Comments on “Potential Vorticity Mixing and Rapid Intensification in the Numerically Simulated Supertyphoon Haiyan (2013)”

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ABSTRACT: In a recent paper, Tsujino and Kuo proposed that the rapid intensification of Supertyphoon Haiyan (2013) is largely attributed to the lower-level warm core primarily induced by the asymmetric potential vorticity (PV) mixing. They also performed a piecewise PV inversion diagnosis to show that the balanced response to the PV mixing accounts for about 50% of the central pressure fall during the rapid intensification onset. This comment presents two major concerns about their methodology and interpretations. Results herein show that they utilized an erroneous method to separate the individual contributions of the upper- and lower-level eye warming to the central pressure decrease, which can exaggerate the impact of the lower-level warm core. In addition, the balanced geopotential height perturbations inverted from the piecewise PV involve significant contributions from the processes irrelevant to the PV mixing. As a result, these defects can lead to problematic diagnosis results and render unwarranted conclusions.

KEYWORDS: Tropical cyclones; Potential vorticity; Dynamics

1. Introduction

A number of observational and numerical studies have found that intense tropical cyclones (TCs) may develop a double warm-core structure, with a lower-level warm core (LWC) generally below 10 km and an upper-level warm core (UWC) above 12 km (Hawkins and Imbembo 1976; Schwartz et al. 1996; Stern and Zhang 2013; Kieu et al. 2016). Using the hydrostatic equation diagnosis, previous studies have shown that the UWC generally has a larger contribution to the central pressure decrease and thus the TC intensification than the LWC (Zhang and Chen 2012; Chen and Zhang 2013; Chang et al. 2020).

Recently, Tsujino and Kuo (2020, hereafter TK20) examined the impact of potential vorticity (PV) mixing on the RI of the simulated Supertyphoon Haiyan (2013). A key conclusion drawn by TK20 is that the positive PV mixing from the eyewall into the eye induces dynamically a LWC that triggers the RI of Haiyan. TK20 showed also the presence of a UWC in the simulated Haiyan (their Fig. 1b). Using a method that they stated to be similar to Zhang and Chen (2012), TK20 diagnosed the TC central pressure falls related to the LWC and UWC, and showed that the LWC dominated the TC central pressure fall, which is in striking contrast to previous studies (Zhang and Chen 2012; Chen and Zhang 2013; Chang et al. 2020). Besides, the authors further employed a piecewise PV inversion (PPVI) algorithm from Kieu and Zhang (2010) to prove that the balanced response to the PV mixing accounts for about 50% of the central pressure fall during the 6-h period following the RI onset of the simulated TC. Although TK20’s finding of the relationship between the PV mixing and eye warming is consistent with previous studies (Yau et al. 2004; Hendricks and Schubert 2010), TK20’s approach of diagnosis has two critical issues. In the following comments, we will demonstrate that their diagnostic method for examining the relative importance of the UWC and LWC in surface pressure fall is incorrect and their PPVI fails to isolate the effect of PV mixing. Given these defects, it is doubtful whether the PV mixing has that large contribution to the RI initiation as they declared. In section 2, we prove that TK20 is mistaken in diagnosing the central pressure decrease induced by the LWC and UWC. In section 3, the other concern with TK20’s PPVI diagnosis is presented. A summary is given in section 4.

2. Concern about diagnostic of the surface pressure fall

To elucidate the defect in the methodology of TK20, the difference in the diagnostic equations of surface pressure in TK20 and other studies (e.g., Zhang and Chen 2012; Chen and Zhang 2013; Chang and Wu 2017) is first illustrated. Let us start with the hydrostatic balance equation in the height coordinate expressed as

$$\frac{dp}{dz} = -\rho g.$$  \hspace{1cm} (1)

After substituting the equation of state for moist air into (1) and integrating from the model top to the surface, we obtain the expression for the hydrostatic surface pressure:

$$p_{\text{sur}} = p_{\text{top}} \exp \left[ \frac{g}{R_d} \int_{z_{\text{top}}}^{1} \frac{1}{T(z)} \, dz \right],$$  \hspace{1cm} (2)

where $p_{\text{sur}}$ is the surface pressure, $p_{\text{top}}$ and $z_{\text{top}}$ are the pressure and height at the model top, $g$ is the gravitational acceleration,
$R_d$ is the dry-air constant, and $T_v(z)$ is the virtual temperature profile. It is clear from Eq. (2) that the surface pressure can be represented as a function of $p_{top}$ and $T_v(z)$ only. Given a constant $p_{top}$, the change of $p_{sur}$ due to the temperature changes can be written as

