The Problem of Diagnosing Jet Waveguidability in the Presence of Large-Amplitude Eddies

VOLKMAR WIRTH^a AND CHRISTOPHER POLSTER^a

^a *Johannes Gutenberg University Mainz, Mainz, Germany*

(Manuscript received 1 October 2020, in final form 25 June 2021)

ABSTRACT: The waveguidability of an upper-tropospheric zonal jet quantifies its propensity to duct Rossby waves in the zonal direction. This property has played a central role in previous attempts to explain large wave amplitudes and the subsequent occurrence of extreme weather. In these studies, waveguidability was diagnosed with the help of ray tracing arguments using the zonal average of the observed flow as the relevant background state. Here, it is argued that this method is problematic both conceptually and mathematically. The issue is investigated in the framework of the nondivergent barotropic model. This model allows the straightforward computation of an alternative “zonalized” background state, which is obtained through conservative symmetrization of potential vorticity contours and that is argued to be superior to the zonal average. Using an idealized prototypical flow configuration with large-amplitude eddies, it is shown that the two different choices for the background state yield very different results; in particular, the zonal-mean background state diagnoses a zonal waveguide, while the zonalized background state does not. This result suggests that the existence of a waveguide in the zonal-mean background state is a consequence of, rather than a precondition for, large wave amplitudes, and it would mean that the direction of causality is opposite to the usual argument. The analysis is applied to two heatwave episodes from summer 2003 and 2010, yielding essentially the same result. It is concluded that previous arguments about the role of waveguidability for extreme weather need to be carefully reevaluated to prevent misinterpretation in the future.

SIGNIFICANCE STATEMENT: Understanding strong undulations of the midlatitude jet stream is important, because these undulations are sometimes associated with extreme weather. Several previous publications have suggested that episodes with strong undulations can be caused by a jet that acts as a circumglobal waveguide. In the current paper we argue that these results are likely to be based on a misinterpretation and that the causality may be almost the other way around: large jet undulations can render the traditional waveguide diagnostic inappropriate and, thereby, fake strong circumglobal waveguidability. We suggest a novel diagnostic that tries to circumvent the problem. Our work implies that the results from previous publications need to be carefully reevaluated in order to avoid misinterpretation in the future.

KEYWORDS: Barotropic flows; Dynamics; Rossby waves; Extreme events; Potential vorticity; Resonance

1. Introduction

Upper-tropospheric flow in the extratropics is often characterized by a narrow band of strong westerlies, the so-called jet stream or, short, jet (Glickman 2000; Vallis 2006; Galperin and Read 2019; Woollings 2020). Usually, there are undulations superimposed on this jet, which are referred to as Rossby waves and which sometimes occur in the form of Rossby wave packets (Rossby et al. 1939; Rossby 1940; Rhines 2002; Wirth et al. 2018). The troughs and ridges associated with these waves are systematically correlated with surface weather, and this is particularly relevant in the case of extreme weather (Schubert et al. 2011; Hanley and Caballero 2012; Pfahl and Wernli 2012; Screen and Simmonds 2014; Lau and Nath 2014; Chen et al. 2015; Röethlisberger et al. 2016; Brunner et al. 2017; Fragkoulidis et al. 2018; Wolf et al. 2018;

Kornhuber et al. 2019; Xie et al. 2019). Correspondingly, it was suggested that observed trends of extreme weather in recent years can at least partly be attributed to changes in the jet and its properties. For instance, it was argued that the decline in the lower-tropospheric meridional temperature gradient owing to Arctic amplification leads to systematic changes in jet waviness, which in turn are responsible for an increased probability of extreme weather (Francis and Vavrus 2012; Screen and Simmonds 2014; Francis and Vavrus 2015; Vavrus et al. 2017; Pfleiderer et al. 2019).

One particular line of arguments in this context invokes the concept of waveguidability (Hoskins and Karoly 1981; Karoly and Hoskins 1982; Held 1983; Hoskins and Ambrizzi 1993; Manola et al. 2013; Wirth 2020b). Waveguidability of a zonal jet quantifies its propensity to duct Rossby waves in the zonal direction, where “ducting” refers to the wave’s group velocity. The concept was developed in the framework of ray tracing, which in turn is based on linear theory and requires eddies to be of small amplitude. It turns out that strong and narrow jets are generally associated with strong waveguidability. To the extent that changes in the atmospheric background flow imply an increase in jet waveguidability, there is less dispersion of

Denotes content that is immediately available upon publication as open access.

Corresponding author: Volkmar Wirth, vwirth@uni-mainz.de

DOI: 10.1175/JAS-D-20-0292.1

© 2021 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the [AMS Copyright Policy](#) (www.ametsoc.org/PUBSReuseLicenses).

Rossby wave energy in the meridional direction, and this may lead to larger wave amplitudes (Branstator 2002) and an increased probability of extreme weather downstream of the wave source. Specific mechanisms that have been proposed in this context are “quasi-resonant amplification” (Petoukhov et al. 2013; Coumou et al. 2014; Petoukhov et al. 2016; Kornhuber et al. 2017a,b; Mann et al. 2017, 2018) and “wave resonance” (Stadtherr et al. 2016; Kornhuber et al. 2017a,b; Mann et al. 2017; Coumou et al. 2018; Kornhuber et al. 2019), where the exact difference between these two mechanisms remained unclear to us. In principle, the notion of resonance as it is known in theoretical physics may apply to midlatitude wave dynamics. Resonance occurs when the stationary wavenumber of free Rossby waves in a zonal channel approaches the dominant wavenumber of the stationary forcing due to orography or diabatic heating (Charney and Eliassen 1949; Held 1983). The mechanism requires Rossby waves to travel around the entire globe in the zonal direction (Yang et al. 1997; Wirth 2020a, 1–3), and this is facilitated in the real atmosphere by a circumglobal jet with strong waveguidability (Branstator 1983, 2002; Manola et al. 2013). Resonance appears attractive as an explanation for extended periods of extreme weather, because it would explain both large wave amplitudes and near-zero phase propagation of the associated troughs and ridges.

Generally, changes in atmospheric dynamics due to climate change are much harder to assess than changes in atmospheric thermodynamics owing to the large natural variability of the atmospheric circulation and its strong nonlinearity (Shepherd 2014; Zappa and Shepherd 2017). Correspondingly, there are numerous open questions regarding the processes and mechanisms in the causal chain linking planetary-scale long-term changes, jet waviness, and the occurrence of extreme weather (Screen and Simmonds 2013; Barnes and Screen 2015; Overland et al. 2016; Francis 2017; Cohen et al. 2020; Blackport and Screen 2020). Given this state of affairs, it appears desirable to obtain a deeper understanding of the connection between waveguidability and the planetary-scale atmospheric background state. The current paper wants to contribute toward this goal by investigating in more detail the assessment of jet waveguidability in the presence of large-amplitude eddies.

A key ingredient in the discussion of waveguidability is the so-called background state. As it is often done, we assume here that the background state is zonally symmetric, which implies a focus on circumglobal waveguidability in this paper. Waves (or eddies) are defined as the deviations from the background state. Conceptually, the background state exists before any waves have come into existence, and its waveguidability quantifies the meridional confinement of the waves that may subsequently develop. In previous work, the zonally symmetric background state has usually been estimated from the Eulerian zonal average of the observed flow, possibly in combination with a time filter (Petoukhov et al. 2013; Coumou et al. 2014; Petoukhov et al. 2016; Stadtherr et al. 2016; Kornhuber et al. 2017a,b; Mann et al. 2017). Yet, other choices are possible, and it is not obvious a priori whether the zonal average is the best choice. For instance, it would be desirable that the background state does not depend on the presence and the properties of the eddies, but this is not guaranteed when using the zonal mean of the observed flow. It transpires that the computation of the

background state is a nontrivial task. The situation is similar to the problem of estimating the stability of a flow with respect to wavelike perturbations, which also invokes the concept of a zonally symmetric background state and where the definition of the background state has been identified as a “quite formidable problem” (section 7.1 in Pedlosky 1987).

Moreover, the concept of ray tracing and the traditional diagnostic for waveguidability break down in the presence of large-amplitude eddies, because one leaves the range of validity of the underlying linear theory. This is unfortunate, since typical episodes of interest with extreme weather are usually associated with large-amplitude eddies. If one, nevertheless, keeps diagnosing waveguidability as if the eddies were small-amplitude, one is bound to venture onto shaky ground. In particular there is no guarantee that the zonal average of the fully perturbed flow has any value as a background state. In fact, as we will show in the present paper, this practice may lead to artifacts that spuriously suggest a waveguide where in reality there is likely to be none.

The foregoing motivates the present work: we aim to elucidate whether and to what extent the analysis of waveguidability from the zonal-mean flow is adversely affected by the presence of large-amplitude eddies. Following previous authors, we investigate the issue in the framework of the barotropic model. For reference, the model and the related diagnostics are briefly reviewed in section 2. We then set the stage for the later discussion with the help of a thought experiment in section 3. In this thought experiment we follow the evolution of a wave from its small-amplitude early stage to its large-amplitude late stage. We argue that linear wave diagnostics, which may predict good waveguidability with the potential of subsequent growth of wave amplitude, should really be applied to the small-amplitude early stage. The essential steps from the thought experiment are quantified in section 4 through numerical calculations, allowing us to show the occurrence of artifacts when applying linear theory to the large-amplitude late stage of the evolution. In the same section we suggest an alternative method for defining a background state based on the idea of the zonalization of potential vorticity contours, which essentially recovers the small-amplitude early stage of the evolution (Nakamura and Zhu 2010; Methven and Berrisford 2015). As we will show, the two background states may yield vastly different results in terms of waveguidability, and it will be argued that the results from the zonal-mean background state have to be considered as artifacts. The analysis is then applied to data from two observed episodes with extreme weather that have been investigated in earlier studies (section 5). We find that, indeed, earlier analyses of these episodes may be subject to misinterpretation owing to the above-mentioned artifacts. Finally, section 6 provides a discussion and our conclusions.

