A Numerical Study of Cirrus Bands and Low-Static-Stability Layers Associated with Tropical Cyclone Outflow

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ABSTRACT: Prominent cirrus cloud banding occurred episodically within a northern cirrus canopy of Typhoon Talim (2017) during its recurvature. The generation mechanisms of the cirrus bands and low-static-stability layers that support the bands are investigated using a numerical simulation with the Advanced Research version of the Weather Research and Forecasting Model. Inspection of model output reveals that thin layers of near-neutral to weakly unstable static stability are persistently present in the upper and lower parts of the upper-level outflow, and shallow convection aligned along the vertical shear vector is prevalent in these layers. The cirrus banding occurs as the lowered outflow from the weakening storm ascends slantwise over a midlatitude baroclinic zone, and updrafts of the preexisting shallow convection in the upper part of the outflow layer become saturated. It is shown that the strong outflow resulting from violation of gradient-wind balance in the core region, by itself, creates the low-static-stability layers. Analyses of potential temperature and static stability budgets show that the low-static-stability layers are created mainly by the differential radial advection of radial thermal gradients on the vertical edges of the outflow. The radial thermal gradients occur in response to the outward air parcel acceleration in the core region and deceleration in the outer region, which, by inducing compensating vertical mass transport into and out of the outflow, act to tilt the isentropes within the shear layers. The effects of environmental flow and cloud-radiative forcing on the cirrus banding are also addressed.

KEYWORDS: Instability; Mesoscale processes; Tropical cyclones; Budgets; Cirrus clouds; Mesoscale models

1. Introduction

It has long been recognized that banding frequently develops in cirrus clouds associated with jet streams (e.g., [Shaefer and Hubert 1955](#); [Whitney et al. 1966](#); [Ellrod 1985](#)). These banded cirrus clouds are commonly referred to as transverse cirrus bands ([American Meteorological Society 2020](#)) because they are oriented nearly perpendicular to a jet stream axis. [Knox et al. \(2010\)](#) provided a review of transverse cirrus bands and presented examples in a wide variety of weather systems, including jet streams, outflow of mesoscale convective systems (MCSs), tropical cyclones (TCs), and extratropical cyclones. [Lenz et al. \(2009\)](#) showed that bands of anvil cirrus extending radially outward from regions of deep convection can be observed in nearly half of the central U.S. MCSs.

Convection-permitting numerical simulations of banded cirrus clouds (hereafter simply referred to “cirrus bands”) by [Trier et al. \(2010\)](#), [Kim et al. \(2014\)](#), and [Trier and Sharman \(2016\)](#) all indicated cirrus bands result from thermal-shear instability (e.g., [Asai 1970, 1972](#)), where shallow convection is aligned along the vertical shear vector within a near-neutral or slightly unstable layer. The mechanism for the formation of the low-static-stability layers supporting the cirrus bands, however, varies from case to case. [Trier et al. \(2010\)](#) argued that the vertical shear in the MCS outflow is important not

only in determining the alignment of the bands but also in facilitating the thermodynamic destabilization through differential thermal advection. [Kim et al. \(2014\)](#) suggested that synoptic-scale vertical shear and cloud-radiative effects act together to produce cirrus bands within the cloud shield of a west Pacific cyclone. [Trier and Sharman \(2016\)](#) investigated mechanisms of two distinct episodes of aviation turbulence and cirrus banding that occurred on the anticyclonic shear side of a jet stream and argued that inertia-gravity waves promote regions of cirrus bands by influencing the vertical shear and static stability.

As noted in [Knox et al. \(2010\)](#), cirrus bands have been observed in numerous TCs, and in most cases banding occurs episodically rather than persistently within the cloud shields associated with upper-tropospheric outflow. The TC outflow is a prominent part of the secondary circulation of a TC, and its thermodynamic structure has been considered to play a key role in determining the potential intensity of TCs (e.g., [Emanuel and Rotunno 2011](#); [Emanuel 2012](#); [Ramsay 2013](#); [Chavas and Emanuel 2014](#)). [Emanuel and Rotunno \(2011\)](#) hypothesized that the stratification of the TC outflow is determined by small-scale turbulence that limits the Richardson number to a critical value near unity, and derived an analytic theory on the steady-state TC intensity and structure, sometimes referred to as the “self-stratification theory.” The validity of their assumption has been supported by recent high-altitude dropsonde observations of TC outflow layer (e.g., [Molinari et al. 2014](#); [Braun et al. 2016](#); [Duran and Molinari 2016](#); [Molinari et al. 2019](#)), which demonstrated that the Richardson number is small and turbulent layers frequently exist in the outflow regions. Analyses of convection-permitting simulations by [Duran and Molinari \(2019\)](#) and [Tao et al. \(2019\)](#)

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FIG. 1. The track of Typhoon Talim (2017) from 1200 UTC 9 Sep to 1200 UTC 18 Sep 2017 (black line) from JMA best track data, overlaid on the outermost computational domain of the WRF simulation (D1). Every 12-h position (0000 and 1200 UTC) of the storm is indicated by solid circles. The dashed line denotes the period in which the storm was identified as extratropical according to JMA. The simulated track from 1200 UTC 11 Sep to 1200 UTC 17 Sep is shown in blue. The boxes indicate the two nested domains (D2 and D3). The color shading denotes the initial SST in the simulation at 1200 UTC 11 Sep 2017. The JMA best track data are available at <http://www.jma.go.jp/jma/jma-eng/jma-center/rsmc-hp-pub-eg/besttrack.html>.

also indicated small Richardson number exists in the outflow region of TCs.

The cirrus bands associated with TC outflow can be regarded as a manifestation of linearly organized, coherent structures of air motion, and their occurrence should be closely related to the kinematic and thermodynamic structure of the outflow. Such coherent structures would play a similar role as the small-scale turbulence, and therefore, might have some impacts on the TC development through the modification of the outflow-layer stratification. Given the presumed importance of the outflow stratification in determining the TC potential intensity, physical understanding of the TC cirrus bands and the outflow supporting these bands will be important in advancing our understanding of TC intensity change, and in improving forecasts of TC intensity change through, for example, improvements in the data assimilation methods proposed for the recent high resolution geostationary satellite observations (e.g., Zhang et al. 2016; Honda et al. 2018; Minamide and Zhang 2018; Zhang et al. 2019).

In this study, a realistic numerical simulation of a western North Pacific TC that exhibited widespread cirrus banding is used to examine the structure and generation mechanisms of the cirrus bands and the outflow layer that support these bands. In the next section, a brief observational summary of the TC is presented. Section 3 describes the configuration of

FIG. 2. (a) The storm central sea level pressure (hPa) and (b) the maximum wind speed (kt) from the JMA best track data (black) and the simulation (gray).

the mesoscale model simulation. The structure and evolution of the simulated cirrus bands and the outflow layer are examined in section 4. Physical processes that determine the outflow-layer stratification are investigated through analyses of thermodynamic and static stability budgets in section 5. The dynamics involved in the destabilization of outflow shear layers are addressed in section 6. Finally, a summary and conclusions are given in section 7.

2. Overview of Typhoon Talim (2017) and cirrus bands

The western North Pacific TC that provided observational basis for this paper is Typhoon Talim (2017). According to the Japan Meteorological Agency (JMA), Typhoon Talim was first identified as a tropical depression on 7 September, forming to the east of the Mariana Islands. The tropical depression intensified to a tropical storm at 1200 UTC 8 September. The storm reached typhoon intensity at 1200 UTC 9 September. The track and intensity of Talim based on the JMA best track data are shown in Figs. 1 and 2, respectively, along with the simulated track and intensity described in section 4. Talim steadily intensified over a sea surface temperature (SST) of 29°–30°C as it moved north-westward toward the East China Sea. Talim attained its peak intensity at 0000 UTC 14 September with a minimum SLP of 935 hPa (Fig. 2a) and a maximum 10-min sustained wind speed of 95 kt ($1 \text{ kt} \approx 0.51 \text{ m s}^{-1}$; Fig. 2b). Talim maintained its peak intensity until 1800 UTC 14 September while recurving sharply to the northeast. Talim subsequently weakened

FIG. 3. Infrared satellite brightness temperature T_b (K) of *Himawari-8* band 13 ($10.4\ \mu\text{m}$) over a $1360\ \text{km} \times 1360\ \text{km}$ area at times given at the top of each panel. (d)–(f) Green rectangles indicate the regions of spectral analysis in Fig. 4.

and transitioned into an extratropical cyclone by 1800 UTC 17 September.

The upper-level clouds of Talim underwent significant changes and a substantial number of cirrus bands were formed during the recurvature. Figure 3 shows a series of $10.4\text{-}\mu\text{m}$ infrared (IR) brightness temperatures (T_b) observed by the JMA's geostationary satellite *Himawari-8* (Bessho et al. 2016). The resolution of IR data is 2 km at the subsatellite point. At 0000 UTC 14 September, when Talim had reached its peak intensity, the low- T_b clouds around the eye exhibited a near-circular shape (Fig. 3a). The storm subsequently interacted with a preexisting, midlatitude baroclinic zone and the distribution of clouds became highly asymmetric (e.g., Klein et al. 2000). The T_b around the eye increased markedly between 0000 and 1700 UTC. Between 1700 and 2200 UTC, the T_b of the northern cloud shield decreased (Figs. 3c,d). The cloud shield formed a sharp edge by 0100 UTC 15 September (Fig. 3e), indicating that confluence occurred between the typhoon outflow and the jet stream (e.g., Bader et al. 1995).

