The Amplification of Madden–Julian Oscillation Boosted by Temperature Feedback

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ABSTRACT: Due to a small Coriolis force in tropics, the theoretical study of Madden–Julian oscillation (MJO) often assumes weak temperature gradient balance, which neglects the temperature feedback (manifested in the temperature tendency term). In this study, the effect of the temperature feedback on the MJO is investigated by using the MJO trio-interaction model, which can capture the essential large-scale features of the MJO. The scale analysis indicates that the rotation effect is strong for the MJO scales, so that the temperature feedback is as important as the moisture feedback (manifested in the moisture tendency term); the latter is often considered to be critical for MJO. The experiments with the theoretical model show that the temperature feedback has significant impact on the MJO’s maintenance. When the temperature feedback is turned off, the simulated MJO cannot be maintained over the warm pool. This is because the temperature feedback could boost the energy generation. Without the temperature feedback, only the latent heat can be generated. With the temperature feedback, not only the latent heat but also the enthalpy (and therefore the available potential energy) can be generated. Therefore, the total energy generation is more efficient with the temperature feedback, favoring the self-maintenance of the MJO. Further investigation shows that this effect of the temperature feedback on MJO amplification can be inferred from observations. The findings here indicate that the temperature feedback could have nonnegligible impacts on the MJO and have implications in the simulation of MJO.

KEYWORDS: Feedback; Instability; Madden-Julian oscillation

1. Introduction

The Madden–Julian oscillation (MJO) is a convectively coupled planetary-scale circulation system, and its convection signal propagates slowly (∼5 m s⁻¹) over the Indo-Pacific warm-pool region (Madden and Julian 1971, 1972). MJO is one of the dominant variabilities over tropics and has significant impacts on global weather and climate systems (Zhang 2013), providing foundation for “seamless prediction” (Hurrell et al. 2009; Brown et al. 2012). Understanding the fundamental physical mechanisms of MJO is one of the most attractive topics in tropical dynamics, yet this remains a great challenge.

The early MJO theories tend to interpretate the MJO as convectively coupled waves, driven by the wave–conditional instability of the second kind (wave-CISK) (Chang 1977; Lau and Peng 1987), the boundary layer (BL) moisture convergence (Wang 1988; Wang and Rui 1990; Wang and Li 1994), or the wind-induced surface heat exchange (WISHE) (Emanuel 1987; Neelin et al. 1987). In these early theories, the interaction between the large-scale wave circulation and the convection plays a key role in generating the intraseasonal oscillation. The convective heating induces wave responses (primarily the Kelvin wave response and the Rossby wave response), causing temperature fluctuations that mediate away from the heating source. The correlation between the temperature fluctuations and the convective heating generates available potential energy (APE) for the development of the system. The temperature feedback, manifested in the temperature tendency term, dominates the thermodynamics in these theories, while the moisture feedback, manifested in the moisture tendency term, was often neglected by assuming the balance between low-level moisture convergence and precipitation [except Emanuel (1987), who used moisture entropy equation that incorporated both temperature and moisture tendencies]. The MJO modes in these early theories often have faster propagation speed than the observation, suggesting that there could be missing physical processes in these theoretical frameworks.

Although the vertical moisture advection (or low-level moisture convergence) is primarily balanced by precipitation, the observations revealed that precipitation is sensitive to the moisture content in tropics (Bretherton et al. 2004; Holloway and Neelin 2009) and the coherence between the column moisture fluctuation and precipitation fluctuation is most prominent on intraseasonal time scale (Yasunaga and Mapes 2012). These indicate that the moisture plays an important role in MJO thermodynamics, which requires the consideration of moisture feedback in the theoretical framework, leading to the development of modern MJO theories that include (i) the convective quasi-equilibrium theory (Neelin and Yu 1994), (ii) the MJO skeleton theory (Majda and Stechmann 2009), (iii) the MJO trio-interaction theory (Wang et al. 2016), (iv) the moisture mode theory (Fuchs and Raymond 2005; Raymond and Fuchs 2007; Sobel and Maloney 2012, 2013; Adames and Kim 2016; Fuchs and Raymond 2017; Raymond and Fuchs 2018), (v) the BL quasi-equilibrium theory (Emanuel 2019), and (vi) MJO multicloud theory (Majda et al. 2007). In these theories, the moisture–convection feedback
plays a key role in generating the MJO mode. The resulting MJO phase speed in the theories reduces and is comparable to the observation. Wang et al. (2016) argued that it is the moisture–convection feedback that reduces the phase speed of MJO.

The improvement in understanding the MJO by considering moisture feedback leads one to think that the relative importance between the moisture feedback and temperature feedback on the MJO thermodynamics. The pertinent questions are whether the moisture feedback is more important than the temperature feedback and whether the effect of temperature feedback is negligible. In tropics, the Coriolis force is small and the Rossby radius of deformation is large, so that the gravity wave could rapidly homogenize the horizontal temperature gradient. Consequently, the diabatic heating is approximately balanced by the adiabatic cooling, leading to the so-called weak temperature gradient (WTG) approximation. There is no temperature feedback in the WTG balance, because the temporal temperature fluctuation is thought to be rapidly dispersed over a large area by the action of gravity waves. The applying of WTG approximation to the MJO study leads to the development of the moisture mode theory under WTG balance (Sobel and Maloney 2012, 2013; Adames and Kim 2016). In this theory, the only prognostic variable is the moisture, while the MJO-related circulation instantaneously adjusts to the convective heating in the form of the Matsuno–Gill (Matsuno 1966; Gill 1980) steady-state response, which eliminates the temperature feedback and momentum feedback (manifested in the momentum tendency terms). The moisture mode theory under WTG balance has successfully explained many aspects of the MJO, such as the eastward phase propagation caused by moisture advection and the planetary-scale instability caused by cloud-radiation feedback. Using scale analysis, Adames et al. (2019) have demonstrated that the moisture feedback dominates the temperature feedback in the MJO thermodynamics, further supporting the moisture mode theory under WTG balance.

Although the WTG balance is a good approximation in tropics, eliminating the temperature feedback is still debatable. In fact, the time required for the circulation to reach steady state is considerable as compared to the period of MJO (Kacimi and Khouider 2018), suggesting the presence of transient temperature variation (i.e., temperature feedback) during the MJO life cycle. Since the correlation between the temperature fluctuation and the diabatic heating is relevant for the generation of APE when the temperature feedback is present, it suggests that the temperature feedback, although weak in tropics, may have impacts on the MJO.