2. Barotropic model

a. Model equations

We consider nondivergent barotropic flow on the sphere. Key variable is absolute vorticity q , which at any given time is defined as

$$q(\lambda, \phi) = 2\Omega \sin\phi + \zeta, \quad (1)$$

where

$$\zeta(\lambda, \phi) = \frac{1}{a \cos\phi} \frac{\partial \zeta}{\partial \lambda} - \frac{1}{a \cos\phi} \frac{\partial}{\partial \phi} (u \cos\phi) \quad (2)$$

is relative vorticity, $\mathbf{v} = (u, v)$ is the horizontal wind, λ is longitude, ϕ is latitude, t is time, a is Earth's radius, and Ω is the angular velocity of Earth's rotation. In the barotropic model, absolute vorticity plays the role of a potential vorticity (PV), and both terms will be used synonymously henceforth. For later reference we note that in the case of zonally symmetric flow the definition of PV reduces to

$$q(\phi) = 2\Omega \sin\phi - \frac{1}{a \cos\phi} \frac{d}{d\phi} (u \cos\phi). \quad (3)$$

Owing to its nondivergence, the wind can be expressed in terms of a streamfunction ψ through

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\frac{1}{a} \frac{\partial \psi}{\partial \phi} \\ \frac{1}{a \cos\phi} \frac{\partial \psi}{\partial \lambda} \end{pmatrix}. \quad (4)$$

It follows that PV may alternatively be written as

$$q = 2\Omega \sin\phi + \nabla^2 \psi, \quad (5)$$

where ∇^2 represents the two-dimensional Laplace operator. Rearranging terms and considering $q(\lambda, \phi)$ as given, the above turns into a Poisson equation for ψ :

$$\nabla^2 \psi = q - 2\Omega \sin\phi. \quad (6)$$

This equation allows one to perform ‘‘PV inversion,’’ i.e., to retrieve the streamfunction ψ and, through (4), the wind $\mathbf{v}$ from a known PV distribution $q(\lambda, \phi)$. In the present work we solve (6) in spectral space using an expansion in spherical harmonics with triangular truncation at total wavenumber $N = 360$. It transpires that the fields $q(\lambda, \phi)$, $\psi(\lambda, \phi)$, and $\mathbf{v}(\lambda, \phi)$ are equivalent in that they all characterize the dynamical state of the system in a unique fashion.

The temporal evolution of the system in the absence of non-conservative processes is governed by material conservation of PV:

$$\frac{Dq}{Dt} = 0, \quad (7)$$

where D/Dt denotes the material derivative.

b. Diagnosing waveguidability

The concept of waveguidability makes use of the theory of ray tracing (Lighthill 1967; Hoskins and Karoly 1981; Hoskins and Ambrizzi 1993), which in turn is based on the linearized PV equation:

$$\left(\frac{\partial}{\partial t} + \frac{u_b}{a \cos\phi} \frac{\partial}{\partial \lambda} \right) q' + \frac{dq_b}{ad\phi} v' = 0. \quad (8)$$

Here, $u_b(\phi)$ and $q_b(\phi)$ represent the zonal wind and the PV, respectively, of the zonally symmetric background state, and

the primed variables are the perturbations, i.e., the deviations from the background state. The right hand side of (8) is zero, which is tantamount to assuming conservative eddy dynamics. Important subsequent steps in the development of ray tracing are a Mercator projection of the sphere and assuming the validity of WKB theory (Bender and Orszag 1978). After some manipulations, the so-called stationary wavenumber $K_s(\phi)$ appears as a key diagnostic. Following previous work (Hoskins and Karoly 1981; Wirth 2020b), the square of K_s is defined as

$$K_s^2(\phi) = a^2 \frac{\beta_M}{u_M}, \quad (9)$$

with

$$u_M(\phi) = \frac{u_b}{\cos\phi} \quad (10)$$

and

$$\beta_M(\phi) = \cos\phi \frac{dq_b}{ad\phi}. \quad (11)$$

Incidentally, the stationary wavenumber is a variant of the refractive index, which closes the analogy with ray tracing in geometric optics.

The theoretical basis for the definition in (9) and the details of its interpretation were discussed in the above quoted papers. The essential aspect in the present context is that stationary Rossby waves are refracted toward larger values of K_s as long as the underlying assumptions are satisfied (Wirth 2020b). It follows that the theory of ray tracing diagnoses a zonal waveguide at those latitudes where the profile of $K_s(\phi)$ has a relative maximum. For waves with nonzero phase speed c , the numerator in (10) must be replaced by $u_b - c$ (Yang and Hoskins 1996). However, in the current paper we follow previous authors and restrict our attention to waves with very small phase speed, because they are most conducive to extreme weather; we, therefore, stick to u_M and β_M as defined in (10) and (11).

Diagnosing waveguidability with the help of the stationary wavenumber must be taken with a grain of salt, because the WKB assumption underlying the method of ray tracing is often violated in observed flows (Wirth 2020b). However, this is not the topic of the present paper; rather we will focus on the question of whether and to what extent the presence of the waves impacts the profile of the stationary wavenumber and the resulting interpretation.

3. A thought experiment

We now carry through a thought experiment which maps the causal sequence of events implicit in the argument of Petoukhov et al. (2013) and subsequent authors onto an initial value problem. Their argument posits that waveguidability is enhanced during certain episodes, which causes the waves to become circumglobal and subsequently leads to resonant amplification. Our thought experiment allows us to illuminate the

FIG. 1. Illustration of the thought experiment through idealized wind fields that are meant to represent the upper-tropospheric flow. (a) Zonal wind profile of the zonally symmetric reference state (solid line) and (b) two-dimensional flow field obtained by locally superimposing a strong eddy in the region $\lambda \approx 0^\circ$ onto the reference state. The solid blue contours in (b) are streamlines and the red arrows represent the wind. The dashed line in (a) represents the zonally averaged zonal wind from the flow field in (b).

impact of growing wave amplitudes onto the analysis of waveguidability. Figure 1 serves for illustration.

Starting point is a zonally symmetric initial state, which we refer to as “reference state” in the following. Let us assume that this reference state is characterized by a midlatitude westerly jet as shown in Fig. 1a (solid line). Onto this reference state we superimpose small-amplitude waves which are assumed to grow and reach large amplitudes later on. Such temporary growth of wave amplitude has been observed in numerous episodes in the real atmosphere, and it may be due to a variety of different mechanisms. Let us, furthermore, assume that we want to test a hypothesis according to which the mechanism underlying the growth of wave amplitude involves enhanced zonal wave ducting. In the thought experiment it would, therefore, be natural to diagnose the waveguidability of the reference state and investigate whether and to what extent it is correlated with large wave amplitudes at some later stage. This approach would be conceptually sound, because the reference state represents an early stage and can, therefore, be causally related to the ensuing evolution with potentially large wave amplitudes at the later stage. In addition, the approach would be mathematically safe, because the small wave amplitudes during the early stage guarantee that linear theory can be applied and the concept of waveguidability is on solid ground. To the extent that strong waveguidability of the reference state then turns out to be associated with large wave amplitudes at the later stage, this could be taken as circumstantial evidence for the validity of the hypothesis.

Once the waves have reached large amplitudes, they may break and eventually lead to a dipole-like blocking pattern as depicted in Fig. 1b. In the center of this eddy, the ambient westerly flow has been replaced by easterlies. When defining the background state through a simple zonal average of this highly perturbed flow, the resulting profile (dashed line in Fig. 1a) is affected by the local anomaly from the large-amplitude eddy and differs from the reference state. The “contamination” of the so-diagnosed background state may

result in a spurious modification of the profile $K_s(\phi)$, and this may possibly lead to deviations of the diagnosed waveguidability in comparison with what one would have obtained from the underlying reference state.

The foregoing indicates that in the thought experiment one should base the diagnosis of waveguidability on the reference state instead of the Eulerian zonal mean of the fully perturbed flow. To the extent that the two methods yield different results, the results from the Eulerian zonal mean have to be considered as artifacts; this is because the zonal-mean background state is drastically modified during the formation of large-amplitude eddies, which destroys its diagnostic value regarding the *preceding* wave development. Obviously, the problem is that — in contrast to our thought experiment — the reference state is hardly ever realized in observed flows due to the omnipresence of eddies. This is the “quite formidable problem” mentioned before.

