The Circulation Response of a Two-Dimensional Frontogenetic Model to Optimized Moisture Perturbations

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ABSTRACT: An analysis of the influence and sensitivity of moisture in an idealized two-dimensional moist semigeostrophic frontogenesis model is presented. A comparison between a dry (relative humidity RH = 0%) version and a moist (RH = 80%) version of the model demonstrates that the impact of moisture is to increase frontogenesis, strengthen the transverse circulation (u_{ag}, w), generate a low-level potential-vorticity anomaly, and develop a low-level jet. The idealized model is compared with a real case simulated with the full-physics three-dimensional Coupled Ocean–Atmosphere Mesoscale Prediction System (COAMPS) model, establishing good agreement and thereby confirming that the idealized model retains the essential physical processes relevant for improving understanding of midlatitude frontogenesis. Optimal perturbations of mixing ratio are calculated to quantify the circulation response of the model through the computation of singular vectors, which determines the fastest-growing modes of a linearized version of the idealized model. The vertical velocity is found to respond strongly to initial-condition mixing-ratio perturbations such that small changes in moisture lead to large changes in the ascent. The progression of physical processes responsible for this nonlinear growth is (in order) jet/front transverse circulation → moisture convergence ahead of the front → latent heating at mid- to low elevations → reduction in static stability ahead of the front → strengthening of the transverse circulation, and the feedback cycle repeats. Together, these physical processes represent a pathway by which small perturbations of moisture can have a strong impact on a forecast involving midlatitude frontogenesis.

KEYWORDS: Ageostrophic circulations; Frontogenesis/frontolysis; Mesoscale processes; Nonlinear dynamics; Jets; Singular vectors

1. Introduction

Improvement in prediction of atmospheric rivers (AR) is important to water-resource managers in many locations, perhaps most prominently California because of the state's large population and its dependence on the arrival, or lack thereof, of just a handful of precipitation events per year (Dettinger 2013; Ralph et al. 2013; Dettinger and Cayan 2014; Ralph 2017; Lamjiri et al. 2017). DeFlorio et al. (2018) performed an assessment of global landfalling ARs at a 2-day lead time, finding that fewer than 75% of the European Centre for Medium-Range Weather Forecasts (ECMWF) ensemble members predicted AR landfall within 250 km of the actual location. This error provides motivation to work toward improving

not only the magnitude of the water-vapor content in the AR, but also its position. Improvements in AR prediction can be subdivided into two broad categories, improving (i) numerical-prediction systems, and (ii) the data used to initialize the prediction systems; this study is concerned with category ii.

Advances in research on initial conditions are being made through the AR Reconnaissance (AR-Recon) program, which targets ARs using airborne instruments with emphasis on in situ dropsondes (Ralph et al. 2020). Choosing optimal aircraft tracks and dropsonde locations, however, is difficult because it is not always clear where additional observations are most likely to improve the analyses and subsequent forecasts. Initial-condition sensitivity analyses provide guidance toward designing the flight tracks by objectively determining where small errors can grow the largest for each storm individually.

Reynolds et al. (2019) used the Naval Research Laboratories (NRL) moist adjoint modeling system to evaluate the sensitivity of AR forecasts from January to February 2017. They found that the accumulated precipitation forecasts were most sensitive to errors in and around the AR with the largest sensitivities being associated with the moisture field followed by temperature and winds. These results are consistent with the results of Doyle et al. (2014, 2019) for Atlantic Ocean extratropical cyclones that, also using the NRL moist-adjoint tool, demonstrated that perturbations in regions having high

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moisture sensitivity lead to a coupling of the upper- and lower-level potential vorticity (PV) through a process known as PV unshielding. These studies were followed by [Demirdjian et al. \(2020a\)](#), which investigated other dynamical mechanisms leading to the growth of moisture perturbations using the NRL moist-adjoint system for a strong landfalling Pacific Ocean AR. The optimal perturbations in this case led to a substantial growth in the low-level latent heating that (i) amplified the diabatically driven low-level PV anomaly leading to the formation of a low-level jet (LLJ), and (ii) intensified the jet/front transverse circulation leading to substantially stronger updrafts. These dynamical interpretations of the sensitivity fields are consistent with [Cannon et al. \(2020\)](#), who investigated the “importance of frontally forced precipitation on atmospheric heating tendencies.” The adjoint tool provides a reliable method to quantitatively determine the forecast sensitivities, but it is left to the forecaster and researcher to interpret the dynamical origins and impacts of these sensitivity fields, especially if they are to be used to inform observing strategies and design flight-planning operations.