In fact, previous studies have indicated that the temperature feedback could have impacts on the maintenance of the MJO. It was shown that neglecting the temperature feedback (or applying the WTG approximation) will reduce the instability of MJO mode (Fuchs and Raymond 2017; Liu and Wang 2017; Emanuel 2019). Chen and Wang (2019) showed that including wave feedback (i.e., the temperature feedback and the momentum feedback) could enhance instability of the MJO mode on the planetary scale (their Fig. 5). However, the mechanism by which the temperature feedback affects the MJO is still unclear. Therefore, the goal of this study is to explore the relative importance of the temperature feedback, and to reveal the mechanism of how temperature feedback affects the MJO.

The rest of the paper is organized as follows. Section 2 introduces the theoretical model that will be used to investigate the role of the temperature feedback. Section 3 presents the scale analysis of the model equations and discusses the associated implications on relative importance of the temperature feedback. Section 4 shows the impacts of the temperature feedback on the MJO using the theoretical model and explores the corresponding physical mechanisms. The implications from the observation will be presented in section 5. Section 6 gives the conclusions and discussion.

2. The theoretical model
a. Model formulation

The MJO trio-interaction model (Wang and Chen 2017) is used here for studying the effect of temperature feedback, as the model could capture the essential large-scale features of the MJO. Figure 1 illustrates the concept of this model. The model consists of two layers for the free atmosphere and a barotropic BL. The detailed derivation can be found in Wang and Chen (2017), and here we summarize the dimensional governing equations:

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \left( u \right) - \beta y v = - \frac{\partial \Phi}{\partial x}, \tag{1}
\]

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial y} \left( v \right) + \beta u y = - \frac{\partial \Phi}{\partial y}, \tag{2}
\]

\[
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \left( \Phi + C^2_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) + C^2_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = - MQ - \text{Rad}, \tag{3}
\]
The value of $\bar{q}$ according to Wang and Chen (2017). Also, the underlying background sea surface temperature (SST) related to the temperature by the hydrostatic balance; i.e.,

$$\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \bar{\mathbf{u}} = \frac{\partial \bar{H_b}}{\partial x} + \frac{\partial \bar{H_b}}{\partial y} + \mathbf{V} \cdot \nabla \bar{Q} = Ev - Pr,$$

(4)

$$\frac{\partial \bar{H_b}}{\partial t} - \beta y \bar{H_b} = - \frac{\partial \Phi}{\partial x} - Ev \bar{H_b},$$

(5)

$$\frac{\partial \bar{H_b}}{\partial t} + \beta y \bar{H_b} = - \frac{\partial \Phi}{\partial y} - Ev \bar{H_b},$$

(6)

$$\frac{\Delta \bar{p}}{\gamma} \bar{Q}_c = L_c Pr.$$

(7)

Here $u$, $v$, and $\Phi$ represent the free-tropospheric low-level zonal wind, meridional wind, and geopotential of the first baroclinic mode, respectively; note that the geopotential is related to the temperature by the hydrostatic balance; i.e., $\Phi = - (RVp/2p_2)z$; $d = (\Delta p_b/\Delta p) = 0.25$ is the nondimensional BL depth; $\bar{Q}$ represents the Rayleigh friction and Newtonian cooling coefficients, and here we assume that they have the same value; $C_0 = 50$ m s$^{-1}$ is the speed of gravity wave; $Q_c$ is the condensational heating at level 2 and $M = (R \Delta p/2 C_p p_2)$ is the coefficient for $Q_c$; $Ev$, $Pr$, and $Rad$ are the evaporation, precipitation rate, and cloud-radiation heating, respectively, $Q_c$ is related to precipitation by Eq. (7); $q$ is the column-integrated specific humidity anomaly from surface to tropopause; $\bar{Q} = (\Delta \bar{p}/\gamma) (\bar{q}_3 - \bar{q}_1)$ is the difference of background specific humidity between the lower and upper layer; and $\bar{Q}_b = (\Delta \bar{p}/\gamma) (\bar{q}_b - \bar{q}_1)$ is the difference of background specific humidity between the BL and the upper layer. The value of $\bar{Q}$ and $\bar{Q}_b$ are assumed to be controlled by the underlying background sea surface temperature (SST) according to Wang and Chen (2017). Also, $u_b$ and $v_b$ are BL winds, and $E$ is the Ekman coefficient. See Table 1 for more details of the model parameters.

### b. Parameterization

The simplified Betts–Miller type scheme (Bretherton et al. 2004; Frierson et al. 2004; Wang and Chen 2017) is used for precipitation parameterization:

$$Pr = q/\tau,$$

(8)

where $\tau$ is the convective adjustment time scale. Here we have neglected the contribution of temperature on the precipitation for simplicity, based on the observed feature that the MJO’s precipitation is well correlated with the column-integrated moisture (Chen and Wang 2018b). Note that both moisture and temperature in precipitation parameterization could affect the behaviors of the tropical variability (Ahmed et al. 2021). The effect of the temperature in precipitation parameterization is given in appendix B, which suggests that dropping temperature in precipitation parameterization will not affect the conclusions of this study.

According to Adames (2017), $\tau$ for tropical precipitation anomaly is reversely related to the climatological mean precipitation. Therefore, we assume that $\tau = \tau_0/\eta(\varphi)$, where $\tau_0 = 10$ h and $\eta(\varphi) = \exp[-(\varphi/\varphi_w)^2]$. Here, $\varphi$ denotes latitude, $\varphi_w = 11^\circ$, and $\eta(\varphi)$ is the idealized meridional distribution of the climatological mean precipitation over tropics.

Note that different value of $\tau$ has been used in previous studies. In some studies, $\tau$ has been assumed to be on the order of $\sim 2$ h (Neelin and Yu 1994; Wang and Chen 2017), while in other studies $\tau$ was assumed to be on the order of $\sim 0.5–1$ day (Majda et al. 2007; Adames and Kim 2016; Fuchs and Raymond 2017; Chen and Wang 2020). The observational evidence indicates that $\tau$ may be on the order of $\sim 0.5–1$ day for large-scale phenomenon like MJO (Bretherton et al. 2004; Adames et al. 2019). The sensitivity of the model results to the $\tau$ is given in appendix C.