4. Investigation with the barotropic model

As a next step we quantify the predictions and implications of our thought experiment through numerical computations in the framework of the barotropic model. At the same time we suggest an alternative background state which largely avoids the problems related to the occurrence of large-amplitude eddies.

a. Basic procedure

Essentially, we carry through a sequence of steps as outlined in Fig. 2. Point of departure is the zonally symmetric reference state (u_0, v_0) with $v_0 = 0$ and with the zonal component being defined as the superposition of four terms:

$$u_0(\phi) = u_{\text{SB}}(\phi) + u_J(\phi) + u_T(\phi) + L(\phi), \quad (12)$$

where

$$u_{\text{SB}}(\phi) = U_{\text{SB}} \cos \phi \quad (13)$$

FIG. 2. Schematic to illustrate the computational steps carried through in order to quantify the predictions and implications of the thought experiment.

is a solid-body rotation (with $U_{SB} = 10 \text{ m s}^{-1}$),

$$u_J(\phi) = U_J e^{-(\phi - \phi_0)^2 / (2\sigma_J^2)} \quad (14)$$

represents a midlatitude jet (with $\phi_0 = 45^\circ\text{N}$, $U_J = 10 \text{ m s}^{-1}$ and $\sigma_J = 12^\circ$), and

$$u_T(\phi) = -U_T e^{-\phi^2 / (2\sigma_T^2)} \quad (15)$$

is a low-latitude component (with $U_T = 9 \text{ m s}^{-1}$ and $\sigma_T = 10^\circ$) that reduces the flow in the tropics. Generally, the sum $u_{SB}(\phi) + u_J(\phi) + u_T(\phi)$ would be nonzero at the poles, which is undesirable. We, therefore, included a linear function $L(\phi) = a + b\phi$ on the right hand side of (12) with the parameters a and b chosen such that the resulting total profile $u_0(\phi)$ turns zero at both poles. The reference profile thus obtained (Fig. 3b) has a prominent westerly jet in the Northern midlatitudes and weak westerlies in the tropics. The PV of the reference state $q_0(\phi)$ is obtained from $u_0(\phi)$ through (3) (Fig. 3a); this profile shows a monotonic increase from the tropics to the North Pole.

As argued in the previous section, any statement about waveguidability should be based on the reference state.

Correspondingly, the stationary wavenumber K_s should be computed using $u_b = u_0$ and $q_b = q_0$ in (10) and (11). The resulting profile (Fig. 3c) shows a monotonic equatorward increase, which is consistent with the general tendency of stationary Rossby waves to be refracted toward the subtropics (Hoskins and Karoly 1981; Plumb 1985). Depending on the details of the flow at low latitudes, these waves are at least partly dissipated on their way toward the equator, and this can be considered as a sink for midlatitude Rossby wave activity (Held 1983). In other words, our reference state does not indicate any tendency for zonal wave ducting.

We now assume that Rossby waves are generated on this reference state through conservative dynamics and that these waves subsequently develop into large-amplitude eddies. The result of this evolution may be a complex flow pattern at later times with large deviations from zonal symmetry. We could try to simulate this evolution through numerical integration. However, since we are only interested in diagnosing the state of the atmosphere at a given time, we choose a more straightforward approach and simply specify the final state through a procedure called “conservative PV redistribution.” This method simulates the exchange of air parcels between (possibly remote) locations resulting from the nonlinear dynamics. It is implemented by swapping the PV values in a zonally aligned strip of grid boxes with the PV values from another strip of grid boxes at some other latitude, where the surface areas of the respective strips are identical. The procedure is repeated until the desired PV anomaly is obtained. Figure 4b illustrates the procedure, where we chose three equidistant strips of finite length at each latitude between 30° and 60°N . Figure 4a shows the resulting new PV field $q_{rd}(\lambda, \phi)$ after PV redistribution. The corresponding PV perturbation,

$$q_e(\lambda, \phi) = q_{rd}(\lambda, \phi) - q_0(\phi), \quad (16)$$

represents a series of three PV dipoles (Fig. 4b).

In the next step, the wind field $\mathbf{v}(\lambda, \phi)$ corresponding to $q_{rd}(\lambda, \phi)$ is obtained through PV inversion as explained in connection with (6). The result is shown in Fig. 4a. Apparently, the three PV dipoles are associated with three large-amplitude eddies, each of which resembles a blocking flow pattern; on the other hand, the wind between the eddies is approximately zonal and resembles the reference wind profile. We consider this entire wind field as an idealized representation of a strongly perturbed upper-tropospheric flow in the Northern Hemisphere. If such a flow were realized in the atmosphere, it would be associated with noticeable weather anomalies and possibly extreme weather in the region of the eddies. Note that the wind field in Fig. 4a is strongly nonlinear in the sense that it cannot be approximated by any zonally symmetric wind field; in other words, no matter what background state one chooses to use, linear theory is badly violated and the concepts of ray tracing and waveguidability are bound to be inappropriate.

We note in passing that our PV dipoles are not stationary solutions of the equations of motion. Rather, when using $q_{rd}(\lambda, \phi)$ as initial condition for a model integration, the three dipoles would radiate Rossby waves and, hence, decay after a few days. In the real atmosphere, it requires additional factors (such as

FIG. 3. Analysis of the flow from a specific realization of the thought experiment (see text for details). The three panels show latitudinal profiles of (a) potential vorticity q , (b) the zonal wind u , and (c) the stationary wavenumber K_s . The profiles of K_s are slightly smoothed through two passes of a 1–2–1 gridpoint filter. In all panels the blue line represents the zonally symmetric reference state, the dashed red line represents the corresponding variables obtained from a zonal average of the full flow with strong eddies, and the solid red line depicts the reconstruction using the zonalized background state; the latter is hardly visible as it practically overlaps with the blue line.

transient eddies or nonconservative processes) to provide longevity to such a flow configuration.

b. Background state from the zonally averaged flow

We now analyze the perturbed flow in the traditional manner, i.e., by computing the background wind through

$$u_b(\phi) = \overline{u(\lambda, \phi)}, \quad (17)$$

where $u(\lambda, \phi)$ is the perturbed flow field from section 4a and the overbar denotes the Eulerian zonal average. The result is depicted as the dashed red line in Fig. 3b. Apparently, the perturbations due to the eddies are so strong that the westerly jet at 43°N in the reference profile has vanished and been

replaced by much weaker westerlies in the zonal-mean profile. As a consequence, $\bar{u}(\phi)$ features two pronounced relative maxima, one at 30°N and one at 60°N. The sharpness of these two jets is related to the two discontinuities of $\bar{q}(\phi)$ at these latitudes (Fig. 3a), which in turn are due to our PV redistribution procedure; less idealized profiles of $q_{rd}(\lambda, \phi)$ would be associated with smoother jet profiles. Interestingly, $\bar{u}(\phi)$ in Fig. 3b seems to be identical to the reference flow $u_0(\phi)$ outside the range of latitudes $30^\circ \leq \phi \leq 60^\circ$ N. In fact, it can be shown that this is a necessary result of our PV redistribution procedure (see appendix B).

Note that $\bar{u}(\phi)$ and $u_0(\phi)$ would be very close to each other for small-amplitude waves. In other words, the large difference between the two corresponding profiles in Fig. 3b indicate that we have left the range of validity of linear theory. Note also that the occurrence of two jets in $\bar{u}(\phi)$ is reminiscent of the “double-jet pattern,” which plays an important role in the analysis of Kornhuber et al. (2017b) in their investigation of the European heat wave in 2003 and the Russian heat wave in 2010.

The differences between the profiles $\bar{u}(\phi)$ and $u_0(\phi)$ imply large differences in the corresponding profiles of $K_s(\phi)$ (Fig. 3c): the strictly monotonic behavior associated with the reference profile has been replaced by a nonmonotonic behavior with three relative maxima when using the zonal-mean profile. Two narrow maxima in K_s at 30° and 60°N, respectively, coincide with the two jet maxima (cf. Fig. 3b); they are due to the cusp-like behavior of $\bar{u}(\phi)$ at these latitudes, resulting in a near discontinuous second derivative of $\bar{u}(\phi)$ and, hence, a peak in $K_s(\phi)$ [see (9), (11) and (3)]. The third relative maximum of $K_s(\phi)$ at around 47°N is associated with the pronounced minimum in $\bar{u}(\phi)$ at that latitude, which arises due to (9) in combination with (10).

These qualitative differences in $K_s(\phi)$ lead to a drastically different interpretation in terms of waveguidability compared to what we have obtained earlier: the three relative maxima in $K_s(\phi)$ represent three waveguides for a range of synoptic-scale wavenumbers. This would imply that at least part of the wave activity is channeled in the zonal direction, and one would

FIG. 4. Idealized flow configuration with strong eddies: (a) total flow, where the color depicts the PV field $q_{rd}(\lambda, \phi)$ and the arrows show the associated horizontal wind; (b) PV perturbation $q_e(\lambda, \phi)$ corresponding to the PV distribution $q_{rd}(\lambda, \phi)$ in (a). Both panels show the Northern Hemisphere in a Mollweide projection, which is area conserving.

conclude that such a background state should lead to increased downstream wave amplitudes. However, in our thought experiment it is clear that these “waveguides” do not represent a property of the underlying reference state; rather, they are an imprint left by the large-amplitude eddies on the zonal-mean background state and, in this sense, artifacts. The stark difference between the two background states regarding their waveguidability is not a big surprise since it is related to the strong nonlinearity of the fully perturbed flow. As pointed out before, this nonlinearity implies that one cannot expect the method of ray tracing to be viable and the concept of waveguidability to be meaningful.

c. A better alternative: The zonalized background state

By design, there is a true underlying background state in our thought experiment, namely, the reference state. Eddy generation is assumed to be conservative and, hence, PV contours are simply advected by the flow [see (7)]. Whenever the reference PV profile is monotonically increasing with latitude (implying barotropic stability), the reference state can be recovered from any subsequent perturbed state by conservative rearrangement of PV contours such that they become purely zonal. We refer to this method as “zonalization” and the so-obtained background state as the “zonalized background state” (cf. Nakamura and Zhu 2010; Methven and Berrisford 2015). No matter how strong or how nonlinear the eddies are, the zonalized background state is always identical to the underlying eddy-free reference state as long as the dynamics are conservative. Note that the zonalized background state can be viewed as a modified Lagrangian mean state (McIntyre 1980; Solomon et al. 2012).