By 2200 UTC 14 September, remarkably regular banded features became apparent in the cirrus cloud shield on the north and west flanks of the storm (Fig. 3d). The spacings between the

cirrus bands were estimated to be 10–20 km, similar to the typical spacings of the observed MCS-induced cirrus bands (Lenz et al. 2009; Trier et al. 2010). Each cirrus band became clearer, and the coverage of the cirrus bands increased by 0100 UTC 15 September (Fig. 3e). By 0500 UTC 15 September, the cloudiness near the outer edge of the cloud shield decreased, and the cirrus bands became diffuse (Fig. 3f). Note also that banded regions of low T_b , oriented nearly perpendicular to the cirrus bands and spaced roughly 100 km apart, are also recognized in the regions of cirrus banding (Figs. 3d–f). This point will also be addressed later.

For a quantitative understanding of the evolution of cirrus band characteristics, one-dimensional (1D) power spectral densities (PSDs) of T_b are calculated for the $256\ \text{km} \times 80\ \text{km}$ regions inside the green rectangles in Fig. 3. The T_b data in the rectangles were interpolated into 128×40 grid points with 2-km grid spacing. The PSD was first calculated for each 128-point cross section line that is nearly perpendicular to the cirrus bands, using a 1D fast Fourier transform with a linear detrending and a Welch window. The 1D PSD at each time in Fig. 4 was obtained by averaging 40 different PSDs. As suggested by the T_b field in

FIG. 4. One-dimensional power spectra of the infrared satellite brightness temperature T_b as functions of horizontal wavenumber k and wavelength for the $256 \text{ km} \times 80 \text{ km}$ regions inside the green rectangles in Fig. 3. The $k^{-5/3}$ spectral density dependence on wavenumber is shown by the gray line for reference.

Fig. 3, enhanced power occurs at wavelengths between 10 and 20 km at 2200 UTC 14 September, and the peak moves to larger scales between 20 and 35 km by 0100 UTC 15 September. The overall power increases and the spectral peak becomes less clear by 0500 UTC 15 September, consistent with the diffusive, irregular shapes of dissipating cirrus bands.

3. Numerical model and experimental design

A numerical simulation of Typhoon Talim (2017) is performed using the Advanced Research Weather Research and Forecasting Model, version 3.9.1 (ARW-WRF; Skamarock et al. 2008). The model domain consists of three interactive domains (denoted D1–D3) with domain sizes of $3780 \text{ km} \times 4320 \text{ km}$ (D1), $2160 \text{ km} \times 2700 \text{ km}$ (D2), and $1440 \text{ km} \times 2160 \text{ km}$ (D3) and horizontal grid spacings of 18, 6, and 2 km, respectively (Fig. 1). The model time steps for D1, D2, and D3 are 30, 10, and 3.3 s, respectively. All domains have a model top of 20 hPa ($\sim 26 \text{ km}$) with 108 uneven η levels. The vertical grid spacing increases from approximately 100 to 200 m in the lowest $\sim 4.5 \text{ km}$ and remains almost constant at approximately 200 m from $z \approx 4.5$ to 18 km. Above $z \approx 18 \text{ km}$, the grid spacing increases linearly up to approximately 1000 m near the model top. A sponge layer with Rayleigh damping is applied to the uppermost 5 km.

The model physical parameterizations include the WRF single-moment 6-class microphysics (Hong and Lim 2006), the Yonsei University planetary boundary layer scheme (Noh et al. 2003; Hong et al. 2006), the Rapid Radiative

Transfer Model for an application to general circulation models (Iacono et al. 2008) for both shortwave and longwave radiation, the Kain–Fritsch cumulus scheme for subgrid deep and shallow convection (Kain 2004) activated in the 18- and 6-km grid domains, and the Noah land surface scheme (Ek et al. 2003). The surface flux calculation employs the Monin–Obukhov similarity hypothesis with bulk exchange coefficients.

The initial and boundary conditions are obtained from the National Centers for Environmental Prediction (NCEP) Global Data Assimilation System (GDAS) final analysis (FNL) data, with 0.25° horizontal and 6-h temporal resolutions. The NCEP Real-Time Global Sea Surface Temperature High Resolution (RTG-SST-HR) with $1/12^\circ$ horizontal and 1-day temporal resolution is used for the SST. A bogus vortex is not included in the initial conditions. The 18-km grid domain (D1) is initialized at 1200 UTC 11 September 2017, approximately 3 days before Talim attains its maximum intensity, and is integrated for 144 h until 1200 UTC 17 September 2017. To save computational resources, the 6-km grid (D2) and the 2-km grid (D3) domains are activated at 0000 and 1200 UTC 13 September 2017, respectively, and integrated until 0600 UTC 15 September 2017.

Except where noted, the analysis in this study uses the output on the 2-km grid (D3) that was saved every 5 min. Following the method of Braun (2002), the storm center is determined by minimizing the azimuthal variance of the surface pressure at all radii between the center and the outer portion of the eyewall (75-km radius). In the following, the x and y components of wind relative to the storm are denoted as u and v , respectively, and w represents the vertical wind velocity. The radial and tangential winds derived from the horizontal winds relative to the storm motion (relative to the model domain) are denoted as v_r and v_t (V_r and V_t), respectively. The output fields from D3 are also interpolated onto a cylindrical coordinate system (r, λ, z) , with the origin at the center of the storm at the surface. Here, r is the radius, λ is the azimuth with $\lambda = 0^\circ$ and 90° for the positive x and y directions, respectively. The radial, azimuthal, and vertical grid spacings of the interpolated data are 2 km, 0.5° , and 0.25 km, respectively.

4. Simulation results

a. Model verification

The track and intensity of the simulated storm are compared with those from the JMA best track data in Figs. 1 and 2, respectively. The simulated storm closely follows the observed track for the first 2 days. The timing of subsequent recurvature of the storm is also consistent with the observation, though the simulated northward movement during the recurvature is faster than that observed. The simulation also reproduces the storm intensity change reasonably well (Fig. 2). The simulated storm reaches its peak intensity with a central SLP of 932 hPa and a maximum 10-m wind speed of 110 kt at 0600 UTC 14 September.

Figure 5 shows the infrared brightness temperatures (T_b) derived from the output of outgoing longwave radiation (OLR) at the top of atmosphere for the same times as in Fig. 3.

FIG. 5. Model-derived infrared brightness temperature T_b (K) for the same times as in Fig. 3 for a $1360 \text{ km} \times 1360 \text{ km}$ portion of D3. Vectors represent storm-relative wind vectors at the height of maximum radial velocity. (e) The yellow rectangle locates the region of analysis in Fig. 8. (d)–(f) Green rectangles indicate the regions of spectral analysis in Fig. 6.

The vectors represent storm-relative wind vectors at the level of the maximum storm-relative radial velocity described in the next section. The model-derived T_b also exhibits an evolution broadly consistent with that found in the satellite images. The low- T_b area around the eye exhibits a near-circular shape at 0000 UTC 14 September (Fig. 5a), with wider cloud area and stronger outflow on the east side of the storm than those on the west side. The T_b around the eye increases after 0000 UTC 14 September. The distributions of outflow and clouds subsequently become highly asymmetric, weighted in the northeast quadrant of the storm by 1700 UTC (Figs. 5b,c). It appears that the environmental weak inertial stability on the anticyclonic shear side of the midlatitude jet appears to provide favorable condition for the development of asymmetric outflow weighted in the northeast quadrant of the storm (e.g., Rappin et al. 2011). The cloud shield formed a well-defined edge in its northern and western sectors by 2200 UTC (Fig. 5d).

Prominent cirrus banding occurs inside the edge of the cloud shield by 2200 UTC (Fig. 5d). The spacings between the bands increase as the time elapses, and new cirrus bands

develop near the edge of the inner cloud area with lower T_b (Fig. 5e). As in the satellite images (Figs. 3d–f), banded regions of low T_b , with orientation nearly perpendicular to the cirrus bands, are also recognized in the region of cirrus banding (Figs. 5d–f). The spacing between the cirrus bands further increase and the bands become shorter and ill-defined by 0500 UTC 15 September (Fig. 5f). Unlike the satellite observation, diffusive, small-scale cirrus bands are absent in the simulation at this time.

Figure 6 shows the 1D PSDs of the simulated T_b for the regions inside the green rectangles in Fig. 5, calculated in the same manner as those in Fig. 4. A spectral peak occurs at a wavelength of $\sim 14 \text{ km}$ at 2200 UTC 14 September and the peak moves to larger scales at $\sim 22 \text{ km}$ by 0100 UTC 15 September, consistent with the PSDs for the satellite-derived T_b . However, in contrast with the satellite-derived T_b , the overall power in the shorter wavelengths subsequently decreases and the dominant spectral peak shifts to larger scales at $\sim 50 \text{ km}$ by 0500 UTC 15 September. Skamarock (2004) showed that features and phenomena smaller than 7 times the horizontal grid interval (14 km in the current

FIG. 6. One-dimensional power spectra of the model-derived T_b as functions of horizontal wavenumber k and wavelength for the $256 \text{ km} \times 80 \text{ km}$ regions inside the green rectangles in Fig. 5. The $k^{-5/3}$ spectral density dependence on wavenumber is shown by the gray line for reference.

case) tend to be damped in the WRF Model. The absence of diffusive, small-scale bands and the low PSD in the shorter wavelengths at 0500 UTC 15 September in the simulation may be attributable to the insufficient model resolution adopted in this study.

