the CPLE, as in the CTRL, and no longer plays a role. This elevated buoyant acceleration lifts high-\(\theta_e\) air available above the MABL into the updraft, as the buoyancy gradient in the lowest 500-m decreases and surface-based ascent decays [Figs. 16b(3),c(3)].
Processes are briefly compared for the storm moving over the 1500-m-deep, −5-K-θ’ MABL and the less cold −3-K-θ’ MABL with the same depth, to highlight the influence of MABL temperature on storm evolution and differences in precipitation noted in Fig. 4. The cold pool remains less buoyant than the −3-K-θ’ MABL following its descent downslope, allowing for continued convergence and surface-based upward acceleration at the CPLE at sea level [denoted by ‘A’ in Figs. 17b(1)–b(3)], as for the CTRL storm; however, elevated buoyant acceleration forms above the MABL [denoted by ‘B’ in Figs. 17b(1)–b(3)], as for the storms over the coldest MABLs. Therefore, when MABLs are more buoyant/warmer than the incoming cold pool, surface-based ascent remains due to the continued presence of a low-level θ_e gradient, while elevated ascent develops due to the relative stability of the layer helping to support bore activity. The conjunction of these two maxima yields deep vertical motion at the CPLE [Figs. 17c(1)–c(3)]. Though the lower-θ_e MABL air is lifted into the updraft by the surface-based inertial acceleration (a_i), any potentially negative impact on storm survival may be mitigated by the influx of high-θ_e from above the MABL by the elevated buoyant process (a_b), contributing to the maintenance of the storm at the plateau base.

6. Discussion

a. The development and role of bores

Bores have been shown to form following the collision of a squall line’s cold pool and a MABL in coastal regions with no terrain relief, capable of supporting convection offshore over the stable layer MABL (LK18; Lombardo 2020). For squall lines over mountainous coastal regions, similar processes occur following the cold pool–MABL collision. Lifting is supported by an internal bore once storms arrive at sea level and move over the full depth of the MABL, evidenced by elevated buoyant accelerations above the MABL east of the CPLE [Figs. 15b(2,3), 16b(2,3), 17b(2,3)], and a semipermanent dome in buoyancy (θ not shown) west of the CPLE [Figs. 15a(2–3), 16a2–3]. While these bores can provide a mechanism to lift high-θ_e air located above the MABL into the updraft, storm maintenance is, in part, determined by the availability of this elevated unstable air. Deeper, colder MABLs reduce and can eliminate the elevated high-θ_e layer, promoting storm decay, despite the presence of bore-generated elevated lift.

Bores have also been shown to form when squall lines descend the lee slope of an idealized bell-shaped mountain and encounter a stationary, nocturnal stable layer (Letkiewicz and Parker 2011). With no stable layer downstream of the mountain, convection was suppressed over the lee slope, leading to a precipitation shadow, as in our CTRL simulation. With a lee-side nocturnal stable layer included, convection initiated over the slope as bore-driven lifting developed, and the lee precipitation shadow diminished. Once the nocturnal storms moved away from the mountain, those over the coldest nocturnal layers decayed fastest due to the reduction in available instability, as for our storms over the colder, deeper MABLs. In contrast, for our coastal storms, bores only form once the storms move away from the slope, and the leeside precipitation enhancements are due to the cold-pool-driven processes rather

---

**FIG. 9.** As in Fig. 7, but for the experiment with a 1000-m-deep, −5-K-θ’ MABL.
than the bore-induced lifting. Such differences may be attributed to the motion of stable layer (stationary nocturnal versus moving MABL), the thermal deficit and depth of the stable layer, the terrain characteristics (height, slope angle and distance, bell versus plateau concavity), and the base-state environment (CAPE, vertical wind shear). One commonality between the studies is that an enhancement in precipitation over the downslope does not indicate a longer-lived squall line.

The presence of coastal terrain impacts the generation and utility of bores in supporting offshore deep convection, thus the lifetime of a storm offshore, compared to storms over flat coastal land. Lombardo (2020) showed for squall lines exclusively at sea level, initiated in the same base-state thermodynamic-kinematic environment and with the same CI forcing at t = 0 as the present study, the collision of a storm’s cold pool and a −6-K-θ’

1500-m-deep MABL produced relatively strong bore-induced ascent capable of supporting an intense storm hundreds of kilometers offshore. This is in direct contrast to the storm colliding with the −5-K-θ’, 1500-m-deep MABL in the present study, which decayed within 20 km of the terrain base. We hypothesize that such differences may be a consequence of 1) differences in MABL depth at the time of collision, which is larger over flat-land than over the sloping terrain, 2) differences in cold pool buoyancy and depth at the time of collision, with shallower, more buoyant cold pools over the downward sloping terrain, 3) the angle of interaction between the cold pool and MABL, and 4) differences that arise between simulations performed in 2D (this study) and 3D (Lombardo 2020). Additional research is required to more deeply understand differences in coastal storm processes for those in the presence and absence of coastal terrain relief.