The algorithm used for PV zonalization is detailed in appendix A. In essence, given an arbitrary PV distribution $q(\lambda, \phi)$, the procedure yields the zonalized PV profile $Q(\phi)$. The result for our PV field $q_{rd}(\lambda, \phi)$ is shown in Fig. 3a. As expected, the zonalized PV profile is so close to the reference PV profile that these two can practically not be distinguished from each other on the plot.

As a next step we perform one-dimensional PV inversion, i.e., we compute the zonal wind profile $U(\phi)$ corresponding to the zonalized PV profile $Q(\phi)$. The equation for PV inversion is obtained from (3), yielding

$$\frac{d}{a d\phi} \left[\frac{1}{\cos\phi} \frac{d}{a d\phi} (U \cos\phi) \right] = \frac{2\Omega}{a} \cos\phi - \frac{dQ}{a d\phi}, \quad (18)$$

which is a one-dimensional elliptic differential equation. We specify $U = 0$ at both poles and solve the equation using second order finite differences.

Incidentally, there is an alternative, but equivalent method to compute $U(\phi)$, namely, through the following relation

$$U(\phi) = \bar{u} + \mathcal{A}, \quad (19)$$

where $\mathcal{A}(\phi)$ is the finite amplitude wave activity (short: FAWA, Nakamura and Zhu 2010). The expression for $\mathcal{A}(\phi)$ in spherical coordinates is given in Eq. (4) of Ghinassi et al. (2020). We also implemented this method and found that it yields practically indistinguishable results in comparison with

PV inversion. Relation (19) describes the impact of (conservative) eddies on the zonal-mean flow in the framework of the barotropic model. As $\mathcal{A}$ is nonnegative everywhere by definition, this relation implies $\bar{u} \leq U$. It follows that the presence of eddies is associated with a reduction of $\bar{u}$ in comparison with the underlying reference state $U(\phi)$.

The zonalized wind profile $U(\phi)$ in Fig. 3b obtained from (18) is very close to the reference profile $u_0(\phi)$. In particular, $U(\phi)$ does not show any indication for a double-jet structure. As a consequence, the profile $K_s(\phi)$ associated with $U(\phi)$ is monotonic and practically identical to the profile for the reference state. This is in stark contrast to the profile of $K_s(\phi)$ associated with the zonal-mean wind $\bar{u}(\phi)$, which (as pointed out before) diagnoses the existence of three separate waveguides. We conclude that in the present context the zonalized background state is superior to the zonal-mean background state, because it exactly recovers the underlying reference state and is not fraught by artifacts arising from the presence of the eddies.

To be sure, the concept of the zonalized background state makes only sense in the idealized framework of conservative dynamics. By contrast, the evolution of growing eddies in the real atmosphere may be affected by nonconservative processes to some extent (Pfahl et al. 2015). We, therefore, need to assume that these nonconservative effects are of minor importance. However, this does not imply any additional restriction, because the concepts of ray tracing and waveguidability assume conservative eddy dynamics anyways [see (8)]. Our argument is, therefore, self-consistent.

d. Reference state with a waveguide

So far, we deliberately considered a reference state that does not contain any waveguide in terms of the associated K_s profile. This scenario illustrates how the zonal average in the presence of strong eddies may lead to artifacts. However, it is also interesting to consider a scenario where the reference state does have a well-defined waveguide in the sense of a relative maximum in $K_s(\phi)$, and this is what we are going to do now.

As is well known, large waveguidability is usually obtained in the case of a strong and narrow jet (Manola et al. 2013; Wirth 2020b). We, therefore, specify a new reference profile $u_0(\phi)$ using $U_J = 40 \text{ m s}^{-1}$ and $\sigma_J = 9^\circ$ in (12) (Fig. 5b). In fact, the associated K_s profile (Fig. 5c) indicates a broad relative maximum at the latitude of the jet, which is equivalent to a zonal waveguide. The PV profile $q_0(\phi)$ associated with the new reference state is nonmonotonic with gradient reversals around 60°N (Fig. 5a). This behavior suggests that the new reference state is barotropically unstable (which we verified through numerical integration).

We now follow the same procedure as before to perform a spatial redistribution of PV and, thus, obtain $q_{rd}(\lambda, \phi)$. The resulting zonally averaged PV profile (red dashed line in Fig. 5a) looks similar as in the previous example with discontinuities at 30° and 60°N . By contrast, the zonalized PV profile $Q(\phi)$ (red solid line in Fig. 5a) is continuous and monotonically increasing with latitude and, hence, differs from the reference profile $q_0(\phi)$ poleward of 50°N . As a consequence, the corresponding zonalized wind $U(\phi)$ (Fig. 5b) shows slight deviations

FIG. 5. As in Fig. 3, but for a reference state that contains a zonal waveguide in the form of a strong narrow jet. Negative values in (c) represent minus the imaginary part of K_s .

from the reference wind $u_0(\phi)$ at the northern flank of the jet. Nevertheless, in terms of $K_s(\phi)$ the zonalized background state provides a better approximation to the reference state than the zonal-mean background state (Fig. 5c). In particular, the zonalized background state (solid red line) predicts a waveguide at the center of the jet similar to the reference state. On the other hand, it does not show the two additional waveguides (one at each flank of the jet) that are spuriously diagnosed from the zonal-mean background state (dashed red line). We conclude that—at least in the present example—our zonalization procedure allows one to faithfully diagnose a waveguide from the perturbed wind field as long as it is present in the underlying reference state and the reference state is not subject to strong barotropic instability.

5. Analysis of observed episodes

We now aim to transfer the insight from section 4 to the real atmosphere. For this purpose we consider two episodes, which were associated with severe weather and which have been analyzed in terms of waveguidability in previous studies (Petoukhov et al. 2013; Coumou et al. 2014; Petoukhov et al. 2016; Stadtherr et al. 2016; Kornhuber et al. 2017b). While doing so, we stick to the barotropic framework for two reasons. First, this strategy guarantees consistency with these earlier studies, which are all based on horizontal ray tracing in the barotropic model. Second, it avoids ambiguities in the definition of the zonalized state that arise for three-dimensional flow (Nakamura and Solomon 2010). The problem here is that the zonalization procedure in three dimensions requires a choice to be made at the lower boundary. Various equally justified approaches yield rather different results, and it is not clear at this point which one is the best.

It is noted, again, that the analysis of waveguidability of an observed flow with large-amplitude eddies is on thin ice, because the concept of waveguidability is based on linear theory requiring wave amplitudes to be small. This issue did not prevent previous authors from diagnosing such episodes, which must have required a certain leap of faith. Motivated by our findings from section 4, we take here a more cautious approach: to the extent that the zonalized background state and the

zonal-mean background state yield different results, we take this as an indication that the eddies are strongly nonlinear. The only way to keep talking about waveguidability in a meaningful way is to assume that these large-amplitudes eddies have evolved from small amplitudes, and then to diagnose waveguidability during the early small-amplitude stage—quite like in our thought experiment. Obviously, in reality the omnipresence of large-amplitude waves prevents one from actually performing such an analysis by simply considering an earlier point in time. However, our zonalized background state allows us to retrieve at least a hypothetical earlier stage of the evolution in accordance with the thought experiment. This is what we are going to do in the current section. When the two background states turn out to be different, we give preference to the zonalized background state, because it is the “correct” choice in the thought experiment and because the thought experiment represents the sequence of events in the argument of Petoukhov et al. (2013). In other words, any differences between the two background states are interpreted as an indication that the zonal-mean background state is subject to artifacts owing to the presence of the eddies.

Our analysis is based on data obtained from the ERA5 project (Hersbach et al. 2020), which were processed on a regular latitude–longitude grid with 1.5° resolution. Barotropic PV (i.e., absolute vorticity) was calculated from the horizontal wind components on the 300-hPa pressure level every 6 h (0000, 0600, 1200, and 1800 UTC) via a spectral representation with spherical harmonics. In addition, a triangular truncation at total wavenumber $N = 36$ was applied to the PV field to reduce the noise that occurs when computing the finite differences in the expression for K_s . At the same time, this truncation removes mesoscale features that are undesirable when analyzing the synoptic-scale flow. We have verified that our results do not depend on the exact choice of the truncation threshold and data resolution. The smoothing has negligible effect on the magnitude of zonal means, but it does affect the zonalization procedure to some extent. However, the qualitative behavior seen in our setup, in particular that of the stationary wavenumber $K_s(\phi)$, is consistent with other resolution-truncation combinations that are appropriate for the representation of the synoptic-scale flow.