In deep convection zone, the elevated cloud top reduces the loss of longwave radiation, leading to warming in troposphere and the so-called cloud-radiation feedback. Thus, the cloud-radiation heating anomaly is assumed to be proportional to convective heating (Fuchs and Raymond 2002; Peters and Bretherton 2005):

$$\text{Rad} = rMQ_c,$$

(9)

where $r$ is the radiation coefficient and is assumed to be a constant.

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<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Typical scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>Gravity wave speed</td>
<td>50 m s$^{-1}$</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Half pressure depth of the free atmosphere</td>
<td>400 hPa</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant for dry air</td>
<td>287 J K$^{-1}$ kg$^{-1}$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat of dry air at constant pressure</td>
<td>1004 J K$^{-1}$ kg$^{-1}$</td>
</tr>
<tr>
<td>$L_v$</td>
<td>Latent heat of condensation at 0°C</td>
<td>$2.5 \times 10^6$ J kg$^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Radiation coefficient</td>
<td>0.15</td>
</tr>
<tr>
<td>$E$</td>
<td>Ekman coefficient</td>
<td>(0.5 day)$^{-1}$</td>
</tr>
<tr>
<td>$e$</td>
<td>Damping coefficient</td>
<td>(10 days)$^{-1}$</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity acceleration</td>
<td>9.8 m s$^{-2}$</td>
</tr>
<tr>
<td>$\Delta p_b$</td>
<td>Pressure depth of the boundary layer</td>
<td>100 hPa</td>
</tr>
<tr>
<td>$\bar{q}_1$</td>
<td>Background specific humidity at layer 1</td>
<td>Depends on SST</td>
</tr>
<tr>
<td>$\bar{q}_3$</td>
<td>Background specific humidity at layer 3</td>
<td>Depends on SST</td>
</tr>
<tr>
<td>$\bar{q}_b$</td>
<td>Background specific humidity at boundary layer</td>
<td>Depends on SST</td>
</tr>
</tbody>
</table>
When the background low-level zonal flow is easterly, the surface evaporation can cause eastward propagation of MJO mode (Emanuel 1987; Neelin et al. 1987; Fuchs and Raymond 2017; Emanuel 2019). Since this effect of the surface evaporation is similar to the effect of the BL moisture convergence, as they both moisten the lower troposphere to the east of the enhanced MJO deep convection, the surface evaporation is neglected in this study for simplicity. It should be noted that surface evaporation could be important for the MJO in numerical models (Khairoutdinov and Emanuel 2018; Shi et al. 2018).

3. Scale analysis and its implications

In this section, the scale analysis will be applied to the model equations to explore the relative importance of the temperature feedback on the MJO.

a. Scale analysis

Following Adames et al. (2019), we assume the basic characteristic scales are

\[ x \sim L_x, y \sim L_y, u \sim U' , v \sim V' , t \sim \tau_w , C = L_x / \tau_w , \]

where \( L_x \) and \( L_y \) are the zonal and meridional length scales; \( U \) and \( V \) are the zonal and meridional velocity scales, respectively. Using the continuity equation, the meridional velocity scale can be expressed as \( V = U L_x / L_y \); and \( \tau_w \) is the wave’s time scale and \( C \) is the waves’ phase speed. The nondimensional variable are denoted by symbols with primes.

Using above basic characteristic scales, the zonal and meridional momentum equations can be written as

\[ UC \left( \frac{\partial u'}{\partial t} - \frac{1}{Ro} y' v' \right) = - [\Phi] \frac{\partial \Phi'}{\partial x} - UC \frac{\tau_w}{\tau_e} u' , \]

(10)

\[ UC \left( \frac{L_x^2}{L_y^2} \frac{\partial v'}{\partial t} + \frac{1}{Ro} y' u' \right) = - [\Phi] \frac{\partial \Phi'}{\partial y} - UC \frac{L_x^2}{L_y^2} \frac{\tau_w}{\tau_e} v' , \]

(11)

where \( \tau_e \) is the damping time scale and \( Ro = C / (BL_y^2) \) is the Rossby number. Note that \( Ro = Re / L_x \), where \( Re = C / f = C / (BL_y) \) is the Rossby radius of deformation.

Since the MJO has larger zonal length scale than the meridional scale (i.e., \( L_z / L_x \sim 10^{-1} \)), the leading balance in Eq. (11) is between the Coriolis force and the meridional pressure gradient, which is the so-called long wave approximation. Therefore, the scale for the geopotential is

\[ [\Phi] \sim UC / Ro . \]

(12)

It is indicated by Eq. (12) that when \( Ro = Re / L_y \ll 1 \), the scale of the geopotential (or temperature) increases. This is because when the rotational effect dominates (i.e., the scale of the motion is much larger than the Rossby radius of deformation), the geopotential field can be maintained against the damping due to dispersion of fast gravity waves.

Using the scale of geopotential, the nondimensional momentum equations can be expressed as

\[ Ro \frac{\partial u'}{\partial t} - y' v' = - \frac{\partial \Phi'}{\partial x} - Re \frac{\tau_w}{\tau_e} u' , \]

(13)

\[ Ro \frac{L_x^2}{L_y^2} \frac{\partial v'}{\partial t} + y' u' = - \frac{\partial \Phi'}{\partial y} - Ro \frac{L_x^2}{L_y^2} \frac{\tau_w}{\tau_e} v' . \]

(14)

To nondimensionalize the moisture equation and thermodynamic equation, we need to determine the moisture scale. We consider that the leading balance of the moisture equation in tropics is between the free atmospheric moisture convergence and precipitation:

\[ \frac{\partial q}{\partial t} - \nabla \cdot \left( \frac{u' q'}{\tau_e} \right) \sim - \nabla \cdot \frac{\tilde{Q} u' + \tilde{Q} \tilde{v}}{\tau_e} . \]

(15)

where \( \tau_e \) is the scale for the convective adjustment time scale. Therefore, we can assume the moisture scale has the form

\[ [q] = d_0 \frac{\Delta \rho N_c U}{gC} . \]

