The Terminal Velocity of Axisymmetric Cloud Drops and Raindrops Evaluated by the Immersed Boundary Method

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ABSTRACT: The terminal velocity of cloud drops and raindrops used in numerical model calculations can significantly affect weather predictions. Current formulations rely on laboratory experiments made in the 1940s and 1960s. Because these experiments were performed only at typical environmental conditions of 20°C and 1013 hPa, parameterizations have been introduced to deduce the terminal velocity aloft without rigorous evaluation. In this study, an incompressible two-phase flow direct numerical simulation model is used to calculate the free-falling motion of axisymmetric drops with diameters between 0.025 and 0.5 mm to study the terminal fall velocity. Simulated terminal fall velocities of free-falling drops at 20°C and 1013 hPa agree within 3.2% with the previous empirical parameterization (Beard formula), and 4.5% with existing laboratory data in the diameter range between 0.3 and 0.5 mm. The velocities converge to the analytic Hadamard–Rybczynski solution within 2% for small Reynolds numbers, demonstrating the robustness of our simulations. Simulations under various atmospheric conditions show that existing empirical parameterizations that account for the air density dependence of the terminal velocity have errors up to 11.8% under the conditions examined in this study. We propose a new empirical formula that describes the air density dependence of the terminal velocity. It is also shown that the falling speed of a small drop is not sensitive to shape oscillation, and the terminal velocity decreases by only less than 1.3% when the axis ratio increases by 12% with reduced surface tension. Internal circulation within falling drops is also presented and compared with previous studies.

KEYWORDS: Cloud droplets; Cloud microphysics; Numerical analysis/modeling; Parameterization

1. Introduction

All cloud drops and raindrops¹ experience an acceleration toward the ground by the pull of the gravitational force. Due to the existence of the viscosity, they attain a constant falling speed when the gravitational force is balanced by the viscous frictional force. This constant speed at which drops fall is called the terminal velocity. The terminal velocity of raindrops is one of the major factors that determine the rate of removal of liquid water from the atmosphere, which can strongly affect weather predictions (Parodi and Emanuel 2009; Hagos et al. 2018). An accurate prediction of the terminal velocity is therefore one of the necessary conditions for improving the accuracy of weather and climate simulations. Because the general equation governing the free-falling motion of water drops of even moderate size (on the order of 10^{-1} mm) is

nonlinear, an analytic solution of the terminal velocity exists only for drops that are small enough (on the order of $1\ \mu\text{m}$) for the Stokes law to be applicable. In atmospheric modeling, the terminal velocity is commonly retrieved from parameterizations via some empirical formulas derived from experimental data.

There have been numerous experimental reports (Gunn and Kinzer 1949; Beard and Pruppacher 1969; Yu et al. 2016; Chowdhury et al. 2016) on the measurement of the terminal velocity of water drops. The laboratory experiment conducted by Gunn and Kinzer (1949) produced detailed data on the terminal velocity of drops with diameters ranging from 0.0783 to 5.8 mm at a temperature of 20°C and pressure of 1013 hPa. In their experiment, droplets were electrically charged and allowed to fall freely in a rain tower. Two electrode detectors that could detect the passing of charged droplets at separate heights were used to calculate the time span, and thus the terminal velocity could be obtained. Beard and Pruppacher (1969) pointed out that the environmental condition of 50% relative humidity in Gunn and Kinzer's experiment might have generated considerable errors in the measurement of the terminal velocity, especially for smaller drops, because some water might have evaporated from the drop before its mass was measured. Instead, they used a wind tunnel with 100% relative humidity to measure the terminal velocity, and their results indeed suggested a slight overestimation of the terminal velocity of the smallest droplets measured by Gunn and Kinzer (1949). Although wind tunnel method might be prone to measurement error due to non-uniform velocity profile in the tunnel, the measured values for larger droplets agreed well with Gunn and Kinzer's experimental data (Beard 1976).

¹ We follow the AMS *Glossary of Meteorology* to define drops of diameters smaller than 0.2 mm as cloud drops in this study. While drops with diameters between 0.2 and 0.5 mm are defined as drizzle drops and drops larger than 0.5 mm are called raindrops in the AMS *Glossary of Meteorology*, we call all precipitation drops larger than 0.2 mm raindrops for simplicity. The word “drops” refers to water drops of all sizes.

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These values were used in the Beard (1976) parameterization of the terminal velocity.

More recent experimental studies (Yu et al. 2016; Chowdhury et al. 2016) used a high-speed camera to measure the terminal velocity. However, it is prone to errors due to pixel resolution and determination of centroid, and which may lead to larger measurement errors for cloud drops and small raindrops (Yu et al. 2016). The experimental data of Gunn and Kinzer (1949) and Beard and Pruppacher (1969) have remained as the reference solution of the terminal velocity under standard conditions (20°C and 1013 hPa) up to the present day. Although these data are rather complete, there is a severe limitation to the range of applicability since the experiments were performed only at one particular set of environmental conditions. As cloud drops or raindrops usually fall from high altitudes at temperatures and pressures that are far different from the standard conditions of 20°C and 1013 hPa, an extrapolation is necessary to deduce the terminal velocity at arbitrary temperatures and pressures.

Finding a fixed functional relationship among dimensionless numbers is a common way to predict the terminal velocity at high altitudes. As is shown in section 2, four dimensionless numbers are required to completely describe the solution to the free-falling droplet problem, and the Reynolds number is the only dimensionless number that contains the terminal velocity. However, it is a daunting task to find an equation that relate the Reynolds number to the other dimensionless numbers; therefore, assumptions are usually made to reduce their number. For example, Beard and Pruppacher (1969) attacked this problem by assuming that there exists a fixed relationship between the Reynolds number and the drag coefficient. This relationship does not depend on environmental conditions if the drop is a solid object. In other words, experiments of a free-falling solid object conducted at arbitrary temperatures and pressures will lead us to the same relationship. Nonetheless, drops have finite viscosity and surface tension. Two dimensionless numbers are insufficient to fully describe the solution, thus the relationship actually depends on environmental conditions. This dependency has not been studied in a quantitative way.

Most microphysical schemes adopt a much simpler method to formulate the effect of the reduced air density on the terminal velocity at high altitudes. A simple function of the ratio of the corresponding air densities or pressures is directly related to the ratio of the terminal velocity aloft to that at the surface. However, the accuracy of this method has not been rigorously examined. It is worth mentioning that Yu et al. (2016) compared their measurements with Gunn and Kinzer's data modified by this method at nonstandard environmental conditions of 30.2°C and 956.4 hPa, and they found a mean measurement error of approximately 6% (1.86%) for drops of diameters smaller (larger) than 1 mm. Due to measurement uncertainties of a similar magnitude as the mean measurement error, it is unclear whether this method is the primary cause of the error.

Although the terminal velocity of pure water drops was thoroughly studied in previous experiments, uncertainties about the influence of varying environmental conditions on the terminal velocity have not been well investigated. The purpose of the present work is to revisit this problem and tackle it with a

numerical approach. We adopted the recently developed numerical solver of the two-phase flow Navier–Stokes equation (Ong and Miura 2018, 2019), which can explicitly resolve capillary waves, to investigate the motion of an axisymmetric free-falling drop by direct numerical simulations (DNS), with the aim of shedding light on the relationship between the terminal velocity and environmental conditions. We also compared existing empirical formulas with our simulation results.

Besides temperature and pressure, there are other factors that can also affect the falling speed and terminal velocity. Examples are reduction of the surface tension coefficient by aerosols and shape oscillation by capillary force. It is known that chemical compounds that are commonly observed in the atmosphere can act as surfactants to reduce the surface tension coefficient σ (Gill et al. 1983; Taraniuk et al. 2008). Although no experimental evidence had been given to indicate that there exists a significant fluctuation in falling speed due to shape oscillation (Beard 1976) and terminal velocity due to reduced surface tension (Müller et al. 2013), we can examine the correctness of the above statement by simulations with a two-phase flow model.

Since our two-phase flow model is currently limited to two-dimensional simulations, only free-falling drops of diameters smaller than 0.5 mm without unstable higher modes of asymmetric oscillations are considered in this study. Attempts to apply the axisymmetric model to larger raindrops lead to nonphysical diverging solutions. This size limitation is certainly a major drawback of the present study, it may serve as the first example of successful applications of the two-phase flow model in the atmospheric science. Once it is verified that this approach can be successfully applied on small raindrops, we will step forward to 3D calculations of raindrops with diameters larger than 0.5 mm, the difficulty of which is associated with their chaotic motion. For this purpose, a new surface reconstruction scheme for 3D simulations has recently been developed (Ong and Miura 2018), but a development of a full 3D two-phase flow model warrants further investigations. Simulations of large raindrops with unsteady wakes are inevitably left for future work.