While sensitivity tools do provide guidance toward observing strategies, the impacts of the observations can only be quantified after the event. [Stone et al. \(2020\)](#) examined the impact of assimilating dropsonde observations into the Navy Global Environmental Model (NAVGEM) finding that the dropsondes had an overall relatively large beneficial impact with the per dropsonde impact being more than double that of the North American radiosonde network. [Zheng et al. \(2021\)](#) documented the existing observational data gaps within ARs finding that, due to cloud cover, much of the satellite data measured for ARs are not used, which may increase the impacts of dropsondes consistent with the findings of [Stone et al. \(2020\)](#). The present work complements these studies by focusing on the basic understanding of dynamical mechanisms that govern the growth of initial-state perturbations, which has an important influence on later-stage error growth.

The goals of the present study are to (i) investigate the dynamics responsible for moist-perturbation growth, and (ii) quantify the relation between the moisture-perturbation amplitude and the strength of the circulation response. While other studies have looked at the dynamical mechanisms for perturbation growth associated with ARs, this study is unique because it uses an idealized two-dimensional model that simplifies the physical processes and makes the interpretation of the moisture-perturbation impacts to the system dynamics much more transparent. Furthermore, the method of optimal perturbations explored here makes it possible to isolate the impacts of the moisture perturbations. An increased understanding of the dynamics associated with the growth of moist perturbations in a simplified system will lead to a more complete understanding of AR dynamics and forecast challenges.

2. Methods

a. Frontal model

This study explores a variant of the idealized semi-geostrophic two-dimensional frontogenetic confluence model developed by [Hoskins and Bretherton \(1972\)](#) and used later in

[Hoskins \(1975\)](#), [Hoskins and West \(1979\)](#), [Hoskins \(1982\)](#), [Thorpe and Emanuel \(1985\)](#), and [Snyder et al. \(1991\)](#). The model is based on the geostrophic coordinate transformation described by

$$X = x + (V_g/f), \quad Z = z, \quad \text{and} \quad T = t, \quad (1)$$

where V_g is the geostrophic along-frontal wind component, and (x, z, t) are the Cartesian coordinates in physical space. The model was evolved forward in time by solving prognostic equations for the potential vorticity and mixing ratio given by

$$\frac{Dq}{DT} = \frac{\zeta}{\rho} \frac{\partial S}{\partial Z} \quad \text{and} \quad \frac{Dr}{DT} = -\frac{c_p}{L_v} S, \quad (2)$$

where q is the potential vorticity, ζ is the relative vorticity, ρ is the density, r is the mixing ratio, c_p is the dry specific heat capacity, L_v is the latent heat of water vaporization, S is the latent heating rate, and $D/DT = \partial/\partial t + U_g \partial/\partial X + w \partial/\partial Z$. After the evolution of these fields, diagnostic quantities were calculated using the geopotential-height and Sawyer–Eliassen equations given respectively by

$$\frac{\partial^2 \Phi}{\partial X^2} + \frac{\theta_0 f^3}{g \rho q} \frac{\partial^2 \Phi}{\partial Z^2} = f^2 \quad \text{and} \quad (3)$$

$$\rho \frac{\partial}{\partial Z} \left(\frac{1}{\rho} \frac{\partial \psi}{\partial Z} \right) + \frac{\partial}{\partial X} \left(\frac{\rho q g}{f^3 \theta_0} \frac{\partial \psi}{\partial X} \right) = -2Q - \frac{g \rho}{f^2 \theta_0} \frac{\partial S}{\partial X}, \quad (4)$$

where ψ is the ageostrophic streamfunction, g is the gravitational acceleration, f is the Coriolis parameter, θ_0 is the base-state potential temperature, Q is geostrophic forcing of frontogenesis, and Φ is the geopotential height. The remaining diagnostic quantities were calculated using the following relations:

$$(u_{ag}, w) = \left(\frac{1}{\rho} \frac{\partial \psi}{\partial Z}, -\frac{1}{\rho f} \frac{\partial \psi}{\partial X} \right), \quad (5)$$

$$\zeta = f \left(1 - \frac{1}{f} \frac{\partial V}{\partial X} \right)^{-1}, \quad (6)$$

$$V_g = \frac{1}{f} \frac{\partial \Phi}{\partial X}, \quad (7)$$

$$S = \frac{1}{2} (q - q_e) \left(\frac{\rho}{\zeta} w + \left| \frac{\rho}{\zeta} w \right| \right), \quad (8)$$

$$q_e = q + \frac{L_v \zeta}{c_p \rho} \frac{\partial r}{\partial Z}, \quad \text{and} \quad (9)$$

$$Q = \frac{\rho \alpha}{f} \frac{\partial V}{\partial Z}, \quad (10)$$

where u_{ag} is the ageostrophic across-front wind component, w is the vertical wind component, and q_e is the equivalent potential vorticity. [Thorpe and Emanuel \(1985\)](#) observed that the latent heating S is dependent on the solution of ψ and accounted for this by rearranging the Sawyer–Eliassen equation [Eq. (4)] as follows:

$$\rho \frac{\partial}{\partial Z} \left(\frac{1}{\rho} \frac{\partial \psi}{\partial Z} \right) + \frac{\partial}{\partial X} \left(\frac{\rho q g}{f^3 \theta_0} \frac{\partial \psi}{\partial X} \right) = -2Q, \quad (11)$$