(16)

where \( d_0 = 2 \pi C_{p \rho} C_0^2 / (\Delta \rho RL_e) \) is a nondimensional number following Wang and Chen (2017), and \( N_c = \tau_e / \tau_w \) is a nondimensional number following Adames et al. (2019). With the aid of Eqs. (12) and (16), the nondimensional thermodynamic equation and moisture equation can be written as

\[ \frac{F_r^2}{Ro} \frac{\partial \Phi'}{\partial t} + \left( \frac{\partial u'}{\partial x} + \frac{\partial u'}{\partial y} \right) + d \left( \frac{\partial \tilde{u_b}}{\partial x} + \frac{\partial \tilde{u_b}}{\partial y} \right) = - (1 + r) \frac{F_r^2}{Ro} \frac{\tau_w}{\tau_e} \Phi' ; \]

(17)

\[ N_c \frac{\partial q'}{\partial t} + \tilde{Q} \left( \frac{\partial u'}{\partial x} + \frac{\partial u'}{\partial y} \right) + d \tilde{Q} \left( \frac{\partial \tilde{u_b}}{\partial x} + \frac{\partial \tilde{u_b}}{\partial y} \right) + \mu V' \cdot \nabla \tilde{Q} = - \frac{q'}{\tau_e} . \]

(18)

where we have assumed that \( \tilde{\delta \tilde{Q}} \sim \mu \tilde{\tilde{Q}} \) and \( \mu \sim 0.1 \), given that the background moisture content varies slowly in spatial domain. \( \tilde{Q} = (\tilde{q}_3 - \tilde{q}_1) / d_0 \) and \( \tilde{Q}_b = (\tilde{q}_b - \tilde{q}_1) / d_0 \) are the nondimensional background moisture stratification coefficient. The definitions of \( \tilde{Q} \) and \( \tilde{Q}_b \) are the same as those defined in Wang and Chen (2017) and Chen and Wang (2019). \( F_r^2 = C^2 / C_0^2 \) is the Froude number.

b. Implications

Equations (13), (14), (17), and (18) consist of the nondimensional equation set. The magnitudes of the nondimensional numbers measure the relative importance of the corresponding terms in the equation. Table 2 summarizes the values of the basic characteristic scales for MJO and the corresponding characteristic nondimensional numbers. Given these values, the first-order balances of the system are
Table 2. Basic characteristic scales for the MJO and the corresponding nondimensional numbers.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Typical scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_x )</td>
<td>Zonal length scale</td>
<td>( 10^7 ) m</td>
</tr>
<tr>
<td>( L_y )</td>
<td>Meridional length scale</td>
<td>( 10^6 ) m</td>
</tr>
<tr>
<td>( C )</td>
<td>Characteristic wave (phase) speed</td>
<td>( 5 ) m s(^{-1})</td>
</tr>
<tr>
<td>( U )</td>
<td>Characteristic wind speed</td>
<td>( 10 ) m s(^{-1})</td>
</tr>
<tr>
<td>( \tau_w = L_x / C )</td>
<td>Characteristic wave time scale</td>
<td>25 days</td>
</tr>
<tr>
<td>( \tau_r )</td>
<td>Damping time scale</td>
<td>10 days</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Meridional gradient of Coriolis force at equator</td>
<td>( 2.3 \times 10^{-11} )</td>
</tr>
<tr>
<td>( \tau_m )</td>
<td>Scale of convective adjustment time scale</td>
<td>1 d</td>
</tr>
<tr>
<td>( R_0 = C / (BL_x^2) )</td>
<td>Rossby number</td>
<td>0.2</td>
</tr>
<tr>
<td>( P_0^2 / C_0^2 )</td>
<td>Froude number</td>
<td>( 10^{-2} )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Fraction of background moisture change</td>
<td>0.1</td>
</tr>
<tr>
<td>( N_C = \tau_c / \tau_w )</td>
<td>Ratio of convective adjustment time scale and wave time scale</td>
<td>( 4 \times 10^{-2} )</td>
</tr>
<tr>
<td>( N_m = P_0^2 / (R_0 N_C) )</td>
<td>Mode number under rotation</td>
<td>1.25</td>
</tr>
<tr>
<td>( \tilde{N}_m = P_0^2 / N_C )</td>
<td>Mode number without rotation</td>
<td>0.25</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
-y' u' &= - \frac{\partial \Phi'}{\partial x'} - R_0 \frac{\tau_w}{\tau_r} u', \\
\cdot y' u' &= - \frac{\partial \Phi'}{\partial y'}, \\
\left( \frac{\partial u'}{\partial x'} + \frac{\partial u'}{\partial y'} \right) &= -(1 + r) \frac{q'}{\tau_r}, \\
\dot{Q} \left( \frac{\partial u'}{\partial x'} + \frac{\partial u'}{\partial y'} \right) + d \dot{Q}_b \left( \frac{\partial h}{\partial x'} + \frac{\partial h}{\partial y'} \right) &= - \frac{q'}{\tau_r}.
\end{align*}
\]

Equations (19)–(22) state that at first order the circulation (including winds and geopotential) can be considered as steady response to the convective heating. Note that the meridional momentum equation is in long wave approximation [i.e. Eq. (20)] and the thermodynamic equation is in WTG balance [i.e. Eq. (21)]. In moisture equation [Eq. (22)], the precipitation is primarily balanced by the moisture convergence.

The system at the first-order balance is stationary. To predict the evolution of MJO, the tendency terms should be included at the next order balance. The questions are which tendency term should be included and which one can be neglected. Note that the nondimensional numbers in the tendency terms of Eqs. (13), (14), (17), (18) cannot be compared to each other directly, because different equations have different dimensions. To compare them, we considered the column-integrated kinetic energy equation, enthalpy equation, and latent heat equation, as they have the same dimension (i.e., kg s\(^{-2}\) or J m\(^{-2}\)).