As previously mentioned, the evolution of the overall pattern of T_b can be related to the interaction of the TC outflow with a preexisting midlatitude baroclinic zone. Figure 7 shows north–south cross sections through the storm center of storm-relative meridional winds and potential temperature θ from D1. At 0000 UTC 14 September (Fig. 7a), upper-tropospheric outflow is roughly symmetric about the storm center and concentrated in a $\sim 3\text{-km}$ -deep layer beneath the cold-point tropopause. The northward-directed outflow is located above the baroclinic zone, which is characterized by θ surfaces tilted upward with increasing latitude below 12-km height. The eyewall clouds reach the tropopause at 17-km height, and the cloud tops descend outward from the storm center.

In association with the weakening of the eyewall convection, the outflow shifts downward and becomes favorably directed northward. The cross section at 2200 UTC 14 September shows that the northward-directed outflow ascends slantwise over the baroclinic zone (Fig. 7b). The cloud formation within the upper and outer portion of the outflow causes the northward expansion of the cloud shield in Fig. 5.

Although some details such as the size of eye are different, the model reproduced reasonably well the observed evolution of the cloud pattern and the widespread cirrus banding, as well as the track and intensity of the storm. Thus, the simulation

FIG. 7. Vertical north–south cross sections through the storm center of storm-relative meridional wind velocity (shaded and contoured every 5 m s^{-1}) and potential temperature θ from D1 at (a) 0000 UTC 14 Sep and (b) 2200 UTC 14 Sep 2017. Thick black contours enclose ice cloud regions where the cloud ice mixing ratio is more than $1 \times 10^{-6} \text{ kg kg}^{-1}$. Thick magenta line in each panel represents the cold-point tropopause height. Location of the jet stream axis in the plane of the cross section is denoted by “J.”

results are used in the following to investigate the structure of cirrus bands and their formation mechanisms.

b. Shallow convection in the outflow layer

The structure of the simulated cirrus bands at 0100 UTC 15 September in Fig. 5e is analyzed in Figs. 8 and 9. The cirrus bands oriented in the radial direction in Fig. 5e are identified in the cloud ice mixing ratio at $z = 12.5 \text{ km}$, slightly above the level of outflow maximum (Fig. 8a). The radial velocity field exhibits linear patterns oriented parallel to the vertical shear vectors (Fig. 8b). Stripes of strong radial winds are collocated with the cloud bands and upward motions, indicating that the linear patterns in the radial velocity result from the vertical transport of radial momentum in the strongly sheared layer associated with the outflow.

Linear patterns are also identified in the cloud and radial velocity fields in the lower part of the outflow layer (Figs. 8c,d). The banded features at $z = 11.0 \text{ km}$ are oriented clockwise relative to those at $z = 12.5 \text{ km}$, indicating that the disturbances in the upper and lower parts of the outflow layer are independent of each other. Note that the cloud ice mixing ratio is locally high along the orange dashed lines both in the upper and lower parts of the outflow layer (Figs. 8a,c), and enhanced upward motions occur just outside (i.e., northwest side) the

FIG. 8. Horizontal cross sections of (a) cloud ice mixing ratio at $z = 12.5$ km (shaded, $10^{-6} \text{ kg kg}^{-1}$), (b) storm-relative radial velocity at $z = 12.5$ km (shaded and contoured every 2 m s^{-1}), (c) cloud ice mixing ratio at $z = 11.0$ km (shaded, $10^{-6} \text{ kg kg}^{-1}$), and (d) storm-relative radial velocity at $z = 11.0$ km (shaded and contoured every 2 m s^{-1}) at 0100 UTC 15 Sep 2017 over the yellow rectangular region in Fig. 5e. Thin red solid (black dashed) contours in (a) and (c) represent vertical velocity of 0.1 (-0.1) m s^{-1} . Vectors in (b) and (d) represent vertical shear vectors between $z = 12.0$ and 13.0 km and between $z = 10.5$ and 11.5 km, respectively. The orange dashed lines in (a) and (c) denote regions of locally high cloud ice mixing ratio.

lines. As will be shown later, these features are caused by gravity waves trapped below the outflow layer.

Figure 9 shows vertical cross sections approximately normal to the cirrus bands (along line AB in Fig. 8). Here, the X and Y axes are taken in the direction parallel to and normal to the line AB, as shown in Fig. 8a, and the X and Y components of the storm-relative horizontal wind are denoted as U and V , respectively. The horizontal wind component normal to the cross section (V) and the vertical velocity show substantial variations within the layers above and below the outflow maximum (Figs. 9a,b). As seen from the isentropes in Fig. 9b, the static stability is small in these layers. Each updraft in these layers is warmer than the adjacent downdrafts, indicating that the vertical motions are driven by buoyancy. The horizontal and vertical variations of the horizontal wind component parallel to the cross section (U) are small compared to those of V in the outflow layer between $z = 10.5$ and 13 km (Fig. 9c). The vertical velocity perturbations in the upper and lower outflow layer generate distinct cloud bands separated by cloud-free regions (Fig. 9d), but a horizontally continuous sheet of cloud exists in the stably stratified layer at the level of maximum outflow.

Thus, the cirrus bands in the lower outflow layer are obscured in the model-derived T_b field. The Y component of vorticity ($\partial U/\partial z - \partial W/\partial X$) superposed on Fig. 9d exhibits alternating series of positive and negative vorticity at $z \sim 11.0$ and 12.3 km, indicating the presence of counterrotating vortices within the low-static-stability layers. The vorticity field also shows peaks or isolated patches of positive vorticity in the stably stratified layers at $z \sim 10.3$ and 11.7 km. Although the vertical shear approximately normal to the cirrus bands ($\partial U/\partial z$) is larger than that approximately parallel to the bands ($\partial V/\partial z$) in these layers, it is unlikely that these vorticity anomalies and/or disturbances in the low-static-stability layers are created as a result of Kelvin–Helmholtz (K–H) instability; the axes of locally enhanced vorticity are persistently tilted downshear (to the right) with height as in Fig. 9d, indicating that the vertical shear normal to the bands ($\partial U/\partial z$) acts to damp these vorticity anomalies. The variations of vorticity in the stably stratified layers appear to be caused by the buoyancy-driven vertical motions in the near-neutral layers, which may induce the variations in shear-layer depth in the adjacent stably stratified layers. These results indicate that the cirrus bands in the present

FIG. 9. Vertical cross sections along the line AB in Fig. 8. (a) Storm-relative horizontal wind component in the direction normal to the cross section V (shaded and contoured every 2 m s^{-1}), (b) vertical velocity (shaded, m s^{-1}) and potential temperature (contoured every 0.5 K), (c) storm-relative horizontal wind component in the direction parallel to the cross section U (shaded and contoured every 2 m s^{-1}), and (d) cloud ice mixing ratio (shaded, $10^{-6} \text{ kg kg}^{-1}$) and horizontal vorticity component in the direction normal to the cross section ($\partial U/\partial z - \partial w/\partial X$; contoured every $2.5 \times 10^{-3} \text{ s}^{-1}$ with negative values dashed and the zero contours thickened). The thick solid line segments in (d) denote the axes of locally enhanced vorticity in the stably stratified layers.

case are manifestations of shear-parallel shallow convection resulting from thermal-shear instability, as with the cirrus bands investigated in previous numerical case studies (Trier et al. 2010; Kim et al. 2014; Trier and Sharman 2016).

The cirrus bands become apparent in the model-derived T_b field around 2200 UTC 14 September (Fig. 5). Inspection of the model output revealed, however, that thin layers of reduced static stability are persistently present on the vertical edges of the outflow where the vertical shear is strong, and that shallow convection occurs widely within these layers.

Next, the distribution and evolution of the shallow convection are examined. The height of the shallow convection shows considerable temporal and spatial variations in association with the evolution of the outflow. Thus, the heights of outflow shear layers supporting the bands are determined and then amplitudes of the disturbances in those layers are obtained following the procedure described below.

First, the model output fields from the 2-km grid domain (D3) are smoothed by a cowbell spectral filter described by Barnes et al. (1996) as

$$F(x, y) = \left[\frac{\sin(\pi x/D)}{(\pi x/D)} \right] \left[\frac{\sin(\pi y/D)}{(\pi y/D)} \right], \quad (1)$$

where $D = 30 \text{ km}$. The cutoff wavelength of this filter is 30 km . As noted in Barnes et al. (1996), the amplitudes and phases of larger-scale features are affected little by this filtering. Any field variable A are decomposed into two components as $A = \bar{A} + A'$, where the overbar denotes the smoothed value and the prime denotes the deviation from the smoothed value.

Then, the maximum storm-relative radial velocity $\bar{v}_{r\max}(x, y, t)$ and its height $z_{\text{mrv}}(x, y, t)$ are obtained by the $\bar{v}_r(x, y, z, t)$ between $z = 10 \text{ km}$ and the tropopause level ($z \approx 18 \text{ km}$) and outside of the 150-km radius from the storm center. The height of upper (lower) outflow shear layer $z_{\text{st}}(x, y, t)$ [$z_{\text{sb}}(x, y, t)$] is determined as the height of a local minimum (maximum) of $\partial \bar{v}_r/\partial z$ in height closest to z_{mrv} .