where

$$\begin{aligned}\tilde{q} &= q_e & \text{if } w > 0 \\ \tilde{q} &= q & \text{if } w \leq 0.\end{aligned}$$

b. Numerical setup

The model is on an A grid extending from $-\pi L_R l^{-1}$ to $\pi L_R l^{-1}$ (approximately from -2500 to 2500 km) with 60 grid points (~ 80 -km grid interval) in X and from 0 to 10 000 m with 45 grid points (~ 200 -m grid interval) in Z where $L_R = NH/f$ is the Rossby radius of deformation and $l = 1.1358$ is a horizontal scaling factor. The prognostic equations were solved using a leapfrog integration with a 15-min time step and the Robert–Asselin filter (Robert 1966; Asselin 1972) applied to reduce numerical instabilities. The initialization parameters used are $\theta_0 = 292$ K, $N^2 = 10^{-4}$ (Brunt–Väisälä frequency), $\rho = 1.23$ kg m $^{-3}$, $f = 10^{-4}$, $\alpha = 10^{-5}$, a prescribed pressure field of $p = 1000e^{-0.75Z/H}$, and constant initial potential vorticity with $q_0 = fN^2\theta_0/(g\rho) \cong 2.42 \times 10^{-1}$ PVU (1 PVU $\equiv 10^{-6}$ m 2 s $^{-1}$ K kg $^{-1}$). A convergence check was performed by integrating the model with doubled resolution that gave similar results.

The boundary conditions for the Sawyer–Eliassen partial differential in Eq. (4) are simply $\psi = 0$ on all boundaries, physically meaning that $w = 0$ on the top and bottom boundaries and $u_{\text{ag}} = 0$ on the left and right boundaries. The boundary conditions for the geopotential-height partial differential are

$$\frac{d\phi}{dZ} = \frac{g}{\theta_0} \left[\Delta\theta_{\text{bot}} \sin(X_l l) + \frac{1}{2} X_l \Delta\theta_{\text{bot}} \right] \quad (12)$$

on the bottom,

$$\frac{d\phi}{dZ} = \frac{g}{\theta_0} \left[\Delta\theta_{\text{top}} \sin(X_l l) + \frac{1}{2} X_l \Delta\theta_{\text{top}} \right] + N^2 Z \quad (13)$$

on the top,

$$\phi = \frac{gH}{\theta_0} \left\{ \frac{1}{2} \frac{X}{L_R} \left[\Delta\theta_{\text{bot}} \frac{Z}{H} + \frac{1}{2} (\Delta\theta_{\text{top}} - \Delta\theta_{\text{bot}}) \left(\frac{Z}{H} \right)^2 \right] \right\} + \frac{1}{2} N^2 Z^2 \quad (14)$$

for the left boundary, and

$$\phi = \frac{gH}{\theta_0} \left\{ \frac{1}{2} \frac{X}{L_R} \left[\Delta\theta_{\text{bot}} \frac{Z}{H} + \frac{1}{2} (\Delta\theta_{\text{top}} - \Delta\theta_{\text{bot}}) \left(\frac{Z}{H} \right)^2 \right] \right\} + \frac{1}{2} N^2 Z^2 \quad (15)$$

for the right boundary. Here $X_l = X e^{\alpha T/L_R}$, $\Delta\theta_{\text{bot}} = 3$ K is the bottom-boundary potential-temperature gradient, $\Delta\theta_{\text{top}} = 6$ K is the top-boundary potential-temperature gradient, and $H = 10$ km. The method of successive overrelaxation was used to solve the Sawyer–Eliassen and geopotential height partial differential equations that were set to converge when the difference between each successive step ($x_{k+1} - x_k$) reached $\psi = 10^{-2}$ kg m $^{-1}$ s $^{-1}$ and $\phi = 10^{-8}$ m 2 s $^{-2}$, respectively. The model was also validated using the same boundary conditions presented in Thorpe and Emanuel (1985), which resulted in figures

identical to theirs, thereby providing confidence in our numerical routines.

c. Optimal perturbation method

A quantitative method was used to determine the optimal initial-condition perturbations of any field (water-vapor mixing ratio in this study) that maximizes a given response function (vertical velocity). The method determines the fastest-growing modes of a linearized version of the idealized model by computing a singular value decomposition. Following Moore and Kleeman (1997) and Peng and Reynolds (2006), the full nonlinear model described in section 2b may be written as

$$\mathbf{x}_t = \mathbf{M}(\mathbf{x}_0), \quad (16)$$

where $\mathbf{x}_t$ is the evolved state vector at time t , $\mathbf{M}$ is the nonlinear model, $\mathbf{x}_0$ is the initial-condition state vector, and boldface font represents a vector. A perturbation at the initial time takes the form