Inspired by Adames et al. (2019), we combine enthalpy and latent heat equations into moist enthalpy (ME) equation. The column-integrated ME is \( h = \int_0^1 \int_{L_y} \left[ (C_p T + L_v q) \right] dp \). In the trio-interaction model, the column-integrated ME can be expressed as \( h = - \left[ 2 P_0 C_p / (R_0 g) \right] (\Phi + L_v q) \). Note that the column-integrated ME in the trio-interaction model is approximately equal to the column-integrated moist static energy (MSE), since the geopotential has first baroclinic structures in free troposphere and the column-integrated geopotential is vanishingly small compared to the column-integrated ME. Using dimension \( C_p^0 \left[ 2 P_0 C_p / (R_0 g) \right] (U / L_x) \), it can be shown that the nondimensional ME equation is

\[
N_C \frac{\partial h'}{\partial t} = (1 - \dot{Q}) \left( \frac{\partial u'}{\partial x'} + \frac{\partial u'}{\partial y'} \right) + d \left( 1 - \dot{Q}_b \right) \left( \frac{\partial h}{\partial x'} + \frac{\partial h}{\partial y'} \right) - \mu V' \cdot V - r' + N_C N_m \frac{\tau_w}{\tau_r} H',
\]

where \( h' = q' - N_m \Phi' \) is the nondimensional form of ME and \( N_m = P_0^2 / (R_0 N_C) \). The relative importance between the moisture and the temperature feedback is therefore measured by the nondimensional number \( N_m \), which is referred to mode number by Adames et al. (2019). Using the same dimension [i.e., \( C_p^0 \left[ 2 P_0 C_p / (R_0 g) \right] (U / L_x) \)], it can be shown that the nondimensional column-integrated kinetic energy \( \{ \text{i.e.} (2dP/g) \} \) can be expressed as

\[
N_C \frac{\partial}{\partial t} \left( \gamma_K \tilde{N}_m K' \right) = - N_C \gamma_K N_m V' \cdot V \Phi' - 2 \frac{\tau_w}{\tau_r} N_C \gamma_K \tilde{N}_m K' - 2 N_C \gamma_K \gamma_m \tilde{N}_{m'} K',
\]

where \( \gamma_K \tilde{N}_m K' \) is the nondimensional form of the column-integrated kinetic energy, \( \tilde{N}_m = P_0^2 / N_C \) is the mode number without rotation effect (Adames et al. 2019), \( \gamma_K = UR \Delta p / (C_p^2 C_p) \) is a nondimensional number, and \( K' = [u^2 + \left( L_x^2 / L_y^2 \right) v^2 / 2]. \)

From Eqs. (23) and (24), the relative importance of the tendency term is measured by \( N_m \) and \( \gamma_K \tilde{N}_m \). Given the characteristic scales of the MJO (Table 2), it can be shown that \( N_m \sim 1 \). Therefore, the scale analysis here suggests that the temperature feedback is as important as the moisture feedback for the MJO. Consequently, the temperature tendency should be included if the moisture feedback is retained.

Note that the scale of \( N_m \) in this study is larger than that estimated by Adames et al. (2019). This is because they estimated a larger Rossby number \( R_0 \sim 10^4 \), by assuming larger wave speed scale \( C \sim 10^3 \) m s\(^{-1}\) and smaller scale of \( \beta \) \( (1 \times 10^{-11} \) s\(^{-1}\) m\(^{-1}\)). In the present study, the wave speed...
scale is assumed to be $5 \text{ m s}^{-1}$ and the value of $\beta$ at the equator ($2.3 \times 10^{-11} \text{ s}^{-1} \text{ m}^{-1}$) is used. Consequently, the estimated Rossby number has scale $Ro \sim 0.2$. The increasing importance of temperature feedback for smaller $Ro$ is due to the fact that the temperature fluctuation is less damped by the dispersion of fast gravity waves when the rotational effect is strong.

On the other hand, $g\tilde{K}N_{m} \sim 0.1$, indicating that the kinetic energy is much smaller than the ME. This is because the kinetic energy only accounts for about 10% of the APE (Lorenz 1955), and the smallness of $\gamma_k\tilde{N}_m$ just reflects this fact. However, neglecting the momentum feedback (i.e., the momentum tendency) may lead to unbalance in the momentum equations. This can be shown by considering the balance of wave dynamics, as the MJO is interpreted in terms of Kelvin–Rossby structure. For the Rossby wave component ($\nu \neq 0$), the momentum tendencies may be dropped at leading order for small Rossby number and the Rossby component is balanced (i.e., meridional wind is balanced by zonal pressure gradient). On the other hand, for the Kelvin wave component ($\nu = 0$), the flow is unbalanced if damping is small (i.e., $\tau_v/\tau_s \to 1$). Thus, the zonal momentum tendency cannot be dropped.

In short, the scale analysis indicates that the temperature feedback is as important as the moisture feedback, and they are the leading terms for depicting the MJO evolution. The zonal momentum feedback, on the other hand, may not be neglected, although it is small.

4. Effect of the temperature feedback on the maintenance of MJO

The scale analysis has suggested that the temperature feedback can be as important as the moisture feedback. However, the results from scale analysis are only qualitative. In this section, we will give quantitative measures of how temperature feedback affects the MJO, as well as to reveal the corresponding mechanism.

a. The MJO mode with temperature feedback

To explore the effect of the temperature feedback, we will conduct several experiments using the trio-interaction model. Since the horizontal moisture advection is found to play a role in MJO propagation, we will use idealized warm-pool settings to conduct initial value problems, so that the horizontal moisture advection can be included. This is different from Chen and Wang (2019), who used uniform SST distribution to obtain eigen solutions. Here we assume SST has simple form:

$$\text{SST} = A(\lambda)B(\varphi),$$  \hspace{1cm} (25)

where $\lambda$ is longitude and $\varphi$ is latitude. An idealized Indo-Pacific warm pool is specified by the function $A(\lambda)$, which is shown in Fig. 2a. $B(\varphi) = \exp[-(\varphi/21^\circ)^2/4]$ is the meridional distribution of SST. The horizontal distribution of SST specified by Eq. (25) is shown in Fig. 2b.

Let us first check how the theoretical model simulates the MJO mode. In the control experiment, the full model
equations [Eqs. (1)–(7)] are used and the parameters are set to default values (Table 1). The initial value problem is solved numerically. The numerical scheme is identical to that of Wang and Chen (2017), which is also given in appendix A.

The initial condition is a Kelvin wave low pressure perturbation, which is centered at 90°E with a zonal width of 120°. Note that the initial condition does not need to be a Kelvin wave low pressure, a random initial condition will also generate MJO mode.