Finally, the signed column maximum values of the perturbation variables in the upper shear layer are extracted from the data within a 2-km-deep layer between $z = z_{\text{st}} - 1.0 \text{ km}$ and $z_{\text{st}} + 1.0 \text{ km}$ (if $z_{\text{mrv}} < z_{\text{st}} - 1.0 \text{ km}$) or between $z = z_{\text{mrv}}$ and $z = z_{\text{mrv}} + 2.0 \text{ km}$ (if $z_{\text{mrv}} \geq z_{\text{st}} - 1.0 \text{ km}$). Likewise, the signed column maximum values in the lower shear layer are extracted from the data between $z = z_{\text{sb}} - 1.0 \text{ km}$ and $z_{\text{sb}} + 1.0 \text{ km}$ (if $z_{\text{mrv}} > z_{\text{sb}} + 1.0 \text{ km}$) or between $z = z_{\text{mrv}} - 2.0 \text{ km}$ and $z = z_{\text{mrv}}$ (if $z_{\text{mrv}} \leq z_{\text{sb}} + 1.0 \text{ km}$).

The maximum radial velocity perturbation $v'_{r\max}$ for the upper shear layer is shown in Fig. 10 for the same times as in

FIG. 10. Signed column maximum values of the radial velocity perturbation $v'_{r,\max}$ (shaded, m s^{-1}) within the 2-km-deep outflow shear layer above the level of maximum storm-relative outflow for the same times and areas as in Fig. 5. The region within 150-km radius from the storm center is masked white. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. Thick black lines indicate locations where smoothed value of the maximum storm-relative radial velocity ($\bar{v}_{r,\max}$) is zero.

Fig. 5. The perturbation radial velocity field is examined here because it is less noisy than other perturbation fields and most clearly exhibits linear patterns associated with the shallow convection. At 0000 UTC 14 September (Fig. 10a), radially oriented stripes associated with the shallow convection are prevalent on the north and south flanks of the storm. The shallow convection subsequently becomes apparent on the northeast side, while those on the west side become insignificant (Figs. 10b,c). The shallow convection becomes apparent also on the northwest side of the storm by 2200 UTC (Fig. 10d) and subsequently increases its radial extent (Fig. 10e). In association with the dissipation of the cirrus bands near the outer edge of the cloud shield (cf. Fig. 5f), the linear patterns of $v'_{r,\max}$ become less distinct and the spacing between the positive and negative $v'_{r,\max}$ increases by 0500 UTC 15 September (Fig. 10f).

Figure 11 shows $v'_{r,\max}$ for the lower outflow shear layer. At 0000 UTC 14 September, stripes of positive and negative values associated with the shallow convection are present in nearly all directions, with their magnitudes generally larger

than those in the upper shear layer (Fig. 11a). The signal of shallow convection subsequently decreases its intensity and extent, and mostly disappears by 1700 UTC (Figs. 11b,c). By 2200 UTC 14 September, the signal of shallow convection becomes apparent just inside the edge of the northern cloud shield (Figs. 11d,e). The linear patterns of $v'_{r,\max}$ subsequently become less distinct and almost dissipate by 0500 UTC 15 September (Fig. 11f).

Previous numerical studies of cirrus banding have suggested that convectively generated gravity waves play roles in generating cirrus bands by helping to release the thermodynamic instability that is generated by other processes (Trier et al. 2010) and by directly influencing the static stability and the vertical shear (Trier and Sharman 2016). The deep convection in TCs also disturbs the surrounding atmosphere and generate gravity waves that propagate outward and upward from their source (e.g., Sato 1993; Pfister et al. 1993; Kim et al. 2009; Nolan and Zhang 2017; Jewtoukoff et al. 2013; Horinouchi et al. 2020; Nolan 2020).

FIG. 11. As in Fig. 10, but for $v'_{r\max}$ within the 2-km-deep outflow shear layer below the level of maximum storm-relative outflow.

In the present case, cloud features suggestive of gravity waves radiating outward from the storm are recognized in Figs. 5d–f and 8c as lines of lower T_b and enhanced cloud ice mixing ratio perpendicular to the cirrus bands.

The structure of the wavelike disturbances that cause the radial variations of T_b and cloud ice in the cirrus banding region is examined in Figs. 12a and 12b, which are constructed using the smoothed data. The vertical velocity field at $z = 9.0$ km, beneath the outflow layer in the cirrus banding region, exhibits spiraling wave pattern with horizontal wavelengths of ~ 60 km in the region of cirrus banding (Fig. 12a). The time–height section of the vertical velocity and potential temperature perturbations from their 4-h running-mean values, taken at the point indicated by a cross mark in Fig. 12a indicates that time variations below the outflow cloud layer are dominated by vertical wavenumber-1 patterns, which oscillate with periods of roughly 1.3 h. The constant phase lines of the vertical velocity and potential temperature perturbations are oriented nearly vertically and are in quadrature, indicating that the disturbances beneath the outflow cloud layer are trapped gravity waves with vertical wavelengths

of about 11 km. The gravity waves are trapped probably because of the strong vertical wind shear and the low static stability at the bottom of the outflow layer. At the location of the time–height section, the gravity waves propagate outward with storm-relative phase velocities of $12\text{--}14\text{ m s}^{-1}$ (not shown). The cloud ice mixing ratio in the outflow layer is maximized just above negative potential temperature perturbations. This indicates that the radial variations of T_b and cloud ice in the cirrus banding region are generated by the vertical displacement of the outflow layer by the trapped gravity waves.

The effects of gravity waves on the shallow convection in the outflow shear layers are examined in azimuth–time diagrams of $v'_{r\max}$ and smoothed vertical velocity perturbations at $z = 9.0$ km (Figs. 12c,d). The cross sections are taken at the 560-km radius for the azimuthal range between 70° and 130° . The gravity waves beneath the outflow layer are represented by leftward-propagating positive and negative vertical velocity perturbations. Alternating lines of positive and negative $v'_{r\max}$ associated with shallow convection are widely present and retain their coherent structure both in the upper and lower outflow shear layers (Figs. 12c,d), and the magnitudes of $v'_{r\max}$ are affected little by the propagation of gravity

FIG. 12. (a) Vertical velocity at $z = 9.0$ km at 2200 UTC 14 Sep 2017 (shaded every 0.02 m s^{-1}). (b) Time–height section of the perturbations of smoothed vertical velocity (shaded every 0.02 m s^{-1}) and smoothed potential temperature (thin black contours every 0.15 K starting at $\pm 0.05 \text{ K}$, with negative values dashed) from their 4-h running-mean values, and cloud ice mixing ratio (green contours at 1, 20, and $40 \times 10^{-6} \text{ kg kg}^{-1}$), taken at the point indicated by a thick cross mark in (a) (560-km radius from the storm center and at 115° azimuth). Azimuth–time diagrams of the signed column maximum values of the radial velocity perturbation within the (c) upper and (d) lower outflow shear layers (shaded, m s^{-1}) and the perturbation of smoothed vertical velocity at $z = 9.0$ km (contoured at 0.04 and -0.04 m s^{-1} with solid and negative lines, respectively), taken at the 560-km radius from the storm center for the azimuthal range between 70° and 130° . Dashed circles in (a) represent the 300- and 600-km range rings.

waves. Thus, the gravity waves do not appear to be essential to the destabilization of outflow layer and the generation of shallow convection, though the waves exert substantial influence on the appearance of cirrus cloud bands.

Several studies showed that moist convection in or near the TC eyewall is modulated by low-wavenumber dynamical asymmetries associated with mesovortices that cyclonically rotate around the eyewall, and convection is activated as the mesovortices move through the downshear-left quadrant of the sheared TC (e.g., Black et al. 2002; Braun et al. 2006; Braun and Wu 2007; Reasor et al. 2009). Nolan (2020) showed that dominant source for the spiral gravity waves that can be seen in observations and numerical simulations of TCs is the pulsation of such cyclonically rotating convective asymmetries. The eyewall convection in the present simulation also exhibited time variations consistent with those described in the studies cited above, with a marked pulsation in the northeast quadrant (not shown). Although the generation mechanisms of the gravity waves are beyond the scope of this study, this

suggests that the waves are generated by the similar mechanisms as proposed by Nolan (2020).

c. Structure of the outflow layer

In the remainder of this paper, the analysis is focused on the stage where the simulated storm is intense and the low T_b area around the eye exhibits a near-circular shape (hereafter referred to as the mature stage), and on the stage where the storm exhibits prominent cirrus banding (hereafter referred to as the banding stage). Temporal and/or azimuthal averaging is applied to field variables in a vortex-following sense, to smooth out small-scale, high-frequency variabilities associated with gravity waves and the shallow convection. For the mature stage, fields averaged over the 4-h period from 2000 UTC 13 September to 0000 UTC 14 September are examined, while those averaged over the 4-h period from 2000 UTC 14 September to 0000 UTC 15 September are examined for the banding stage.

Hereafter, the term “core region” is used to denote the broad region of mean ascent that includes the eyewall convection and

FIG. 13. Radius–height cross sections of temporally and azimuthally averaged fields for the mature stage. (a) Storm-relative radial velocity (shaded every 4 m s^{-1}) and potential temperature θ (thin black contours every 2 K for $\theta < 380 \text{ K}$ and thick black contours every 10 K for $\theta \geq 380 \text{ K}$), (b) radial mass flux (ρv_r) per radian (shaded and contoured every $4 \times 10^5 \text{ kg m}^{-1} \text{ s}^{-1}$) and equivalent potential temperature θ_e (thin black contours every 2 K for $\theta_e < 380 \text{ K}$ and thick black contours every 10 K for $\theta_e \geq 380 \text{ K}$), (c) vertical velocity (shaded and contoured, cm s^{-1}), (d) cloud ice mixing ratio (shaded and contoured, $10^{-6} \text{ kg kg}^{-1}$), (e) squared Brunt–Väisälä frequency N^2 (shaded every $0.25 \times 10^{-4} \text{ s}^{-2}$), and (f) percentage frequency of averaged data points with $N^2 < 0$ (shaded, %). All fields are averaged in a vortex-following sense over the 4-h time period from 2000 UTC 13 Sep to 0000 UTC 14 Sep 2017 and the 30° -wide azimuthal range between 90° and 120° . Thin black contours in (c)–(f) represent storm-relative radial velocity at 10 m s^{-1} intervals, with positive values solid and negative values dashed. Thick green line in each panel represents the cold-point tropopause height. The gray dashed line in (b) represents the level above which the difference between θ_e and θ is less than 1 K.

weaker but deep convection just outside it. The core region is within a radius of about 300 km from the storm center in the present case, and the region outside the core region is referred to as “outer region.”