**b. RKW approach to evaluate squall-line structure in a mountainous coastal environment**

Previous studies have applied a vorticity budget analysis to determine the vertical orientation of ascent over the depth of the cold pool, thus updraft orientation and storm lifetime

---

**Fig. 10.** As in Fig. 7, but for the experiment with a 500-m-deep, −5-K-θ’ MABL.

**TABLE 3.** Statistics of the MABL parcels released when the cold pools collide with the coldest MABLs (−5-K θ’) with varied depths (indicated by the black contours in Figs. 11a–c), including buoyancy (B, 10⁻² m s⁻²), convective available potential energy (CAPE, J kg⁻¹), and convective inhibition (CIN, J kg⁻¹) at the release times (112, 109, and 107 min for 500-, 1000-, and 1500-m depths, respectively), parcel height (z₁₀, km) and vertical velocity (w, m s⁻¹) 10 min after release. Results are averaged over all MABL air parcels with B < −0.01 m s⁻².

<table>
<thead>
<tr>
<th>Expt</th>
<th>B (10⁻² m s⁻²)</th>
<th>CAPE (J kg⁻¹)</th>
<th>CIN (J kg⁻¹)</th>
<th>z₁₀ (km)</th>
<th>w (m s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500m_−5K</td>
<td>−4.45</td>
<td>1052.54</td>
<td>23.68</td>
<td>2.65</td>
<td>2.65</td>
</tr>
<tr>
<td>1000m_−5K</td>
<td>−5.16</td>
<td>705.38</td>
<td>19.25</td>
<td>3.26</td>
<td>3.42</td>
</tr>
<tr>
<td>1500m_−5K</td>
<td>−5.96</td>
<td>455.22</td>
<td>18.86</td>
<td>3.71</td>
<td>4.23</td>
</tr>
</tbody>
</table>
Here, we illustrate the connection between this approach and the pressure perturbation analysis presented above, toward a unified theory on squall-line evolution in complex environments, namely in the presence of orographic and low-level atmospheric variability. We begin with RKW’s Eq. (5) (Rotunno et al. 1988; see also Eq. (5) in Weisman 1992), which is based on the two-dimensional vorticity and mass-continuity equations integrated over a box centered on the CPLE in a coordinate system that moves at the speed of the CPLE:

\[
\frac{\partial}{\partial t} \int_L^R \int_0^H \eta \, dz \, dx = \int_L^R \int_0^H (u \eta)_x \, dz \, dx - \int_L^R \int_0^H (u \eta)_y \, dz \, dx - \int_0^R (w \eta)_L \, dx + \int_0^H (B_L - B_R) \, dz ,
\]

where \( L \) is the left boundary, \( R \) is the right boundary, \( H \) is the top of cold pool, \( u \) is the horizontal velocity relative to the

\( \eta \) is the buoyancy, \( \theta \) is the potential temperature, \( B \) is the buoyancy, and \( \omega \) is the vertical velocity. The terms on the right-hand side of the equation represent the tendency, the flux at left, the flux at right, the flux at top, and the net baroclinic generation, respectively.

![Figure 11](image-url)

**Fig. 11.** Trajectory centroids of MABL parcels (defined by buoyancy \( < -0.01 \) m s\(^{-2} \)) released 4–6 km in front of the CPLE (area shown by black closed contour) (a)–(c) at the initial collision times between the cold pools and a \(-5\)-K-\( \theta \) (a) 500-, (b) 1000-, and (c) 1500-m-deep MABL; (d)–(f) as in (a)–(c), but for parcels released at 170 min. Parcels with buoyancy of \([-0.04, -0.01] \) m s\(^{-2} \) are contoured solid black, \([-0.07, -0.04] \) m s\(^{-2} \) are dashed black, \([-0.10, -0.07] \) m s\(^{-2} \) are dashed dotted black, and \( < -0.10 \) m s\(^{-2} \) are dotted black. Black circles indicate the selected parcels’ centroid positions after 10 min (note that the trajectory paths extend beyond 10 min). Vertical displacements (km) after 10 min for all parcels released in the full domain displayed in the figure panels are shaded. The MABL is contoured blue, defined by a buoyancy value of \(-0.01 \) m s\(^{-2} \). Terrain is shaded gray. The \( x \) axis is distance with respect to the CPLE. Storm motion is not removed from the parcel trajectories.
The speed of the CPLE, \( w \), is the vertical velocity, \( \eta \) is the 2D horizontal vorticity, and \( B_L(B_R) \) is the buoyancy west (east) of the CPLE. A steady balance is assumed, and the horizontal vorticity tendency is set to zero (as in Rotunno et al. 1988):

\[
0 = \left( \frac{u_L^2}{2} - \frac{u_R^2}{2} \right) - \left( \frac{u_L^2}{2} - \frac{u_R^2}{2} \right) - \int_L^R (\mathbf{w}\eta) \, dx + \int_0^H (\dot{B}_L - \dot{B}_R) \, dz.
\]

Table 4. Statistics of the MABL parcels released when the squall lines are at the plateau base (indicated by the black contours in Figs. 11d–f), including buoyancy \( (B, 10^{-2} \text{ m s}^{-2}) \), CAPE \( (\text{J kg}^{-1}) \), and CIN \( (\text{J kg}^{-1}) \) at the release time (170 min for all experiments), parcel height \( (z_{20}, \text{km}) \), and vertical velocity \( (w, \text{m s}^{-1}) \) 20 min after release. Results are averaged over all MABL parcels with \( B \) of \([-0.1, -0.01) \) m \( \text{s}^{-2} \). Note that the MABL parcels with \( B < -0.1 \) m \( \text{s}^{-2} \) are not included in the statistics to evaluate only parcels that are lifted to the updrafts (Figs. 11d–f).