Figure 3a shows the Hovmöller diagram of the simulated equatorial precipitation in the control experiment. It shows that the trio-interaction model could well capture many essential features of MJO, i.e., the slow eastward propagation (~5 m s^{-1}), the amplification over warm pool and decay over cold tongue, and the planetary zonal scale. The system is amplified with time, as manifested by the increasing amplitude (black contours), indicating the existence of linear instability. This linear instability could help the self-maintenance of the MJO against dissipation. In real world, the nonlinear effect and the convection saturation could limit the linear intensification when the MJO amplitude is large. Note that changing the zonal scale of the initial perturbation does not qualitatively change the results (Fig. 3b), especially the zonal scale of the MJO which is comparable to the warm-pool size. This indicates that the most unstable mode has planetary zonal scale that is restrained by the warm-pool size.

Several sensitivity experiments are conducted to reveal mechanisms relating to the MJO propagation and amplification. Since the horizontal moisture advection is found to play a role in the MJO’s eastward propagation (Andersen and
Kuang 2012; Kim et al. 2014; Jiang 2017), the effects of meridional and zonal moisture advection are examined by eliminating the corresponding terms in the moisture equation [i.e. Eq. (4)]. Figure 3c shows the result when the advection of background moisture by MJO meridional wind is removed. Comparing Figs. 3a and 3c, it shows that the meridional moisture advection could accelerate the eastward propagation of MJO, consistent with Kim et al. (2014). Figure 3d shows the result when the advection of background moisture by MJO zonal wind is removed. It reveals that the advection of background moisture by MJO zonal wind could reduce the zonal scale and the amplitude of MJO, because the drying at both ends of warm pool due to zonal moisture advection could suppress the convection anomalies, therefore reducing the zonal scale and hindering the amplification of MJO. Note that here we only considered the advection of background moisture by the MJO winds. The observational studies have shown that the horizontal moisture advection by synoptic eddy also plays a role for MJO propagation (Maloney 2009; Andersen and Kuang 2012; Hsu and Li 2012).

Other sensitivity experiment has been conducted with both zonal and meridional moisture advection turned off. It reveals that the MJO mode could still emerge (not shown), indicating that MJO mode in the trio-interaction model does not depend on the horizontal moisture advection. Note that there are other modern MJO theories that do not rely on the horizontal moisture advection, including the MJO skeleton theory (Majda and Stechmann 2009), the WISHE-moisture mode theory (Fuchs and Raymond 2017), and the gravity wave theory (Yang and Ingersoll 2013).

The effect of the BL moisture convergence feedback is examined by turning off the coupling between the BL and the free atmosphere [by setting $d$ to zero in Eqs. (3) and (4)]. Comparing Figs. 3a and 3e, it shows that the BL moisture convergence feedback is responsible for the maintenance of the MJO. When the BL moisture convergence feedback is turned off, the system decays quickly after initialization. The BL moisture convergence feedback is also a driver for the MJO eastward propagation. The system is quasi standing when there is no BL convergence feedback (Fig. 3e). This effect of BL convergence feedback on MJO propagation is supported by the previous observational studies (Hendon and Salby 1994; Maloney and Hartmann 1998; Matthews 2000; Chen and Wang 2018b), and the recent studies on moisture mode theory that emphasize the role of BL moisture convergence (Wang and Li 2020; Hu et al. 2021). Note that neglecting the surface evaporation could be one of the reasons for this damped standing feature when the BL convergence feedback is removed.

Finally, the effect of the cloud-radiation feedback is examined by turning off the cloud-radiation heating [by setting $r$ to zero in Eq. (3)]. Consistent with previous studies (Fuchs and Raymond 2005; Sobel and Maloney 2013; Adames and Kim 2016; Chen and Wang 2019), the major effect of the cloud-radiation feedback is to maintain the MJO, which is manifested by the quick decay of MJO mode (Fig. 3f).

The horizontal structures of the MJO mode in the control experiment are presented in Fig. 4. The circulation exhibits a convectively coupled Kelvin–Rossby structure, with Kelvin wave response to the east of the convection anomaly and Rossby wave response to the west. This structure is to some extent consistent with the observed MJO features (Rui and Wang 1990; Matthews 2000; Adames and Wallace 2014; Wang et al. 2019). The evolution of the MJO mode is also realistic, compared to observations. The MJO’s convection anomaly and its associated circulation propagate eastward slowly over the warm pool. When its convection anomaly propagates into the eastern edge of the warm pool, the Kelvin wave response decouples with the convection (days 135–140) and propagates eastward as dry Kelvin wave (days 140–145). As the dry Kelvin wave propagates into the western edge of the warm pool, it helps reinitiating the MJO convection over there (days 145–150).

In summary, the trio-interaction model could well capture many essential large-scale features of the MJO with reasonable physical processes. Therefore, it is valid to use the trio-interaction model to study the effect of temperature feedback.

b. The MJO mode without temperature feedback

To see how temperature feedback affects the MJO mode, we will conduct experiments neglecting the temperature (geopotential) tendency. As shown by previous section, this is equivalent to assume $N_m \to 0$. Since $R_0 < 1$, $N_m \to 0$ leads to $\tilde{N}_m \to 0$ in Eq. (24), indicating that the momentum feedback (i.e., momentum tendency) in the momentum equation [e.g., Eqs. (1) and (2)] could be neglected when the temperature feedback is turned off. Note that the damping term should be retained in the zonal momentum equation for the balance of dynamics. Therefore, turning off the temperature feedback requires to treat the circulation as steady response to the MJO convection. This treatment amounts to the assumption that the time scale for the steady response to be established is short compared to the MJO frequency (Sobel and Maloney 2012). Instead of applying strict WTG approximation as suggested by Eq. (21), we include the Newtonian cooling in the thermodynamic equation, as applying WTG balance will require additional compensate heating for the mass continuity (Bretherton and Sobel 2003). For simplicity, we use numerical integration to obtain quasi-steady responses rather than solve them analytically as done by Gill (1980). This is because we use warm-pool SST configuration rather than uniform SST, which makes it difficult to solve analytically. The initial condition is a dipole precipitation perturbation. The detail of the numerical scheme can be found in appendix A, and the sensitivity to monopole initial precipitation condition is presented in appendix D.

