A Thermodynamic Perspective on Steady-State Tropical Cyclones

RAPHAËL ROUSSEAU-RIZZI,^a RICHARD ROTUNNO,^b AND GEORGE BRYAN^b

^a *Lorenz Center, Department of Earth Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts*

^b *Mesoscale and Microscale Meteorology Laboratory, National Center for Atmospheric Research, Boulder, Colorado*

(Manuscript received 11 May 2020, in final form 25 November 2020)

ABSTRACT: Theories for the maximum intensity of tropical cyclones (TCs) assume steady state. However, many TCs in simulations that run for tens of days tend to decay considerably from an early steady state in the core (CS), before stabilizing at a final equilibrium steady state (ES). This decay raises the question of whether CS or ES should be used as a comparison to the maximum intensity theories. To understand the differences between CS and ES, we investigate why TCs decay and attempt to simulate a TC with steady intensity over a 100-day period. Using the axisymmetric Cloud Model 1, we find that the CS TC decay is due to a large-scale drying of the subsidence region. Such a drying is very pronounced in axisymmetric models because shallow-to-midlevel convection is not represented accurately enough to moisten air in the subsidence region. Simulations with an added moisture relaxation term in the subsidence region and dry cyclones without any moisture both remain in a steady state for over 100 days, without decaying appreciably after the spinup period. These simulations indicate that the decay in TC simulations is due to the irreversible removal of precipitation combined with the lack of a moistening mechanism in the subsidence region. Once either of these conditions is removed, the decay disappears and the CS and ES intensities become essentially equivalent.

KEYWORDS: Atmosphere; Tropics; Tropical cyclones; Numerical analysis/modeling

1. Introduction

Our understanding of complex atmospheric phenomena, like tropical cyclones (TCs), evolves as the result of a synergy between analytical theories, numerical modeling, and observations. Each branch of the research process informs the other two. Interestingly enough, simple analytical assumptions like that of an inviscid free troposphere, or of a system in steady state, can be challenging to understand when taken in the context of numerical modeling. For comparison with analytical theories derived using steady state assumptions, idealized numerical studies of TCs that focus, for example, on maximum winds or TC structure, conduct their analyses on a part of the simulation considered to be in steady state (e.g., Tang and Emanuel 2012; Chavas and Emanuel 2014; Persing and Montgomery 2003). Theories of potential intensity (PI), a thermodynamic bound on maximum TC velocity that depends on properties of the environment, rely on such assumptions of steady state, and so do the numerical studies of PI (Rotunno and Emanuel 1987; Bryan and Rotunno 2009a; Hakim 2011; Rousseau-Rizzi and Emanuel 2019).

For all the instances where steady state is invoked in modeling studies, there does not seem to be a generally accepted rule for what constitutes a steady state in TCs, and as a result, different studies have quite different definitions. For example, Rotunno and Emanuel (1987) simulate the TCs' evolution for 6 days before they consider having reached a steady state,

Bryan and Rotunno (2009a) runs simulations out to 12 days, and Hakim (2011) runs them for hundreds of days. Hakim (2011) performs axisymmetric numerical simulations of TCs using Cloud Model 1 (CM1) (Bryan and Fritsch 2002; Bryan and Rotunno 2009b) and notices that, after being quasi steady for a few days, the intensity of the storm, defined here as the maximum wind speed, decays over a period of order 10 days before becoming quasi steady again at a much lower value. This newly achieved intensity then remains essentially constant for over a hundred days. Hakim (2011) argues that this latter intensity represents the physical steady state of TCs to which PI should be compared, and that the higher intensities reached earlier on in the simulation are transient fluctuations due to either internal dynamics of the TC, or to unbalanced initial conditions. A similar behavior is noted by Chavas and Emanuel (2014). Smith et al. (2014) note that simulated TCs also decay in 3D model simulations. Their explanation is that angular momentum sinks far outweigh the sources and that the TC runs out of angular momentum. In this paper, we wish to elucidate why decays such as those noted by Hakim (2011) and Smith et al. (2014) occur, and what differentiates various definitions of steady state. These questions are important because they inform both our general understanding of TC energetics, and the applicability of PI theories to real TCs, which are relatively short lived.

In general, the various definitions of steady-state intensity introduced in the literature can be grouped into two phenomenologically inspired categories, which we will label core steady state (CS), and equilibrium steady state (ES). As shown by the schematic of Fig. 1, CS is a definition that describes the state of the storm where the intensity varies slowly around peak intensity, which can occur after just a few days, depending on the numerical setup. ES describes the final state of a storm

Denotes content that is immediately available upon publication as open access.

Corresponding author: Raphaël Rousseau-Rizzi, rrizzi@mit.edu

FIG. 1. Conceptual schematic of maximum hurricane intensity over time in a typical long (>10 days) axisymmetric simulation. Core steady state (CS) is identified by the dashed blue line and equilibrium steady state (ES) is identified by the dashed orange line. The decay period is identified by dashed gray lines.

where the intensity becomes statistically steady for as long as anyone cares to run a simulation. It occurs later in time and is usually separated from CS by a decay period of a few tens of days. The steady state definitions of [Hakim \(2011\)](#) and [Chavas and Emanuel \(2014\)](#) fall in the ES category, while those of [Rotunno and Emanuel \(1987\)](#) or [Bryan and Rotunno \(2009a\)](#) belong to the CS category.