<table>
<thead>
<tr>
<th>Expt</th>
<th>( B (10^{-2} \text{ m s}^{-2}) )</th>
<th>CAPE (J kg(^{-1}))</th>
<th>CIN (J kg(^{-1}))</th>
<th>( z_{20} ) (km)</th>
<th>( w ) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>500m_5K</td>
<td>-3.75</td>
<td>912.91</td>
<td>8.51</td>
<td>6.21</td>
<td>4.51</td>
</tr>
<tr>
<td>1000m_5K</td>
<td>-4.89</td>
<td>685.52</td>
<td>13.61</td>
<td>5.66</td>
<td>4.03</td>
</tr>
<tr>
<td>1500m_5K</td>
<td>-6.16</td>
<td>196.74</td>
<td>32.78</td>
<td>5.16</td>
<td>3.34</td>
</tr>
</tbody>
</table>

Fig. 12. CTRL storm 10-min-averaged (from the collision time to 10 min after) (a) vertical acceleration \( \frac{dw}{dt} \) (10\(^{-2}\) m s\(^{-2}\), shaded), inertial acceleration \( a_i \) (intervals of 0.02 m s\(^{-2}\) contoured black), buoyant acceleration \( a_b \) (intervals of 0.02 m s\(^{-2}\) contoured cyan), \( u-w \) wind (reference vector: 6 m s\(^{-1}\)); \( u \) wind is relative to the CPLE speed; and vertical velocity (8 m s\(^{-1}\) contoured magenta); (b) nonhydrostatic pressure perturbation \( p_{nh} \) (hPa, shaded), inertial pressure perturbation \( p_{inh} \) (nh) (intervals of 0.3 hPa contoured black), and buoyant pressure perturbation \( p_{bih} \) (nh) (intervals of 0.3 hPa contoured cyan); (c) \( p_{nh} \) (hPa), \( p_{inh} \) (nh) (intervals of 0.3 hPa contoured black), and \( p_{bih} \) (nh) (intervals of 0.3 hPa contoured cyan); (d) the sum of the splat and spin terms (10\(^{-2}\) s\(^{-2}\), shaded), and \( u-w \) wind (reference vector: 4 m s\(^{-1}\)); and (f) baroclinic term \( -\dot{B}/\dot{\alpha} \) (10\(^{-4}\) s\(^{-2}\), shaded) and horizontal vorticity (intervals of 8 \times 10\(^{-3}\) s\(^{-1}\) contoured black). All results are spatially averaged relative to the CPLE (gray dashed line). Positive (negative) values are indicated by solid (dotted) lines, and terrain is shaded gray.
Because an optimal state is sought such that the ambient low-level flow at the CPLE is turned vertically by the cold pool and exits the top of the cold pool as a symmetric, vertically orientated jet, the integral of $w_h$ across the jet is set to zero (Rotunno et al. 1988; Weisman 1992). With no further assumptions to perform a rigorous evaluation of the vorticity budget, the fundamental RKW concept is obtained, namely a balanced state between ambient-wind-shear-generated and baroclinically generated horizontal vorticity:

$$\left(u^1_{R,0} - u^1_{L,H}\right) + \int_0^H -\left(B_L - B_R\right) \, dz = u^2_{R,D} - u^2_{R,H}. \tag{3}$$

The first term on the lhs of Eq. (3) is the vorticity associated with storm-relative flow at the left boundary, and second is...
The rhs of Eq. (3) is the vorticity associated with environmental shear on the right boundary over the depth of the cold pool. Physically, Eq. (3) includes contributions from the environmental wind shear (Fig. 1), the storm-relative flow associated with the moving MABL (Fig. 6), the MABL modified ambient buoyancy (i.e., $B_R \neq 0$; Figs. 8c–10c, 14a–17a), and the storm-relative flow within the cold pool (Figs. 8a–10a, 14a–17a), all necessary features to consider a mountainous coastal environment. Note that over the sloping terrain, the near-surface relative flow within the cold pool is gravitationally accelerated downward (i.e., $u_{L,0} > 0$; Figs. 12a and 13a,d,g), indicating that the cold pool is in transition from deep (subcritical) to shallow (supercritical) flow (Frame and Markowski 2006). We define the net baroclinic generation of horizontal vorticity $C^2$ as

$$C^2 = 2 \int_0^H - (B_L - B_R) \, dz = C_L^2 - C_R^2$$

where $C_L^2$ ($C_R^2$) is negative (positive) vorticity generated by cold pool (environmental) buoyancy. We also define the negative (positive) vorticity associated with shear at the left (right) boundary as,