$$\mathbf{M}(\mathbf{x}_0 + \Delta\mathbf{x}_0) = \mathbf{M}(\mathbf{x}_0) + \mathbf{L}\Delta\mathbf{x}_0 + O(\Delta\mathbf{x}_0)^2, \quad (17)$$

where $\Delta\mathbf{x}_0$ is the perturbation at the initial time, $\mathbf{L}$ is the tangent linear model (TLM; also called the linear forward propagator), and $O(\Delta\mathbf{x}_0)^2$ represents the higher-order terms of the model at time t . Assuming that the higher-order terms are negligible, the TLM may be written as

$$\mathbf{L} = (\Delta\mathbf{X}_t \Delta\mathbf{X}_0^T) (\Delta\mathbf{X}_0 \Delta\mathbf{X}_0^T)^{-1}, \quad (18)$$

where the superscript T represents the transpose. The $\mathbf{X}$ represents the ensemble matrix, not to be confused with the coordinate X in section 2b. This formulation is necessary in the case where the ensemble perturbation matrix $\Delta\mathbf{X}_0$ is not orthonormal so the inverse is not trivial. If it is not full rank, then its inverse is approximated by a pseudoinverse, keeping only the eigenvalues and eigenvectors that cumulatively explain 99% of the ensemble covariance matrix. Equation (18) allows for the approximation of the linear tangent model with an ensemble set of runs without the requirement of finding it analytically, which can be difficult.

The ensemble was composed of a set of two-dimensional Gaussian mixing ratio perturbations, each given by

$$\Delta\mathbf{x}_0 = A \exp \left[-\frac{(X - X_0)^2}{2\sigma_X^2} - \frac{(Z - Z_0)^2}{2\sigma_Z^2} \right], \quad (19)$$

where $A = 0.5$ g kg $^{-1}$, $\sigma_X = 1000$ km, and $\sigma_Z = 1$ km. The ensemble was formed by spacing each member by a two- e -folding distance that amounts to spacing the perturbation center locations X_0 and Z_0 approximately every 800 km and 800 m, respectively. This required a 30-member ensemble be run to form the full set of $\Delta\mathbf{X}_0$, and $\Delta\mathbf{X}_t$. The estimated optimal perturbation vectors are restricted to being linear combinations of these perturbations.

Last, the optimal perturbations were determined by calculating the fastest-growing linear modes of $\mathbf{L}$ that was done through a singular value decomposition given by

$$\mathbf{L} = \mathbf{S}\mathbf{\Lambda}\mathbf{W}^T, \quad (20)$$

where $\mathbf{S}$ is a matrix containing the left singular vectors, $\mathbf{\Lambda}$ is a diagonal matrix containing the singular values, and $\mathbf{W}$ is a matrix containing the right singular vectors. Descriptively, the right singular vector represents the optimal perturbations, with the left singular vector being the evolved linear state of those perturbations and the singular values being the amplification factor of the perturbation when compared with the relative humidity (RH) = 80% control run [see Diaconescu and Laprise (2012) for a thorough background of singular vectors].

d. COAMPS model

The NRL Coupled Ocean–Atmosphere Mesoscale Prediction System (COAMPS; Hodur 1997; Doyle et al. 2014) model is used to compare the physical processes contained within the solution of the two-dimensional idealized model with the realistic case of a regional weather prediction model. COAMPS is a nonlinear, nonhydrostatic, compressible, terrain-following model run on a 221×161 grid with a grid spacing of 40 km on a Lambert conformal grid with 70 vertical levels. For a full description of the parameterizations and physics packages, see Doyle et al. (2012, 2019). For a detailed description of the specific run used in this study see Demirdjian et al. (2020a).

3. Results

a. The moist control simulation

The control simulation was initialized with an RH of 80% everywhere, resulting in a mixing ratio maximum of 15.4 g kg^{-1} and a minimum of 1.1 g kg^{-1} . The evolution of the control simulation is shown in Fig. 1 from forecast hour 6 and ending at hour 42. The model was initialized with a geostrophically balanced upper-level jet (Fig. 1a), a constant potential vorticity field (Fig. 1b), a weak transverse circulation (Fig. 1c) that is forced entirely by the quasigeostrophic $\mathbf{Q}$ vector at early times, and a moisture field with greater content weighted toward the warm sector (Fig. 1d). All fields were transformed from geostrophic to Cartesian coordinates using $x = X - V_g/f$.

Twelve hours later at 18 h (Figs. 1e–h), the baroclinicity has strengthened, the alongfront geostrophic wind has intensified, the vertical circulation has strengthened substantially, the moisture begins to converge toward the center, and a weak but visible surface-PV anomaly has developed below the maximum in latent heating. The evolution up until this time is driven primarily by the “dry” prescribed confluence rate since too little time has passed for moist processes to act strongly.