To understand the difference between the meaning of CS and that of ES, it is useful to understand the decay that separates them. To do so, we consider the evolution of conserved variables along the secondary circulation. The secondary circulation is the component of the TC circulation in radius–height (r – z) cylindrical coordinates. It captures most of the changes in the parcel properties that are relevant to TC energetics ([Eliassen and Kleinschmidt 1957](#), hereafter [EK57](#)). It is most easily understood in terms of the evolution of conserved or nearly conserved quantities that do not arise as the residual of large cancelling terms. Angular momentum (M) and pseudoadiabatic entropy (s) are such variables. For an air parcel, time tendencies of M can only arise due to turbulence, while time tendencies of s can only arise due to turbulence or departure from the pseudoadiabatic assumptions (e.g., [Bryan 2008](#)). This makes it easier to identify the physical causes of changes in s and M along the secondary circulation.

a. Eliassen and Kleinschmidt theory

We consider the early theory of TC energetics introduced by [EK57](#). The theory of [EK57](#) was the first one to our knowledge to represent the steady state TC as a closed thermodynamic cycle. This cycle is illustrated in [Fig. 2](#), from [EK57](#). In leg 1–2, air with environmental properties gains moist entropy in the form of a large moisture gain, and a small temperature increase, until the temperature is similar to the sea surface temperature (SST). Afterward, the air ascends in the eyewall and into the outflow along leg 2–3, which is a moist adiabat, where by definition moist entropy is constant. Finally, the air

FIG. 2. Figure 30 from [EK57](#), presenting a conceptual vertical cross section through a hurricane. The solid line with arrows denotes the shape and direction of the secondary circulation. Numbers indicate the transition between important segments, as discussed in the text.

cools radiatively, which causes it to subside along leg 3–1, back to its starting point. They suggest that an estimate of the storm velocity can be obtained by integrating the work done along that cycle. This is a method that has been used for some potential intensity (PI) theories (e.g., [Emanuel 1988](#)).

One key fact is not made explicit in the model of [EK57](#). The subsiding air will be much drier than the initial environmental air at the surface, unless the system obeys moist reversible thermodynamics (i.e., water condensed along leg 2–3 then evaporates along leg 3–1), or unless moisture is somehow regained through convective or environmental fluxes. Without regaining moisture, there would be a much lower moist entropy at the end of circuit 1–2–3–1 than at the beginning. So while the series of processes 1–2–3–1 is represented as a closed thermodynamic cycle, it must assume, perhaps implicitly in [EK57](#), that the system is reversible or that entropy is regained by mixing at some point along the subsiding leg of the TC. The inflowing leg 1–2 and the ascent leg 2–3 each take ~ 1 day to complete, while the radiatively driven subsidence leg takes tens of days to complete. Similar to the evolution of s , [EK57](#) mention that M is lost to the sea surface in leg 1–2 and conserved in leg 2–3. To have a closed cycle, M must be regained in leg 3–1.

b. Distinction between CS and ES

Given a conserved variable c , the conservation equation is written

$$\frac{dc}{dt} = \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D_c, \quad (1)$$

where $\mathbf{u}$ is the velocity vector in the r – z plane and D_c is the turbulent mixing tendency of c . More comprehensively, CS is defined such that in the inflow and ascent 1–2–3,

$$\left| \frac{\partial c}{\partial t} \right| \ll |\mathbf{u} \cdot \nabla c|, \quad (2)$$

and the subsidence leg is assumed to evolve slowly from the initial value. That is, the local rate of change is small with

respect to the balance between advection and turbulence, and the environment through which the air subsides is taken to be nearly fixed. ES, on the other hand, is defined as

$$\oint_{\text{sc}} \frac{1}{u_l} \frac{dc}{dt} dl \approx 0, \quad (3)$$

where sc denotes the fact that the integral is performed along the secondary circulation, l is a position along sc, and u_l is the velocity along the secondary circulation, at position l . ES essentially requires the value of c of a parcel to be the same at the beginning and at the end of a loop along the secondary circulation. Hence, ES requires the integral of the local tendencies along sc, weighted by the inverse r - z velocity, to vanish. This means that low-velocity regions, such as the subsidence region, must have correspondingly smaller time tendencies to satisfy ES. For now, we surmise that the difference (i.e., the decay) between CS and ES is due to a change in the environment of the storm. The conserved variables we chose to compare between the definitions of steady state are angular momentum M and pseudoadiabatic entropy s because their evolution and distribution capture, we believe, the most important dynamics (via M) and thermodynamics (via s).

c. Hypothesis

We further hypothesize that the decay noted by previous studies is due to a decrease of the subsidence region pseudoadiabatic entropy, on the subsidence time scale. Indeed, as we discussed, s of the air parcel after subsiding down to the surface will be much lower than the original environmental value, unless the parcel can somehow regain water vapor when subsiding. There are essentially only two ways in which this can happen. By vertical mixing, that is to say moistening by convection, or by horizontal mixing with environmental air. However, it happens that, in axisymmetric models like that of Hakim (2011), or in coarsely resolved subsidence regions like that of Smith et al. (2014), shallow-to-midlevel convection is strongly suppressed. In addition, these models have closed boundary conditions so that no air “renews” the environment as would in effect happen with a real TC moving into a new environment. The implication of these model limitations is that moisture is unlikely to be regained along the subsidence leg.

d. Goal

The aim of this paper is twofold: 1) to compare the two different definitions of TC intensity steady state defined above and 2) to quantify the physical processes required to obtain a closed EK57 type thermodynamic cycle. First, section 2 presents the model, simulation setup and methods to investigate the difference between CS and ES and the closure of the thermodynamic cycle. Then, section 3 shows the results of these computations and section 4 discusses the implications of the results and evaluates an alternative interpretation based on angular momentum. Finally, section 5 sums up the study and concludes.

2. Methodology

In this paper, we are using CM1, a compressible atmospheric model in axisymmetric configuration (Bryan and Fritsch 2002;

Bryan and Rotunno 2009b). The model’s equation sets conserve mass and internal energy in saturated air and includes dissipative heating. The domain outer radius is 1500 km and the height is 25 km. The grid is radially uniform with a 2 km grid spacing in the inner 64 km and then stretches to 4 km at the outer edge of the domain. The grid spacing is 100 m in the vertical in the lower 500 m of the domain, stretching to 500 m grid spacing at the height of 6000 m. The vertical grid spacing is uniform above 6000 m.