FIG. 6. Latitude–time Hovmöller diagrams for an episode in August 2010 showing the (a)–(c) zonal wind and (d)–(f) the stationary wavenumber K_s . The three different columns refer to three different procedures to obtain the background state: (a), (d) using a simple zonal average; (b), (e) using a zonal average plus a 7-day centered running mean; and (c), (f) using zonalization. Values of the zonal wind in excess of 50 m s^{-1} in (c) are depicted through contours with a contour spacing of 5 m s^{-1} . The red markers at the bottom of (e) and (f) indicate the date selected for Figs. 7a–c.

The zonal-mean zonal wind and the corresponding stationary wavenumber K_s for the episode in July and August 2010 are shown in the left column of Fig. 6. During this time, a large-amplitude perturbation of the midlatitude flow persisted over Russia and has been linked to a severe heat wave (Schneider et al. 2012) and downstream flooding (Hong et al. 2011). The zonal-mean zonal wind on the Northern Hemisphere during this period (Fig. 6a) exhibits a double-jet structure with a polar jet located at around 75°N and a subtropical jet located at around 45°N . The jets' strengths vary with time in the range from 10 to 30 m s^{-1} . Between the two jets the wind minimum reaches values below 5 m s^{-1} , and south of 30°N the wind is westward. Overall, the temporal evolution of the stationary wavenumber K_s associated with this zonal-mean state is rather erratic (Fig. 6d). Yet, coherent local maxima can be detected in the vicinity of the subtropical jet extending for several consecutive days, corresponding to waveguides for synoptic-scale waves up to zonal wavenumber 9. K_s turns imaginary equatorward of the zero-wind contour.

The double-jet pattern and the subtropical waveguide are prevalent features during the considered period. This motivates the application of a temporal filter in an attempt to reduce the variability of the background state. The result is

shown in the middle column of Fig. 6 using a 7-day centered running mean as time filter. As expected, there is less temporal variability in the double-jet structure of $\bar{u}$ and in the subtropical waveguide as diagnosed from K_s . As a consequence, the previously intermittent subtropical waveguide has turned into a coherent waveguide for wavenumbers between 6 and 8 during the entire episode, broadly consistent with Fig. 1 in Kornhuber et al. (2017b). The addition of a temporal filter has, thus, resulted in properties that are more in line with the idea of a background state. However, it comes with some caveats: the choice of the filter and its parameters (e.g., the window width in case of a running mean) affects the resulting background state and requires some justification. In addition, a running centered time mean represents a noncausal filter, for which the future state of the atmosphere influences the current state. This property prevents a robust interpretation in terms of causal relationships as well as the use of such a filter for real-time observations.

Finally, the right column of Fig. 6 shows u and K_s of the background state derived from our zonalization procedure. Instead of a double-jet structure, the background state now features a single midlatitude jet centered at 65°N with westerly winds exceeding 70 m s^{-1} . Although the maximum jet speed

Aug 04 2010 12Z

Aug 04 2003 12Z

FIG. 7. Latitudinal profiles of (a),(d) potential vorticity; (b),(e) zonal wind; and (c),(f) stationary wavenumber K_s at (a)–(c) 1200 UTC 4 Aug 2010 and (d)–(f) 1200 UTC 4 Aug 2003. The solid red lines depict the zonalized background state; the dashed red lines represent a background state obtained from a combination of a 7-day centered running mean and a zonal average; and the dashed black line in (b) and (e) depicts the 7-day running-mean finite-amplitude wave activity (FAWA).

varies over time, this variation is smaller in relative terms than in the zonal average–based background states (Figs. 6a,b). However, what is more important in the present context, there is no sign of a waveguide whatsoever. The stationary wavenumber in Fig. 6f increases monotonically toward the equator and becomes imaginary at about 15°N, where the winds turn easterly. The temporal variation in K_s is small and can be neglected for the purpose of waveguide analysis. The reason for the absence of a waveguide despite the strong jet in Fig. 6c lies in the fact that this jet is very broad.

To understand the key differences between the zonal-mean background state and the zonalized background state, consider Figs. 7a–7c, where a single time step from the 2010 episode is analyzed in more detail using the same plot conventions as in Figs. 3 and 5. In the zonalized background state, high values of PV have been preserved and redistributed to the pole instead of being averaged out as in the zonal-mean state (Fig. 7a). This results in a stronger meridional PV gradient overall and,

therefore, stronger zonal winds in the midlatitudes (Fig. 7b). The 7-day-averaged finite-amplitude wave activity (FAWA) maximizes between the two jets from the zonal-mean state (Fig. 7b). This is consistent with our thought experiment above, where we demonstrated that a double-jet pattern can emerge in the zonal-mean wind profile when strong eddies (measured here in terms of FAWA) develop on a single jet of the reference state. The zonalized wind profile (Fig. 7b) is consistent with (19), which explains that the high values of the zonalized wind result from simply adding finite amplitude wave activity $\mathcal{A}$ to the zonally averaged zonal wind $\bar{u}$, where the former dominates the latter. Note that the relation (19) is only approximately satisfied in Fig. 7 owing to the temporal filter applied to $\bar{u}$ and $\mathcal{A}$.

The situation is very similar for the episode from August 2003, when a long-lasting heat wave associated with a large-amplitude perturbation of the midlatitude flow impacted Europe severely (Black et al. 2004). The corresponding plots are presented in Figs. 7d–f. Like in the 2010 episode, the time-averaged

FIG. 8. As in Figs. 7b and 7c, but with ad hoc modifications to the zonalized wind that aim to account for the effect of the residual circulation in a three-dimensional framework (see text for details).

zonal-mean zonal wind was characterized by a double-jet structure and the southern jet coincides with a waveguide for wavenumbers 7 and 8. This diagnosed “waveguide” played an important role for earlier claims that this episode was affected by quasi-resonance (Petoukhov et al. 2013; Kornhuber et al. 2017b). The wave activity maximizes at the latitude of the zonal-mean zonal wind minimum, and the zonalized state shows a single jet only. As before, there is no indication of a waveguide anywhere on the hemisphere when using the zonalized background state (Fig. 7f).

As we have seen in Figs. 6 and 7, the zonalized background state in the two observed episodes is associated with zonal winds that are more than 3 times larger than those from the zonal-mean background state. This issue indicates one of the limitations of the barotropic model, namely, the fact that it does not allow a zonally symmetric residual circulation in the meridional plane owing to its reduced dimensionality. In a more realistic model with three-dimensional geometry, the eddy forcing would induce such a residual circulation, which in turn would partly compensate the impact of the eddies on the mean flow (Andrews et al. 1987; Nakamura and Solomon 2010). It is impossible to realistically represent the effect of the residual circulation in the barotropic framework, but we try to estimate the modifications to be expected using an ad hoc procedure. In three-dimensional quasigeostrophic theory the zonalized wind $U(\phi, z)$ is a function of both latitude ϕ and altitude z , and (19) must be replaced by

$$\mathcal{L}U = \text{RHS}[(\bar{u} + \mathcal{A}), \bar{u}], \quad (20)$$

where $\mathcal{L}$ is a linear elliptic differential operator in ϕ and z and the right-hand side (rhs) involves meridional derivatives of $(\bar{u} + \mathcal{A})$ as well as vertical derivatives of $\bar{u}$ [see Eq. (11) in Nakamura and Solomon 2010]. The solution of (20) effectively reduces the strength of U in comparison to what one would obtain from (19). To broadly represent this effect in the framework of the barotropic model, we follow Nakamura and Huang (2018) and Nakamura et al. (2020) and replace (19) by

$$U = \bar{u} + \alpha \mathcal{A} \quad (21)$$

with a constant $\alpha < 1$. The two quoted studies motivated their choice of α through a linear regression of observed $\bar{u}$ against finite amplitude wave activity and found $\alpha \approx 0.4$. Correspondingly, we take $\alpha = 0.4$, and the so-obtained profile $U(\phi)$ is subsequently subject to four passes of a 1–2–1 grid point smoother. This filter accounts for the fact that the solution of an elliptic partial differential equation such as (20) generally yields a rather smooth solution. The result is shown in Fig. 8 for the episode from 2010. Wind speeds in the zonalized background state are smaller than before by up to 35 m s^{-1} (Fig. 8a, to be compared with Fig. 7b). There is still a single jet, although its maximum looks more like a broad plateau between 50° and 80°N . As before, the profile of the stationary wavenumber for the zonalized background state differs considerably from that for the zonal-mean background state (Fig. 8b); although it is not strictly monotonic in midlatitudes any longer (red solid line), its relative maxima are much weaker than for the corresponding profile from the zonal-mean background state and shifted toward lower wavenumbers. A literal interpretation in terms of ray tracing predicts a waveguide for wave-number 4 at $\phi = 45^\circ\text{N}$, but not for wavenumbers 6–8 which played the key role in the argument of Kornhuber et al. (2017b). In addition, it was demonstrated by Wirth (2020b) that a weak relative maximum in K_s (corresponding to a waveguide for a single wavenumber only) is associated with poor waveguidability. In summary, the ad hoc argument suggests that our primary result would also be valid in a three-dimensional framework, namely, the lack of prominent waveguidability when using the zonalized instead of the zonal-mean background state.