Another 12 h later at 30 h (Figs. 1i–l), the latent heating has further intensified, causing the strengthening of the surface-PV anomaly. A cyclonic circulation has formed centered at $x = 400 \text{ km}$ in response to the PV anomaly as can be seen by the development of the LLJ on the south (right) side of the PV and a weakening of the wind of the north (left) side. The sustained latent heating also leads to a weakening of the static stability thereby allowing for the vertical velocity to continue to intensify. By this time, the moist processes have grown to sufficient strength to begin to affect the evolution of the

circulation in a noticeable way (described in more detail in section 3b).

At the final time 42 h (Figs. 1m–p), a continued progression of the same processes is observed leading to a strong LLJ with peak amplitude of $\sim 20 \text{ m s}^{-1}$ at the surface, a large surface PV anomaly having amplitude of 1 PVU, a strong latent heating rate ahead of the front having amplitude of 8 K day^{-1} , a convergence of moisture along the frontal zone, and a sharp narrow updraft ahead of the front having amplitude of about 3 cm s^{-1} . The control simulation demonstrates that the idealized model can reproduce the physical processes consistent with the basic meteorological understanding of frontogenesis.

b. Dry versus moist run

An analysis of a dry versus a moist version of the model was performed to isolate the impacts of moisture on the solution. The dry version set RH = 0% while the moist version set RH = 80% everywhere in the model domain (same run as in Fig. 1). The impact of moisture is clearly shown in Fig. 2 for the alongfront geostrophic wind at 42 h. The solution in the moist run (Fig. 2b) results in a stronger frontal temperature gradient and the development of a LLJ. The dry case develops only a weak LLJ consistent with the hypothesis that the LLJ is a direct response of the surface PV anomaly (Lackmann 2002) that has much stronger development in the moist case (Fig. 1). Further support of this hypothesis is found in the fact that the dipole in the alongfront geostrophic-wind differences (Fig. 2a) is centered exactly on the surface-PV anomaly seen in Fig. 1n.

Similarly, a comparison of the across-front ageostrophic wind for the dry and moist cases is shown in Fig. 3. The moist run exhibits noticeably stronger ageostrophic wind magnitudes as well as a strong x gradient of the ageostrophic wind ($\partial u_{ag}/\partial x$). Perhaps the greatest difference between the two runs is seen in the vertical velocity field (Fig. 4); the moist run exhibits an updraft that is much narrower and approximately 3 times as strong as the dry runs. This is a direct result of the strong latent heating that acts through the moisture term of Eq. (4). The role of latent heating within the Sawyer–Eliassen transverse circulation is to reduce static stability that reduces the resistance to vertical motion thereby allowing for a more vigorous updraft to develop and a stronger across-front wind component. In turn, the more vigorous transverse circulation leads to stronger latent heating [Eq. (8)] and a simple feedback process is established that allows the system to intensify quickly.

c. Comparison with COAMPS

It is important that the physical processes contained within this idealized two-dimensional model have realistic solutions so that concepts learned here may be applied to real cases with greater complexities. A comparison between the two-dimensional idealized run with the full physics COAMPS model was performed for validation purposes. The case selected for the validation is the same one used in Demirdjian et al. (2020a) because it (i) has strong frontogenesis, (ii) is a strong AR case, and (iii) was readily available at the high temporal output (15 min) required for the trajectory analysis described below.

The two-dimensional idealized model implicitly assumes a transect that follows the mean flow of the system.

FIG. 1. The control simulation from forecast hour 6 to 42 h every 12 h for (a),(e),(i),(m) the alongfront geostrophic wind (color shades; m s^{-1}); (b),(f),(j),(n) the potential vorticity (color shades; PVU) and latent heating rate (black contours; contour interval $\text{CI} = 2 \text{ K day}^{-1}$); (c),(g),(k),(o) vertical velocity (color shades; cm s^{-1}); and (d),(h),(l),(p) water vapor mixing ratio (color shades; g kg^{-1}). All plots include the potential temperature (gray contours; $\text{CI} = 2 \text{ K}$).

To facilitate a fair comparison between the two models, a Lagrangian air-parcel trajectory analysis was performed on the COAMPS model to follow a transect along as it is advected by the mean flow. Backward trajectories were calculated using the same algorithm used in Demirdjian et al. (2020a) starting from a point along the final time (42 h) cold front (Fig. 5b). The sensitivity of the backward trajectory to the starting location was investigated by shifting the

starting location up and down by 2° along the transect with the resulting changes in trajectory and subsequent figures showing little change. The backward trajectory initialized at the center of the transect in Fig. 5b leads to the transect center shown in Fig. 5a. Feature tracking between Figs. 5a and 5b of the integrated vapor transport (IVT) object from 42 to 0 h is misleading because it appears as though the transect crosses the IVT object maximum at 42 h but not at

FIG. 2. A comparison at forecast time 42 h for (a) the dry (RH = 0%) and (b) moist (RH = 80%) versions of the model. Both plots have the alongfront geostrophic wind (color shades; m s^{-1}), and potential temperature (gray contours; CI = 2 K); the moist-minus-dry alongfront geostrophic wind (black contours; CI = 2 m s^{-1}) is also shown in (a).