$$\Delta C_L^2 = u_{L,0}^2 - u_{L,H}^2$$
respectively. Equation (3) can then be reduced to
\[ \Delta u_R^2 = u_L^2 - u_R^2, \]
where the sum of vorticity tendencies west of the CPLE is denoted by \( C_j \). A vertically oriented jet at the CPLE is obtained when \( C_j/\Delta u_R > 1 \), while the vertical jet tilts upshear when \( C_j/\Delta u_R > 1 \) and downshear when \( C_j/\Delta u_R < 1 \), following Rotunno et al. (1988). In general, for our squall lines, Eq. (7) implies that the import of the relative flows within the cold pool (\( \Delta u_L^2 > 0 \)) intensifies the negative vorticity west of the CPLE, so an optimal state requires a larger ambient wind shear east of the CPLE over the depth of cold pool. In contrast, the negatively buoyant MABL (\( C_R^2 > 0 \)) will decrease the integrated buoyancy gradient (\( C_R^2 \)), requiring a smaller ambient wind shear.

Here, we present the terms in the vorticity budget and the resulting balance predicted by the analysis while storms are over the sloping terrain and over the marine layer at sea level, two important stages in the storms’ life cycles. For storms over the terrain, calculations are performed at the time of collision over a 20-km horizontal domain centered on the CPLE, integrated from the height of the terrain surface where the CPLE is located (\( z \sim 0.75 \) km) to the top of the cold pool (\( z \sim 2.25 \) km) where the buoyancy is near zero [Figs. 7c(1)–c(1)]. For the CTRL storm with no MABL, the downslope accelerating surface flow within the cold pool (\( u_L \)) and relatively large horizontal buoyancy gradient across the CPLE (\( C^2 = C_j^2 - C_R^2 \), Table 6) leads to an imbalance of vorticity across the CPLE; Negative vorticity west of the CPLE has a larger magnitude than that of positive vorticity on the east side, dictating an upshear-tilted updraft (\( C_j/\Delta u_R = 1.77 \), Table 6). This is consistent with the orientation produced in our simulations [Figs. 7a(1)–c(1)], and our pressure perturbation analysis; The baroclinically generated negative pressure perturbation above the cold pool results in the downward acceleration of parcels.
following their initial ascent at the CPLE, limited parcel ascent, weaker precipitation, and an upshear tilted updraft (Figs. 12a–c).

For progressively deeper MABLs, $C_j/\Delta u_{R}$ values decrease from 1.77 to 1.17 (Table 6), indicating more upright ascent with increasing MABL depth. This is primarily due to a reduction in the buoyancy gradient across the CPLE ($C_j$) due to the presence of the relatively dense MABL, and an increase in the ambient wind shear east of the CPLE ($\Delta u_{R}$) associated with stronger upslope flows generated by the moving MABL (Fig. 6). The vertically integrated buoyancy is lowest and the magnitude of the upslope flow is greatest for the deepest MABL. Though fluctuations in the buoyancy of the cold pools ($C_j^2$) and the relative flow within the cold pools ($\Delta u_{R}^2$) also exist, they do not systematically contribute to the resulting updraft orientations. These results are consistent with those of our pressure perturbation analysis: For progressively deeper MABLs, the reduction in the baroclinically generated elevated negative pressure perturbation (Figs. 13b,e,h) allows for parcels to accelerate vertically (Figs. 11a–c), leading to more intense precipitation and upright updrafts (Figs. 8–10).

Once the cold pools reach sea level, they deepen, intensify, and slow down [Figs. 14a(2)–17a(2)], reflected by the larger $C_j^2$ and smaller $\Delta u_{R}^2$ (cf. Table 6 and Table 7). As such, the dominant contributor to negative vorticity within the cold pool transitions from kinematic ($\Delta u_{R}^2$) to thermodynamic ($C_j^2$; Table 7), as in our pressure-decomposition analysis. For the CTRL squall line, the negative vorticity behind the CPLE ($C_j$) continues to overwhelm the environmental wind shear ($C_j/\Delta u_{R} = 1.63$; Table 7), and the upshear tilted structure is maintained. This behavior is consistent with the continued presence of the elevated negative vertical acceleration associated with the baroclinically generated negative pressure perturbation (Fig. 14). The marine layer helps to “correct” this imbalance, as the buoyancy gradient across the CPLE is diminished ($C_j^2$) and the ambient wind shear is larger ($\Delta u_{R}$; Table 7). The relative contributions of the vorticity budget terms, thus $C_j/\Delta u_{R}$, are sensitive to MABL depth and density, as for the vertical accelerations and pressure perturbations. Comparing the shallow, cold (500-m-deep, $-5-K$) MABL.

Fig. 16. As in Fig. 14, but for the experiment with a 500-m-deep, $-5-K$ MABL.
and deep, less cold (1500-m-deep, $2-3\,^\circ\mathrm{C}$) MABLs, values of the buoyancy terms are similar ($C^2$), given that the buoyancy term is the vertical integral over the depth of the cold pool, i.e., the contribution of a less cold MABL distributed through a greater depth is similar to that of a cold MABL concentrated over a shallow depth. However, the deep MABL drives a stronger near-surface wind ($D_uR$; Fig. 6), which contributes to more upright ascent ($CJ/D_uR = 0.98$; Table 7; Fig. 17) compared to the shallow MABL ($CJ/D_uR = 1.36$; Table 7; Fig. 16). This is consistent with the greater surface-based inertial acceleration ($a_i$) observed at the CPLE due to enhanced convergence in the presence of the deep, but less cold MABL (cf. Figs. 16 and 17).