In summary, we prepared five sets of numerical experiments to achieve our objectives of 1) verification and validation of our numerical model for free-falling water drops by comparing numerical solutions with existing experimental data and numerical data, e.g., Gunn and Kinzer (1949), Wang and Pruppacher (1977), and LeClair et al. (1972); 2) investigating the influence of environmental conditions on the terminal velocity at high altitudes and evaluating the accuracy of the existing empirical formulas for the terminal velocity aloft; 3) examining the dependency of the relationship between the Reynolds number and the drag coefficient ($Re-C_D$) on environmental conditions by comparing Re and C_D of free-falling small raindrops under standard conditions and at high altitudes; 4) determining the influence of oscillatory motion on the falling speed; and 5) determining the shape deformation effect on the motion of a small raindrop with a reduced surface tension coefficient. The detailed setup of these five sets of numerical experiments is given in section 3. Based on the results of the second and third set of experiments, we will propose a new

empirical formula of the terminal velocity aloft to account for reduced air density.

A simple description of the mathematical model of a free-falling drop and existing parameterizations of terminal velocity at any temperature and pressure are given in [section 2](#). The numerical setup is described in [section 3](#). Simulation results are presented in [section 4](#). The results and findings are summarized in [section 5](#).

2. Terminal velocity

To facilitate the elaboration of the experimental methods and computation of the terminal velocity in later sections, we briefly describe the fundamental equation of the incompressible two-phase flow and the four dimensionless numbers that determine its solution in the first half of this section. The empirical formulas of the terminal velocity aloft that are commonly used in atmospheric models are briefly explained in the second half of this section.

a. Governing equation

The Navier–Stokes momentum equation for the incompressible two-phase flow is given by

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u} \mathbf{u} = -\nabla p + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \sigma \kappa \hat{\delta} - \Delta \rho g \hat{\mathbf{k}}, \quad (1)$$

where $\mathbf{u}$ is the flow velocity, p is the pressure, μ is the viscosity, superscript T denotes transpose of a tensor, σ is the surface tension coefficient, κ is the interfacial curvature, $\hat{\delta}$ is the normalized delta vector of surface tension force ([Ong and Miura 2019](#)), $\Delta \rho$ is the difference between the local density and the density of air, g is the gravitational constant, and $\hat{\mathbf{k}}$ is the unit vector pointing in the direction opposite to that of the gravitational force. The density ρ and viscosity μ are continuous Heaviside functions:

$$\rho = \rho_a + H(\rho_w - \rho_a) \quad \text{and} \quad (2)$$

$$\mu = \mu_a + H(\mu_w - \mu_a), \quad (3)$$

where H is the Heaviside function, ρ_w is the density of water, ρ_a is the density of air, μ_w is the viscosity of water, and μ_a is the viscosity of air. We can nondimensionalize the above equations by using the following scale factors:

$$x^* = \frac{x}{D}, \quad z^* = \frac{z}{D}, \quad (4)$$

$$\mathbf{u}^* = \frac{\mathbf{u}}{U}, \quad (5)$$

$$t^* = \frac{t}{D/U}, \quad (6)$$

$$p^* = \frac{p}{\rho_a U^2}, \quad (7)$$

where x and z represent the spatial coordinates, D is the equivalent diameter, $\mathbf{u}$ is the velocity, U is the characteristic fall speed of the droplet, t is time, and p is the pressure. The superscript asterisk (*) denotes that the variables are

dimensionless. Substituting Eqs. (4) to (7) into the fundamental equation, Eq. (1) then becomes

$$\begin{aligned} \frac{\partial \rho^* \mathbf{u}^*}{\partial t^*} + \nabla^* \cdot \rho^* \mathbf{u}^* \mathbf{u}^* \\ = -\nabla^* p^* + \frac{1}{\text{Re}} \nabla^* \cdot [\mu^* (\nabla^* \mathbf{u}^* + \nabla^* \mathbf{u}^{*T})] + \frac{1}{\text{We}} \kappa \nabla H - \frac{H}{\text{Fr}} \hat{\mathbf{k}}, \end{aligned} \quad (8)$$

$$\text{Re} = \frac{\rho_a U D}{\mu_a}, \quad (9)$$

$$\text{We} = \frac{\rho_a U^2 D}{\sigma}, \quad (10)$$

$$\text{Fr} = \frac{\rho_a U^2}{g D (\rho_w - \rho_a)}, \quad (11)$$

$$R_\rho = \frac{\rho_w}{\rho_a}, \quad (12)$$

$$R_\mu = \frac{\mu_w}{\mu_a}, \quad (13)$$

where We is the Weber number, Fr is the Froude number, $\rho^* = 1 + H(R_\rho - 1)$, and $\mu^* = 1 + H(R_\mu - 1)$. The Froude and Weber numbers can be rewritten by using the Reynolds number, the Eötvös number $\text{Eo} = g D^2 \rho_a (R_\rho - 1) / \sigma$, and the Morton number $\text{Mo} = g \mu_a^4 (R_\rho - 1) / \rho_a \sigma^3$ as follows:

$$\text{We} = \text{Re}^2 \text{Eo}^{-1/2} \text{Mo}^{1/2} \quad (14)$$

$$\text{Fr} = \text{Re}^2 \text{Eo}^{-3/2} \text{Mo}^{1/2}. \quad (15)$$

By substituting Eqs. (14) and (15) into Eq. (8), we see that its solution is determined by five dimensionless numbers: Re , Eo , Mo , R_ρ , and R_μ . It is notable that only the Reynolds number includes the characteristic speed U . Since the free-falling motion is considered here, it is possible to find a correlation between the Reynolds number and the remaining four dimensionless numbers. For instance, we can take the characteristic speed U to be the Hadamard–Rybczynski ([Hadamard 1911](#); [Rybczynski 1911](#)) terminal velocity:

$$U = \frac{1}{18} \frac{g \rho_a D^2 (R_\rho - 1)}{\mu_a} \frac{1 + R_\mu}{2/3 + R_\mu}. \quad (16)$$

In this form of U , surface tension does not appear because the surfactants are not included and the droplet is assumed to be spherical in the Hadamard–Rybczynski solution. Substituting Eq. (16) into Eq. (9), the Reynolds number can be written as

$$\text{Re} = \frac{1}{18} \text{Eo}^{3/2} \text{Mo}^{-1/2} \frac{1 + R_\mu}{2/3 + R_\mu}. \quad (17)$$

This equation shows an explicit relationship between the Reynolds number and the physical properties of the ambient fluid and the droplet that can only be applied to the creeping motion of a spherical droplet. In general, there is no analytic solution to Eq. (1). Experiments or simulations must be performed to obtain the relationship.

b. Formulations in previous studies

It is commonly assumed in microphysics parameterizations that the terminal velocity is not sensitive to changes in water viscosity and surface tension coefficient. This assumption may be regarded as valid for the creeping flow since the percentage difference between Hadamard–Rybczynski and Stokes solutions is less than 1% under standard conditions. Under these assumptions, the two-phase flow equation can be replaced by the single-fluid Navier–Stokes equation describing flow around a free-falling solid object. As a result, only two dimensionless numbers remain. Any two numbers among the drag coefficient C_D , the Best number $Be = C_D Re^2$, and the Reynolds number can be chosen. This approach was adopted in the work of [Beard and Pruppacher \(1969\)](#). They assumed a unique functional relationship between the Reynolds number and the drag coefficient. This functional relationship is commonly approximated by a piecewise power-law function to simplify the calculation of the bulk terminal velocity:

$$C_D = aRe^b, \quad (18)$$

where a and b are constants that can be determined from experimental data. We can rewrite Eq. (18) to express the velocity in terms of physical properties:

$$U = \frac{2}{\sqrt{3}} \sqrt{\frac{gD\Delta\rho}{\rho_a C_D}}, \quad (19)$$

substituting Eq. (18) into Eq. (19), we get

$$U = \left(\frac{4g\Delta\rho}{3a}\right)^{1/(2+b)} \mu_a^{b/(2+b)} \rho_a^{-(1+b)/(2+b)} D^{(1-b)/(2+b)} \quad (20)$$

$$= AD^B.$$

If the densities and viscosities of air and water are given, the terminal velocity of water drops can be determined at any temperature and pressure by the above equation.