The conversion of water vapor to hydrometeors is represented by the simple liquid water scheme of Rotunno and Emanuel (1987). The terminal velocity of liquid water is 7 m s^{-1} , which was shown by Bryan and Rotunno (2009b) to yield intensities close to the pseudoadiabatic limit. The advection scheme for both scalars and momenta is a fifth-order weighted essentially nonoscillatory (WENO) scheme. The turbulence parameterization is similar to that of Rotunno and Emanuel (1987) and is based on Smagorinsky (1963). The horizontal mixing length is fixed at a value of $l_h = 1000 \text{ m}$ and the asymptotic vertical mixing length is $l_v = 100 \text{ m}$. Both of these are typical values in simulated axisymmetric TCs. The simulations use a surface exchange coefficients parameterization based on Fairall et al. (2003), Donelan et al. (2004), and Drennan et al. (2007). The simulations use a constant Coriolis parameter $f = 5 \times 10^{-5} \text{ s}^{-1}$. Radiation is parameterized by a Newtonian relaxation of potential temperature to the initial environmental state. The setup comprises two Rayleigh damping layers, within 5 km of the top boundary and within 100 km of the outer boundary. The top damping layer acts on all three components of velocity and on potential temperature, while the lateral damping layer only acts on vertical velocity. The outer boundary is closed.

The initial vortex, in all simulations, is defined to have a maximum wind speed of 15 m s^{-1} at the surface and at a radius of 100 km. The vertical extent of the vortex is 15 km, and the radius where the winds go to zero is 500 km. The initial thermodynamic profile in all simulations is exactly moist neutral to vertical displacement, using the model’s equations, and is nearly saturated throughout the troposphere (Fig. 1 of Bryan and Rotunno 2009a). It is useful for the purpose of this paper to have a model sounding that is initially neutral to the model equations, as initial CAPE adds to the unsteadiness of the solution (not shown). The sea surface temperature is constant at 301 K, with an air–sea temperature difference of 3 K.

a. Simulations

To test our hypothesis that drying in the subsidence region is responsible for the decaying intensity of simulated TCs, we run three main simulations. The first one is a control case (CTRL), which uses the settings described above. The second one is almost identical, except that moisture in the subsidence region is relaxed to the initial value (RELAX) with a time scale of 2 days, as described below. The third one is a dry simulation (DRY) which has three differences with the CTRL: 1) a dry adiabatic troposphere in the environment, with a potential temperature lapse rate of 10 K km^{-1} in the stratosphere, 2) no moisture included in the simulation, and 3) a large air–sea temperature difference (12 K). Everything else is kept

identical to the CTRL. Then, in order to further test the sensitivity of the intensity to the drying, we also run two additional simulation named REL1 and REL4 that are similar to RELAX but use relaxation time scales of 1 day and 4 days, respectively.

b. Role of the moistening

In RELAX, REL1 and REL4, water vapor mixing ratio is relaxed to the initial environmental state q_{v0} using

$$\frac{\partial q_v}{\partial t} = [\dots] + \mu \left(\frac{q_{v0} - q_v}{\tau} \right), \quad (4)$$

where the constant τ is 2 days in RELAX, 1 day in REL1 and 4 days in REL4; and $\mu = \mu(r, z)$ is a mask that determines the region over which this relaxation term is applied. The mask excludes the core of the storm, the boundary layer, and the upper-tropospheric outflow of the storm. It is applied only to the midlevels in the subsidence region. We note that this relaxation of water vapor is analogous to the Newtonian relaxation parameterization for radiation introduced in Rotunno and Emanuel (1987) and used in our study. Rotunno and Emanuel (1987) introduced the radiation parameterization to balance the impact that a simulated TC would have on the environmental stratification inside a closed domain. The q_v relaxation term introduced here serves to balance excessive drying in the subsidence region. A similar moisture relaxation term (albeit applied to the entire domain) was used by Frisius (2015), who tested the sensitivity of TC size to varying simultaneously the time scale of both the Newtonian relaxation of temperature and that of moisture. The intensity in the simulations of Frisius (2015) decayed by about 10 to 20 m s⁻¹ from day 10 to 50 but remained remarkably steady afterward.

c. Trajectory integral

To understand the evolution of the entropy of a parcel along the secondary circulation, we define a method to integrate budget equations for pseudoadiabatic entropy s along the parcel trajectory. First, following Bryan (2008), we define the differential form of the pseudoadiabatic entropy equation

$$Tds = c_p dT + L_0 dq_v - \frac{1}{\rho_d} dp, \quad (5)$$

and the integrated form

$$s = c_p \ln(T) - R_d \ln(p_d) + \frac{L_0 q_v}{T} - R_v q_v \ln \mathcal{H}, \quad (6)$$

where c_p is the heat capacity of dry air at constant pressure, T is the temperature, R_d is the dry air gas constant, p is the pressure, p_d is the dry air pressure, ρ_d is the dry air density, L_0 is a constant latent heat of vaporization, q_v is the water vapor mixing ratio, R_v is the gas constant of water vapor and $\mathcal{H}$ is the relative humidity. Then, since CM1 outputs accurate budgets of potential temperature (θ) and q_v , we write the budget equation for entropy in terms of θ and q_v . Following a trajectory, we have

$$\frac{ds}{dt} = \frac{c_p}{\theta} \frac{d\theta}{dt} + \frac{L_0}{T} \frac{dq_v}{dt}, \quad (7)$$

which we can break down into budget terms

$$\frac{ds}{dt} = \frac{c_p}{\theta} (D_\theta + N_\theta + R_\theta + \epsilon_\theta) + \frac{L_0}{T} (D_{q_v} + N_{q_v}), \quad (8)$$

or equivalently,

$$\frac{ds}{dt} = D_s + N_s + R_s + \epsilon_s, \quad (9)$$

where D is parameterized turbulence, N is implicit diffusion, R is radiation/relaxation and ϵ is dissipative heating. The subscripts indicate the variable to which the tendency term applies. By implicit diffusion N , we mean an estimate of the diffusive component in the advection scheme, computed using higher-order expansion of the advection scheme, similar to the method of Wicker and Skamarock (2002). This N needs to be taken into account in the budget because along-trajectory budgets require an accurate computation of the Lagrangian derivative, which by definition does not include source terms or diffusive effects, either parameterized or implicit. The next step is to average the equation in time to filter out high-frequency variability like gravity waves. The Lagrangian derivative of time-averaged entropy $\bar{s}$ in a time averaged flow $\bar{\mathbf{u}}$ is

$$\frac{d\bar{s}}{dt} = \frac{\partial \bar{s}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{s}. \quad (10)$$