![Fig. 17. As in Fig. 14, but for the experiment with a 1500-m-deep, $-3\,^\circ\mathrm{C}$ MABL.](image)

Table 6. Vorticity budget analysis following Rotunno et al. (1988), where $C^2$ ($\mathrm{m}^2 \, \mathrm{s}^{-2}$) is the net baroclinic generation of vorticity calculated from $C_L^2 - C_R^2$; $C_L^2$ and $C_R^2$ are the baroclinically generated vorticity to the left and right of the CPLE; $\Delta u_r^2$ ($\mathrm{m}^2 \, \mathrm{s}^{-2}$) are contributions from flows within the cold pool; $C_I$ ($\mathrm{m} \, \mathrm{s}^{-1}$) represents the negative vorticity within the cold pool, including contributions from $C^2$ and $\Delta u_r^2$; $\Delta u_R$ ($\mathrm{m} \, \mathrm{s}^{-1}$) is the ambient wind shear east of the CPLE; and $C_J/\Delta u_R$ is the balance approximation. Values of $C_L^2$ and $\Delta u_r^2$ ($C_R^2$ and $\Delta u_R$) are averaged over 10 km west (east) of the CPLE from the terrain height ($z = 0.75$ km) to the top of the cold pool ($z = 2.25$ km) when the cold pool collides with the upslope flow/moving MABL (107–113 min) on the slope, for the CTRL storm and the those encountering a 500-m-deep $-5\,^\circ\mathrm{C}$ MABL, a 1000-m-deep $-5\,^\circ\mathrm{C}$ MABL, and a 1500-m-deep $-5\,^\circ\mathrm{C}$ MABL.

<table>
<thead>
<tr>
<th>Expt</th>
<th>$C_L^2$ ($\mathrm{m}^2 , \mathrm{s}^{-2}$)</th>
<th>$C_R^2$ ($\mathrm{m}^2 , \mathrm{s}^{-2}$)</th>
<th>$C^2$ ($\mathrm{m}^2 , \mathrm{s}^{-2}$)</th>
<th>$\Delta u_r^2$ ($\mathrm{m}^2 , \mathrm{s}^{-2}$)</th>
<th>$C_I$ ($\mathrm{m} , \mathrm{s}^{-1}$)</th>
<th>$\Delta u_R$ ($\mathrm{m} , \mathrm{s}^{-1}$)</th>
<th>$C_J/\Delta u_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTRL</td>
<td>86.33</td>
<td>-32.41</td>
<td>118.74</td>
<td>315.77</td>
<td>20.84</td>
<td>11.76</td>
<td>1.77</td>
</tr>
<tr>
<td>MABL_500m_5K</td>
<td>102.53</td>
<td>-23.82</td>
<td>126.35</td>
<td>345.59</td>
<td>21.72</td>
<td>12.24</td>
<td>1.77</td>
</tr>
<tr>
<td>MABL_1000m_5K</td>
<td>99.11</td>
<td>22.15</td>
<td>76.96</td>
<td>215.50</td>
<td>17.10</td>
<td>14.30</td>
<td>1.20</td>
</tr>
<tr>
<td>MABL_1500m_5K</td>
<td>117.30</td>
<td>64.54</td>
<td>52.76</td>
<td>269.29</td>
<td>17.95</td>
<td>15.30</td>
<td>1.17</td>
</tr>
</tbody>
</table>
For the squall line encountering the coldest, deepest MABL, the cold pool becomes elevated \[\text{Figs. 8a(2)–c(2); e.g., Hitchcock and Schumacher 2020; Parsons et al. 2019; Grasmick et al. 2018; Haghi et al. 2019}\], requiring an upward shift in the bottom boundary of the vorticity budget analysis (e.g., \(z = 0.5\) km above sea level), with the MABL serving as a lower boundary. Similar conclusions were reached using a bottom boundary of \(0.4\)–\(0.7\) km, i.e., within the moving MABL. Regardless of the vertical displacement, the reduction in the horizontal buoyancy gradient across the CPLE and wind shear within the cold pool limits the negative vorticity production west of the CPLE \((C_J = 9.11)\), leading to a downstream tilted structure \([C_J/\Delta u = 0.93, \text{Table 7; Fig. 15a(2)}]\). However, at this time, the CI mechanism transitions from a surface-based cold-pool to an elevated bore.