6. Discussion and conclusions

The present paper addresses the problem of how to diagnose jet waveguidability in the presence of large-amplitude eddies. Our focus is on the choice of a suitable eddy-free background state, which forms the basis for the subsequent analysis. In past

applications, the background state was often defined through an Eulerian zonal average. Here, we argue that this zonal-mean background state suffers from two serious issues. First, there is a conceptual issue. Consider an argument that hypothesizes a causal chain of events involving the growth of wave amplitudes as a consequence of enhanced waveguidability. In this situation one should really diagnose waveguidability at an early, low-amplitude stage rather than at the fully perturbed late stage, because it is the properties of the early stage background state that make the waves grow later on. Second, there is a mathematical issue. The traditional concept of waveguidability is based on linear theory and can, therefore, not be expected to provide meaningful results if applied to fully perturbed flows with large-amplitude eddies. Incidentally, the latter issue has been pointed out before in a somewhat different context by Solomon et al. (2012). We then proposed the use of an alternative background state, namely, the so-called zonalized background state, which is obtained by rearranging PV contours such that they become purely zonal. The zonalized background state can be viewed as a modified Lagrangian mean (McIntyre 1980) representing the early stage of a notional evolution from small to large eddy amplitudes; it is superior to the Eulerian zonal average because it avoids both the conceptual and the mathematical issue.

We investigated the problem in a barotropic framework starting with an idealized model configuration. It turned out that the two background states can yield vastly different results in terms of waveguidability in the presence of large-amplitude eddies. In particular, the zonal-mean background state may suggest the existence of waveguides in situations that resemble blocking-like flows. These waveguides were identified as artifacts, because they were absent when the analysis was based on the zonalized background state. To be sure, we only considered a small number of idealized examples. However, these examples served to illustrate the issues (in particular the occurrence of artifacts with the zonal-mean background state) and allowed us to understand the underlying reasons. We, therefore, believe that the effects discovered here are generic rather than coincidental.

We then went on to analyze two past episodes of extreme weather, namely, the heatwaves from summer 2003 and summer 2010. Both episodes have been analyzed before and associated with the mechanism of quasi-resonance, which requires enhanced waveguidability as a prerequisite (Petoukhov et al. 2013; Kornhuber et al. 2017b). Consistent with these studies, we found a waveguide in both episodes when the analysis was based on the zonal-mean background state, but these waveguides vanished upon the use of the zonalized background state. This result suggests that previous diagnostics of waveguidability in cases of extreme weather are likely to be affected by the above-described artifact and that some of the earlier conclusions have to be considered as misinterpretations. In particular, there is no causal link from large waveguidability (as diagnosed from the zonal-mean of the fully perturbed flow) to the occurrence of large-amplitude eddies. Rather, the opposite seems to be true: the spuriously diagnosed waveguides are likely to be a consequence of the existence of large-amplitude eddies.

Interestingly, this reversed interpretation in terms of cause and effect is consistent with the basic part of the analysis in Kornhuber et al. (2017b). Their Fig. 1 indicates the onset of a double jet structure in both summers *before* the formation of a waveguide in the zonal-mean background state. Assuming that the double jet structure results from a strongly meandering jet—a possibility mentioned by the authors themselves—and considering what we found in the present paper, this suggests that large-amplitude waves were, indeed, the cause for the diagnosed waveguide rather than its effect.

It would be desirable to transcend the barotropic framework and use a fully three-dimensional zonalized background state as a basis for diagnosing waveguidability. A step up in the hierarchy of models would be the three-dimensional quasigeostrophic model, which still allows one to compute the zonalized background state with a moderate amount of effort (Nakamura and Solomon 2010). However, we anticipate that this model may not be appropriate for the present purpose, because quasigeostrophic theory tends to produce artifacts in the subtropics and tropics (Nakamura and Solomon 2010, 2011). Instead, we believe that a suitable three-dimensional zonalized background state requires the primitive equations framework (Nakamura and Solomon 2011; Methven and Berrisford 2015). In that framework, too, we expect that the zonal wind from the zonalized state is stronger than the one from the zonal-mean state, with a tendency toward a smooth single jet even when the zonal-mean background state indicates a double-jet structure (see Fig. 2 in Methven and Berrisford 2015). If this turned out to be true, the results from the current work would be put on a firmer basis. Exploring this option is technically quite challenging and left for future investigation.

For clarity we reiterate that both the analysis in terms of the zonal-mean background state and the analysis in terms of the zonalized background state search—by design—for circumglobal waveguides on a zonally symmetric background state. This makes perfect sense when investigating the potential for quasi-resonance, since quasi-resonance requires the wave activity to travel around the entirety of Earth in order to constructively interfere with itself. However, real waveguides are often not circumglobal; rather, they usually extend only over a limited sector in longitude (Hoskins and Ambrizzi 1993). Such sectoral waveguides are unlikely to be properly diagnosed by a method that implicitly assumes a zonally symmetric background state. This issue calls for an appropriate “sectoral generalization” of the zonalization procedure in order to stick to the concepts developed in this paper. However, such a generalization does not appear to be straightforward and an investigation along these lines is beyond the scope of the present paper.

Our work has implications regarding earlier claims that the occurrences of extreme weather may be due to “quasi-resonant amplification” or “wave resonance” (Petoukhov et al. 2013; Coumou et al. 2014; Petoukhov et al. 2016; Stadtherr et al. 2016; Kornhuber et al. 2017a,b; Mann et al. 2017; Huntingford et al. 2018; Coumou et al. 2018; Mann et al. 2018; Kornhuber et al. 2019). In these studies, strong waveguidability was argued to be a *precondition* for quasi-resonance and ensuing large wave amplitudes, and the analysis of waveguidability was based on the Eulerian zonal-mean

background state. However, we have shown that prominent waveguidability in such an analysis may in fact be just a *consequence* of the large-amplitude eddies. This would imply that an important piece of circumstantial evidence supporting quasi-resonance in the original argument is lost.

We conclude that diagnosing the waveguidability of a midlatitude jet in the presence of large-amplitude eddies is a tricky task. To the extent that one leaves the range of validity of linear theory, one is bound to obtain misleading results. Our analysis suggests that this was, indeed, the case in recent analyses of heat-wave episodes. In particular, the mechanism of (quasi-) resonance conceivably plays a less important role for extreme weather (if any) than claimed by earlier publications.

Acknowledgments. We thank M. Riemer and G. Frangkoulidis for constructive comments on an earlier version of this manuscript. We also thank the three anonymous reviewers, whose thought-provoking comments were very helpful for improving the clarity of our arguments. Part of the research leading to these results has been done within the subproject “Dynamics and predictability of blocked regimes in the Atlantic-European region” of the Transregional Collaborative Research Center SFB/TRR 165 “Waves to Weather” (www.wavestoweather.de) funded by the German Research Foundation (DFG).

APPENDIX A

Computation of the Zonalized PV Profile

We briefly outline the procedure to compute the zonalized PV profile $Q(\phi)$ from a given PV distribution $q(\lambda, \phi)$. For further details the reader is referred to Nakamura and Zhu (2010). The algorithm includes the following steps:

- 1) Select a set of PV contours, i.e., values of PV with equivalent spacing between the lowest and the highest value in $q(\lambda, \phi)$;
- 2) for each PV contour labeled with the value Q : consider the region with PV values higher than Q and determine its area A using a trapezoidal quadrature scheme for the sphere;
- 3) associate each contour Q with an “equivalent latitude” ϕ which is defined as the latitude for which the poleward surface area equals A ; this step results in a unique relation between any given Q value and a corresponding latitude, denoted as $Q \leftrightarrow \phi$;
- 4) obtain the profile $Q(\phi)$ for a set of specified latitudes ϕ_i from the relation $Q \leftrightarrow \phi$ of the previous step through linear interpolation.

APPENDIX B

Zonal-Mean Wind Profile in Case of Local PV Redistribution

Our PV redistribution procedure described in section 4a modifies PV within the range of latitudes $30^\circ \leq \phi \leq 60^\circ\text{N}$ only, which means that $\bar{q}(\phi) = q_0(\phi)$ for all other latitudes. Here we show that this implies $\bar{u} = u_0$ outside the range $30^\circ \leq \phi \leq 60^\circ\text{N}$.

The relation between the full (redistributed) PV and the full wind is given by (1) and (2):

$$q(\lambda, \phi) = 2\Omega \sin \phi + \frac{1}{a \cos \phi} \frac{\partial v}{\partial \lambda} - \frac{1}{a \cos \phi} \frac{d}{d\phi} (u \cos \phi). \quad (\text{B1})$$

Forming the zonal average yields

$$\bar{q}(\phi) = 2\Omega \sin \phi - \frac{1}{a \cos \phi} \frac{d}{d\phi} (\bar{u} \cos \phi). \quad (\text{B2})$$

At the same time, the reference state satisfies (3):

$$q_0(\phi) = 2\Omega \sin \phi - \frac{1}{a \cos \phi} \frac{d}{d\phi} (u_0 \cos \phi) \quad (\text{B3})$$

due to its zonal symmetry. Mathematically, the latter two equations are identical. Given that q_0 and $\bar{q}$ are identical outside the range $30^\circ \leq \phi \leq 60^\circ\text{N}$ and that both u_0 and $\bar{u}$ have to vanish at the poles (in order to avoid a singularity in vorticity), the foregoing implies $\bar{u} = u_0$ outside the range $30^\circ \leq \phi \leq 60^\circ\text{N}$.