We note that the time mean of the advection is not the same as the advection by the time mean flow. As we take the time average of the advection budget term, we need to account for the eddy mixing term,

$$\overline{\mathbf{u} \cdot \nabla s} = \bar{\mathbf{u}} \cdot \nabla \bar{s} + \overline{\mathbf{u}' \cdot \nabla s'}, \quad (11)$$

where the overbar denotes a time average, and the prime a perturbation with respect to that average. For example, $s(r, z, t) = \bar{s}(r, z) + s'(r, z, t)$. On average, we are therefore left with an additional eddy term which needs to be treated like a source term,

$$\frac{d\bar{s}}{dt} = \bar{D}_s + \bar{N}_s + \bar{R}_s + \bar{\epsilon}_s + \bar{E}_s, \quad (12)$$

where $\bar{E}_s = -\overline{\mathbf{u}' \cdot \nabla s'}$. The eddy term needs to be computed with the model's advection scheme (because of the large higher-order spatial derivatives near the eyewall of the hurricane, a lower-order advection scheme yields large errors). Since this budget is applied to time-averaged flow, hereafter, "trajectories" will refer to nominal trajectories in r - z space that are computed from the mean flow by integrating

$$\frac{d\bar{r}}{dt} = \bar{u}, \quad \frac{d\bar{z}}{dt} = \bar{w}. \quad (13)$$

The different terms contributing to the budgets of θ and q_v are interpolated onto the trajectory and used to compute the entropy budget.

3. Results

Since we are concerned with the evolution of intensity and its steady state, in Fig. 3, we compare time series of the

FIG. 3. Time series of the maximum instantaneous azimuthal velocity for CTRL (gray), RELAX (blue), and DRY (yellow). The time series are filtered with a Lanczos filter and a cutoff frequency of 1 day.

maximum instantaneous tangential velocity for 1) the CTRL case (gray), 2) the RELAX case (blue), and 3) the DRY TC simulation (yellow). The CTRL case reaches its peak intensity around day 9, remains quasi steady at about 73 m s^{-1} for 3 to 4 days and then decays to 37 m s^{-1} over the course of about 50 days. The CTRL TC eventually dies out completely without reaching ES, like a similar simulation in Hakim (2011). In the RELAX case, the simulation reaches peak intensity around day 15, remains quasi steady at about 85 m s^{-1} for a few days, and then decays slowly by about 10 m s^{-1} before reaching ES around day 80, which holds for as far as we have run that

simulation (200 days). Finally, the DRY case reaches a maximum around 20 days and then remains statistically steady for the rest of the simulation.

Figure 3 provides evidence that either relaxing moisture in the subsidence region or removing moisture altogether removes most of the decaying behavior of the TC and essentially makes ES equivalent to CS. The tentative conclusion from that result, combined with the discussion in the introduction, is that moisture changes in the environment cause CS to depart from ES. To further test that hypothesis, an additional simulation was run with reversible thermodynamics (no fallout of precipitation), which also removes the decay (not shown).

Figure 4 compares three Hovmöller (radius–time) diagrams of the evolution of the entropy perturbation near the sea surface. The plots extend from a radius of 0 to 900 km, to encompass all of the inflow branch of the secondary circulation, and from a time of 0 to 40 days, to encompass both intensification and decay in the CTRL case. Extending the plots further in radius or time does not change the conclusions. The color maps extend from -50 to $50 \text{ J kg}^{-1} \text{ K}^{-1}$. For reference, $50 \text{ J kg}^{-1} \text{ K}^{-1}$ is roughly the difference of entropy between the near-surface layer at the initial time and the outflow layer of CTRL at maximum intensity. A negative value means that the entropy has decreased with respect to the initial value. In the CTRL simulation, the moist entropy increases for about the first 10 days, with the largest increase occurring in the eyewall region. Then s decreases substantially under the subsidence region, to values well below the initial conditions. The radial wind-induced increase in s as the air spirals inwards from a radius of 400 km is not sufficient to counteract that massive local decrease. This decrease in s is due to moisture decreases, as temperature and pressure cannot depart much from the initial values. Surface and low- to midtropospheric

FIG. 4. Day 0–40 Hovmöller diagrams of the near-surface ($z = 100 \text{ m}$) pseudoadiabatic entropy perturbation ($\text{J kg}^{-1} \text{ K}^{-1}$) for (left) CTRL, (center) RELAX, and (right) DRY.

TABLE 1. Summary of the time (days) it takes a parcel to travel along different segments of the secondary circulation. The total time is the sum of the inflow time, the outflow time, and the subsidence time.

Simulation	Inflow time	Outflow time	Subsidence time
CTRL	1.5	1.5	39
RELAX	2	2	37
DRY	2	0.8	1.7

mixing ratio exhibit similar signals (not shown). In the RELAX simulation, the entropy increases everywhere near the surface, as the storm intensifies, and remains higher than in the initial conditions. The DRY case is similar to the RELAX case in the sense that the near-surface entropy remains everywhere higher than in the initial conditions, after intensification. Consistent with the time series, the DRY case takes longer (≈ 20 days) to become steady on average than the RELAX case (≈ 10 days), and has higher variability afterward. Figure 4 along with Fig. 3 makes a strong case that the drying of the subsidence region is responsible for the TC decay in the CTRL case.