**7. Conclusions**

Coastal midlatitude squall-line evolution was evaluated in the presence of plateau-shaped orography, a common coastal terrain configuration in regions with squall-line activity. Most importantly, this work identified and quantified the dominant

<table>
<thead>
<tr>
<th>Expt</th>
<th>(C_J^2) (m(^2)s(^{-2}))</th>
<th>(C_K^2) (m(^2)s(^{-2}))</th>
<th>(C_L^2) (m(^2)s(^{-2}))</th>
<th>(\Delta \sigma_1^2) (m(^2)s(^{-2}))</th>
<th>(C_J) (m s(^{-1}))</th>
<th>(\Delta u_R) (m s(^{-1}))</th>
<th>(C_J/\Delta u_R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTRL</td>
<td>290.58</td>
<td>-7.03</td>
<td>297.61</td>
<td>30.80</td>
<td>18.12</td>
<td>11.10</td>
<td>1.63</td>
</tr>
<tr>
<td>MABL_500m_5K</td>
<td>359.61</td>
<td>164.10</td>
<td>195.51</td>
<td>-7.27</td>
<td>13.72</td>
<td>10.06</td>
<td>1.36</td>
</tr>
<tr>
<td>MABL_1500m_5K</td>
<td>318.59</td>
<td>159.98</td>
<td>158.61</td>
<td>41.09</td>
<td>14.13</td>
<td>14.42</td>
<td>0.98</td>
</tr>
<tr>
<td>MABL_1500m_3K</td>
<td>210.18</td>
<td>160.93</td>
<td>49.25</td>
<td>33.76</td>
<td>9.11</td>
<td>9.83</td>
<td>0.93</td>
</tr>
</tbody>
</table>

**Table 7.** As in **Table 6**, but at 160–170 min when the cold pool arrives at the plateau base, for the CTRL storm and the those encountering a 500-m-deep \(-5\text{-K}\) MABL a 1500-m-deep \(-3\text{-K}\) MABL, and a 1500-m-deep \(-5\text{-K}\) MABL. For the experiment with a 1500-m-deep \(-5\text{-K}\) MABL, results are calculated from \(z = 0.5\)–\(2.25\) km due to the presence of an elevated cold pool.

**Fig. 18.** Schematic diagram of a squall line descending a plateau slope and encountering an upslope-moving (a) 500-m-deep, \(-5\text{-K}\) MABL, where cold-pool-lifted air parcels are accelerated downward due to a baroclinically generated \(p_{0i}^\text{up}(\text{nh})\), (b) 1500-m-deep, \(-5\text{-K}\) MABL, where cold-pool-lifted parcels rise unimpeded into the updraft due to the smaller, weaker \(p_{0i}^\text{up}(\text{nh})\). Squall lines moving away from the terrain over a (c) 500-m-deep, \(-3\text{-K}\) MABL, where a surface-based cold pool lifts low-\(\theta_e\) air from within the MABL and an elevated bore lifts high-\(\theta_e\) air from above the MABL into the updraft, and (d) 1500-m-deep, \(-5\text{-K}\) MABL, where only a thin layer of high-\(\theta_e\) air is available for bore-induced lift, with no surface-based cold pool ascent.
physical mechanisms responsible for precipitation enhancement over the plateau slope as storms descend from the plateau top toward the moving MABL, contributing to observed coastal rain maxima. A summary schematic of the associated processes is provided in Fig. 18. Specifically, the collision of a storm’s cold pool and a slope-ascending MABL drives an upward acceleration of lower-buoyancy marine boundary layer air parcels, whose vertical displacement directly impacts precipitation amount and intensity. A parcel’s vertical displacement is determined by the magnitude and location of a negative nonhydrostatic inertial pressure perturbation \( p'_{\text{nh}} \) behind the gust front, which forms through the baroclinic generation of horizontal vorticity behind the CPLE. For large horizontal gradients in buoyancy across the gust front (e.g., 500-m-deep MABL), an elevated, but low-level, negative \( p'_{\text{nh}} \) induces a downward acceleration of air parcels following the initial cold pool lift, leading to shallow ascent and less precipitation (Fig. 18a). Presence of a cold, deep MABL reduces the horizontal buoyancy gradient across the gust front, leading to smaller baroclinically generated horizontal vorticity behind the gust front, thus a smaller pressure perturbation and weaker downward vertical acceleration (Fig. 18b). Consequently, parcels ascend to higher altitudes along more upright trajectories leading to overall more precipitation and a greater rain rate. The magnitude of this horizontal vorticity is sensitive to MABL depth, with less vorticity, a smaller negative \( p'_{\text{nh}} \) and a smaller negative vertical acceleration for deeper MABLs. Therefore, precipitation over the slope is more intense in the presence of deep MABLs, in spite of the smaller surface-based instability.

As the squall lines move away from the terrain, those over the deepest MABLs decay most rapidly, while those over the shallowest decay last, predominantly related to the availability of elevated instability. MABL depth dictates the height of the elevated forcing that forms as storms move over a MABL, and thus the amount of warm, moist air above the MABL that is drawn into the storm (Figs. 18c,d). Low-level \( \theta_e \) is reduced in the presence of the MABL, and more importantly, deeper MABLs diminish larger amounts of elevated instability available to the storm, promoting more rapid storm decay. Further, MABL temperature determines whether a storm retains a surface-based component (e.g., \(-3\text{-K} \theta') or becomes completely elevated (e.g., \(-5\text{-K} \theta') . For less cold MABLs, surface-based convergence persists, which allows MABL air to ascend into the storm, which overtime may be detrimental (Fig. 18c). For the coldest MABLs, cold pools become elevated due to the large stability of the lowest-layers (i.e., the moving MABLs), and storms are supported by bores with no surface-based forced ascent, which lifts the limited supply of warm, moist air from above the MABL into the storm (Fig. 18d).