A much simpler assumption is used to account for the effect of air density on the terminal velocity in other microphysics parameterization schemes such as [Lin et al. \(1983\)](#), [Rutledge and Hobbs \(1984\)](#), and [Heymsfield et al. \(2007\)](#). In these microphysics schemes, the variables A and B in Eq. (20) are treated as constants, and the equation is multiplied by an extra correction term α to take into account the change in environmental conditions:

$$U = AD^B \alpha. \quad (21)$$

There is no unified parameterization of this correction term. In [Lin et al. \(1983\)](#), α is set to $(\rho_o/\rho)^{0.5}$. Here, the subscript o means that the variable is measured on the ground. This parameterization is based on the assumption that the drag coefficient and $\Delta\rho$ in Eq. (20) are constant for water drops of all diameters. In [Rutledge and Hobbs \(1984\)](#), the correction term is an approximated form of the empirical formula of [Foote and du Toit \(1969\)](#), $(p_o/p)^{0.4}$. In [Heymsfield et al. \(2007\)](#), the form is slightly modified to $(\rho_o/\rho)^{0.54}$, which is derived from the ensemble fall speed of snow crystals in ice clouds and has been

applied to drops in, for instance, CAM3 model ([Morrison and Gettelman 2008](#)). However, the drag coefficient is known to vary with the Reynolds number even under standard conditions ([Gunn and Kinzer 1949](#)), the accuracy of the empirical formula of [Foote and du Toit \(1969\)](#) has not been investigated (see below), and the validity of the correction term in [Heymsfield et al. \(2007\)](#) has not been explicitly demonstrated for drops. Without any sufficient theoretical or experimental justifications, these forms of the correction term are widely used in atmospheric models.

A more complicated empirical formulation of terminal velocity under nonstandard conditions was provided in [Foote and du Toit \(1969\)](#):

$$U_f(D) = U_0(D) 10^Y \left[1 + 0.0023 \left(1.1 - \frac{\rho}{\rho_0} \right) (T - T_0) \right]$$

$$Y = 0.43 \log_{10} \left(\frac{\rho_0}{\rho} \right) - 0.4 \left[\log_{10} \left(\frac{\rho_0}{\rho} \right) \right]^{2.5}, \quad (22)$$

where $U_0(D)$, $\rho_0(D)$, and T_0 are the terminal velocity, density, and temperature, respectively, at surface level. As the experimental procedure and data are unpublished, the accuracy of this formula is unclear and need to be examined rigorously.

In the next section, our numerical model and procedure to simulate free-falling water drops for the purpose of this study are concisely explained.

3. Numerical method

An improved immersed boundary method (IBM) ([Ong and Miura 2018, 2019](#)) was adopted for DNS of a free-falling water drop. This two-phase flow model was chosen because it has been shown to be robust even when the pressure difference due to surface tension and distortion of the interface are large ([Ong and Miura 2019](#)). The drop diameters considered in this study ranged from 0.025 to 0.5 mm. In particular, free-falling water drops of diameters 0.025, 0.05, 0.1, 0.2, 0.3, 0.4, and 0.5 mm with an initial velocity of zero were simulated. The reason for the lower limit of 0.025 mm is that creeping motion (Stokes flow) is a good assumption for droplet flow with diameters as small as 0.025 mm ([Gunn and Kinzer 1949](#)). The terminal velocity can be deduced analytically and compared with the numerical solution. Capillary oscillations appear to magnify and numerical solutions become unstable when the raindrop diameter is larger than approximately 0.5 mm.

When a water drop falls through the air in non-Stokes flow, it takes a relatively large distance, compared to its diameter, before the falling speed approaches its asymptotic value, i.e., the terminal velocity. To reduce the required size of the computational domain, the noninertial domain was used ([Komrakova et al. 2013; Ong and Miura 2019](#)). In the non-inertial domain, the drop is kept approximately at a fixed position by adjusting the inflow speed at the lower boundary such that the downward acceleration of the drop is zero, with the gravitational force acting vertically toward the lower boundary. Ideally, the domain size should be infinitely large so that the flow field around the drop is not affected by the existence of the wall. In our model, we set the computational

FIG. 1. Schematic diagrams of the computational grids used for the simulations of free-falling water drops with diameters of (a) 0.025 mm; (b) 0.05 mm; (c) 0.01, 0.02, 0.03, 0.04, and 0.05 mm.

domain size to be at least twice the thickness of the boundary layer (see appendix A for the validity of this choice). Based on the boundary layer theory of Tomotika (Tomotika 1935; Abraham 1970), the boundary layer thicknesses are approximately $25D$ and $10D$ for free-falling spheres with diameters of 0.025 and 0.05 mm, respectively. The thickness decreases as the diameter increases. Therefore, three different domain sizes were used for simulations of free-falling water drops with diameters of 0.025, 0.05, and above 0.1 mm, respectively, as shown in Fig. 1. Grid cells with the highest resolution were placed in the vicinity of the drop (yellow region in the figure). We set their grid size to $0.0125D$ in all simulations. All physical quantities were nondimensionalized. Further details regarding the numerical schemes can be found in Ong and Miura (2019).

The physical properties of the drop water and the surrounding air in the numerical experiments are given in Table 1. The viscosity of dry air at different temperatures was deduced from Eq. (3) in Kadoya et al. (1985). The viscosity of pure water was obtained based on the work of Sengers and Watson

(1986). The surface tension coefficient was interpolated from tables provided in the handbook of Lide (2005). The density of air was derived from the gas law, and the density of water was calculated from the interpolation equation of Tanaka et al. (2003).

As explained in the introduction, five sets of experiments were prepared to achieve the objectives of this study. These experiments and their corresponding dimensionless numbers are listed in Table 2. The diameter was nondimensionalized to 1 in all simulations. In the first set of experiments A–G, the environmental temperature and pressure were set to the standard conditions. We compared the simulated terminal velocity with the experimental data of Gunn and Kinzer (1949) to validate the numerical model. We also examined the convergence of the numerical solution to the Hadamard–Rybczynski solution at small Reynolds numbers, which is an important indication of whether the numerical solution can converge properly to the reference solution.

In the second set of experiments A1–G3, a fixed lapse rate of $-6.5^{\circ}\text{C km}^{-1}$ was assumed in order to observe the change in

TABLE 1. Physical properties of air and water at altitudes of 1000, 2000, and 3000 m assuming a fixed lapse rate of $-6.5^{\circ}\text{C km}^{-1}$, surface temperature of 20°C , and pressure of 1013 hPa. The density, viscosity, and surface tension coefficient are rounded to four significant figures.

Altitude (m)	Temperature ($^{\circ}\text{C}$)	Pressure (hPa)	Viscosity of air (Pa s)	Density of air (kg m^{-3})	Viscosity of water (Pa s)	Density of water (kg m^{-3})	Surface tension coefficient (N m^{-1})
0	20	1013.25	1.824×10^{-5}	1.204	1.002×10^{-3}	998.2	0.072 75
1000	13.5	900.493	1.792×10^{-5}	1.094	1.185×10^{-3}	999.3	0.073 71
2000	7	798.122	1.759×10^{-5}	0.9925	1.428×10^{-3}	999.9	0.074 65
3000	0.5	705.387	1.727×10^{-5}	0.8980	1.762×10^{-3}	999.9	0.075 57

TABLE 2. Summary of numerical experiments conducted in this work and the corresponding dimensionless numbers. All values are rounded to four significant figures.