1) THE MATURE STAGE

Figure 13 shows radius–height sections of the temporally and azimuthally averaged fields in the northern sector of the storm. The fields are averaged over the 30° -wide range between 90° and 120° , where the shallow convection is prominent (cf. Figs. 10a and 11a). Note that characteristic features of the outflow described below were also recognized in the fields constructed by using full azimuthal data, but sharp vertical gradients of variables were strongly damped (not shown). During the mature stage, the axis of maximum outflow slopes downward with radius, with maximum radial velocity exceeding 30 m s^{-1} (Fig. 13a). The vertical spacings of the

superposed isentropes within the outflow decrease with radius for $r < 300 \text{ km}$, while the spacings increase with radius for $350 < r < 650 \text{ km}$, forming negative and positive radial θ gradients in the upper and lower parts of the outflow layer, respectively. The vertical shear of the radial wind acts to tilt the sloped isentropes more vertical and reduces the static stability in these layers, as discussed by Trier and Sharman (2009) for the upper-level outflow of an MCS.

In the outflow layer, the radial mass flux per radian (ρv_r), where ρ is the air density, increases outward within 400-km radius, beyond which the mass flux decreases outward (Fig. 13b). Because the mixing ratio of water vapor is small in the upper troposphere, the equivalent potential temperature with respect to ice θ_e superimposed on the radial mass flux is nearly equal to θ in the outflow layer, with the difference between θ_e and θ being less than 1 K.

The distribution of vertical motions is consistent with the radial variation of the radial mass flux, and vertical mass transport into (out of) the outflow occurs across the $v_r = 0$ contour for $r < 400$ km ($r > 400$ km) (Fig. 13c). These vertical motions cause the isentropes to tilt upshear within the outflow shear layers. A 1.5-km-deep layer of downward motion above the inner-core updrafts slopes downward into the outflow layer, making the upper part of the outflow cloud-free (Fig. 13d).

Figure 13e shows averaged values of the squared Brunt–Väisälä frequency N^2 . Here, N^2 in subsaturated areas is given as the squared dry Brunt–Väisälä frequency $N_d^2 = g(\partial\theta/\partial z)/\theta$, whereas N^2 in saturated areas is given as the squared moist Brunt–Väisälä frequency N_m^2 defined by Durran and Klemp (1982) but modified to account for ice latent heating/cooling. However, at and above the outflow level ($z > 12$ km), the dry and moist adiabatic lapse rates are nearly equal and N_m^2 is nearly the same as N_d^2 . Averaged values of N^2 are small in the strongly sheared layers on the vertical edges of the outflow. Although averaged N^2 values are mostly positive, the isentropes locally and/or temporally tip over in these layers (not shown). Thus, statically unstable regions occur in these layers as shown by the percentage frequency of the averaged data points with $N^2 < 0$ in Fig. 13f. The distribution of N^2 relative to the outflow is consistent with those reported in previous numerical studies of TCs such as Tao et al. (2019, their Fig. 6).

The azimuthal variations of the variables in the upper and lower parts of the outflow are examined in Fig. 14, which provides time-averaged values of w , θ at $z = 16.0$ and 13.0 km and percentage frequency of $N^2 < 0$ in the 2-km deep shear layers defined previously. The arrow in the bottom-left of Fig. 14a shows environmental vertical wind shear between 850 and 200 hPa, averaged over a circle of 300-km radius following Braun and Wu (2007). In the upper part, at $z = 16.0$ km, a roughly annular zone of upward motions and local θ minima is found at about 150-km radius (Figs. 14a,b). These features are associated with the convection overshooting its level of neutral buoyancy. The updrafts and cold anomaly tend to be enhanced on the downshear to downshear-left-hand side of the storm. The overshooting convection is surrounded by subsiding warm air. The direction of the environmental vertical shear and the region of enhanced updrafts as well subsequently undergo counterclockwise rotation, roughly at a rate of 5° h^{-1} (not shown).

In the lower part, an annular zone of local θ minima is found just outside the core region updrafts (Figs. 14d,e). On the other hand, downward motions and associated warming are prevalent in the outer region, especially in the northwest quadrant of the storm. At this level, a band of upward motions extending southward from the east side of the core region updrafts is evident (Fig. 14d). During the mature stage, the storm is situated in the upper-tropospheric sheared transition zone between the tropical easterly and the midlatitude westerly. The band of upward motions is formed between the southward-directed outflow and the environmental easterly and is not identified in the middle and lower troposphere (not shown).

The vectors superimposed on Fig. 14 represent local vertical shear vectors at the level of the cross section. The reduction of static stability by the differential horizontal advection of θ gradients occurs when the shear vectors point in the same direction as the θ gradients. The frequency of $N^2 < 0$ in the

upper shear layer (Fig. 14b) is large in regions where vertical shear vectors point toward warmer air in Fig. 14b, while data points with $N^2 < 0$ are nearly absent in regions where shear vectors point toward colder air or are nearly parallel to isentropes. This is also true for the frequency of $N^2 < 0$ in the lower shear layer (Fig. 14f), which shows large frequency in regions of large positive radial θ gradients in Fig. 14e. These results strongly suggest that the low-static-stability layers supporting the shallow convection are created mainly by the differential horizontal advection of θ .

2) THE BANDING STAGE

The outflow structure for the banding stage of the storm is depicted in radius–height sections (Fig. 15) and horizontal sections (Fig. 16). The fields in Fig. 15 are azimuthally averaged over the 30° -wide range between 100° and 130° . As previously noted in Fig. 7b, the air in the lower part of the outflow ascends slantwise over the tilted θ_e surfaces associated with the baroclinic zone (Figs. 15a–c). The outflow layer is filled with clouds (Fig. 15d), making the banded updrafts of preexisting shallow convection visible as cirrus bands in the T_b field (cf. Figs. 5d,e). Since the moisture in the outflow layer is larger than that during the mature stage, the θ_e profile is different than that of θ in the lower part of the outflow layer and exhibits a moist absolutely unstable layer (Bryan and Fritsch 2000; Bryan et al. 2007) in which θ_e decreases with height in saturated air (Fig. 15b).

Unlike in the mature stage, upward motions are prevalent within the outflow. However, a 1-km-deep sloping layer of weak downward motions or reduced upward motions is present between 200- and 450-km radius and between 12- and 14-km height (Fig. 15c). As in the mature stage, this airflow transports potentially warm air into the outflow, thereby forming negative radial θ gradients on the right of the downdraft and positive θ gradients on the left of the downdraft. The distributions of averaged N^2 and percentage frequency of $N^2 < 0$ relative to the outflow are qualitatively similar to those in the mature stage (Figs. 15e,f). In this stage, N^2 in the lower part of the outflow below is considerably smaller than N_d^2 (not shown).

The w and θ fields in the upper and lower parts of the outflow layer in Fig. 16 exhibit spiraling patterns, with local minima (maxima) of θ located just outside (i.e., downstream side) of local maxima (minima) of w . In this stage, the enhanced updrafts and cold anomaly associated with the overshooting convection are evident to the north-northeast of the storm center at $z = 13.5$ km (Figs. 16a,b). Just outside the overshooting convection, a zone of weak downward motions noted in Fig. 15c is present. In the outer region on the north side of the storm, a band of cold (warm) anomaly is evident in the upper (lower) part of the outflow layer (Figs. 16b,e). As in the mature stage, the percentage frequency of $N^2 < 0$ is large in regions of negative (positive) radial θ gradients in the upper (lower) part of the outflow layer (Figs. 16c,f).

5. Thermodynamic and static stability budgets

The analysis of the outflow structure suggests that the low-static-stability layers associated with the outflow are forced by the differential thermal advection. In this section, physical

FIG. 14. Horizontal cross sections of temporally averaged fields in the (top) upper and (bottom) lower parts of the outflow layer for the mature stage. (left) Vertical velocity at (a) $z = 16.0$ km and (d) $z = 13.0$ km (shaded, cm s^{-1}), (center) potential temperature at (b) $z = 16.0$ km and (e) $z = 13.0$ km (shaded and contoured every 0.5 K), and (right) percentage frequency of averaged data points with $N^2 < 0$ (shaded) within the 2-km-deep shear layers (c) above and (f) below the level of maximum storm-relative outflow. The region within 150-km radius from the storm center, where the level of maximum storm-relative outflow is not defined, is masked white in (c) and (f). Vectors in (b) and (c) represent vertical shear vectors between $z = 15.5$ and 16.5 km. Vectors in (e) and (f) represent vertical shear vectors between $z = 12.5$ and 13.5 km. All fields are averaged in a vortex-following sense over the 4-h period from 2000 UTC 13 Sep to 0000 UTC 14 Sep 2017. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. The bold arrow and adjacent number in the bottom left of (a) provide vertical wind shear direction and magnitude between 850 and 200 hPa averaged over the circle of 300-km radius.

processes that determine the stratification of the outflow are investigated in detail through analyses of thermodynamic and static stability budgets.