0 h. However, a closer examination of the feature tracking (see the first figure in the online supplemental material) demonstrates that the trajectory algorithm does indeed track the IVT maximum and that the reason for this confusion is due

to the development of a secondary cyclone and reinvigoration of the AR sometime between 0 and 42 h.

The COAMPS Lagrangian transects are shown in Fig. 6 along with the moist (RH = 80%) control run of the

FIG. 3. As in Fig. 2, but for the across-front ageostrophic wind.

FIG. 4. As in Fig. 2, but for the vertical velocity.

two-dimensional idealized model for comparison. At time 0 h, the COAMPS model has a zone of weak baroclinicity accompanied by a weak upper-level jet (Figs. 5a and 6a). Similarly, at time 0 h the idealized model has approximately the same magnitude baroclinicity but with a moderately stronger upper-level jet (Fig. 6b). By time 42 h, the COAMPS run (Figs. 5b and 6c) has undergone midlevel frontogenesis (centered on ~6-km elevation) leading to a substantial increase in the upper-level jet strength. Furthermore, a low-level cyclonic circulation is seen to develop with an LLJ on the southeast (right) side and a

reduction in the wind speed on the northwest (left) side centered at approximately $x = 2200$ km. Similarly, by 42 h the idealized simulation (Fig. 6d) has undergone frontogenesis from top to bottom that has led to the intensification of the upper-level jet, though not as strongly as in the COAMPS jet. A low-level cyclonic circulation is also observed to develop causing the formation of an LLJ of similar magnitude and relative location (relative to the front and upper-level jet) as in the COAMPS run. In both simulations, this low-level cyclonic development is a response to the development of the low-level

Cross Sections and Trajectories

FIG. 5. A COAMPS plan view of the sea-level pressure (black contours; CI = 4 hPa) and IVT (color shades; kg m s⁻¹) at times (a) 0 and (b) 42 h. The black dotted lines are the same in each plot and are the backward trajectories with a dot every 3 h. The straight diagonal black lines are the transects used in Fig. 6.

FIG. 6. Cross sections of the alongfront geostrophic wind (color shades; m s^{-1}) and potential temperature (gray contours; $\text{CI} = 2 \text{ K}$) for (a) 0-h COAMPS, (b) 0-h idealized model, (c) 42-h COAMPS, and (d) 42-h idealized model. Panel (a) corresponds to the transect in Fig. 5a, and (c) corresponds to the transect in Fig. 5b.

PV anomaly (see the second figure in the online supplemental material) as discussed in sections 3a and 3b. The midlevel frontogenesis (centered on $\sim 6\text{-km}$ elevation) is observed to be of similar strength between the two simulations as witnessed in Figs. 6c and 6d. The discrepancy in surface frontogenesis between the two models is explained by the fact that the idealized model does not have a boundary layer while the COAMPS model is equipped with complex boundary layer physics that are not represented in the idealized model.

The good agreement between the idealized model with the full-physics nonhydrostatic COAMPS model suggests that the simplified physics contained within the idealized model is sufficient for the study of the basic frontogenetical processes within ARs.

d. Optimal perturbations

The method of objectively determining the optimal perturbations described in section 2c was used by solving for the initial-condition mixing-ratio perturbation (optimal perturbation) that maximizes the final-time vertical velocity (i.e., the response function). This specific form of the TLM is created by writing the generalized form of Eq. (18) into the form

$$\mathbf{L} = (\Delta \mathbf{w}_t \Delta \mathbf{q}_0^T)(\Delta \mathbf{q}_0 \Delta \mathbf{q}_0^T)^{-1}, \quad (21)$$

where $\mathbf{w}_t$ is the final time ($t = 42 \text{ h}$) vertical velocity and $\mathbf{q}_0$ is the initial condition mixing ratio. In terms of the singular-vector terminology (section 2c), the mixing ratio is the leading right singular vector and vertical velocity is the leading left singular

FIG. 7. The left and right singular vectors scaled by $10\times$ and dimensionalized for (a) the left singular vector vertical velocity (color shades; cm s^{-1}), and (b) the right singular vector mixing ratio (g kg^{-1}).

vector as shown in Fig. 7 on the full model domain that is larger than that shown in Figs. 1–4 and 6. In physical terms, Fig. 7 describes a linearized version of the two-dimensional idealized nonlinear model in which mixing-ratio perturbations at 0 h of the form shown in Fig. 7b are mapped onto the evolved vertical velocity perturbations at 42 h shown in Fig. 7a. It is important to emphasize that “perturbation” is used here as meaning the difference of the perturbed run from the moist control run used in sections 3a–c. In this context, positive values of any of the difference fields shown in the following figures indicate that the perturbed-run field is greater than that in the control.