Expt	Diameter (mm)	Altitude (m)	Temperature (°C)	Pressure (hPa)	Viscosity ratio	Density ratio	Eötvös number	Morton number
A	0.025	0	20	1013.25	54.95	829.0	8.400×10^{-5}	1.937×10^{-12}
B	0.05						3.360×10^{-4}	
C	0.1						1.344×10^{-3}	
D	0.2						5.376×10^{-3}	
E	0.3						1.210×10^{-2}	
F	0.4						2.150×10^{-2}	
G	0.5						3.360×10^{-2}	
A1	0.025	1000	13.5	900.493	66.14	913.1	8.300×10^{-5}	2.102×10^{-12}
A2	0.025	2000	7	798.122	81.16	1007.4	8.201×10^{-5}	2.289×10^{-12}
A3	0.025	3000	0.5	705.387	102.0	1113.4	8.102×10^{-5}	2.501×10^{-12}
B1	0.05	1000	13.5	900.493	66.14	913.1	3.320×10^{-4}	2.102×10^{-12}
B2	0.05	2000	7	798.122	81.16	1007.4	3.281×10^{-4}	2.289×10^{-12}
B3	0.05	3000	0.5	705.387	102.0	1113.4	3.241×10^{-4}	2.501×10^{-12}
C1	0.1	1000	13.5	900.493	66.14	913.1	1.328×10^{-3}	2.102×10^{-12}
C2	0.1	2000	7	798.122	81.16	1007.4	1.312×10^{-3}	2.289×10^{-12}
C3	0.1	3000	0.5	705.387	102.0	1113.4	1.296×10^{-3}	2.501×10^{-12}
D1	0.2	1000	13.5	900.493	66.14	913.1	5.312×10^{-3}	2.102×10^{-12}
D2	0.2	2000	7	798.122	81.16	1007.4	5.249×10^{-3}	2.289×10^{-12}
D3	0.2	3000	0.5	705.387	102.0	1113.4	5.185×10^{-3}	2.501×10^{-12}
E1	0.3	1000	13.5	900.493	66.14	913.1	1.195×10^{-2}	2.102×10^{-12}
E2	0.3	2000	7	798.122	81.16	1007.4	1.181×10^{-2}	2.289×10^{-12}
E3	0.3	3000	0.5	705.387	102.0	1113.4	1.167×10^{-2}	2.501×10^{-12}
F1	0.4	1000	13.5	900.493	66.14	913.1	2.125×10^{-2}	2.102×10^{-12}
F2	0.4	2000	7	798.122	81.16	1007.4	2.100×10^{-2}	2.289×10^{-12}
F3	0.4	3000	0.5	705.387	102.0	1113.4	2.074×10^{-2}	2.501×10^{-12}
G1	0.5	1000	13.5	900.493	66.14	913.1	3.320×10^{-2}	2.102×10^{-12}
G2	0.5	2000	7	798.122	81.16	1007.4	3.281×10^{-2}	2.289×10^{-12}
G3	0.5	3000	0.5	705.387	102.0	1113.4	3.241×10^{-2}	2.501×10^{-12}
H	0.32	0	20	1013.25	54.95	829.0	1.376×10^{-2}	1.937×10^{-12}
I	0.315						1.334×10^{-2}	
J	0.31						1.292×10^{-2}	
K	0.305						1.250×10^{-2}	
L	0.295						1.170×10^{-2}	
M	0.29						1.150×10^{-2}	
N	0.285						1.092×10^{-2}	
O	0.28						1.054×10^{-2}	
P	0.495						3.293×10^{-2}	
Q	0.49						3.227×10^{-2}	
R	0.485						3.161×10^{-2}	
S	0.48						3.096×10^{-2}	

the terminal velocity when water drops fall at different altitudes. Free-falling water drops at three specific altitudes of 1000, 2000, and 3000 m were simulated, with corresponding environmental temperatures of 13.5°, 7°, and 0.5°C, respectively. The ratio of the simulated terminal velocity aloft to terminal velocity under standard conditions for each drop was then compared with existing empirical formulas [Eqs. (21) and (22)].

In the third set of experiments H–S, two separate $\text{Re}-C_D$ power-law curves [Eq. (18)] were obtained within the diameter ranges of 0.28–0.32 mm and 0.48–0.5 mm, respectively. Two more similar curves were obtained from experiments E1–E3 and G1–G3. Then, the variation in the exponent b of deduced from these four curves were compared to examine the error in the computation of terminal velocity aloft when a fixed $\text{Re}-C_D$ curve is assumed.

Experiment G was repeated with the raindrop initially perturbed to an oblate spheroidal shape of eccentricity 0.8062 to study the influence of the oscillatory motion. The diameters along the long a and short c semiaxes were $1.3D$ and $D/1.3$, respectively. The grid and four dimensionless numbers remained unchanged. Finally, experiments E–G were repeated with σ artificially reduced by factors of 2, 10, and 100 ($\sigma = 3.638 \times 10^{-2}$, 7.275×10^{-3} , and $7.275 \times 10^{-4} \text{ N m}^{-1}$) to examine the impact of the shape deformation on the terminal velocity and tangential velocity in the fifth set of experiments. σ in experiment E of a free-falling 0.3 mm raindrop was further reduced to $7.275 \times 10^{-5} \text{ N m}^{-1}$ by a factor of 1000 to gain insight into the influence of large deformation on the terminal velocity. Note that the viscosity and density ratios are unaffected by simply changing σ ,

FIG. 2. Falling speed of free-falling water drops plotted against (a) falling distance and (b) falling time under standard conditions. The plus signs are simulation results. The solid curves are predicted by the empirical formula of Wang and Pruppacher (1977). It should be noted that all drops fell from rest (initial velocity is zero) in simulations, and the falling distance and time were obtained by measuring the distance traveled and time elapsed by the drops.

while the Eötvös and Morton numbers are modified according to their definition.

4. Results

a. Terminal velocity at 20°C and 1013 hPa

The falling speed profiles of water drops computed from experiments A–G, with diameters ranging from 0.025 to 0.5 mm, are shown in Fig. 2. All numerical solutions asymptotically reached steady state solutions. Note that the falling distance traveled until the terminal velocity was reached was less than about 1.5 m for a 0.5 mm raindrop and, as expected, was smaller for smaller drops. The time required for these drops to reach their terminal velocity was less than 1 s. The conversion formula from dimensionless time to physical time can be found in the appendix of Ong and Miura (2018).

For comparison, the falling speed profiles predicted by the empirical formula of Wang and Pruppacher (1977) are drawn as solid curves on the same figure. Clearly, the growth pattern of simulated falling speeds agreed with the empirical formula. Let $Z_{99\%}$ and $T_{99\%}$ respectively be the distance and time traveled by a water drop to reach its 99% terminal velocity from rest. Table 3 compares $Z_{99\%}$ and $T_{99\%}$ obtained from current simulation results and the empirical formula. The simulation results were in good agreement with the empirical formula when

$D \geq 0.1$ mm, with the maximum percentage difference of 5.1%. However, the percentage difference was larger than 9% when $D = 0.025$ and 0.05 mm. Considering that the maximum error between the simulated terminal velocity and Beard's (1976) terminal velocity was only 3.2%, this large percentage difference was possibly due to the slow acceleration of small drops. In other words, a small change in the terminal velocity leads to large changes in $Z_{99\%}$ and $T_{99\%}$.

Figure 3 shows a plot of the terminal velocity against the drop diameter. A comparison between the numerical and interpolated experimental data of the terminal velocity with various parameterizations (Gunn and Kinzer 1949; Beard 1976; Atlas et al. 1973) is summarized in Table 4. The discrepancy from Gunn and Kinzer's experimental data was below 4.5% for diameters larger than 0.3 mm, increasing rapidly to more than 10% for a 0.1 mm water drop. There are two main reasons for this overestimation and the decreasing trend of the discrepancy with the diameter. The first reason is discretization error, which is the main source of error in numerical methods. It is shown in appendix B that the magnitude of the error is 1.9%. The second reason is measurement error in Gunn and Kinzer's experiment. According to the speculation of Beard and Pruppacher (1969), the drops might have evaporated slightly before their diameters were determined at the end of fall because the relative humidity of the air in their experiment

TABLE 3. A comparison of the simulated falling distance ($Z_{99\%}$) and falling time ($T_{99\%}$) obtained from numerical simulations and the empirical formula of Wang and Pruppacher (1977). $Z_{99\%}$ ($T_{99\%}$) is defined as the distance (time) traveled until the drop reaches its 99% terminal velocity. The value in parentheses beside each numerical result denotes its percentage difference from the corresponding value predicted by the empirical formula. For comparison, numerical time steps are also shown in the second column of the table. We decreased the time step for smaller drops to avoid unstable solutions because smaller drops have larger water surface tension and higher capillary wave speeds.

Diameter (mm)	Δt (s)	Numerical results		Wang and Pruppacher (1977)	
		$Z_{99\%}$ (m)	$T_{99\%}$ (s)	$Z_{99\%}$ (m)	$T_{99\%}$ (s)
0.025	2.746×10^{-8}	1.892×10^{-4} (50.71%)	1.208×10^{-2} (40.70%)	1.255×10^{-4}	8.588×10^{-2}
0.05	1.165×10^{-7}	2.072×10^{-3} (11.83%)	3.589×10^{-2} (9.059%)	1.852×10^{-3}	3.291×10^{-2}
0.1	4.394×10^{-7}	2.026×10^{-2} (0.5944%)	1.055×10^{-1} (0.05778%)	2.038×10^{-2}	1.055×10^{-1}
0.2	1.243×10^{-6}	1.331×10^{-1} (5.104%)	2.573×10^{-1} (2.344%)	1.403×10^{-1}	2.634×10^{-1}
0.3	1.712×10^{-6}	3.434×10^{-1} (4.770%)	4.041×10^{-1} (1.926%)	3.606×10^{-1}	4.121×10^{-1}
0.4	1.758×10^{-6}	6.511×10^{-1} (2.986%)	5.510×10^{-1} (0.7303%)	6.711×10^{-1}	5.551×10^{-1}
0.5	2.456×10^{-6}	1.060×10^0 (0.6856%)	7.001×10^{-1} (0.7162%)	1.068×10^0	6.951×10^{-1}

was only 50%. Therefore, the terminal velocity was possibly overestimated, especially for small drops.