Though this work has advanced our understanding of squall-line dynamics over orographic coastal regions, future expansion of this work should include moving to 3D, as well as exploring a larger parameter space of ambient conditions (instability, vertical wind shear, moisture, which impact cold pool characteristics) and orographic configurations (heights, slopes, concavity) to further verify the robustness of this work, while seeking out deviations from these processes. A parallel manuscript is under development exploring the dynamics of moving MABLs as they ascend sloping terrain, and the impact of squall lines on these coastal circulations. Additional work should also continue toward bridging the gap between coastal and nocturnal squall lines, highlighting their many shared physical processes, but also emphasizing their differences.

Acknowledgments. This research would not have been possible without the generous support from the National Science Foundation Grant AGS-1514115, Office of Naval Research Grant N00014-16-1-3199, and Office of Naval Research Grant N00014-17-1-2478. Computational support for this research was provided by the Pennsylvania State University’s Institute for Computational and Data Sciences’ Roar supercomputer. The authors greatly thank Dr. George Bryan for the development of and continued improvements to the CM1. The authors also thank the Editor Dr. David Mechem and two anonymous reviewers for their insightful comments.

APPENDIX

Decomposition of Vertical Accelerations and Pressure Perturbations

The anelastic momentum equation (JR15) can be written as

\[
\frac{d\mathbf{v}}{dt} = -\nabla p - pgk, \tag{A1}
\]

where \( \bar{p} \) is the base-state density including all hydrometeors, \( \mathbf{v} = (u, v, w) \) is the wind field, \( p \) is pressure, \( g \) is the gravitational acceleration, and \( \nabla p \) is the pressure gradient. Total \( p \) and \( \rho \) can be divided into a horizontally homogeneous base-state value and a deviation from the base state:

\[
p(x, y, z, t) = \bar{p}(z) + p'(x, y, z, t) \tag{A2}
\]

and

\[
\rho(x, y, z, t) = \bar{\rho}(z) + \rho'(x, y, z, t), \tag{A3}
\]

where the base state (perturbation) is denoted with an overbar (a prime).

Introducing Eqs. (A2) and (A3) to Eq. (A1) and applying the hydrostatic balance, the anelastic momentum equation can be rewritten in the form of perturbation as

\[
\frac{d\mathbf{v}'}{dt} = -\nabla p' - \rho'gk, \tag{A4}
\]

which can be expressed as

\[
\frac{d\mathbf{v}'}{dt} + \mathbf{v}' \cdot \nabla \mathbf{v}' = -a_0 \nabla p' + Bk, \tag{A5}
\]

where \( B = -(\rho'/\bar{\rho})g \) is buoyancy and \( a_0 = 1/\bar{\rho} \) is specific volume.

The pressure perturbation can be decomposed into a hydrostatic \( p'_h \) and nonhydrostatic \( p'_{\text{nh}} \) component:

\[
p' = p'_h + p'_{\text{nh}}. \tag{A6}
\]
where $p'_b$ can be related to $\rho'$ as $\partial p'_b/\partial z = -\rho' g$. Plugging Eq. (A6) into Eq. (A5) yields
\[
\frac{\partial v}{\partial t} + v \cdot \nabla v = -\alpha_0 (\nabla p'_{nh} + \nabla p'_b).
\] (A7)
Therefore, the vertical component of Eq. (A7) is
\[
\frac{dw}{dt} \bigg|_{\mu = \pi} = -\alpha_0 \frac{\partial p'_{nh}}{\partial z}.
\] (A8)
which indicates that vertical acceleration is from the non-hydrostatic perturbation pressure gradient force (NPPGF), which can be solved for by taking the divergence of Eq. (A7) and applying the anelastic mass continuity $\nabla \cdot (\rho v) = 0$, yielding the Poisson equation (PE) about $p'_b$:
\[
-\nabla^2 p'_b = \nabla \cdot (\rho v \cdot \nabla v) + \nabla^2 p'_0.
\] (A9)
where $\nabla^2 (\nabla^2)$ is the 3D (2D) Laplace operator. Plugging $-\partial_z$ from Eq. (A9) into Eq. (A8) yields a similar form of the vertical equation to JR15:
\[
-\nabla^2 \left( \frac{dw}{dt} \right) = -\partial_z \nabla \cdot (\rho v \cdot \nabla v) + g \nabla^2 p'_0.
\] (A10)
Following JR15, we define the buoyant acceleration ($a_b$) as the vertical acceleration if the wind field is instantaneously zero:
\[
a_b = \left. \frac{dw}{dt} \right|_{v = 0}.
\] (A11)
and the inertial acceleration ($a_i$) if the horizontal density anomalies are instantaneously zero:
\[
a_i = \left. \frac{dw}{dt} \right|_{\mu = \pi}.
\] (A12)
Applying Eq. (A11) to Eq. (A10) yields the PE about $a_b$:
\[
-\nabla^2 (\rho a_b) = g \nabla^2 p'_0.
\] (A13)
and applying Eq. (A12) to Eq. (A10) yields the PE about $a_i$:
\[
-\nabla^2 (\rho a_i) = -\partial_z \nabla \cdot (\rho v \cdot \nabla v).
\] (A14)
As $w$ maintains zero at bottom and top boundaries, the PEs of Eqs. (A10), (A13), and (A14) have the same Dirichlet boundary conditions (BCs). Since $\rho (dw/dt)$ and $(\rho a_b + \rho a_i)$ obey the same PE and BCs:
\[
\left. \frac{dw}{dt} \right|_{\mu = \pi} = a_b + a_i.
\] (A15)
Therefore, a similar decomposition of vertical acceleration as in JR15 can be derived by using perturbation variables.