$$\delta p_{sur} = p_{top} \left\{ \exp \left[ \frac{g}{R_d} \int_0^{z_{sur}} \frac{1}{T_v(z, t)} dz \right] - \exp \left[ \frac{g}{R_d} \int_0^{z_{sur}} \frac{1}{T_v(z, 0)} dz \right] \right\},$$

where $\delta p_{sur} = p_{sur}(t) - p_{sur}(0)$ denotes the surface pressure change, $T_v(z, t) = T_v(z, t) - T_v(z, 0)$ is the difference in the virtual temperature profile between a specific time and the reference time (which is generally defined as the initial time of model integration). Then the $\delta p_{sur}$ induced by the UWC (LWC) can be evaluated with Eq. (3) by setting $T_v(z, t)$ to zero at lower (higher) levels. This diagnostic equation has been adopted by several studies unambiguously supporting the importance of UWC in surface pressure decrease (Zhang and Chen 2012; Chang and Wu 2017; Chang et al. 2020).

By comparison, TK20 separated the contribution to $\delta p_{sur}$ from the LWC and UWC in a different way. Specifically, the hydrostatic surface pressure is obtained by directly integrating the density from the model top to the surface:

$$p_{sur} = p_{top} + \int_0^{z_{sur}} \rho(z) dz,$$

According to Eq. (4), $\delta p_{sur}$ may be written as

$$\delta p_{sur} = g \int_0^{z_{sur}} \rho(z, t) dz,$$

where $\rho(z, t) = \rho(z, t) - \rho(z, 0)$ denotes the change in the density profile between a specific time and the reference time. Then TK20 calculated the $\delta p_{sur}$ induced by the LWC and UWC as the column integration of $\rho(z, t)$ from the surface to $z = 11$ km and from $z = 11$ km to the model top, respectively. It is apparent that this approach assumes that the $\delta p_{sur}$ related to either UWC or LWC is only determined by the air density deviation in the layer where the warm core resides.

The method in TK20 is based on the assumption that the warm-core-induced density anomalies are only confined within the warm-core layer. However, this is not true. Once a warm core forms aloft, the air density would be decreased not only at the warm-core layer but also through the deep level below as a response to the corresponding pressure drop. Therefore, TK20’s method cannot evaluate accurately the actual contributions of the UWC and LWC to the central pressure fall.

To better illustrate this argument, the surface pressure falls induced by idealized warm cores are examined in a quiescent atmosphere. The idealized warm core profile is given by the following equation:

$$T_v(z, r) = A \exp \left[ -\left( \frac{z - z_c}{H} \right)^4 \right] \exp \left[ -\left( \frac{r}{R} \right)^2 \right],$$

where $z$ is the altitude and $r$ is the radius relative to the TC center; $A$, $z_c$, $H$, and $R$ are four parameters controlling the amplitude, altitude, vertical scale, and horizontal scale of the warm core, respectively. In this study, $A$, $H$, and $R$ are set to 10 K, 2.5 km, and 25 km. To explore how the estimated surface pressure fall depends on the height of warm core, two warm cores at upper and lower levels are constructed with $z_c = 15$ and 5 km, respectively. The background temperature and mixing ratio, which are used to calculate the reference-state $T_v$ profile (Fig. 1a), are interpolated from the tropical mean sounding of Dunion (2011). Our calculation is performed between the 0–50-km radii with a radial resolution of 1 km and from the surface to the 20.5-km height with a vertical grid spacing of 500 m. The $p_{top}$ is set to be 50 hPa. Here, the upper bound of 20.5 km is chosen because the uppermost level of the Dunion sounding is at 20.7 km.

It is evident from Eq. (2) that $p_{sur}$ only depends on $\int_0^{z_{sur}} 1/T_v(z) dz$ when $p_{top}$ is specified. To illustrate how the different heights of the warm cores impact $\int_0^{z_{sur}} 1/T_v(z) dz$, the deviations of $1/T_v$ from the reference state induced by the UWC and LWC at the TC center are first examined (Fig. 1b). Clearly, the decrease in $1/T_v$ induced by the UWC is more significant than that induced by the LWC. This is because the background $T_v$ decreases with increasing height such that the $1/T_v$ at higher altitudes would decrease more significantly than that at lower altitudes when the denominator of $1/T_v$ is increased by the same value. The results in Fig. 1b indicate that a warm core at higher altitudes can lead to more decrease in $\int_0^{z_{sur}} 1/T_v(z) dz$ and thus more surface pressure fall than its lower-altitude counterpart.