A much better agreement was achieved when comparing the simulated terminal velocities with Beard (1976) parameterization. The overall discrepancy was below 3.2%. Because it is known that Atlas et al. (1973) formula has a better accuracy when the raindrop diameter is larger than ~ 0.5 mm, a large discrepancy from the simulated terminal velocities was observed.

The red dashed line in Fig. 3 represents the terminal velocity predicted by the Hadamard–Rybczynski solution of the creeping flow of a spherical droplet in a uniform flow. The discrepancies between the numerical results and these theoretical values are also summarized in Table 4. We see that the discrepancy of the simulated terminal velocity from the Hadamard–Rybczynski solution Δ_{HR} was below 1.9% when the diameter was as small as 0.025 mm. The creeping flow assumption for a spherical droplet is approximately valid for diameters smaller than 0.1 mm. Note that the number of grid cells is almost the same in the experiments A and B, whereas the domain is larger in the experiment A. This might have led to a larger discretization error in the numerical solution of the experiment A. The Stokes solution for a free-falling spherical solid object is also shown in the last two columns of Table 4. The discrepancy between the Stokes velocity and simulated terminal velocity is larger than Δ_{HR} because of the nonzero viscous stress exerted from the internal circulation.

The wake and internal circulation of water drops with diameters 0.025, 0.4, and 0.5 mm at terminal velocity are shown in Fig. 4. At low Reynolds numbers, there was no visible wake or separation of the boundary layer behind the droplet (Fig. 4a), and thus the Hadamard–Rybczynski solution may be valid with good accuracy. A stationary wake appeared behind 0.4 and 0.5 mm raindrops with moderate Reynolds numbers. Moreover, the axis ratio of the 0.5 mm raindrop, which is defined as the ratio of the semimajor axis over the semiminor axis, increased merely to 1.001.

We further examined quantitatively the maximum and averaged tangential velocity on drop surface, eddy's location inside the drop, and wake's size behind the drop. These data are compiled in Table 5. All angles are measured from the front

stagnation point, and a velocity vector is positive if it points rearward. At low Reynolds numbers, the center of the internal circulation and maximum tangential velocity were located approximately on the plane perpendicular to the falling direction. This is in accordance with the Hadamard–Rybczynski theory. The internal circulation was pushed forward (into falling direction) slightly as the Reynolds number increased, and its distance from the drop center was nearly constant. These results are in agreement with the simulation results of a spherical water sphere by LeClair et al. (1972). A similar trend was observed for the maximum tangential velocity, with its angular position decreasing from 67.15° when $Re = 8.88$ to 57.21° when $Re = 65.4$. Nevertheless, LeClair et al. (1972) did not observe a notable change in the location of the maximum tangential velocity in their simulations when the Reynolds

FIG. 3. Plot of terminal velocity against drop diameter under standard conditions. The solid black line was obtained from the experimental work of Gunn and Kinzer (1949). The dotted line and dashed line represent Beard (1976) and Atlas et al. (1973) parameterizations of terminal velocity, respectively. The red dashed line was computed from the Hadamard–Rybczynski solution (Hadamard 1911; Rybczynski 1911).

TABLE 4. Terminal velocities of axisymmetric water drops of different diameters obtained from the numerical simulations and theoretical solution, and their discrepancies. U_H and U_S are the Hadamard–Rybczynski and Stokes terminal velocities, respectively. The discrepancies from Gunn and Kinzer (1949) and the empirical formula of Beard (1976) are shown in the third and fourth columns. The terminal velocities of Gunn and Kinzer (1949) at these particular diameters were estimated by direct interpolation of their data points. The smallest drop they measured had a diameter of 0.0783 mm, and thus the simulated terminal velocities of drops with diameters of 0.025 and 0.05 mm were not compared with their data. All values are rounded to four significant figures.

Diameter (mm)	Numerical results U (m s^{-1})	Discrepancy from Gunn and Kinzer (1949) (%)	Discrepancy from Beard (1976) (%)	U_H (m s^{-1})	$ U - U_H /U_H \times$ 100 (%)	U_S (m s^{-1})	$ U - U_S /U_S$ $\times 100$ (%)
0.025	1.908×10^{-2}	—	3.156	1.873×10^{-2}	1.886	1.862×10^{-2}	2.497
0.05	7.367×10^{-2}	—	2.581	7.491×10^{-2}	1.653	7.447×10^{-2}	1.064
0.1	2.478×10^{-1}	14.64	2.237	2.997×10^{-1}	17.30	2.979×10^{-1}	16.80
0.2	6.726×10^{-1}	6.336	2.804	1.199×10^0	43.91	1.191×10^0	43.57
0.3	1.109×10^0	4.524	2.913	2.697×10^0	58.87	2.681×10^0	58.62
0.4	1.545×10^0	3.803	2.391	4.794×10^0	67.79	4.766×10^0	67.59
0.5	1.981×10^0	3.572	1.604	7.491×10^0	73.56	7.447×10^0	73.40

number is between 10 and 100. Note that the wake started to appear when the diameter was larger than 0.3 mm.

Figure 5a shows the ratios of tangential velocities $v_{i_{75^\circ}}$, $v_{i_{90^\circ}}$, $v_{i_{150^\circ}}$, and $v_{i_{165^\circ}}$ at 75° , 90° , 160° , and 175° , respectively, to the terminal velocity. It can be seen that $v_{i_{90^\circ}}/U$ (black curve) approached the value predicted by the Hadamard–Rybczynski theory, indicating the convergence of the numerical solution of the internal circulation to the Hadamard–Rybczynski solution at small Reynolds numbers. However, tangential velocities predicted by LeClair et al. (1972) are significantly lower than the current results. Szakáll et al. (2009) also compared their experimental data of the averaged tangential velocity with the simulation results of LeClair et al. (1972). Based on their Fig. 11, the results of LeClair et al. (1972) appear to be slightly underestimated when $D < 1$ mm.

Interestingly, the wake that appeared behind the 0.4 and 0.5 mm raindrops was not strong enough to cause a flow separation on the surface, because $v_{i_{150^\circ}}/U$ and $v_{i_{165^\circ}}/U$ (blue and green curves in Fig. 5a) still remained positive. Therefore, the

secondary internal circulation did not appear in the raindrop. This is in line with the observation by LeClair et al. (1972) that a flow separation did not occur when the Reynolds number is below 100. Figure 5b is a plot of the dimensionless tangential velocity in terms of angles. We again can see that the symmetric pattern of the tangential velocity for the 0.025 mm cloud drop indicates the solution converging to the Hadamard–Rybczynski solution. An inflection point appeared when the $D \geq 0.3$ mm due to the presence of the wake.

b. Terminal velocity aloft

With the assumption of a fixed lapse rate of $-6.5^\circ\text{C km}^{-1}$, the ratio of the terminal velocity aloft to the terminal velocity under standard conditions, R_{tv} , at the three altitudes of 1000, 2000, and 3000 m obtained from simulations A1–G3 are summarized in Table 6. The terminal velocity increased with the altitude, mainly because the air density was lower at higher altitudes while the density of water was nearly unchanged. R_{tv} increased with diameters when $D < 0.3$ mm; however,

FIG. 4. Streamline plots of free-falling water drops with diameters of (a) 0.025, (a) 0.4, and (b) 0.5 mm at terminal velocity. The red dot represents the location of the center of the internal circulation.

TABLE 5. Numerical data of the averaged tangential velocity on drop surface ($v_{i,ave}$), ratio of maximum tangential velocity on drop surface ($v_{i,max}$) to the terminal velocity (U), location of the center of internal circulation, and wake's size. All angles are measured from the front stagnation point. R_{eddy} is the distance from the center of internal circulation to the drop center and θ_{eddy} is its angular position. L_{wake} is the vertical length of the wake and θ_{wake} is its minimum angular position on the interface.

Diameter (mm)	Reynolds number	$v_{i,ave}$ (cm s ⁻¹)	$v_{i,max}/U$	$\theta(v_{i,max}/U)$ (°)	$R_{eddy}/0.5D$	θ_{eddy} (°)	L_{wake}/D	θ_{wake} (°)
0.025	0.031 46	1.121×10^{-2}	9.026×10^{-3}	91.43	0.3563	88.99	—	—
0.05	0.2432	4.481×10^{-2}	9.364×10^{-3}	85.71	0.3563	88.99	—	—
0.1	1.636	1.783×10^{-1}	1.128×10^{-2}	75.72	0.3567	86.99	—	—
0.2	8.880	6.899×10^{-1}	1.746×10^{-2}	67.15	0.3506	78.69	—	—
0.3	21.98	1.484	2.434×10^{-2}	62.89	0.3563	74.74	—	—
0.4	40.80	2.509	3.104×10^{-2}	57.19	0.3564	77.15	0.3684	151.5
0.5	65.40	3.707	3.681×10^{-2}	57.21	0.3684	75.26	0.6550	141.3

it remained almost constant when $D \geq 0.3$ mm. The increment of the terminal velocity was 4.6% for a 0.025 mm cloud drop, and it grew to 10.1% for a 0.5 mm raindrop at 3000 m. Comparison with existing parameterizations of the correction term in Eq. (21) is also shown in the same table.