As mentioned previously, at the outflow level, the difference between θ and θ_e and the effects of moisture on the static stability are small except in the lower part of the outflow layer during the banding stage. Thus, for simplicity, budgets of θ and the dry static stability $\partial\theta/\partial z$ are examined here. The procedure for the calculation of the budget terms is similar to those described in [Kepert et al. \(2016\)](#) and [Duran and Molinari \(2019\)](#). All budget terms are calculated using the data interpolated onto a cylindrical coordinate system. The budget equation for θ can be written as

$$\frac{\partial\theta}{\partial t} = \text{ADVT} + \text{MP} + \text{RAD} + \text{DIFFT}. \quad (2)$$

In this equation, ADVT represents the advection of θ given by

$$\text{ADVT} = -v_r \frac{\partial\theta}{\partial r} - \frac{v_\theta}{r} \frac{\partial\theta}{\partial \lambda} - w \frac{\partial\theta}{\partial z}. \quad (3)$$

MP and RAD represent diabatic heating from microphysics and radiation schemes, respectively. DIFFT represents the subgrid-scale diffusion, which is the sum of vertical diffusion obtained from the planetary boundary layer scheme and horizontal diffusion obtained from the first-order Smagorinsky scheme. The budget equation for $\partial\theta/\partial z$ can be obtained by differentiating (2) with respect to z :

$$\frac{\partial}{\partial t} \frac{\partial\theta}{\partial z} = \text{ADVT}_z + \text{MP}_z + \text{RAD}_z + \text{DIFFT}_z, \quad (4)$$

where the subscript z denotes the partial differentiation of θ budget terms with respect to z . All budget terms are averaged over the same time periods and azimuthal ranges adopted in the analysis of outflow structure in the previous section.

Figure 17 compares the θ tendencies derived from the actual model change in θ and those derived from the sum of budget terms on the right-hand side of Eq. (2) for both the mature and banding stages. The tendencies derived from the budget terms (Figs. 17c,d) agree well with those derived from the actual

FIG. 15. As in Fig. 13, but for the temporally and azimuthally averaged fields for the banding stage. All fields are averaged in a vortex-following sense over the 4-h period from 2000 UTC 14 Sep to 0000 UTC 15 Sep 2017 and the 30°-wide azimuthal range between 100° and 130°. (b) θ_e is contoured every 2 K by thin solid contours for $\theta_e \leq 348$ and $\theta_e \geq 350$ K, while θ_e is contoured every 0.5 K by thin dashed contours for $348.5 \leq \theta_e \leq 349.5$ K. Regions of moist absolute instability are enclosed by green lines in (b).

model change in θ (Figs. 17a,b) for both the mature and banding stages. Although nonnegligible differences are recognized (Figs. 17e,f) in regions of large spatial gradient of θ tendency, the magnitudes of the differences are less than 10% of the magnitudes of adjacent maxima or minima of the θ tendency. Thus, the budget calculations seem sufficiently accurate and can be used to determine the relative importance of the various terms in the maintenance and evolution of the low-static-stability layers.

a. The mature stage

Figure 18 shows radius–height sections of θ and $\partial\theta/\partial z$ budget terms for the mature stage. The advective θ tendency exhibits alternating layers of positive and negative values (Fig. 18a). The downward transport of the potentially warm air into the outflow noted in Fig. 13 is represented by a layer of warm advection extending from the tropopause level at about 150-km radius to 550-km radius, above which cold advection occurs by the inward radial flow. As suggested previously, this differential θ advection creates a negative $\partial\theta/\partial z$ tendency at the top of the outflow (Fig. 18b). A downward-sloping layer of cold advection beneath the warm advection layer ($100 < r < 380$ km)

represents the outward-directed branch of the subsiding airflow from the top of overshooting convection. This cold advection and the warm advection above provide forcing for stabilization, creating the high-static-stability layer within the outflow (cf. Fig. 13e). The lower part of the outflow is dominated by cold advection outside of 250-km radius, and warm advection dominates beneath the outflow. The differential θ advection acts to destabilize an extensive layer near the bottom of the outflow.

The θ and $\partial\theta/\partial z$ tendencies from MP are large in magnitude in the core-region updrafts (Figs. 18c,d) and largely cancel the advective tendencies there (Figs. 18a,b). The θ tendency from RAD exhibits a downward-sloping layer of cooling near the top of the cloud within 250-km radius (Fig. 18e). The diabatic warming due to the absorption of shortwave occurs in this layer during the daytime, but its magnitude is at most about 50% of the longwave cooling (not shown). The net radiative cooling largely balances with the warm advection between 100- and 250-km radius and between the 14- and 15.5-km height. In the outflow layer outside of 250-km radius, the θ and $\partial\theta/\partial z$ tendencies from microphysical and radiative diabatic heating are much smaller than those from θ advection.

FIG. 16. Horizontal cross sections of temporally averaged fields in the (top) upper and (bottom) lower parts of the outflow layer for the banding stage. (left) Vertical velocity at (a) $z = 13.5$ km and (d) $z = 10.5$ km (shaded, cm s^{-1}), (center) potential temperature at (b) $z = 13.5$ km and (e) $z = 10.5$ km (shaded and contoured every 0.5 K), and (right) percentage frequency of averaged data points with $N^2 < 0$ (shaded) within the 2-km-deep outflow shear layers (c) above and (f) below the level of maximum outflow. The region within 150-km radius from the storm center, where the level of maximum storm-relative outflow is not defined, is masked white in (c) and (f). Vectors in (b) and (c) represent vertical shear vectors between $z = 13.0$ and 14.0 km. Vectors in (e) and (f) represent vertical shear vectors between $z = 10.0$ and 11.0 km. All fields are temporally averaged in a vortex-following sense over the 4-h period from 2000 UTC 14 Sep to 0000 UTC 15 Sep 2017. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. The bold arrow and adjacent number in the bottom left of (a) provide vertical wind shear direction and magnitude between 850 and 200 hPa averaged over the circle of 300-km radius.

The θ and $\partial\theta/\partial z$ tendencies from subgrid-scale diffusion exhibit alternating layers of positive and negative values (Figs. 18g,h). Positive $\partial\theta/\partial z$ tendencies from diffusion occur in the low-static-stability layers, while negative $\partial\theta/\partial z$ tendencies occur immediately above and below the low-static-stability layers (Figs. 18h). Within 300-km radius and near the tropopause level, the tendencies from diffusion largely offset those from θ advection. The relationship between the advective and diffusive stability tendencies in the core region is consistent with that found by Duran and Molinari (2019, their Figs. 6 and 7).

To examine the contributions of the shallow convection in the θ and $\partial\theta/\partial z$ budgets, tendencies from the convergence of eddy heat fluxes are also calculated and shown in Figs. 18i and 18j. The convergence of eddy heat fluxes (HFC) is given by

$$\text{HFC} = -\frac{1}{r} \frac{\partial(rv_r'\theta'')}{\partial r} - \frac{1}{r} \frac{\partial(v_r'\theta'')}{\partial \lambda} - \frac{\partial(w_r'\theta'')}{\partial z}, \quad (5)$$

where the double prime indicates the deviation from an azimuthally smoothed profile that is obtained by applying a

10° -wide running mean. In the outflow region, the third term on the right-hand side of (5) (i.e., vertical heat flux convergence) is one order of magnitude greater than other two terms (not shown). Note that HFC is implicitly included in ADVT, though its magnitude is much smaller than that of ADVT. Since the shallow convection transports heat upward, HFC shows cooling (warming) at the bottom (top) of the low- N^2 layers (Fig. 18i), and HFC_z shows stabilizing tendencies in those layers (Fig. 18j). The tendencies from HFC are similar in pattern to those from DIFFT, though their magnitudes are much smaller in the upper part of the outflow layer.

b. The banding stage

Figure 19 shows the budget terms for the northern sector of the storm for the banding stage. As in the mature stage, a thin layer of warm advection occurs in the upper part of the outflow layer between 260- and 580-km radius (Fig. 19a). Because of the dominance of upward motions above this layer

FIG. 17. Radius–height cross sections of (top) θ tendencies derived from the actual model change in θ and (middle) θ tendencies derived from the sums of the budget terms on the right-hand side of Eq. (2), and (bottom) the difference of θ tendencies computed by subtracting the tendencies in the middle panels from those in the top panels for the (left) mature and (right) banding stages (shaded, 10^{-5} K s^{-1}). Thin black contours in each panel represent storm-relative radial velocity at 10 m s^{-1} intervals, with positive values solid and negative values dashed. Thick black line in each panel represents the cold-point tropopause height. All fields in the left (right) column are averaged over the same time period and azimuthal range as employed in Fig. 13 (Fig. 15).

(cf. Fig. 15c), negative θ tendency occurs above this warm advection layer. Thus, the θ advection provides forcing for destabilization in an extensive layer near the top of the outflow (Figs. 19b).