REFERENCES

- Andrews, D. G., J. R. Holton, and C. B. Leovy, 1987: *Middle Atmosphere Dynamics*. Academic Press, 489 pp.
- Barnes, E. A., and J. A. Screen, 2015: The impact of Arctic warming on the midlatitude jet-stream: Can it? Has it? Will it? *Wiley Interdiscip. Rev. Climate Change*, **6**, 277–286, <https://doi.org/10.1002/wcc.337>.
- Bender, C. M., and S. A. Orszag, 1978: *Advanced Mathematical Methods for Scientists and Engineers*. McGraw-Hill Book Company, 593 pp.
- Black, E., M. Blackburn, G. Harrison, B. Hoskins, and J. Methven, 2004: Factors contributing to the summer 2003 European heat-wave. *Weather*, **59**, 217–223, <https://doi.org/10.1256/wea.74.04>.
- Blackport, R., and J. A. Screen, 2020: Insignificant effect of Arctic amplification on the amplitude of midlatitude atmospheric waves. *Sci. Adv.*, **6**, eaay2880, <https://doi.org/10.1126/sciadv.aay2880>.
- Branstator, G., 1983: Horizontal energy propagation in a barotropic atmosphere with meridional and zonal structure. *J. Atmos. Sci.*, **40**, 1689–1708, [https://doi.org/10.1175/1520-0469\(1983\)040<1689:HEPIAB>2.0.CO;2](https://doi.org/10.1175/1520-0469(1983)040<1689:HEPIAB>2.0.CO;2).
- , 2002: Circumglobal teleconnections, the jet stream waveguide, and the North Atlantic Oscillation. *J. Climate*, **15**, 1893–1910, [https://doi.org/10.1175/1520-0442\(2002\)015<1893:CTTJSW>2.0.CO;2](https://doi.org/10.1175/1520-0442(2002)015<1893:CTTJSW>2.0.CO;2).
- Brunner, L., G. C. Hegerl, and A. K. Steiner, 2017: Connecting atmospheric blocking to European temperature extremes in spring. *J. Climate*, **30**, 585–594, <https://doi.org/10.1175/JCLI-D-16-0518.1>.
- Charney, J. G., and A. Eliassen, 1949: A numerical method for predicting the perturbations of the middle latitude westerlies. *Tellus*, **1**, 38–54, <https://doi.org/10.3402/tellusa.v1i2.8500>.
- Chen, G., J. Lu, D. A. Burrows, and L. R. Leung, 2015: Local finite-amplitude wave activity as an objective diagnostic of midlatitude extreme weather. *Geophys. Res. Lett.*, **42**, 10 952–10 960, <https://doi.org/10.1002/2015GL066959>.
- Cohen, J., and Coauthors, 2020: Divergent consensus on Arctic amplification influence on midlatitude severe winter weather. *Nat. Climate Change*, **10**, 20–29, <https://doi.org/10.1038/s41558-019-0662-y>.
- Coumou, D., V. Petoukhov, S. Rahmstorf, S. Petri, and H. J. Schellnhuber, 2014: Quasi-resonant circulation regimes and

- hemispheric synchronization of extreme weather in boreal summer. *Proc. Natl. Acad. Sci. USA*, **111**, 12331–12336, <https://doi.org/10.1073/pnas.1412797111>.
- , G. Capua, S. Vavrus, L. Wang, and S. Wang, 2018: The influence of Arctic amplification on mid-latitude summer circulation. *Nat. Commun.*, **9**, 2959, <https://doi.org/10.1038/s41467-018-05256-8>.
- Fragkoulidis, G., V. Wirth, P. Bossmann, and A. H. Fink, 2018: Linking Northern Hemisphere temperature extremes to Rossby wave packets. *Quart. J. Roy. Meteor. Soc.*, **144**, 553–566, <https://doi.org/10.1002/qj.3228>.
- Francis, J. A., 2017: Why are Arctic linkages to extreme weather still up in the air? *Bull. Amer. Meteor. Soc.*, **98**, 2551–2557, <https://doi.org/10.1175/BAMS-D-17-0006.1>.
- , and S. J. Vavrus, 2012: Evidence linking Arctic amplification to extreme weather in mid-latitudes. *Geophys. Res. Lett.*, **39**, L06801, <https://doi.org/10.1029/2012GL051000>.
- , and —, 2015: Evidence for a wavier jet stream in response to rapid Arctic warming. *Environ. Res. Lett.*, **10**, 014005, <https://doi.org/10.1088/1748-9326/10/1/014005>.
- Galperin, B., and P. Read, Eds., 2019: *Zonal Jets: Phenomenology, Genesis, and Physics*. Cambridge University Press, 550 pp.
- Ghinassi, P., M. Baumgart, F. Teubler, M. Riemer, and V. Wirth, 2020: A budget equation for the amplitude of Rossby wave packets based on finite amplitude local wave activity. *J. Atmos. Sci.*, **77**, 277–296, <https://doi.org/10.1175/JAS-D-19-0149.1>.
- Glickman, T. S., Ed., 2000: *Glossary of Meteorology*. 2nd ed. Amer. Meteor. Soc., 855 pp., <http://glossary.ametsoc.org/>.
- Hanley, J., and R. Caballero, 2012: The role of large-scale atmospheric flow and Rossby wave breaking in the evolution of extreme windstorms over Europe. *Geophys. Res. Lett.*, **39**, L21708, <https://doi.org/10.1029/2012GL053408>.
- Held, I. M., 1983: Stationary and quasi-stationary eddies in the extratropical troposphere: Theory. *Large Scale Dynamical Processes*, B. J. Hoskins and R. P. Pearce, Eds., Academic Press, 127–168.
- Hersbach, H., and Coauthors, 2020: The ERA5 global reanalysis. *Quart. J. Roy. Meteor. Soc.*, **146**, 1999–2049, <https://doi.org/10.1002/qj.3803>.
- Hong, C.-C., H.-H. Hau, N.-H. Lin, and H. Chiu, 2011: Roles of European blocking and tropical-extratropical interaction in the 2010 Pakistan flooding. *Geophys. Res. Lett.*, **38**, L13806, <https://doi.org/10.1029/2011GL047583>.
- Hoskins, B. J., and D. J. Karoly, 1981: The steady linear response of a spherical atmosphere to thermal and orographic forcing. *J. Atmos. Sci.*, **38**, 1179–1196, [https://doi.org/10.1175/1520-0469\(1981\)038<1179:TSLROA>2.0.CO;2](https://doi.org/10.1175/1520-0469(1981)038<1179:TSLROA>2.0.CO;2).
- , and T. Ambrizzi, 1993: Rossby wave propagation on a realistic longitudinally varying flow. *J. Atmos. Sci.*, **50**, 1661–1671, [https://doi.org/10.1175/1520-0469\(1993\)050<1661:RWPOAR>2.0.CO;2](https://doi.org/10.1175/1520-0469(1993)050<1661:RWPOAR>2.0.CO;2).
- Huntingford, C., D. Mitchell, K. Kornhuber, D. Coumou, S. Osprey, and M. Allen, 2018: Assessing changes in risk of amplified planetary waves in a warming world. *Atmos. Sci. Lett.*, **20**, e929, <https://doi.org/10.1002/asl.929>.
- Karoly, D. J., and B. J. Hoskins, 1982: Three dimensional propagation of planetary waves. *J. Meteor. Soc. Japan*, **60**, 109–123, https://doi.org/10.2151/jmsj1965.60.1_109.
- Kornhuber, K., V. Petoukhov, D. Karoly, S. Petri, S. Rahmstorf, and D. Coumou, 2017a: Summertime planetary wave resonance in the Northern and Southern Hemispheres. *J. Climate*, **30**, 6133–6150, <https://doi.org/10.1175/JCLI-D-16-0703.1>.
- , —, S. Petri, S. Rahmstorf, and D. Coumou, 2017b: Evidence for wave resonance as a key mechanism for generating high-amplitude quasi-stationary waves in boreal summer. *Climate Dyn.*, **49**, 1961–1979, <https://doi.org/10.1007/s00382-016-3399-6>.
- , S. Osprey, D. Coumou, S. Petri, V. Petoukhov, S. Rahmstorf, and L. Gray, 2019: Extreme weather events in early summer 2018 connected by a recurrent hemispheric wave-7 pattern. *Geophys. Res. Lett.*, **14**, 054002, <https://doi.org/10.1088/1748-9326/ab13bf>.
- Lau, N.-C., and M. J. Nath, 2014: Model simulation and projection of European heat waves in present-day and future climates. *J. Climate*, **27**, 3713–3730, <https://doi.org/10.1175/JCLI-D-13-00284.1>.
- Lighthill, M. J., 1967: Waves in fluids. *Commun. Pure Appl. Math.*, **20**, 267–293, <https://doi.org/10.1002/cpa.3160200204>.
- Mann, M. E., S. Rahmstorf, K. Kornhuber, B. A. Steinman, S. K. Miller, and D. Coumou, 2017: Influence of anthropogenic climate change on planetary wave resonance and extreme weather events. *Sci. Rep.*, **7**, 45242, <https://doi.org/10.1038/srep45242>.
- , —, —, —, —, S. Petri, and D. Coumou, 2018: Projected changes in persistent extreme summer weather events: The role of quasi-resonant amplification. *Sci. Adv.*, **4**, eaat3272, <https://doi.org/10.1126/sciadv.aat3272>.
- Manola, I., F. Selten, H. de Vries, and W. Hazeleger, 2013: “Waveguidability” of idealized jets. *J. Geophys. Res.*, **118**, 10 432–10 440, <https://doi.org/10.1002/jgrd.50758>.
- McIntyre, M., 1980: Towards a Lagrangian-mean description of stratospheric circulations and chemical transport. *Philos. Trans. Roy. Soc. London*, **296**, 129–148, <https://doi.org/10.1098/rsta.1980.0160>.
- Methven, J., and P. Berrisford, 2015: The slowly evolving background state of the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **141**, 2237–2258, <https://doi.org/10.1002/qj.2518>.
- Nakamura, N., and A. Solomon, 2010: Finite-amplitude wave activity and mean flow adjustments in the atmospheric general circulation. Part I: Quasigeostrophic theory and analysis. *J. Atmos. Sci.*, **67**, 3967–3983, <https://doi.org/10.1175/2010JAS3503.1>.
- , and —, 2011: Finite-amplitude wave activity and mean flow adjustments in the atmospheric general circulation. Part II: Analysis in the isentropic coordinate. *J. Atmos. Sci.*, **68**, 2783–2799, <https://doi.org/10.1175/2011JAS3685.1>.
- , and D. Zhu, 2010: Finite-amplitude wave activity and diffusive flux of potential vorticity in eddy–mean flow interaction. *J. Atmos. Sci.*, **67**, 2701–2716, <https://doi.org/10.1175/2010JAS3432.1>.
- , and C. S. Y. Huang, 2018: Atmospheric blocking as a traffic jam in the jet stream. *Science*, **361**, 42–47, <https://doi.org/10.1126/science.aat0721>.
- , J. Falk, and S. W. Lubis, 2020: Why are stratospheric sudden warmings sudden (and intermittent)? *J. Atmos. Sci.*, **77**, 943–964, <https://doi.org/10.1175/JAS-D-19-0249.1>.
- Overland, J. E., and Coauthors, 2016: Nonlinear response of mid-latitude weather to the changing Arctic. *Nat. Climate Change*, **6**, 992–999, <https://doi.org/10.1038/nclimate3121>.
- Pedlosky, J., 1987: *Geophysical Fluid Dynamics*. 2nd ed. Springer, 710 pp.
- Petoukhov, V., S. Rahmstorf, S. Petri, and H.-J. Schellnhuber, 2013: Quasiresonant amplification of planetary waves and recent Northern Hemisphere weather extremes. *Proc. Natl. Acad. Sci. USA*, **110**, 5336–5341, <https://doi.org/10.1073/pnas.1222000110>.