It is evident that a simple correction term consisting of the density or pressure ratio with a fixed exponent, or even the more complicated empirical formula of Foote and du Toit (1969), is not an ideal solution for the accurate prediction of the terminal velocity at arbitrary temperatures and pressures. The discrepancy between simulated R_{tv} and existing parameterizations generally increased with altitude. It is also clear that this discrepancy became larger as the Reynolds number decreased toward the range in which the Hadamard–Rybczynski solution is valid. The largest discrepancy was about 10% at an altitude of 3000 m for cloud drops with diameters smaller than 0.05 mm. It can be shown that R_{tv} is linearly proportional to the ratio of the density difference in

the creeping flow, whereas the exponent of the density or pressure ratio in the parameterizations considered in this study is between 0.4 and 0.54. The underestimation of this exponent thus leads to a large discrepancy for low Reynolds numbers.

The empirical formula of Foote and du Toit (1969) [Eq. (22)] yields values of the terminal velocity aloft with the smallest discrepancies. However, the discrepancy is still larger than 4% at the high altitude of 3000 m. Even though the discretization error in the simulated terminal velocity is 1.9%, the ratio of the terminal velocities is relatively insensitive to the grid refinement operation (see appendix B). Thus, a discrepancy as small as 4% cannot be solely due to numerical errors. It is likely that this discrepancy is a result of imperfections in the empirical formula.

c. New empirical formula

As explained in section 2b, a more rigorous way to predict the terminal velocity aloft is through the functional relationship

FIG. 5. (a) Plot of dimensionless tangential velocity on the drop surface at 75° (red solid line), 90° (black solid line), 150° (blue solid line), and 165° (green solid line). Numerical data of LeClair et al. (1972) computed at Reynolds numbers 10, 30, and 57 are shown as cross markers. Their colors correspond to the colors of the lines obtained from the current simulation results. Theoretical tangential velocity at 90° predicted by the Hadamard–Rybczynski theory is shown as dotted line. (b) Plot of dimensionless tangential velocity against angle at terminal velocity under standard conditions.

TABLE 6. Ratios R_{tv} of the terminal velocity at altitudes of 1000, 2000, and 3000 m with a fixed lapse rate of $-6.5^\circ\text{C km}^{-1}$ over the terminal velocity at the surface (standard conditions). $\Delta_1 = [(\rho_0/\rho)^{0.5} - R_{tv}]/R_{tv} \times 100\%$, $\Delta_2 = [(P_0/P)^{0.4} - R_{tv}]/R_{tv} \times 100\%$, $\Delta_3 = [(\rho_0/\rho)^{0.54} - R_{tv}]/R_{tv} \times 100\%$, and $\Delta_4 = [(U_f/U_0) - R_{tv}]/R_{tv} \times 100\%$. Δ_5 is the percentage error of the proposed parameterization in which the velocity is calculated from Eq. (23), and the exponent b is taken from Table 7. All values are rounded to four significant figures.

Diameter (mm)	Altitude (m)	R_{tv}	Δ_1 (%)	Δ_2 (%)	Δ_3 (%)	Δ_4 (%)	Δ_5 (%)
0.025	1000	1.017	3.184	3.123	3.579	2.756	0.2515
	2000	1.033	6.589	6.463	7.417	5.825	0.5036
	3000	1.046	10.49	10.29	11.79	9.379	0.9798
0.05	1000	1.019	2.976	2.916	3.370	2.549	0.04944
	2000	1.037	6.211	6.086	7.036	5.449	0.1471
	3000	1.056	9.651	9.453	10.94	8.548	0.2124
0.1	1000	1.024	2.398	2.338	2.790	1.973	0.2915
	2000	1.049	4.970	4.846	5.785	4.217	0.5772
	3000	1.074	7.783	7.589	9.055	6.699	0.8182
0.2	1000	1.030	1.812	1.753	2.202	1.390	0.2850
	2000	1.062	3.747	3.624	4.552	3.003	0.5729
	3000	1.094	5.822	5.632	7.071	4.758	0.8613
0.3	1000	1.032	1.615	1.555	2.004	1.193	0.09611
	2000	1.066	3.330	3.208	4.132	2.589	0.1987
	3000	1.101	5.161	4.972	6.402	4.104	0.3060
0.4	1000	1.033	1.583	1.523	1.972	1.161	0.005877
	2000	1.067	3.250	3.128	4.051	2.509	0.02848
	3000	1.103	5.014	4.825	6.254	3.958	0.06857
0.5	1000	1.032	1.660	1.600	2.049	1.238	0.06310
	2000	1.065	3.395	3.272	4.197	2.653	0.09701
	3000	1.101	5.211	5.022	6.453	4.153	0.09655

between the Reynolds number and the drag coefficient, as was done in the previous studies of Beard and Pruppacher (1969), Beard (1976), and Böhm (1992). This assumption is based on the approximation that the dynamical effect of the internal circulation on the outer flow can be safely neglected (i.e., the water drop is treated as a solid object), so a one-to-one relationship exists between these two dimensionless numbers at terminal velocity. From Eq. (20), R_{tv} can then be expressed by the following equation:

$$R_{tv} = \left[\frac{\Delta\rho}{\Delta\rho_0} \left(\frac{\mu}{\mu_0} \right)^b \left(\frac{\rho_0}{\rho} \right)^{1+b} \right]^{1/(2+b)}. \quad (23)$$

The error introduced by this approximation can be demonstrated by comparing $\text{Re}-C_D$ curves generated from the numerical experiments H–O, in which only the diameter was slightly perturbed to values between 0.28 and 0.32 mm, and experiments E1–E3, in which the 0.3 mm raindrop fell at different altitudes. The exponent b in Eq. (23), or Eq. (18), is the gradient of the $\text{Re}-C_D$ curve on a logarithmic scale. A plot of $\text{Re}-C_D$ curves obtained from these experiments is shown in Fig. 6a. These two curves diverge, and thus b is not of the same value. By performing a least squares fitting of the power law [Eq. (18)] to each curve, b is equal to -6.265×10^{-1} and -7.211×10^{-1} for the solid (experiments E1–E3) and dashed (experiments H–O) curves, respectively. This amounts merely to about 1% error in the estimation of the terminal velocity at the altitude of 3000 m. Recall that the discretization error in the simulated terminal velocity is around 1.9%, the error above is considered small. Nevertheless, The error increased gradually to about 1.5% for the 0.5 mm raindrop at

3000 m (Fig. 6b). $b = -5.767 \times 10^{-1}$ as deduced from the numerical results of experiments P–S and $b = -7.252 \times 10^{-1}$ from experiments G1–G3.

In fact, the error in the estimation of the terminal velocity by treating a water drop as a solid object increases with Reynolds number when $D \leq 0.5$ mm. This can be observed in Figs. 7a and 7b. Figure 7a is a plot of the $\text{Re}-C_D$ linear spline curve by joining the Reynolds numbers and drag coefficients at terminal velocity obtained from numerical experiments A–G under standard conditions. Since b is a gradient of the $\text{Re}-C_D$ curve in logarithmic scale, we can estimate its value by calculating the gradient of line segments at $D = 0.05, 0.1, 0.2, 0.3, 0.4$, and 0.5 mm. Let us denote this approximation of b from the curve in Fig. 7a by b' . On the other hand, the true value of b for each diameter can be derived from the gradient of the least squares fitting curves obtained in the experiments A1–G3 (e.g., see the dashed lines in Fig. 6). Note that b' is a rather crude approximation because the data points are scarce. However, the motivation here is to illustrate the trend and compare it with the true values of b . This comparison is shown in Fig. 7b.

The linear spline curve of b in Fig. 7b can be used to create a new parameterization of the correction term α in Eq. (21) with improved accuracy by replacing α with the right-hand side of Eq. (23). The downside of using this complicated correction term is that it cannot be integrated analytically over the entire mass spectrum to obtain the total mass flux in closed form. An alternative approach is to approximate the exponent b by a piecewise constant function of the diameter by taking averaged values between all adjacent data values in Fig. 7b. The new piecewise b function is presented in Table 7. The maximum error in the estimation of the terminal velocity aloft at each

FIG. 6. Plots of the drag coefficient against the Reynolds number. (a) The solid line represents the results of numerical experiments H–O in which the diameter varied between 0.28 and 0.32 mm while other physical constants were kept constant. The dashed line represents the results of numerical experiments E1–E3 in which 0.3 mm raindrops fell at altitudes of 1000 m (13.5°C), 2000 m (7.0°C), and 3000 m (0.5°C). (b) The solid line represents the results of experiments P–S with the diameter varying from 0.48 to 0.5 mm, and the dashed line is interpolated from the results of experiments G1–G3 in which 0.5 mm raindrops fell at altitudes of 1000 m (13.5°C), 2000 m (7.0°C), and 3000 m (0.5°C).

diameter is shown in Table 6. We see that the maximum error is smaller than 1% for all diameters. The new piecewise function can thus be a good replacement for a smooth b function with the advantage of straightforward integration over the mass spectrum. Nevertheless, this alternative approach is not without clear disadvantages. The terminal velocity predicted from

the piecewise function becomes discontinuous and may produce numerical errors (Khvorostyanov and Curry 2002).