a. Trajectory integral results

Using the trajectory integral method introduced earlier, we now look at the time it takes an air parcel to travel along the different sections of the secondary circulation. In Table 1 we see that for both the CTRL and RELAX cases, the parcel takes about 10 times longer to subside as it takes to travel from the inflow to the outflow of the storm. In CTRL, the subsidence time scale of 39 days is essentially the time it takes for all moisture to be removed from the subsidence region in the absence of some process to replenish it. The drying starts as soon as air begins to subside, and the environment becomes completely dry ($\text{RH} \approx 0$) around 39 days later. In addition, the profile of moisture is exponential in temperature, which means that, for fixed subsidence, the relative humidity at a given point in the subsidence region decreases faster earlier in the simulation than later, shifting the impacts of the drying earlier in the

simulation. This is likely responsible for the maximum surface entropy (and intensity) peaking earlier and at a lower value in CTRL than in RELAX (Figs. 3 and 4). In DRY, it takes about the same time to subside as it takes to ascend. These results indicate that the assumption in the CS definition, that the subsidence region varies slowly with respect to the core, is appropriate for CTRL and RELAX. In DRY, however, the subsidence region does not vary slowly with respect to the core. Hence, the slowly varying inflow and outflow conditions required by CS only arise when equilibrium is reached, and ES and CS must become equivalent in this case.

Next, we look at the integrals of s along the cycle to better understand the definition of ES.

To gain insight into the decay of the control case, Fig. 5 shows plots of the T - s cycles of the three main simulations, along the trajectory that passes through the position of maximum winds. The black lines illustrate conditions along the parcel trajectories that cross the position of maximum winds in each simulation, in T - s space. The colored (gray, blue, and yellow) lines are the values of the integrals in time of the Lagrangian entropy budget, from the position of maximum winds onward. That is,

$$\oint_{sc} \frac{1}{u_l} (\overline{D_s} + \overline{N_s} + \overline{R_s} + \overline{\varepsilon_s} + \overline{E_s}) dl. \quad (14)$$

The averages here are performed from days 20 to 100 for both the DRY and the RELAX case, which are quite steady, and performed from days 15 to 25 for the CTRL simulation, which is decaying. In both RELAX and DRY, the s integral computation over the whole cycle matches the model simulated values of s very well, indicating that both RELAX and DRY are essentially in ES over the averaging period. In addition, this indicates that the budget is accurate and quantitatively captures the contribution of various source terms to the entropy variation along the trajectory. In CTRL, one can observe two main differences with respect to the steady cases. First, the budget does not close, which is unsurprising given that the simulation is obviously decaying and not in an equilibrium

FIG. 5. Illustration of thermodynamic conditions along the secondary circulation in T - s space using two approaches: using actual entropy along a trajectory (black) and using the integral of the entropy budget along the same trajectory (colors). The trajectory begins and ends at point 2 (the radius of maximum winds). The red dot indicates the final point of the integrated budget in CTRL, since it differs from the initial point; (left) gray is CTRL, (center) blue is RELAX, and (right) yellow is DRY. Temperature decreases upward and entropy decreases to the right so that the T - s space is oriented in the same way as r - z space.

FIG. 6. RELAX case parcel trajectory along the secondary circulation in (left) T - s and (right) physical space. The trajectory is color-coded to outline the most important contributors to the entropy budget. Blue represent turbulence terms, white represent radiation and microphysics, and yellow represents parameterized moisture relaxation.

steady state. If the tendency terms, averaged over days 15–25 are integrated over the full trajectory, the result is a much lower entropy than the starting value. Second, there is a massive decrease in entropy along the trajectory as the temperature of subsiding air increases. The entropy of the subsiding air decreases to much lower values than the initial values of entropy near sea surface, which once again, points to the lack of a mechanism to regain entropy while the air subsides.

To gain additional insight into the processes involved in reaching ES, we break down the entropy budget into its main components. Figure 6 shows again an integrated T - s cycle for the RELAX simulation, along with the trajectory of the parcel in physical space. The plot is colored as a function of the three-way relative magnitude of terms in the entropy budget: blue for turbulence terms, white for radiation/relaxation and hydrometeor fallout (which is small), and yellow for parameterized moisture relaxation. The interpolation of these terms is presented in the triangular color map. This shows that segment 1–2 is dominated by turbulence, driven by air–sea interaction. Next, segment 2–3 is dominated initially by turbulence and then by radiation. Even though turbulence is the dominant term in the vertical portion of 2–3, it has a small absolute magnitude there. Finally, segment 3–1 is dominated initially by radiation, and then by q_v relaxation, with a large counteracting effect from radiation. This quantification of the mechanisms entering in a hurricane thermodynamic cycle over the time scales of the secondary circulation verifies the requirements for an EK57-type TC. The CTRL case, without q_v relaxation, shows a cycle that does not close because the radiation keeps driving the entropy to lower values without counteracting effects (not shown). We note that the secondary circulation in RELAX does not extend far radially compared to other studies performing long term integrations (e.g., Chavas and Emanuel 2014; Persing et al. 2019). One possible explanation is the choice of Newtonian relaxation to a background sounding rather than more realistic parameterized radiation such as the Rapid Radiative Transfer Model. Newtonian relaxation does not account for cloud radiative feedbacks (CRFs), and as was

shown by Bu et al. (2014), CRFs act to greatly enhance the radial extent of tropical cyclones. The lack of CRFs in our study, as opposed to the relatively small domain (compared to these previous studies), might result in this compact storm. Indeed, there is no evidence that the secondary circulation reaches the outer wall in CTRL and RELAX. We further confirmed that the domain size does not constrain the radial extent of the CTRL and RELAX storms by rerunning those simulations for 80 days in 6000 km domains. We could not find any appreciable difference in the structure and evolution of the CTRL and RELAX storms in the two different domain sizes (not shown). In addition, other studies using Newtonian relaxation also obtained similarly compact storms (e.g., Emanuel and Rotunno 2011; Frisius 2015) for which the secondary circulation does not impinge on the outer boundary. We acknowledge that the midtropospheric drying reported in this study is likely to be sensitive to radiative and microphysical parameterizations via their influence on the secondary circulation.

b. Moisture relaxation time-scale sensitivity

CTRL does not have a clear ES, and while RELAX does, it is not very different from its CS. To better understand the difference between CS and ES, and the evolution from CS to ES, we now turn our attention toward REL1 and REL4, which have different moisture relaxation time scales than RELAX, and should lead to different ES. Figure 7 shows time series of the maximum velocity for REL1, RELAX, and REL4. From CS onward, the smaller the τ , the closer EC becomes to CS. During CS, REL1 is 3 m s^{-1} more intense than RELAX, and 6 m s^{-1} more intense than REL4. The decay is more rapid and lasts longer at large τ so that the intensity differences grow until a clear ES is reached for all three simulations. During ES, REL1 is 6 m s^{-1} more intense than RELAX, and 25 m s^{-1} more intense than REL4.