The optimal perturbation distribution is used to create an ensemble of perturbation runs by scaling the optimal perturbation (Fig. 7b) from $0\times$ to $10\times$ by every $0.5\times$. The purpose of this ensemble is to quantify the idealized model’s circulation response to the perturbation amplitude. The moisture evolution out to 36 h of the ensemble members is shown in Fig. 8.

This end time is chosen because it represents the common time at which none of the members have become numerically unstable due to the generation of negative PV and also provides a sufficient amount of time for the perturbations to grow and affect the model. Only final times are shown in Fig. 8 because prior to 24 h each of the perturbation runs look identical (not shown) highlighting the fact that the initial-condition perturbations only made small changes to the basic state that later became amplified. Figure 8 demonstrates that the optimal moisture perturbation evolves to increase the amplitude of the vertical gradient of mixing ratio ($\partial r/\partial z$) ahead of the front. Latent heating is directly related to the strength of $\partial r/\partial z$ as seen in Eq. (8) and this relationship is clearly observed in Fig. 9. Since the Sawyer–Eliassen transverse circulation is forced by the latent heating, increased latent heating drives an increase in ascent rate by reducing the static stability (Fig. 10). In addition, the vertical velocity

FIG. 8. The evolved water vapor mixing ratio perturbations at 36 h for 3 of the 20 ensemble members. The domain is zoomed in to focus on relevant features.

FIG. 9. As in Fig. 8, but for perturbation latent heating (black contours; CI = 4 K day⁻¹) and full-value potential vorticity (color shades; PVU).

of the nonlinear model (Fig. 10) correlates well having a correlation coefficient of $r_c = 0.6$ with that of the linearized model (Fig. 7a; note the different domain size) indicating that the linearity assumption made in the optimal perturbation analysis is valid.

Another by-product of the ensemble members' latent-heating increase is the intensification of the surface-PV anomaly (Fig. 9). Since the PV anomaly is stronger, the alongfront geostrophic-wind response is also stronger (Fig. 11) leading to an intensification of the LLJ in the perturbation runs when compared with the control. Interestingly, a negative alongfront geostrophic wind field is observed in the strongest ensemble member centered on $x = 700$ km (Fig. 11). The latter feature in the wind field forms as a result of the generation of a negative surface-PV anomaly seen to be centered on

$x = 500$ km that has the opposite circulation response (positive V_g on the left and negative on the right) as the positive PV anomaly. Importantly, the optimal moist perturbations were constructed to maximize only the vertical velocity, the intensification of the alongfront circulation through a stronger LLJ is an interesting by-product.

The question arises as to whether the circulation responds linearly or nonlinearly to the increasing amplitude of the optimal moisture perturbations within the ensemble. The maximum in perturbation vertical velocity at 36 h is plotted as a function of the domain-maximum mixing-ratio perturbation at 0 h demonstrating the clear nonlinear relationship (in Fig. 12a). Doyle et al. (2014) states that the dropsonde uncertainty range due to measurement error for relative humidity is 10%, which, converted to mixing ratio, is 1.5 g kg⁻¹ at 925 hPa.

FIG. 10. As in Fig. 9, but for the perturbation vertical velocity (color shades; cm s⁻¹).

FIG. 11. As in Fig. 8, but for the perturbation alongfront geostrophic wind (color shades; m s^{-1}).

This indicates that the perturbation sizes used here are within a realistic forecast uncertainty range. This nonlinear response to the moisture perturbations is also seen in the third figure in the online supplemental material, in which a linear regression was used to remove the linear component from the total with the residual being the nonlinear component. Given a change of about 10% to the initial condition mixing ratio, the largest increase in the vertical velocity for the optimally perturbed runs is about 140% relative to the moist ($\text{RH} = 80\%$) control run and about 550% relative to the dry ($\text{RH} = 0\%$) control run. The change in the amount of domain-total latent energy due to the optimal perturbations is investigated in Fig. 12b. Overall, the change in the latent energy increases by only a small amount ($<0.2\%$), demonstrating that the rearrangement of

initial-condition energy can have a large impact on the final-time forecast.

4. Conclusions and discussion

This study is focused on the understanding of how relatively small moisture perturbations can affect the evolution of a frontogenetic system simulated using an idealized two-dimensional semigeostrophic model following Hoskins and Bretherton (1972), Hoskins (1975), Hoskins and West (1979), Hoskins (1982), Thorpe and Emanuel (1985), and Snyder et al. (1991). The effects of moisture were first investigated through a comparison between a dry ($\text{RH} = 0\%$) and moist ($\text{RH} = 80\%$) run where the moist frontogenesis and resulting circulation was

FIG. 12. Scatterplots of each ensemble member at 36 h for (a) the maximum vertical velocity percent change (%) relative to the moist control simulation vs the initial condition mixing ratio perturbation maximum (g kg^{-1}) and (b) the domain-integrated latent-energy percent change (%) vs the initial-condition mixing-ratio-perturbation maximum (g kg^{-1}).

found to be substantially stronger, even producing a low-level jet consistent with other studies (Thorpe and Emanuel 1985; Lackmann 2002; Hu and Dominguez 2019). The idealized model was then compared with a real atmospheric-river case using the full physics three-dimensional COAMPS model in which good agreement between the two demonstrated that the idealized model retains the essential physics for the study of midlatitude systems. Optimal perturbations of moisture were calculated using the singular value decomposition method and a 16-member ensemble was created by scaling the perturbations from an amplitude of 0 to $\sim 1.1 \text{ g kg}^{-1}$. The vertical velocity response at 36 h was found to depend nonlinearly on the mixing-ratio-perturbation amplitude at 0 h demonstrating a pathway by which small moisture perturbations can substantially impact a forecast.