- , S. Petri, S. Rahmstorf, D. Coumou, K. Kornhuber, and H. J. Schellnhuber, 2016: Role of quasiresonant planetary wave dynamics in recent boreal spring-to-autumn extreme events. *Proc. Natl. Acad. Sci. USA*, **113**, 6862–6867, <https://doi.org/10.1073/pnas.1606300113>.
- Pfahl, S., and H. Wernli, 2012: Quantifying the relevance of atmospheric blocking for co-located temperature extremes in the Northern Hemisphere on (sub-)daily time scales. *Geophys. Res. Lett.*, **39**, L12807, <https://doi.org/10.1029/2012GL052261>.
- , C. Schwierz, M. Croci-Maspoli, C. M. Grams, and H. Wernli, 2015: Importance of latent heat release in ascending air streams for atmospheric blocking. *Nat. Geosci.*, **8**, 610–614, <https://doi.org/10.1038/ngeo2487>.
- Pfleiderer, P., C.-F. Schleussner, K. Kornhuber, and D. Coumou, 2019: Summer weather becomes more persistent in a 2°C world. *Nat. Climate Change*, **9**, 666–671, <https://doi.org/10.1038/s41558-019-0555-0>.
- Plumb, R. A., 1985: On the three-dimensional propagation of stationary waves. *J. Atmos. Sci.*, **42**, 217–229, [https://doi.org/10.1175/1520-0469\(1985\)042<0217:OTTDPO>2.0.CO;2](https://doi.org/10.1175/1520-0469(1985)042<0217:OTTDPO>2.0.CO;2).
- Rhines, P. B., 2002: Rossby waves. *Encyclopedia of Atmospheric Sciences*, G. R. North, J. A. Pyle, and F. Zhang, Eds., Elsevier, 1923–1939.
- Röethlisberger, M., S. Pfahl, and O. Martius, 2016: Regional-scale jet waviness modulates the occurrence of midlatitude weather extremes. *Geophys. Res. Lett.*, **43**, 10 989–10 997, <https://doi.org/10.1002/2016GL070944>.
- Rossby, C. G., 1940: Planetary flow patterns in the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **66** (Suppl.), 68–97.
- Rossby, C. R., and Coauthors, 1939: Relations between variations in the intensity of the zonal circulation and displacements of the semi-permanent centres of action. *J. Mar. Res.*, **2**, 38–55, <https://doi.org/10.1357/002224039806649023>.
- Schneider, A., S. Schubert, P. Vargin, F. Lunkeit, S. Zhu, D. H. W. Peters, and K. Fraedrich, 2012: Large-scale flow and the long-lasting blocking high over Russia: Summer 2010. *Mon. Wea. Rev.*, **140**, 2967–2981, <https://doi.org/10.1175/MWR-D-11-00249.1>.
- Schubert, S., H. Wang, and M. Suarez, 2011: Warm season sub-seasonal variability and climate extremes in the Northern Hemisphere: The role of stationary Rossby waves. *J. Climate*, **24**, 4773–4792, <https://doi.org/10.1175/JCLI-D-10-05035.1>.
- Screen, J. A., and I. Simmonds, 2013: Exploring links between Arctic amplification and mid-latitude weather. *Geophys. Res. Lett.*, **40**, 959–964, <https://doi.org/10.1002/grl.50174>.
- , and —, 2014: Amplified mid-latitude planetary waves favour particular regional weather extremes. *Nat. Climate Change*, **4**, 704–709, <https://doi.org/10.1038/nclimate2271>.
- Shepherd, T. G., 2014: Atmospheric circulation as a source of uncertainty in climate change projections. *Nat. Geosci.*, **7**, 703–708, <https://doi.org/10.1038/ngeo2253>.
- Solomon, A., G. Chan, and J. Lu, 2012: Finite-amplitude Lagrangian-mean wave activity diagnostics applied to the baroclinic eddy life cycle. *J. Atmos. Sci.*, **69**, 3013–3027, <https://doi.org/10.1175/JAS-D-11-0294.1>.
- Stadtherr, L., D. Coumou, V. Petoukhov, S. Petri, and S. Rahmstorf, 2016: Record Balkan floods of 2014 linked to planetary wave resonance. *Sci. Adv.*, **2**, e1501428, <https://doi.org/10.1126/sciadv.1501428>.
- Vallis, G. K., 2006: *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*. Cambridge University Press, 745 pp.
- Vavrus, S. J., F. Wang, J. E. Martin, J. A. Francis, Y. Peings, and J. Cattiaux, 2017: Changes in North American atmospheric circulation and extreme weather: Influence of Arctic amplification and Northern Hemisphere snow cover. *J. Climate*, **30**, 4317–4333, <https://doi.org/10.1175/JCLI-D-16-0762.1>.
- Wirth, V., 2020a: Interactive comment on “Waveguidability of idealized midlatitude jets and the limitations of ray tracing theory” by Volkmar Wirth. *Wea. Climate Dyn. Discuss.*, <https://doi.org/10.5194/wcd-2020-3-RC2>.
- , 2020b: Waveguidability of idealized midlatitude jets and the limitations of ray tracing theory. *Wea. Climate Dyn.*, **1**, 111–125, <https://doi.org/10.5194/wcd-1-111-2020>.
- , M. Riemer, E. K. M. Chang, and O. Martius, 2018: Rossby wave packets on the midlatitude waveguide—A review. *Mon. Wea. Rev.*, **146**, 1965–2001, <https://doi.org/10.1175/MWR-D-16-0483.1>.
- Wolf, G., D. J. Bryshaw, N. P. Klingman, and A. Czaja, 2018: Quasi-stationary waves and their impact on European weather and extreme events. *Quart. J. Roy. Meteor. Soc.*, **144**, 2431–2448, <https://doi.org/10.1002/qj.3310>.
- Woollings, T., 2020: *Jet Stream*. Oxford University Press, 220 pp.
- Xie, Z., R. X. Black, and Y. Deng, 2019: Planetary and synoptic-scale dynamic control of extreme cold wave patterns over the United States. *Climate Dyn.*, **53**, 1477–1495, <https://doi.org/10.1007/s00382-019-04683-7>.
- Yang, G.-Y., and B. J. Hoskins, 1996: Propagation of Rossby waves of nonzero frequency. *J. Atmos. Sci.*, **53**, 2365–2378, [https://doi.org/10.1175/1520-0469\(1996\)053<2365:PORWON>2.0.CO;2](https://doi.org/10.1175/1520-0469(1996)053<2365:PORWON>2.0.CO;2).
- Yang, S., B. Reinhold, and E. Källen, 1997: Multiple weather regimes and baroclinically forced spherical resonance. *J. Atmos. Sci.*, **54**, 1397–1409, [https://doi.org/10.1175/1520-0469\(1997\)054<1397:MWRABF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<1397:MWRABF>2.0.CO;2).
- Zappa, G., and T. G. Shepherd, 2017: Storylines of atmospheric circulation change for European regional climate impact assessment. *J. Climate*, **30**, 6561–6577, <https://doi.org/10.1175/JCLI-D-16-0807.1>.