Next, Fig. 8 shows the thermodynamic cycles of REL1, RELAX and REL4 over the CS, decay, and ES periods identified in Fig. 7. Figure 8 also shows the thermodynamic cycles of

FIG. 7. Time series of the maximum instantaneous azimuthal velocity for REL1 (black, $\tau = 1$ day), RELAX (gray, $\tau = 2$ days), and REL4 (yellow, $\tau = 4$ days). The time series are filtered with a Lanczos filter and a cutoff frequency of 1 day. CS, decay, and ES time intervals are identified for the purpose of comparing thermodynamic cycles in Fig. 8.

CTRL over comparable CS (days 8 to 12) and decay (days 15 to 25) periods, and right before the storm simulation ends (days 70 to 90). Note that the abscissa in CTRL has the same maximum value as those in the other plots, but extends to much smaller values. The most obvious difference between the plots is that, for a given period (CS, decay, or ES), the entropy in the core of the storm is lower when τ is larger, and that difference grows as time goes on, until it stabilizes at ES. The second thing to notice is that the area encompassed by the thermodynamic diagram itself changes much less for smaller τ . This shows that CS and ES are not only very similar in intensity at small τ , they are also similar in the area within the T - s diagram. At larger τ , or in the CTRL case, the entropy in the core decreases as the near-surface entropy at large radius decreases. REL4 confirms that quasi-steady intensity can exist both when the thermodynamic cycle is still far from equilibrium (CS) and once it has reached equilibrium (ES). Examination of the thermodynamic cycles of CTRL shows why the storm ultimately dies out. As too little entropy is regained along the inflow to compensate that lost in the subsidence, s decreases in the core until it drops below the value in the upper troposphere subsidence region. While the TC streamlines in physical space do not change much in REL1 and RELAX between CS and ES, they do in REL4 and CTRL, where, over the decay period, the outflow leg contracts radially from an extent of about 950 km in both cases to about 650 km in REL4 and 400 km in CTRL (not shown).

4. Discussion

a. CS versus ES

These results indicate that the decay in CTRL is due to the combination of the irreversible precipitation removal in the ascent leg, and the lack of a moistening mechanism in the

subsidence leg. If we either add a rapid moistening mechanism in the subsidence region (as in RELAX or REL1), remove moisture altogether from the model (as in the DRY simulation) or prevent precipitation removal (not shown), the decay mostly disappears. This is further confirmed by the sensitivity of ES to τ , which we would expect from the argument that drying is driving the decay and explains the difference between CS and ES. Interestingly, CS is also sensitive to τ , albeit not as much. For example, the RELAX peak (CS) intensity is higher than that of CTRL by 10 m s^{-1} while lower than that of REL1 by 3 m s^{-1} . This indicates that the CS assumption, that the environment does not have time to change before peak intensity, is not very robust. In CTRL, subsidence drying results in a peak intensity that is lower, even just 10 days after the beginning of the simulation. There remains a small decay in RELAX after CS, which can be due to the moisture and temperature fields slowly reaching equilibrium or to the evolution of the M field (discussed below). In any case, the dominant effect on the decay in CTRL is the drying. The DRY case is also interesting in that its inflow–ascent–outflow time scale is so similar to its subsidence time scale that the theoretical difference between CS and ES disappears. And indeed, DRY does not show any sign of decay after reaching peak intensity.

From our results it seems that the definition of ES would be quite difficult to generalize or to compare with environmental PI theories, that is, PI for a specified and possibly arbitrary environmental profile. Indeed, while CS is very sensitive to a host of model parameters like resolution (Hausman 2001), terminal velocity (Bryan and Rotunno 2009b), or mixing length (Rotunno and Bryan 2012), ES will additionally be sensitive to any parameter that influences the equilibrium state of the model, like τ , or any parameterization for the radiation, surface fluxes, turbulence, or convection. The foregoing sensitivities would defeat the purpose of using PI to predict the intensity of a TC based on environmental parameters, and it seems that PI should be compared to CS. In addition, the intensity is closer to reaching PI (computed following Bryan and Rotunno 2009a) during CS than during ES, because PI does not decrease as much as intensity during the decay (not shown). A possible explanation for the fact that our CTRL simulation decays until it ends while other simulations without moisture relaxation do not (e.g., Hakim 2011; Chavas and Emanuel 2014) is the fact that these simulations used radiation parameterizations representing CRFs, which enhance the radial extent of the cyclone and entail more time for the air spiraling inwards to regain entropy. As was shown by the τ sensitivity experiment, if the entropy in the core does not decrease as much, a smaller decay should occur.

b. Angular momentum

Thus far, we have been silent about the evolution of other conserved variables, and more specifically dynamical variables like M . The goal was to demonstrate the role of moisture before discussing additional possibilities. Previous literature (e.g., Smith et al. 2014) has argued that the decay of simulated storms is primarily due to the loss of angular momentum to the sea surface, which causes the angular momentum, and thus the intensity to decrease at the radius of maximum winds.

FIG. 8. T - s cycles for CS (pale gray), decay (gray), and ES (black) for (top left) REL1, (top right) RELAX, (bottom left) REL4, and (bottom right) CTRL. There is no ES in CTRL, so a T - s cycle taken right as the TC simulation ends (days 70 to 90) is plotted instead.

From the results obtained in this study, it appears that the decrease in angular momentum at the radius of maximum winds in CTRL, concurrent with the decay, is simply a consequence of the TC failing to produce enough work to draw high angular momentum air inwards at small radii.