With NPPGF the only source of vertical acceleration, decomposition of $p'_b$ is more intuitive to quantify the contributions to vertical motion. Definitions of Eqs. (A11) and (A12) are extended to $p'_b$, with the buoyant pressure perturbation $p'_{b(ab)}$ define as
\[
p'_{b(ab)} = p'_{b} \bigg|_{v = 0},
\] (A16)
and the inertial pressure perturbation $p'_{b(ab)}$ defined as
\[
p'_{b(ab)} = p'_{b} \bigg|_{\mu = \pi}.
\] (A17)
Applying Eqs. (A16) and (A17) to Eq. (A9) yields the PEs about $p'_{b(ab)}$ and $p'_{b(ab)}$:
\[
-\nabla^2 p'_{b(ab)} = \nabla \cdot (\rho v \cdot \nabla v).
\] (A18)
and
\[
-\nabla^2 p'_{b(ab)} = \nabla \cdot (\rho v \cdot \nabla v).
\] (A19)
Taking $-\partial_z$ of Eqs. (A18) and (A19) yields
\[
-\nabla^2 (-\partial_z p'_{b(ab)}) = g \nabla^2 p'_0.
\] (A20)
and
\[
-\nabla^2 (-\partial_z p'_{b(ab)}) = -\partial_z \nabla \cdot (\rho v \cdot \nabla v).
\] (A21)
Comparing with Eq. (A20) with Eq. (A13), $-\partial_z p'_{b(ab)}$ and $\rho a_b$ obey the same PE and BC, which implies
\[
a_b = -\alpha_0 \frac{\partial}{\partial z} p'_{b(ab)},
\] (A22)
and comparing Eq. (A21) with Eq. (A14):
\[
a_i = -\alpha_0 \frac{\partial}{\partial z} p'_{b(ab)}.
\] (A23)
Therefore, $p'_{b(ab)} (p'_{b(b)})$ is the part of $p'_b$ responsible for $a_b$ ($a_i$). Plugging Eqs. (A22) and (A23) into Eq. (A15) and applying Eqs. (A8), (A16), and (A17), we obtain the decomposition of $p'_b$:
\[
p'_{b(ab)} = p'_{b(b)} + p'_{b(i)}.
\] (A24)
As $p'_b$ can also be decomposed into a buoyant ($p'_b$) and dynamic ($p'_d$; Markowski and Richardson 2011; i.e., $p'_{b(ab)}$ in this work) pressure perturbation, combining Eqs. (A6) and (A24) yields $p'_{b(ab)} = p'_g - p'_d$, implying that $p'_{b(ab)}$ is the “effective” portion of $p'_b$ due the compensation for $p'_d$.

Following Klemm and Rotunno (1983), the rhs of Eq. (A19) can be expanded as
\[
-\alpha_0 \nabla p'_{b(ab)} = e_{ij} \frac{\omega^2}{2} - \omega \frac{\partial}{\partial x} \frac{\partial \ln p}{\partial z}.
\] (A25)
where $\omega$ is the relative vorticity vector and $e_{ij}$ is the deformation tensor:
\[
e_{ij}^2 = \frac{1}{4} \sum_{j=1}^{3} \left( \frac{\partial u}{\partial x_j} + \frac{\partial u}{\partial x_j} \right)^2.
\] (A26)
The terms on the rhs of Eq. (A25) are the deformation of the wind field (splat term), the rotation of the wind field (spin term), and a minimal term that can be neglected (JR15), respectively. For well-behaved fields [i.e., $\nabla^2 p'_{b(ab)} \propto -p'_{b(ab)}$], the splat (spin) term contributes positively (negatively) to $p'_{b(ab)}$. 
In our calculation, model output is interpolated to a uniform orthogonal grid with horizontal (vertical) grid spacing of 200 (50) m. Equations (A13) and (A14) are solved numerically using the fast Fourier transform method (Fuka 2015), as well as Eq. (A19), though it is somewhat arbitrary to specify the constant added to the \( p'_{\text{inh}} \) solution due to the unknown BCs. Therefore, we use the same constraint as the CM1 to determine the constant, which is that the domain-averaged \( p'_{\text{inh}} \) field at the model top is zero; \( p'_{\text{inh}} \) is solved similarly. The values of \( p'_{i} \) is calculated by subtracting \( p'_{ab} \) from the total \( p' \) output from the model.

REFERENCES


Bryan, G. H., and J. M. Fritsch, 2002: A benchmark simulation for the constant, which is that the domain-averaged \( p'_{\text{inh}} \) is solved similarly. The values of \( p'_{i} \) is calculated by subtracting \( p'_{ab} \) from the total \( p' \) output from the model.