Up to this point, only discrepancies at specific diameters were considered. Whereas in the dynamics calculation of atmospheric models, a statistical distribution of cloud drops and raindrops is defined and bulk quantities are used to predict prognostic

FIG. 7. (a) Plots of the drag coefficient against the Reynolds number. The dots are the numerical solutions of experiments A–G at terminal velocity. The exponent b' of the C_D – Re curve was roughly estimated from adjacent data points. For instance, b for experiment B is $b_B = (\ln C_{D_B} - \ln C_{D_A}) / (\ln Re_B - \ln Re_A)$, where the subscripts A and B denote experiments A and B, respectively. (b) The dashed curve represents exponent b computed from least squares fitting of the C_D – Re curve for each diameter in experiments A1–G3; the solid curve is a linear spline curve interpolating b' estimated in (a).

TABLE 7. Piecewise function of the exponent b .

Diameter range (mm)	Approximated b
<0.05	-1.000×10^0
0.05–0.1	-9.609×10^{-1}
0.1–0.2	-8.425×10^{-1}
0.2–0.3	-7.481×10^{-1}
0.3–0.4	-7.147×10^{-1}
0.4–0.5	-7.168×10^{-1}

variables. Instead of a discrepancy at a particular diameter, discrepancies in the bulk mass flux $[\int_0^{0.5\text{ mm}} U(D)m(D)N(D)dD]$, where $m(D)$ and $N(D)$ are the individual drop mass and number concentration density distribution, respectively] are a more useful guide to measure the impact of inaccurate values of the terminal velocity. To investigate discrepancies in the bulk mass flux, the normalized gamma distribution defined by Testud et al. (2001) was adopted. Typical values of the median drop diameter ($D_m = 1$ mm), shape parameter, and liquid water content (LWC = 0.5 g m^{-3}) for stratiform precipitation (Thompson et al. 2015) were used. Since the maximum diameter examined by the axisymmetric model is 0.5 mm, we set the upper limit of the integral equal to 0.5 mm. Bulk mass fluxes were computed at an altitude of 3000 m with three parameterizations of the correction term: 1) $(\rho_0/\rho)^{0.5}$, 2) Eq. (22), and 3) Eq. (23) with the proposed parameters in Table 7. The results are compiled in Table 8. We see that the discrepancy between the existing and proposed parameterizations varies between 5.1% and 4.1%. The magnitude of the discrepancy is literally determined by the discrepancy in the estimation of the terminal velocity aloft of larger drops (≥ 0.3 mm), because the total mass and terminal velocity of small drops (< 0.3 mm) is much less than that of larger drops in the gamma distribution.

d. The effect of shape on the terminal velocity

The free-falling water drops in all scenarios provided in Table 2 maintained an approximate spherical shape at terminal velocity (see Fig. 4). It was mentioned in the introduction that shape oscillation and deformation due to reduction in the surface tension coefficient are another factors that might affect the falling speed profile and internal circulation. The free-falling 0.5 mm raindrop simulation with its initial shape deformed to an spheroidal shape was performed to quantitatively study the influence of the oscillatory motion to the falling motion.

Figure 8 shows the time series of the axis ratio a/c , where a (c) is the length of the semimajor (semiminor) axis, and the velocity fluctuation, which is defined as the difference between

the falling speeds in experiment G and this experiment. The periodic oscillation was due to capillary waves caused by the surface tension force. This high-frequency wave was quickly dampened within 0.1 s. The maximum magnitude of the velocity fluctuation was only on the order of $1 \times 10^{-4} \text{ ms}^{-1}$. Therefore, the influence of shape oscillation on the falling speed was insignificant. Note that a negative bias was observed after the velocity fluctuation flattened out. This occurred because the boundary of the raindrop was approximated by linear splines in the IBM, which led to a small difference in the raindrop mass due to different initial shapes. Nevertheless, the bias was much less than the maximum velocity fluctuation caused by the oscillation.

In the sensitivity test of the surface tension coefficient, 0.3, 0.4, and 0.5 mm raindrops with reduced surface tension were simulated to examine the influence of shape deformation on the terminal velocity and tangential velocity. The relationship between the ratio of the terminal velocity of raindrops with reduced surface tension to the original terminal velocity and the axis ratio is illustrated in Fig. 9a. There are two notable physical properties we can deduce from this figure. First, the terminal velocity is weakly dependent on the shape. The terminal velocity decreased by less than 1% when the axis ratio increased by more than 40% in the 0.3 mm raindrop simulation where the surface tension coefficient was reduced by a factor of 1000. For the 0.5 mm raindrop, a 12% increase in the axis ratio resulted in only about a 1.3% decrease in the terminal velocity. Therefore, shape is a relatively minor factor in the determination of the terminal velocity for small raindrops. This agrees with the conclusion drawn from the previous result of the free-falling oscillatory raindrop simulation that large deformation does not significantly change the falling speed profile. Second, the change in the terminal velocity increases with the diameter. In other words, the terminal velocity of larger raindrops is more sensitive to the shape deformation.

In the real atmosphere, σ rarely ever decreases by more than a factor of 2 for precipitation water (Seidl and Hänel 1983). Whereas the reduction in the terminal velocity is only about 0.5% when σ is reduced by as much as a factor of 10 based on the current results. Therefore, the effect of reduced surface tension can be safely neglected for small raindrops of diameter up to 0.5 mm.

From Fig. 9b, we see that v_t/U , the ratio of averaged tangential velocity on the raindrop surface to the terminal velocity, is linearly proportional to the axis ratio. Interestingly, this proportionality remained linear even when the axis ratio was as large as 1.4. The tangential velocity increased as raindrop

TABLE 8. Bulk mass fluxes of drops smaller than 0.5 mm in normalized gamma distribution calculated using the terminal velocity aloft with the correction terms (1) $(\rho_0/\rho)^{0.5}$, (2) Eq. (22), and (3) Eq. (23) denoted by f_1^M , f_2^M , and f_3^M , respectively. The unit of the mass flux is $\text{g m}^{-2} \text{ s}^{-1}$. Since the upper limit is 0.5 mm, these are only partial mass fluxes. The mass flux of drops smaller than 0.5 mm is approximately 44% (2%) of total mass flux when $\mu = 1$ (4) if the terminal velocity of Beard (1976) is used for large raindrops. D_m is the mean drop diameter, LWC is the liquid water content, and μ is the shape parameter of the gamma distribution. D_{peak} is the diameter at which the peak of the distribution is found.

D_m (mm)	D_{peak} (mm)	LWC (g m^{-3})	μ	f_1^M	f_2^M	f_3^M	$ f_3^M - f_1^M /f_3^M \times 100$ (%)	$ f_3^M - f_2^M /f_3^M \times 100$ (%)
1	0.2	0.5	1	0.9568	0.9472	0.9099	5.155	4.097
1	0.5	0.5	4	0.049 07	0.048 58	0.046 68	5.121	4.064

FIG. 8. (a) The time series of the axis ratio (see the main text for the definition) of a 0.5 mm, free-falling raindrop with an initial oblate spheroidal shape of eccentricity 0.8062. (b) The time series of the velocity fluctuation (see the main text) of the same raindrop. The output data of the velocity fluctuation was taken at an interval of 3.681×10^{-3} s.

shape became more elliptic because the pressure difference between the front and rear stagnation points increased while the flow remained laminar (Fig. 10). The center of the internal circulation was stretched outward and rearward. The angular position of the maximum tangential velocity was also pushed rearward. Since the raindrop shape became elongated in the direction perpendicular to the falling direction, the wake subsequently increased to a larger size. This can be seen by

comparing the wake's size in Figs. 4b,c and 10b,c. Furthermore, the front part of small raindrops tended to be distorted into a U shape when the surface tension was weak (Fig. 10a).

5. Conclusions

The terminal velocity of water drops under standard conditions has been investigated in previous experimental studies.