The tendencies from microphysical diabatic heating (Figs. 19c,d) are small in the upper part of the outflow layer compared to those from θ advection. In the lower part of the outflow, diabatic warming due to vapor deposition occurs as the outflow air ascends slantwise over the tilted θ_e surfaces (cf. Fig. 15b) and largely offsets the tendencies from advection. Beneath this layer, a 1-km-deep layer of sublimation cooling occurs (Fig. 19c). The sublimation cooling beneath cloud base decreases (increases) the static stability beneath (above) it (e.g., Luce et al. 2010; Kudo 2013; Molinari et al. 2014). In the present case, the sublimation cooling occurs in a high-static-stability layer at the vertical edge of the baroclinic zone (cf. Figs. 15b). Thus, the cooling does not contribute to the reduction of static stability in the low- N^2 layer (Fig. 19d).

Because ADVT is largely offset by MP in the lower part of the outflow layer, the sum of ADVT and MP and its contribution to the $\partial\theta/\partial z$ budget are also examined (Figs. 19e,f). The sum of ADVT and MP shows a thin upward-sloping layer of cooling near the bottom of the outflow, providing a forcing for destabilization in the low- N^2 layer between 450- and 600-km radius. Since ADVT + MP is approximately the same as the advection of θ_e in the saturated layer, this process represents the reduction of $\partial\theta_e/\partial z$ through differential advection of θ_e within the slantwise ascent over the baroclinic zone, which creates the moist absolutely unstable layer noted in Fig. 15b.

During the banding stage, substantial longwave warming occurs at the bottom of the outflow cloud layer, above which net radiative cooling occurs (Fig. 19g). The net radiative cooling provides weak stabilizing tendency in the low- N^2 layer at the top of the outflow (Fig. 19h). On the other hand, the cloud base radiative warming and cooling above provides

FIG. 18. Radius–height cross sections of (left) θ budget terms (shaded, 10^{-5} K s^{-1}) and (right) $\partial\theta/\partial z$ budget terms (shaded, $10^{-8} \text{ K m}^{-1} \text{ s}^{-1}$) for the mature stage. (a) ADVT, (b) ADVT $_z$, (c) MP, (d) MP $_z$, (e) RAD, (f) RAD $_z$, (g) DIFFT, and (h) DIFFT $_z$. (i) convergence of heat flux associated with the resolved, small-scale disturbances (HFC), and (j) HFC $_z$. Thin black contours in each panel represent storm-relative radial velocity at 10 m s^{-1} intervals, with positive values solid and negative values dashed. Red contours in the right panels enclose regions where the averaged values of N^2 are less than $5 \times 10^{-5} \text{ s}^{-2}$. Thick black line in each panel represents the cold-point tropopause height. All fields are averaged over the same time period and azimuthal range as employed in Fig. 13.

FIG. 19. Radius–height cross sections of (left) θ budget terms (shaded, 10^{-5} K s^{-1}) and (right) $\partial\theta/\partial z$ budget terms (shaded, $10^{-8} \text{ K m}^{-1} \text{ s}^{-1}$) for the banding stage. (a) ADVT, (b) ADVT_z, (c) MP, (d) MP_z, (e) ADVT + MP, (f) ADVT_z + MP_z, (g) RAD, (h) RAD_z, (i) DIFFT, and (j) DIFFT_z. Thin black contours in each panel represent storm-relative radial velocity at 10 m s^{-1} intervals, with positive values solid and negative values dashed. Red contours in right panels enclose regions where the averaged values of N^2 are less than $3 \times 10^{-5} \text{ s}^{-2}$. Thick black line in each panel represents the cold-point tropopause height. The budget terms are averaged over the same time period and azimuthal range as employed in Fig. 15.

forcing for destabilization in the low- N^2 layer near the bottom of the outflow. The tendencies from diffusion generally act to offset those from ADVT + MP, though the tendencies are smaller in magnitude than those in the mature stage (Fig. 19i,j). The tendencies from HFC are qualitatively similar to those from diffusion, but their magnitudes are much smaller (not shown).

Previous numerical studies of cirrus banding showed that cloud-radiative forcing had a positive effect on the cirrus band formation (Trier et al. 2010; Kim et al. 2014; Trier and Sharman 2016). To assess the effect of cloud-radiative feedbacks (CRF) deactivated (simulation CRF-off). Figure 20 shows the signed column maximum values of the radial velocity perturbation $v'_{r,max}$ for the upper and lower shear layers at 0100 UTC 15 September for CRF-off. As in the control simulation (cf. Fig. 10e), stripes of positive and negative values associated with the shallow convection are recognized in the upper shear layer (Fig. 20a). However, the radial extent of shallow convection is smaller than that in the control simulation. This is because the outflow in CRF-off was weaker and less extensive than that in the control simulation (not shown), consistent with the results of Fovell et al. (2010) and Bu et al. (2014). On the other hand, signatures of the shallow convection are nearly absent in the lower shear layer (Fig. 20b, cf. Fig. 11e). This indicates that the cloud-radiative forcing during the banding stage promotes the shallow convection in the lower-shear layer, as suggested by the results of budget analysis.

6. Dynamics involved in the formation of low-static-stability layer

The analyses of thermodynamic and stability budgets indicate that the low-static-stability layers are created by the differential thermal advection. In Figs. 13 and 14, the vertical velocity fields in the upper and lower parts of the outflow layer show similar patterns with opposite signs, suggesting that the vertical motions and associated radial thermal gradients form in response to the divergence and convergence in the outflow. In this section, the dynamics involved in the destabilization of the outflow layer are addressed through the analysis of a radial momentum budget.

Following Zhang et al. (2001), the momentum equation for the ground-relative radial velocity can be written as

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_t}{r} \frac{\partial V_r}{\partial \lambda} + w \frac{\partial V_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + fV_t + \frac{V_t^2}{r} + D_r, \quad (6)$$

where p is the pressure and f is the Coriolis parameter. The first three terms on the right-hand side of (6) represent the pressure gradient, Coriolis, and centrifugal forces, respectively. Here, the effects of vertical motion on the Coriolis force are neglected. The gradient wind balance is defined by the sum of these terms equaling zero. The last term represents the tendency due to parameterized turbulence. When the temporal and azimuthal variations are negligible, the left-hand side of

FIG. 20. Signed column maximum values of the radial velocity perturbation (shaded, m s^{-1}) within the 2-km-deep outflow shear layers (a) above and (b) below the level of maximum outflow at 0100 UTC 15 Sep 2017 for CRF-off. The region within 150-km radius from the storm center is masked white. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. Thick black lines indicate locations where smoothed value of the maximum storm-relative radial velocity ($\bar{v}_{r,max}$) is zero.

(6) reduces to the inner product of the wind vector $\mathbf{V} = (V_r, w)$ and the V_r gradient vector $\nabla V_r = (\partial V_r / \partial r, \partial V_r / \partial z)$. In that case, (6) states that $\mathbf{V}$ and ∇V_r make an angle smaller than 90° if the net radial force is positive, which means that mass transport into the outflow will occur across the contours of V_r . Similarly,

FIG. 21. Horizontal cross sections of (a) net radial force (shaded, 10^{-3} m s^{-2}) and storm-relative streamlines, (b) radial pressure gradient force (shaded, 10^{-3} m s^{-2}) and pressure (thin black contours at 0.1-hPa intervals for values greater than 133.5 hPa, with the 134.2-hPa contour highlighted by the thick contour), (c) sum of the Coriolis and centrifugal forces (shaded, 10^{-3} m s^{-2}), and (d) horizontal divergence (shaded, 10^{-5} s^{-1}) for the mature stage of the storm. The horizontal cross sections are taken at $z = 15.0 \text{ km}$, which is approximately the level at which the outflow is maximized. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. All fields are averaged over the same time period as employed in Fig. 14.

mass transport out of the outflow will occur across the V_r contours if the net radial force is negative.

Terms on the right-hand side of (6) and horizontal divergence are averaged in the same manner as in the thermodynamic budget analysis and are depicted in horizontal sections (Fig. 21) and radius–height sections (Fig. 22) for the mature stage. Although not shown, D_r is large in the upper outflow shear layer and acts to reduce the strong vertical gradient of the radial wind there.

Previous studies have shown that supergradient flows and outward acceleration of the outflow occur in the core region through the upward transport of absolute angular momentum by eyewall updrafts (e.g., Zhang et al. 2001; Bryan and Rotunno 2009; Wang et al. 2020). This is also true for the outflow in the present case. The upward transport of the absolute angular momentum by the eyewall updrafts is sufficiently strong and the airflow outside the eyewall becomes inertially unstable (not

shown). The sum of Coriolis and centrifugal forces (Figs. 21c and 22c) overwhelms the pressure gradient force that is generally directed inward (Figs. 21b and 22b). Thus, in the core region ($r < 300 \text{ km}$), the net radial force (i.e., a gradient force) becomes positive in regions of strong updrafts (cf. Fig. 13c) and accelerates the air parcels outward (Figs. 21a and 22a). Note that the pressure gradient force is locally directed outward in regions of strong net outward force. These regions are in a state of gradient wind “nonbalance” termed by Cohen et al. (2017), in which the sum of the first three right-hand-side terms in (6) becomes positive for any V_t because of a sufficiently strong outward pressure gradient force and a small enough radius. The high pressure responsible for the outward pressure gradient force is induced hydrostatically by the cold anomaly at the top of overshooting convection (cf. Fig. 14b).

The roughly axisymmetric outward parcel acceleration induces significant mass divergence in the core region

FIG. 22. Radius–height cross sections of (a) net radial force (shaded, 10^{-3} m s^{-2}), (b) radial pressure gradient force (shaded, 10^{-3} m s^{-2}), (c) sum of the Coriolis and centrifugal forces (shaded, 10^{-3} m s^{-2}), and (d) horizontal divergence (shaded, 10^{-3} s^{-1}) for the mature stage of the storm. Thin black contours in each panel represent ground-relative radial velocity at 10 m s^{-1} intervals, with positive values solid and negative values dashed. All fields are averaged over the same time period and azimuthal range as employed in Fig. 13.