To further demonstrate how the UWC leads to stronger surface pressure fall than the LWC, the pressure and density anomalies that are balanced with the imposed warm cores are examined (Fig. 2). Specifically, the vertical profile of pressure is calculated by integrating the hydrostatic equation from the top level to different heights in a way similar to Eq. (2). Then the density profile is calculated via the equation of state. The pressure and density anomalies are defined as the differences in pressure and density between the warm-core cases and the reference state.

For both the UWC and LWC, the associated negative density and pressure anomalies extend from the warm core to the surface (Fig. 2), which opposes the assumption of TK20 that the warm-core-induced density anomalies are confined within the warm-core layer. This downward extension of the pressure and density anomalies can be explained as follows. When the warming aloft lowers the density therein, there must be negative pressure anomalies at the adjacent lower level due to the reduced air mass above. According to the equation of state, the negative pressure anomalies would lessen the density at this lower level even in the absence of warming. This reduced density further reduces the mass in the air column, leading to stronger pressure fall and thus density decrease at the levels below. As a result, the negative density and pressure anomalies below the warm core acts to be strengthened monotonically with decreasing altitude, causing the minima at the surface. Compared to the UWC, the density and pressure anomalies associated with the LWC are confined in a much shallower layer (Figs. 2b,d). Since the LWC is closer to the surface, there is little depth for the induced pressure and density anomalies to...
be aggregated vertically. Therefore, the surface pressure decrease caused by the LWC is much less than that by the UWC (20 vs 35 hPa) despite the same magnitude of warming. This is consistent with previous studies that suggest a higher warm core corresponds to more significant surface pressure fall (Zhang and Chen 2012; Chen and Zhang 2013; Chang and Wu 2017).

The fact that the density anomalies related to the warm cores extend to the surface hints a critical problem in TK20. Namely, using their method the contribution of the UWC-induced density anomalies beneath the UWC layer to the central pressure drop would be allocated incorrectly to the LWC. As a result, the $\delta p_{\text{sur}}$ contributed by the LWC (UWC) can be overestimated (underestimated) severely. If following the TK20 method, the negative density anomalies below and above $z = 11$ km in our idealized case would induce the central pressure falls of 47 and 8 hPa, respectively, more than doubling the LWC-induced $\delta p_{\text{sur}}$ of 20 hPa calculated according to Eq. (3).

It should be admitted that the results in Fig. 2 are derived simply via the hydrostatic balance and cannot clarify the dynamics through which the eye warming leads to the density anomalies extending to the surface. To answer this problem, one may use the modeled data with high temporal and spatial resolutions to diagnose how the mass is depleted in the air column, which is beyond the scope of the current comment. Nevertheless, the analysis of the idealized warm cores is sufficient to prove that the integration of the density anomalies in the warm-core layer cannot represent the surface pressure fall induced by the warm core and thus TK20’s method for separating the contribution of the UWC and LWC to the surface pressure fall is incorrect. TK20 asserted that the contribution of the UWC to the central pressure fall was insignificant because the UWC depth is much shallower than that of the LWC. However, our analysis indicates that the greater contribution of the LWC to $\delta p_{\text{sur}}$ is, in fact, an artifact of the incorrect diagnostic used in their study. Therefore, we conclude that TK20’s statement regarding the importance of the LWC in Haiyan’s RI cannot be justified.

3. Concern about TK20’s PPVI diagnosis

TK20 performed an azimuthal-mean PV budget and used the asymmetric PV advection (ASADV) term integrated for the 6-h period following the RI onset to represent the PV anomaly induced by the PV mixing. The ASADV is given as

$$\text{ASADV} = -u \frac{\partial P}{\partial r} - v \frac{\partial P}{\partial z} - w \frac{\partial P}{\partial \lambda},$$  \hspace{1cm} (7)

where $u$, $v$, and $w$ represent the three-dimensional wind in the storm-relative cylindrical coordinate; $P$ is the PV; $r$ is the radius; and $\lambda$ is the azimuth. The overbar and prime denote the azimuthal average and deviation, respectively. The authors then inverted the ASADV within about 100-km radius and throughout the troposphere using the PPVI algorithm to obtain the corresponding balanced geopotential height perturbations. By comparing the balanced geopotential height perturbations with the actual change of geopotential height, TK20 concluded that the PV mixing accounted for about 50% central pressure fall during the 6-h period following the RI onset of the simulated TC (see their Fig. 7).