A schematic representation of the physical processes that make up the pathway by which moisture perturbations are able to impact the forecast is shown in Fig. 13. The evolution of the transverse circulation leads to moisture convergence centered at the baroclinic zone. The moisture convergence is followed by ascent and latent heating at a low to midlevel and above the surface. The potential temperature adjusts to the latent heating by decreasing $\partial\theta/\partial Z$ and thereby the static stability. The reduced static stability allows for a strengthening of the transverse circulation and the entire cycle just described is repeated with greater intensity. The optimal perturbation method appears to have targeted this feedback process allowing for the moisture perturbations to have maximum impact on the vertical velocity. This was achieved in a twofold manner: (i) by creating a distribution that increases the vertical derivative of mixing ratio and thereby latent heating from Eq. (8), and (ii) by positioning the distribution on the warm side of the front whereby the background confluence could advect it toward the ascent region and contribute toward the latent heating rate. Another important but incidental circulation response to the optimal moisture perturbations was in the intensification of the LLJ through the strengthening of the surface-PV anomaly.

The physical processes outlined in Fig. 13 demonstrates that the frontogenetic circulation can respond to moisture perturbations by increasing both the vertical and horizontal along-front circulation. Following Alpert (1986) and Lu et al. (2018) the precipitation can be decomposed into a dynamic component (synoptic and mesoscale ascent) and an orographic component (water vapor transport interacting with orography). Using this framework, the idealized model's vertical velocity represents the dynamic component and the horizontal along-front geostrophic wind represents the orographic component. The precipitation of a frontal system is thereby particularly sensitive to initial-condition moisture content because it can simultaneously affect both components of precipitation consistent with other studies (Cannon et al. 2018; Demirdjian et al. 2020a). This physical argument of a strong precipitation-forecast dependence on the initial-condition moisture content highlights the importance of proper observing-system operational planning in regions like the Pacific that otherwise would have vast areas without any vertically resolved moisture data. Additionally, the nonlinear relationship between vertical velocity and initial-condition moisture content suggests that

FIG. 13. A schematic representation of the important physical processes discussed in this study. The potential temperature (gray contours), latent heating (LH; thick red oval), alongfront geostrophic wind (color shades with increasing value from orange to red), and the ageostrophic streamfunction (transverse circulation; large black circle). The direction of the transverse circulation is denoted by the black arrows, with successively smaller arrows indicating regions of convergence. The dashed gray lines denote changes to the potential temperature resulting from latent-heating effects. The order of processes is labeled numerically, with dotted black arrows connecting them for continuity. The Brunt-Väisälä frequency before and after the latent heating is measured by N^2 and N_s^2 , respectively.

even if the orographic precipitation component were to be ignored, the dynamic component of precipitation would alone be sensitive to moisture content.

When interpreting the results of this study, it is important to keep the idealized model's limitations in mind. The first consideration is that the model is statically stable that actually serves to bolster the conclusions formed here since a statically stable model is unable to trigger free convection. In a more sophisticated full-physics model, moisture perturbations can trigger buoyancy instabilities that would drive free convection and potentially a rapid departure from any nonconvective cases. This suggests that the circulation response to moisture sensitivities shown in Fig. 12 are expected to be an underrepresentation of the circulation response to moisture. The next consideration is that the idealized model neglects both a boundary layer that causes vertical mixing in the lower levels of moisture and sensible heat. The surface front is thereby

stronger than expected as seen in observational studies of fronts (Browning and Pardoe 1973; Thorpe and Clough 1991; Wakimoto and Murphey 2008; Demirdjian et al. 2020b). Similarly, another consideration is that the idealized model also neglects frictional drag at the surface that will reduce the horizontal wind components and thereby the strength of the LLJ. The inclusion of the surface drag will also serve to elevate the LLJ maximum above the surface. The last consideration is that one should be careful not to cavalierly apply the sensitivity distributions of this idealized study to a real case. Rather, the sensitive regions of a real event must be determined on a case-by-case basis since they will be fraught with complexities not included here. This means that while it would be interesting to apply these results to determine the best storm relative locations to place dropsondes, it is not possible to comment generally on this for all systems. Despite these simplifications, the idealized model provides a remarkably high-fidelity representation of the mesoscale processes during frontogenesis.

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