As proposed by Smith et al. (2014), the mass-integrated angular momentum in the domain initially decreases as a result of the stresses applied by the TC winds on the lower boundary. However, that decrease is small with respect to the domain-integrated M . Figure 9 shows the domain-integrated M as a function of time (full line), and the initial value of the domain-integrated M plus the time-integrated surface sink of M along the inflow (dashed line). The two would be equivalent if the only sink or source of momentum were along the surface inflow.

Eventually the domain-integrated M stops decreasing after about 80 days in the RELAX case, even as the slope of the momentum sink time-integral remains negative and linear (implying that the surface sink of M remains constant). This leveling off of domain-integrated M is due to the fact that the anticyclone deepens until it starts impinging on the stratospheric damping layer, at which point the imposed damping starts to restore domain-integrated angular momentum by weakening the anticyclone. In the secondary circulation (which is well below the damping layer), examination of budget terms for M shows that M is regained by parameterized mixing with

the environment while air subsides at the largest radius in the storm outflow. There, while velocities are quite small, the radial derivative of angular velocity is large, which causes turbulent mixing. This mixing, which occurs mostly over a thin 1 km layer, around an altitude of 15 km, restores to the secondary circulation essentially all the angular momentum lost to the sea surface by the circulating air, while slightly decreasing angular momentum in the environment.

FIG. 9. Domain-integrated M as a function of time (solid line) and domain-integrated M at $t = 0$ plus the time-integrated surface momentum sink (dashed line) for CTRL (gray) and RELAX (blue). The momentum sink is integrated in time and space in the inner 500 km from the start of the simulation.

Figure 9 shows that while the variables relevant to TC intensity can be steady after just 10–20 days, the TC structure can take much longer to equilibrate. The higher intensity in RELAX explains the larger magnitude of the M sink and the smaller domain-integrated M in that case, which further supports the conclusion that in this study, decay is not caused by a lack of angular momentum. We do not attempt to dispute that domain-wide angular momentum changes are important for the model structure (as evidenced by Chavas and Emanuel 2014; Smith et al. 2014; Persing et al. 2019), but rather to point out that they do not seem to be the main drivers of intensity variations after the spinup period in these simulations.

5. Conclusions

In conclusion, the large decay in TC intensity seen in some very long simulations (e.g., Hakim 2011; Smith et al. 2014) results primarily from a change of the environment by the TC's secondary circulation. This decay is ultimately due to the lack of shallow-to-midlevel convection in axisymmetric models, and the lack of horizontal moisture fluxes due to the TC translation, which lead to excessive drying and low entropy in the subsidence region. The TCs do not appear, in this study, to be decaying because of a lack of angular momentum. The drying occurs on the time scale of the secondary circulation, of order 40 days.

The entropy decrease in the air that subsides into the boundary layer leads to an entropy decrease in the core of the storm, as the source of entropy along the inflow leg is insufficient to compensate the deficit in the subsidence region. This leads to a decrease in the intensity of the TC, which is sensitive to the magnitude of the drying in CS and especially in ES. Adding a moisture relaxation term in the subsidence region reduces the decay. If the time scale of the moisture relaxation is small enough, the difference between ES and CS disappears, both in intensity, and in the area encompassed by the T – s cycle. These results suggest a large sensitivity of the thermodynamic cycle of equilibrium TCs to the model configuration and parameterizations.

In general, for theories of potential intensity that just require steady state in the core, like those of Emanuel (1986) or Bryan and Rotunno (2009a), theories can be compared with CS, which is much less restrictive than ES, and allows examination of the role of the initial environment, with the caveat that the fixed environment assumption is not very robust on time scales of 10 or more days. However, for theories like the Carnot cycle of Emanuel (1988) or EK57, one requires that steady state be maintained on the time scale of a loop through the secondary circulation, because a steady state environment is necessary to a closed thermodynamic cycle. Considering that reaching ES takes longer than the lifetime of most real tropical cyclones, the CS concept is much more applicable to real storms. Since the effects of the drying affects peak intensity, just 10 days after the start of the simulations, it would be interesting to see if such drying limits the intensity of real-life TCs, or if the shallow convection and continuous translation into new environment are sufficient to counteract the effects of subsidence as does the moisture relaxation in RELAX.

Acknowledgments. The authors are grateful to Kerry Emanuel, Tristan Abbott, Rohini Shivamoggi, Tom Beucler, and Jonathan Lin for their advice. The lead author was supported by the National Science Foundation under Grant AGS-1520683, the Office of Naval Research under Grant N00014-18-1-2458, and the Natural Sciences and Engineering Research Council of Canada under Grant PGSD3-490041-2016.

This material is based upon work supported by the National Center for Atmospheric Research, which is a major facility sponsored by the National Science Foundation under Cooperative Agreement 1852977. Any opinions, findings, and conclusions or recommendations expressed in this material do not necessarily reflect the views of the National Science Foundation.