(Figs. 21d and 22d). Although not shown, the first and third left-hand-side terms in (6) are smaller in magnitude by a factor of 5 or higher than the second and last left-hand-side terms. Thus, to satisfy mass continuity, vertical mass transport into the outflow occurs across the outflow shear layers. Outside the region of horizontal divergence (and within the Rossby radius of deformation), horizontal convergence naturally dominates by the effect of Coriolis force acting on the anticyclonic outflow. On the western side of the storm, the inward-directed pressure gradient force between the 200- and 500-km radius also contributes to the convergence within the outflow layer. These forces induce vertical mass transport out of the outflow in the outer region. The vertical mass transport into and out of the outflow tilts the isentropes on the vertical edges of the outflow upshear, creating a destabilizing tendency due to the differential advection of thermal gradients.

The net inward radial force and horizontal convergence are relatively strong on the west and northwest flanks of the storm, probably because the upper-level environmental westerlies on the north side of the storm oppose the outflow there. The associated upward displacements of isentropes above the convergence zone forms a zone of cold anomaly in the upper outflow layer (cf. Figs. 14b), while downward displacements of isentropes below the convergence zone forms a zone of warm anomaly in the lower outflow layer (cf. Figs. 14e). These potential temperature anomalies hydrostatically increase the pressure between them, forming a pressure ridge at the level of maximum outflow (Fig. 21b).

Figure 23 shows horizontal sections of terms on the right-hand side of (6) and horizontal divergence for the banding stage. As in the mature stage, net outward force occurs in regions where the pressure gradient force is directed outward, or

the inward pressure gradient force is locally weak (Figs. 23a,b). The cold anomaly associated with the overshooting convection to the north-northeast of the storm center (cf. Fig. 16b) increases the pressure at the outflow level (see the 231.4-hPa contour), creating the strong outward-directed pressure gradient force there.

In this stage, the outflow of the storm is mostly embedded in the westerlies associated with the midlatitude jet (not shown), which appear to exert significant influence on the destabilization of the outflow. The outflow on the west and northwest flanks of the storm gives way to anticyclonic rotational flow rapidly with radius (Fig. 23a) and forms a narrow spiraling zone of enhanced horizontal convergence (Fig. 23d). The vertical displacements of isentropes along the convergence zone (cf. Figs. 16b,e) induce a well-defined pressure ridge (Fig. 23b) that contributes to the radial deceleration of the outflow.

Knox et al. (2010) noted that a common characteristic of the environment of cirrus banding events in a wide variety of weather systems is strongly anticyclonic flow. Previous numerical studies of cirrus banding noted the existence of inertial instability associated with strong anticyclonic flows in the vicinity of the banding regions (Kim et al. 2014; Trier and Sharman 2016). Molinari et al. (2019) made a case study of widespread turbulence in Hurricane Ivan (2004) and showed that interactions between the hurricane outflow channel and westerlies to the north created a region of largely negative absolute vorticity. In their case, the outflow accelerated from the storm center into the inertially unstable region, and cirrus bands appeared radially inward of the inertially unstable region.

To examine the relationship between the regions of inertial instability and cirrus banding, Fig. 24 shows temporally averaged values of absolute angular momentum $M = rV_t + fr^2/2$

FIG. 23. Horizontal cross sections of (a) net radial force (shaded, 10^{-3} m s^{-2}) and storm-relative streamlines, (b) radial pressure gradient force (shaded, 10^{-3} m s^{-2}) and pressure (thin black contours at 0.2-hPa intervals for values greater than 229.6 hPa, with the 231.4-hPa contour highlighted by the thick contour), (c) sum of the Coriolis and centrifugal forces (shaded, 10^{-3} m s^{-2}), and (d) horizontal divergence (shaded, 10^{-5} s^{-1}) for the banding stage of the storm. The horizontal cross sections are taken at $z = 11.5 \text{ km}$, which is approximately the level at which the outflow is maximized. The storm center is indicated by a cross mark. Dashed circles represent the 300- and 600-km range rings. All fields are averaged over the same time period as employed in Fig. 16.

and absolute vertical vorticity $\zeta_a = f + \partial v / \partial x - \partial u / \partial y$ for the mature and banding stages of the storm. Note that ζ_a is contoured only for regions of inertial instability that is defined as $\zeta_a < 0$. Negative values of ζ_a are broadly found in the relatively strong, northward-directed outflow for both the mature and banding stages. As mentioned earlier, inertial instability is created by the upward transport of large angular momentum by the eyewall updrafts, but the instability near the eyewall is released immediately and continuously by the outward acceleration of the outflow. Thus, regions of negative ζ_a values are found outside the regions of outward parcel acceleration (cf. Figs. 21a and 23a). Large negative ζ_a are also found near the northern boundary of the negative ζ_a region in the northern sector of the storm for both the mature and banding stages. The spatial distribution of negative ζ_a is similar to that found by Molinari et al. (2019, their Fig. 10). However, unlike the case examined by Molinari et al. (2019), this inertially unstable

region is persistently present and is not relaxed to a neutral or stable state during the integration period of the 2-km grid domain examined here (not shown).

As can be seen from the pressure field in Figs. 21b and 23b, the azimuthal pressure gradient force in the northern sector of the storm is negative (i.e., clockwise). Thus, the enhancement of anticyclonic flow (i.e., the decrease of V_θ) and the decrease of M occur in the northern sector, forming a zone of negative ζ_a . This means that the region of strong inertial instability in the outer region in Fig. 24 should be regarded as a result of the confluence between the outflow and the environmental westerlies, rather than a cause for the outward acceleration of the outflow and/or the generation of cirrus bands.

In summary, the strong outflow resulting from the violation of gradient-wind balance, by itself, destabilizes the strongly sheared layers associated with the outflow by inducing radial thermal gradients in these layers. The results above also indicate that the

FIG. 24. Horizontal cross sections of absolute angular momentum M (shaded and contoured every $2 \times 10^6 \text{ m}^2 \text{ s}^{-1}$) and absolute vertical vorticity ζ_a (contoured at -5 , -2.5 , and $0 \times 10^{-5} \text{ s}^{-1}$ with thick, medium, and thin solid black lines, respectively) for (a) the mature stage ($z = 15.0 \text{ km}$) and (b) the banding stage ($z = 11.5 \text{ km}$). Note that only zero and negative values of ζ_a are contoured to avoid dense contours near the storm center. Dashed circles represent the 300- and 600-km range rings. All fields in (a) and (b) are averaged over the same 4-h time periods as employed in Figs. 14 and 16, respectively.

thermal gradients and reduction of static stability should be enhanced where the radial outflow rapidly gives way to strong anticyclonic flow and may provide an explanation for the close relationship between cirrus bands and strongly anticyclonic flows in a wide variety of weather systems.

7. Summary and conclusions

Pronounced cirrus banding was observed in the outflow cloud shield of Typhoon Talim (2017) during its recurvature. In this study, the generation mechanisms of cirrus bands and low-static-stability layers supporting these bands are investigated using the ARW-WRF Model. The timing, position, and orientation of the simulated cirrus bands are consistent with those in satellite images. Although the widespread cirrus banding was episodic, low-static-stability layers are persistently present in the upper and lower parts of the TC outflow, and shallow convection aligned along the vertical shear vector is prevalent in these layers. When the simulated storm is intense, the upper part of the outflow layer is subsaturated and the shallow convection is only locally identified as cirrus bands in the simulated T_b field. The widespread banding occurs as the lowered outflow from the weakening storm ascends slantwise over the midlatitude baroclinic zone and banded updrafts of the preexisting shallow convection in the upper part of the outflow layer become saturated. It is also shown that gravity waves that radiate outward from the storm are trapped beneath the outflow layer and cause mesoscale radial variations of cloud ice mixing ratio of the cirrus bands. These gravity waves, however, do not appear to be essential to the destabilization of outflow layer and the generation of shallow convection.

It is shown that the strong outflow resulting from the violation of gradient-wind balance in the core region dynamically

induces the low-static-stability layers by itself. These layers are created mainly by the differential advection of radial thermal gradients on the vertical edges of the outflow where the vertical shear is strong. The radial thermal gradients occur in response to the outward air parcel acceleration in the core region and deceleration in the outer region, which, by inducing compensating vertical mass transport into and out of the outflow, act to tilt the isentrope's upshear. The upper-level environmental westerlies appear to promote regions of cirrus bands on the west and northwest flanks of the storm during the banding stage, by enhancing horizontal convergence in the outflow and associated thermal gradients. The parameterized turbulence and the shallow convection act to offset the destabilization tendency due to differential advection. During the banding stage, cloud-base longwave warming and in-cloud longwave cooling also contribute to the generation of shallow convection near the bottom of the outflow.

The shallow convection identified in this study may have important roles in the vertical mixing of heat and momentum in the TC outflow layer. In the present simulation, however, the mixing was accomplished largely by the parameterized turbulence. As in recent numerical studies of TC boundary layers (e.g., Nakanishi and Niino 2012; Ito et al. 2017), small-scale disturbances associated with the TC outflow should be investigated more precisely by using large-eddy simulations in which large energy-containing turbulent motions are explicitly resolved with a fine model resolution. Future research should also address the effects that the organized shallow convection has on the structure and intensity of TCs through the modulation of outflow intensity and stratification.

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