FIG. 9. (a) Plot of the ratio of the terminal velocity with reduced surface tension to the terminal velocity under standard conditions. The legend is shown in (b). (b) Plot of averaged v_i/U against the axis ratio, where v_i is the tangential velocity on the drop surface. Dashed line: 0.3 mm; dotted line 0.4 mm; solid line: 0.5 mm. Triangle, square, circle, and asterisk markers denote that the surface tension coefficient was reduced by a factor of 2, 10, 100, and 1000, respectively.

FIG. 10. Streamline plots of (a) 0.3 mm raindrop with $\sigma = 7.275 \times 10^{-5} \text{ N m}^{-1}$, (b) 0.4 mm raindrop with $\sigma = 7.275 \times 10^{-4} \text{ N m}^{-1}$, and (c) 0.5 mm raindrop with $\sigma = 7.275 \times 10^{-4} \text{ N m}^{-1}$ at terminal velocity.

In the present study, the IBM was employed to study the terminal velocity and shape of free-falling water drops numerically at temperatures and pressures that were not restricted to standard conditions. The validity of the IBM as a robust numerical model for free-falling water drop simulations was demonstrated by comparing the simulated terminal velocity of drops smaller than 0.5 mm with the experimental data of Gunn and Kinzer (1949) and the parameterization of Beard (1976). The simulated terminal velocity agreed with the experimental data to within 4.5% when the diameter was larger than 0.3 mm. The discrepancy grew to 14.64% when the diameter was 0.1 mm, and it was possibly due to a combination of errors in simulations (discretization and wall effect) and those in laboratory measurements (evaporation of drops). Nevertheless, the numerical solution agreed with the Hadamard–Rybczynski solution to within 2% for diameters smaller than 0.05 mm. Moreover, the discrepancy with Beard (1976) parameterization was below 3.2% for all diameters examined in this study.

Falling speed profiles and flow geometry were also compared with Wang and Pruppacher (1977) and LeClair et al. (1972). In spite of larger discrepancies in $Z_{99\%}$ and $T_{99\%}$ for cloud drops, which are possible due to the sensitive dependence of the falling distance and time on the terminal velocity, good agreement was achieved for drops larger than 0.5 mm. Together with the result of the oscillating 0.5 mm raindrop simulation, the improved IBM was demonstrated to be a reliable model to simulate a free-falling, deforming water drop under general conditions.

Free-falling water drop simulations at altitudes of 1000, 2000, and 3000 m with a fixed lapse rate of $-6.5^\circ\text{C km}^{-1}$ were conducted to quantitatively examine the accuracy of common parameterizations of terminal velocity aloft [Eqs. (21) and (22)]. It was found that there were significant discrepancies between the terminal velocities predicted by these empirical formulas and the simulated terminal velocity. The discrepancy was between 9.38% and 11.8% when the diameter was 0.025 mm at 3000 m. It generally decreased with increasing diameter, and increased with increasing altitude. The empirical formula of Foote and du

Toit (1969) had the smallest discrepancies of 9.3% and 4.2% for diameters of 0.025 and 0.5 mm, respectively, at 3000 m.

By analyzing the C_D –Re power-law curves computed from the simulations conducted at standard conditions and high altitudes, it was found that the exponent b indeed depends on the environmental conditions because two dimensionless numbers are not sufficient to characterize the free-falling motion of a drop. The terminal velocity deduced from the assumption of a fixed C_D –Re curve derived under standard conditions could contain a percentage error as large as 1.5% for a 0.5 mm raindrop at an altitude of 3000 m. When the drop diameter was smaller than 0.3 mm, this assumption produced a good approximation of the terminal velocity aloft with an error smaller than 1%.

A new parameterization of the terminal velocity aloft with an overall discrepancy from the simulated terminal velocity aloft better than 1% was proposed. This new parameterization was based on the piecewise C_D –Re power-law relationship (Fig. 7b and Table 7). From Table 6, it can be seen that the accuracy of this new formula is better than that of existing empirical formulas [Eqs. (21) and (22)].

The simulations of free-falling 0.3, 0.4, and 0.5 mm raindrops with reduced surface tension showed that the impact of shape deformation on the terminal velocity was relatively small, with less than 1.3% of reduction in the terminal velocity when the axis ratio increased by 12%. The averaged dimensionless tangential velocity was linearly proportional to the axis ratio. Changes in the shape and terminal velocity were negligibly small when the surface tension coefficient was reduced by a factor of 10, suggesting that the influence of reduced surface tension caused by the presence of soluble aerosols on the shape and terminal velocity of small raindrops is negligible. It should be noted that the IBM does not directly simulate surfactants. Instead, it only simulates a liquid drop with lowered surface tension that has the same density and viscosity as water. It was pointed out by Müller et al. (2013) that surfactants have a considerable impact on the internal circulation and shape of large raindrops. The IBM can

be improved by including the surfactant effect to study the influence of surfactants on small raindrops in the future.

In conclusion, the improved IBM was verified by comparing the falling speed, terminal velocity, and internal circulation of axisymmetric cloud drops and raindrops under standard conditions. It was successfully applied to free-falling axisymmetric drops at high altitudes, and with reduced surface tension to quantitatively study the influence of water viscosity and surface tension on the terminal velocity and internal circulation. Since the error in the estimation of terminal velocity aloft generated by using a fixed C_D -Re curve was found to increase with the diameter (Figs. 6 and 7), the terminal velocity aloft of large raindrops ($D > 0.5$ mm) should be revised in the future. To resolve this problem, a full 3D model is necessary because the motion of large raindrops is asymmetric and chaotic sometimes. This will be our next goal with the foundation laid by this work.

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Data availability statement. For readers who are interested in the model or the data used in this work, please email the corresponding author to gain access.

APPENDIX A

Domain Size

For all the simulations that were performed in this work, we set the computational domain size to be at least twice larger than the boundary layer thickness. To show that the domain size is sufficient enough to keep the error in the simulated terminal velocity due to the wall effect below 1%, the free-falling 0.1 mm water drop simulation was repeated with a larger domain. The original size of the domain can be found in Fig. 1 in the main text. Extra grid cells are adjoined to the original domain such that its width and height become 41.4 and 69, which are twice as large as the width and height of the original domain. The simulated terminal velocity of the 0.1 mm drop in this extra-large domain was 0.2483 m s^{-1} . The difference from the terminal velocity obtained in experiment C was merely 0.0005 m s^{-1} , or 0.2%. This shows that the original domain size is sufficient.

APPENDIX B

Grid Refinement Test

A grid refinement test was performed to demonstrate the reliability of the simulated terminal velocity, and provide a possible range of discretization errors for the results presented in this

FIG. A1. Plot of the Reynolds number against the drag coefficient of raindrops of diameters 0.48–0.5 mm. Solid line: standard grid. Dashed line: doubly refined grid. The percentage differences of the terminal velocity of raindrops with diameters of 0.48, 0.485, 0.49, 0.495, and 0.5 mm were 0.94%, 0.91%, 0.88%, 0.85%, and 0.82%, respectively.

study. Because substantial computational resources and time are needed to perform a free-falling raindrop simulation with doubly refined grid resolution, only simulations of free-falling raindrops with diameters from 0.48 to 0.5 mm were chosen for this test. This is because the thickness of the boundary layer of larger raindrops is smaller and consequently the gradient of the velocity field is higher, thus the discretization error is expected to be larger in contrast to smaller raindrops. In these simulations, the grid size in the vicinity of the raindrop (see Fig. 1 in the main text) was reduced by a factor of 2 to 0.00625. In other words, the number of grid cells in the entire domain was quadrupled.

A comparison of the Reynolds number and the drag coefficient of raindrops at terminal velocity obtained from this refinement test and those from simulations with standard grid size is shown in Fig. A1. Although the discretization error in the simulated terminal velocity between low- and high-resolution simulations of free-falling raindrops was as large as 0.94%, the gradient of the Re curve increased by only about 0.18% when the doubly refined grid resolution was used. Since it has been shown that the order of convergence of the improved immersed boundary method is roughly equal to one (Ong and Miura 2019), i.e., the discrepancy between two numerical solutions is reduced by half if the grid spacing is halved, we expect that the discrepancy of the terminal velocity from the true value is at best 1.9%.^{B1} Furthermore, the

^{B1} Since $U_{\Delta x=0.00625D} = [(1 - 0.94)/100]^{-1} U_{\Delta x=0.0125D}$, $U_{\text{truth}}/U_{\Delta x=0.0125D} = \prod_{n=0}^{\infty} (1 - 0.94/100)^{-1}$, where U_{truth} is the true value of the terminal velocity. Thus, $(U_{\text{truth}} - U_{\Delta x=0.0125D})/U_{\text{truth}} = 1.9\%$ in the best-case scenario.

error in the estimation of gradient of the $Re-C_D$ in logarithmic scale, b , is expected to be 0.36% at best in this case.

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