REFERENCES

- Bryan, G. H., 2008: On the computation of pseudoadiabatic entropy and equivalent potential temperature. *Mon. Wea. Rev.*, **136**, 5239–5245, <https://doi.org/10.1175/2008MWR2593.1>.
- , and J. M. Fritsch, 2002: A benchmark simulation for moist nonhydrostatic models. *Mon. Wea. Rev.*, **130**, 2917–2928, [https://doi.org/10.1175/1520-0493\(2002\)130<2917:ABSFMN>2.0.CO;2](https://doi.org/10.1175/1520-0493(2002)130<2917:ABSFMN>2.0.CO;2).
- , and R. Rotunno, 2009a: Evaluation of an analytical model for the maximum intensity of tropical cyclones. *J. Atmos. Sci.*, **66**, 3042–3060, <https://doi.org/10.1175/2009JAS3038.1>.
- , and —, 2009b: The maximum intensity of tropical cyclones in axisymmetric numerical model simulations. *Mon. Wea. Rev.*, **137**, 1770–1789, <https://doi.org/10.1175/2008MWR2709.1>.
- Bu, Y. P., R. G. Fovell, and K. L. Corbosiero, 2014: Influence of cloud–radiative forcing on tropical cyclone structure. *J. Atmos. Sci.*, **71**, 1644–1662, <https://doi.org/10.1175/JAS-D-13-0265.1>.
- Chavas, D. R., and K. Emanuel, 2014: Equilibrium tropical cyclone size in an idealized state of axisymmetric radiative–convective equilibrium. *J. Atmos. Sci.*, **71**, 1663–1680, <https://doi.org/10.1175/JAS-D-13-0155.1>.
- Donelan, M., B. K. Haus, N. Reul, W. Plant, M. Stiassnie, H. C. Graber, O. Brown, and E. Saltzman, 2004: On the limiting aerodynamic roughness of the ocean in very strong winds. *Geophys. Res. Lett.*, **31**, L18306, <https://doi.org/10.1029/2004GL019460>.
- Drennan, W. M., J. A. Zhang, J. R. French, C. McCormick, and P. G. Black, 2007: Turbulent fluxes in the hurricane boundary layer. Part II: Latent heat flux. *J. Atmos. Sci.*, **64**, 1103–1115, <https://doi.org/10.1175/JAS3889.1>.
- Eliassen, A., and E. Kleinschmidt, 1957: Dynamic meteorology. *Geophysik II/Geophysics II*, Springer, 1–154.
- Emanuel, K. A., 1986: An air–sea interaction theory for tropical cyclones. Part I: Steady-state maintenance. *J. Atmos. Sci.*, **43**, 585–605, [https://doi.org/10.1175/1520-0469\(1986\)043<0585:AASITF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1986)043<0585:AASITF>2.0.CO;2).
- , 1988: The maximum intensity of hurricanes. *J. Atmos. Sci.*, **45**, 1143–1155, [https://doi.org/10.1175/1520-0469\(1988\)045<1143:TMIOH>2.0.CO;2](https://doi.org/10.1175/1520-0469(1988)045<1143:TMIOH>2.0.CO;2).
- , and R. Rotunno, 2011: Self-stratification of tropical cyclone outflow. Part I: Implications for storm structure. *J. Atmos. Sci.*, **68**, 2236–2249, <https://doi.org/10.1175/JAS-D-10-05024.1>.
- Fairall, C., E. F. Bradley, J. Hare, A. Grachev, and J. Edson, 2003: Bulk parameterization of air–sea fluxes: Updates and verification for the COARE algorithm. *J. Climate*, **16**, 571–591, [https://doi.org/10.1175/1520-0442\(2003\)016<0571:BPOASF>2.0.CO;2](https://doi.org/10.1175/1520-0442(2003)016<0571:BPOASF>2.0.CO;2).

- Frisius, T., 2015: What controls the size of a tropical cyclone? Investigations with an axisymmetric model. *Quart. J. Roy. Meteor. Soc.*, **141**, 2457–2470, <https://doi.org/10.1002/qj.2537>.
- Hakim, G. J., 2011: The mean state of axisymmetric hurricanes in statistical equilibrium. *J. Atmos. Sci.*, **68**, 1364–1376, <https://doi.org/10.1175/2010JAS3644.1>.
- Hausman, S. A., 2001: Formulation and sensitivity analysis of a nonhydrostatic, axisymmetric tropical cyclone model. Air Force Institute Of Technology Tech. Rep., 227 pp.
- Persing, J., and M. T. Montgomery, 2003: Hurricane superintensity. *J. Atmos. Sci.*, **60**, 2349–2371, [https://doi.org/10.1175/1520-0469\(2003\)060<2349:HS>2.0.CO;2](https://doi.org/10.1175/1520-0469(2003)060<2349:HS>2.0.CO;2).
- , —, R. K. Smith, and J. C. McWilliams, 2019: Quasi steady-state hurricanes revisited. *Trop. Cyclone Res. Rev.*, **8**, 1–17, <https://doi.org/10.1016/j.tcr.2019.07.001>.
- Rotunno, R., and K. A. Emanuel, 1987: An air–sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *J. Atmos. Sci.*, **44**, 542–561, [https://doi.org/10.1175/1520-0469\(1987\)044<0542:AAITFT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1987)044<0542:AAITFT>2.0.CO;2).
- , and G. H. Bryan, 2012: Effects of parameterized diffusion on simulated hurricanes. *J. Atmos. Sci.*, **69**, 2284–2299, <https://doi.org/10.1175/JAS-D-11-0204.1>.
- Rousseau-Rizzi, R., and K. Emanuel, 2019: An evaluation of hurricane superintensity in axisymmetric numerical models. *J. Atmos. Sci.*, **76**, 1697–1708, <https://doi.org/10.1175/JAS-D-18-0238.1>.
- Smagorinsky, J., 1963: General circulation experiments with the primitive equations: I. The basic experiment. *Mon. Wea. Rev.*, **91**, 99–164, [https://doi.org/10.1175/1520-0493\(1963\)091<0099:GCEWTP>2.3.CO;2](https://doi.org/10.1175/1520-0493(1963)091<0099:GCEWTP>2.3.CO;2).
- Smith, R. K., M. T. Montgomery, and J. Persing, 2014: On steady-state tropical cyclones. *Quart. J. Roy. Meteor. Soc.*, **140**, 2638–2649, <https://doi.org/10.1002/qj.2329>.
- Tang, B., and K. Emanuel, 2012: Sensitivity of tropical cyclone intensity to ventilation in an axisymmetric model. *J. Atmos. Sci.*, **69**, 2394–2413, <https://doi.org/10.1175/JAS-D-11-0232.1>.
- Wicker, L. J., and W. C. Skamarock, 2002: Time splitting methods for elastic models using forward time schemes. *Mon. Wea. Rev.*, **130**, 2088–2097, [https://doi.org/10.1175/1520-0493\(2002\)130<2088:TSMFEM>2.0.CO;2](https://doi.org/10.1175/1520-0493(2002)130<2088:TSMFEM>2.0.CO;2).