Figure 5 shows results when temperature feedback is turned off. Comparing Figs. 3a and 5a, it shows that the system is decaying and cannot be self-sustained over the warm pool when there is no temperature feedback. This is
consistent with the previous results that the linear growth rate of MJO mode is reduced if the temperature feedback is turned off (Fuchs and Raymond 2017; Liu and Wang 2017; Chen and Wang 2019; Emanuel 2019). Note that using a larger radiation coefficient \( r \) will not change the fact that the simulated MJO mode is decaying (Fig. 5b). It is interesting to ask whether there is a value of \( r \) when the MJO mode will become unstable and self-sustained. Experiments show that when the cloud-radiation feedback is further enhanced (by increasing \( r \)), the simulated MJO mode will initially amplify and last longer (not shown), but the MJO mode will eventually fade away and give way to an unstable standing mode. In fact, in the simulation when \( r = 0.15 \) and 0.2 (Fig. 5), this unstable standing mode also emerges some days after the MJO mode fades away. This standing feature is in fact consistent with the requirement for dropping the temperature feedback (i.e., \( N_m \rightarrow 0 \) as \( C \rightarrow 0 \)).

Figure 6 shows the horizontal structures corresponding to Fig. 5a. Comparing Figs. 4 and 6, it shows that the transient response has a salient impact on the simulated horizontal structures. When the quasi-steady response is assumed, the Kelvin wave response is relatively weaker, while the Rossby wave response is relatively stronger.
Since the kinetic energy in the model system is much smaller than the ME [indicated by Eqs. (23) and (24)], the amplification or decay of the system is mainly depicted by the amplitude of the ME. Thus, we will analysis the effect of temperature feedback by studying how they change the ME amplitude.

To see how the ME amplitude changes, we multiply $h'$ to both sides of Eq. (23):

$$ N_c \frac{\partial h'^2}{\partial t} = G1 + G2 + G3 + D1 + D2, \quad (26) $$

where $G1 = d [(1 - \tilde{Q}_b) (\partial u'_b / \partial x' + \partial v'_b / \partial y') h'], \quad G2 = r q' h'/r', \quad G3 = -\mu h' V' \cdot \tilde{V} \tilde{Q}, \quad D1 = (1 - \tilde{Q}) (\partial u'/\partial x' + \partial w'/\partial y') h', \quad D2 = \left(F_r^2 / (R_c \tau_c) \right) (\tau_c / \tau_r) \phi'^2.$ Since the $h'$ is the ME perturbed about the climatological radiation–convection equilibrium, the quantity $h'^2/2$ can be viewed as some measure of moist APE.

The terms on the right-hand side of Eq. (26) depicts how the ME amplitude changes. $G1$ is the amplification of ME due to BL moisture convergence feedback. As $(1 - \tilde{Q}_b) < 0$ near equator, this term is positive where the BL convergence $(\partial u'_b / \partial x' + \partial v'_b / \partial y' < 0)$ collocates with enhanced ME $(h' > 0)$. $G2$ is the amplification of ME due to cloud-radiation feedback. $G3$ is the amplification of ME due to BL moisture convergence feedback. As $(1 - \tilde{Q}) < 0$ near equator, this term is positive where the BL convergence $(\partial u'/\partial x' + \partial w'/\partial y' < 0)$ collocates with enhanced ME $(h' > 0)$. $D1$ is the decay of ME due to cloud-radiation feedback. $D2$ is the decay of ME due to BL moisture convergence feedback. $D2$ is the decay of ME due to cloud-radiation feedback. $D2$ is the decay of ME due to BL moisture convergence feedback.

c. Physical mechanism

Since the kinetic energy in the model system is much smaller than the ME [indicated by Eqs. (23) and (24)], the amplification or decay of the system is mainly depicted by the amplitude of the ME. Thus, we will analysis the effect of temperature feedback by studying how they change the ME amplitude.

To see how the ME amplitude changes, we multiply $h'$ to both sides of Eq. (23):

$$ N_c \frac{\partial h'^2}{\partial t} = G1 + G2 + G3 + D1 + D2, \quad (26) $$

where $G1 = d [(1 - \tilde{Q}_b) (\partial u'_b / \partial x' + \partial v'_b / \partial y') h'], \quad G2 = r q' h'/r', \quad G3 = -\mu h' V' \cdot \tilde{V} \tilde{Q}, \quad D1 = (1 - \tilde{Q}) (\partial u'/\partial x' + \partial w'/\partial y') h', \quad D2 = \left(F_r^2 / (R_c \tau_c) \right) (\tau_c / \tau_r) \phi'^2.$ Since the $h'$ is the ME perturbed about the climatological radiation–convection equilibrium, the quantity $h'^2/2$ can be viewed as some measure of moist APE.

The terms on the right-hand side of Eq. (26) depicts how the ME amplitude changes. $G1$ is the amplification of ME due to BL moisture convergence feedback. As $(1 - \tilde{Q}_b) < 0$ near equator, this term is positive where the BL convergence $(\partial u'_b / \partial x' + \partial v'_b / \partial y' < 0)$ collocates with enhanced ME $(h' > 0)$. $G2$ is the amplification of ME due to cloud-radiation feedback. $G3$ is the amplification of ME due to BL moisture convergence feedback. As $(1 - \tilde{Q}) < 0$ near equator, this term is positive where the BL convergence $(\partial u'/\partial x' + \partial w'/\partial y' < 0)$ collocates with enhanced ME $(h' > 0)$. $D1$ is the decay of ME due to cloud-radiation feedback. $D2$ is the decay of ME due to BL moisture convergence feedback. $D2$ is the decay of ME due to cloud-radiation feedback. $D2$ is the decay of ME due to BL moisture convergence feedback.

FIG. 5. Hovmöller diagram of the equatorial precipitation. Shown are precipitation (shading; unit: mm day$^{-1}$) at equator in (a) experiment without temperature feedback ($r = 0.15$), (b) experiment without temperature feedback but having stronger cloud-radiation feedback ($r = 0.2$). The black contour shows the envelope of the precipitation. The thick black line depicts the phase propagation of MJO.