TK20’s PV budget demonstrated that the ASADV dominated the PV change within the eye in the mid- to lower troposphere (see their Fig. 6), which is consistent with the previous finding that the PV mixing plays an important role in increasing
However, that TK20 attributed the total geopotential height anomalies inverted from the ASADV to the PV mixing is unreasonable, since the ASADV involves a large amount of PV tendency irrelevant to the PV mixing. The PV mixing generally refers to the downgradient transport of PV by the asymmetric winds resulting from the dynamical instability of the eyewall (Montgomery and Shapiro 1995; Schubert et al. 1999). Since the asymmetric winds advect high-PV air from the eyewall into the eye and low-PV air from the eye into the eyewall during PV mixing, the relevant PV anomaly should be negative in the eyewall region and positive within the eye. However, TK20’s PPVI for ASADV presents significant positive PV anomalies in the eyewall with one order of magnitude larger than those within the eye. Specifically, there are two positive cores of ASADV greater than 20 PVU (1 PVU $= 10^{-2}$ K kg$^{-1}$ m$^2$ s$^{-1}$) within the 30–60-km radii, with one at $z = 4–6$ km and the other at $z = 8–14$ km (see their Figs. 6e and 7c). Due to lack of detailed examination of three components in ASADV [i.e., $-\bar{u} (\bar{\partial} \bar{P}/\bar{\partial} r)$, $-\bar{v} (\bar{\partial} \bar{P}/\bar{\partial} \lambda)$ and $-\bar{w} (\bar{\partial} \bar{P}/\bar{\partial} z)$], the individual contributions to these positive PV anomalies are uncertain in TK20. Nonetheless, it is clear that the positive ASADV in the eyewall region should not be attributable to the

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**Fig. 2.** Radius–height cross sections of (a), (b) the imposed $T'_\nu$ anomalies (shading; K) and the corresponding density anomalies (contours; g m$^{-3}$) and (c), (d) pressure anomalies (hPa) for (left) the UWC and (right) the LWC.
PV mixing. The inclusion of these PV anomalies in the eyewall for PPVI can lead to the overestimation of the geopotential perturbations induced by the PV mixing.

Furthermore, TK20’s PV budget showed that a noticeable portion of the positive ASADV in the eye is cancelled by the diabatic term and axisymmetric PV advection. As a result, the actual PV increase in the eye can be lower than the PVA indicated by the ASADV. As shown in the Fig. 6a of TK20, the net PV increase in the eye for the 6-h period is only evident between the altitudes of 4–6 km with a magnitude of 4–8 PVU. This PVA core can be viewed as the contribution of the PV mixing to the net PV increase in the eye. It is expected that the geopotential height anomalies inverted from this PVA core would be smaller than those inverted from the ASADV in TK20. In other words, the PV mixing may not have as large contribution to the central pressure fall and thus RI initiation of Haiyan as suggested in TK20.

Meanwhile, it is worth mentioning that the significant PV increase greater than 12 PVU is seen in the eyewall PV tower. Kieu and Zhang (2010) have examined the nonlinear response of a simulated TC to the piecewise PV anomalies using the PPV algorithm. They found that the PV anomalies in the eyewall have a larger contribution to the central pressure decrease than those of the same magnitude in the eye. Furthermore, the positive balanced temperature perturbation induced by the eyewall PVA is located at the altitudes of 7–15 km (see their Fig. 3e). This result further supports the importance of the upper-level warming in surface pressure fall in the context of the PPVI. According to the above facts, we speculate that the eyewall PV likely plays a more important role in triggering Haiyan’s RI than the PV mixing does. This hypothesis can be readily examined by inverting piecewise the positive PV anomalies in the eye and eyewall.

4. Summary

TK20 stressed the importance of the PV mixing to the RI of simulated Supertyphoon Haiyan. They argued that the RI was triggered by a LWC primarily induced by the PV mixing. Based on the PPVI diagnosis, they further proposed that the balanced response to the PV mixing accounted for about 50% central pressure decrease during the 6-h period following the RI onset. In this comment, we point out two critical concerns about the role of the PV mixing in Haiyan’s RI.

1) When deriving the diagnostic equation for the central pressure fall induced by the LWC and UWC, TK20 mistakenly assumed that the warm-core-induced density anomalies are confined in the warm-core layer. In fact, the influence of warm core at a specific level on density perturbation can spread vertically downward to the surface. As a result, the central pressure fall induced by the LWC (UWC) can be overestimated (underestimated) severely by the incorrect diagnostic method.

2) In TK20’s PPVI diagnosis, the geopotential anomalies inverted from the ASADV term in PV budget was interpreted as the balanced response to the PV mixing. However, we argue that the ASADV in TK20 involves a large amount of positive PV tendency irrelevant to PV mixing. Thus, the inversion of the ASADV cannot represent the actual effect of the PV mixing. Moreover, we propose that the contribution to the RI of Haiyan from the PV mixing is less significant than that from the PV enhancement in the eyewall.

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