FIG. 6. Horizontal structures of the MJO mode in the experiment without temperature feedback ($r = 0.15$). Shown are precipitation (shading; unit: mm day$^{-1}$), geopotential height (contours; interval: 0.15 m), and wind vectors (vectors; unit: m s$^{-1}$). The solid (dashed) contours denote positive (negative) values, and zero contour is omitted.
with enhanced ME. G3 is the amplification (or dissipation) of ME by horizontal moisture advection. This term is positive where positive horizontal moisture advection collocates with the enhanced ME. In the theoretical model, G3 is much smaller than G1 and G2, as indicated by Fig. 3 that the horizontal moisture advection has weaker impact on MJO amplification than the BL moisture convergence feedback and cloud-radiation feedback. Since \( \tilde{Q} \neq 0 \) and free atmospheric convergence collocates with enhanced ME anomalies in the convection zone, D1 is the dissipation of ME due to excessive adiabatic cooling over vertical moistening in the deep convection zone, representing the stabilization effect of the deep convection. D2 is the dissipation of ME due to Newtonian cooling.

Figure 7a exhibits the horizontal structures of the ME, precipitation, and BL divergence for the MJO mode in control experiment (i.e., the results with temperature feedback), and Figs. 7d and 7g show the corresponding G1 and G2. It indicates that the amplification of ME due to the BL moisture convergence feedback (G1) has comparable magnitude to that due to the cloud-radiation feedback (G2). The amplification of ME by radiation feedback is more closely collocated with the enhanced convection, while the amplification of ME by BL moisture convergence feedback leads the enhanced convection, as the BL convergence leads the enhanced convection.

If \( N_{tr} \to 0 \), the moisture variation dominates the ME variation, and the temperature variation can be neglected in the ME. The Eq. (26) reduces to

\[
N_C \frac{\partial}{\partial t} \left( \frac{q'^2}{2} \right) = G1' + G2' + G3' + D1' + D2',
\]

where \( G1' = d \left[ 1 - \tilde{Q} \right] \left( \partial \phi / \partial x' + \partial \psi / \partial y' \right) q' \), \( G2' = r q'' / \gamma' \), \( G3' = -\mu q^2 \nabla' \cdot \nabla \tilde{Q} \). D1' = (1 - \tilde{Q}) (\partial u' / \partial x' + \partial v' / \partial y') q', D2' = \left( F_r^2 / R_0 \right) \phi' q'. The major change is that the amplification of ME is mainly due to the correlations of moisture (or latent heat) to BL convergence and radiative heating when the temperature feedback is turned off, while the amplification of ME related to temperature variation vanishes. This indicates that neglecting the temperature feedback will reduce the efficiency for the amplification of ME.

To further illustrate this, Figs. 7b and 7c show the ME variation associated with latent heat (moisture) and enthalpy (temperature) for the MJO mode in the control experiment, respectively. It shows that although the latent heat variation is larger than the enthalpy variation, they have the same order in magnitude. Figures 7e, 7f, 7h, and 7i further show that the ME amplification associated with the enthalpy variation could accounts for about one fourth of the total ME amplification. On the other hand, when the temperature feedback is turned
off, the latent heat fluctuation dominates the ME fluctuation (Figs. 8a,b) and the enthalpy fluctuation due to Kelvin wave response is very weak (Fig. 8c), indicating that the amplification of ME associated with the enthalpy variation is negligible. Therefore, it explains why including the temperature feedback enhances the amplitude of the system.

Fig. 8. Decomposition of ME for the MJO mode at day 3 in the experiment without temperature feedback ($r = 0.15$). Shown are (a) total ME (contours; interval: $2 \times 10^5$ J m$^{-2}$), (b) ME variation due to moisture, and (c) ME variation due to temperature. Also shown are precipitation (shading; unit: mm day$^{-1}$) and low-level winds (vectors; unit: m s$^{-1}$). The solid (dashed) contours denote positive (negative) values, and zero contour is omitted.
The underlying physical difference between the systems with and without temperature feedback is whether the enthalpy (and therefore the APE) can be generated. Without transient temperature responses, the energy of the system is solely supplied by the releasing of latent heat, and there is no APE generated. The energy generated by latent heat release is immediately removed by the Newtonian cooling and Rayleigh friction, which determines the magnitudes of the temperature (geopotential) and winds. With the temperature feedback included, the APE (or enthalpy) can be generated. Thus, the energy generation is more efficient in the system with temperature feedback.

This important difference also partly explains why including the temperature feedback changes the horizontal structures of the simulated MJO mode. The Kelvin wave response induced by the MJO convection propagates eastward, causing temperature fluctuation (manifested as low pressure in lower troposphere). From Eq. (26), the interaction between the BL moisture convergence and temperature fluctuation of Kelvin wave response could amplify the ME. Since this amplification of ME is in the zone of the Kelvin wave low pressure, part of the ME amplification is manifested in the amplification of dry enthalpy (and thus the generation of APE) associated with the Kelvin wave response, therefore amplifying the Kelvin wave response. Without the temperature feedback, the BL moisture convergence cannot feedback to temperature field, leading to weak Kelvin wave response (Fig. 6).

5. Implications from observation

Can the effect of the temperature feedback be inferred from the observations? This section will answer this question. The datasets used in this section include the daily averaged outgoing longwave radiation (OLR) data on a 2.5° squared resolution from NCEP/NOAA interpolated OLR dataset (Liebmann and Smith 1996), for the period of 1979–2018. The daily averaged ERA-Interim data (Dee et al. 2011) are used for the horizontal and vertical MSE budget. In fact, the column-integrated ME in the theoretical model is approximately equal to the column-integrated MSE budget. Instead of exploring the column-integrated ME budget as done in the theoretical analysis, we investigate the column-integrated MSE budget. In fact, the column-integrated ME in the theoretical model is approximately equal to the column-integrated MSE. The column-integrated MSE equation can be written as

$$\frac{\partial}{\partial t}[m']' = - \left[ \frac{\partial m'}{\partial p} \right]' - [\nabla \cdot \nabla m']' + [Q_e'] + F_r + \text{res},$$

where the square brackets denote vertical integration from 1000 to 200 hPa and prime denotes intraseasonal component.

$Q_e$ denotes radiation and $F_r$ surface heat fluxes. $m = C_pT + gz + L_q = s + L_q$ is the MSE and $s = C_pT + gz$ is the dry static energy (DSE). The residual term, which might originate from spatial interpolation and analysis increment in the assimilation system of ERA-Interim, is denoted by “res”.

The MSE, ME tendency, and MSE advection terms are calculated by the daily data (the 2-day central difference is used for calculating the tendency), and then the filtering process described above is applied to obtain the intraseasonal component. The radiation and surface heat fluxes are calculated as $[Q_e'] + F_r + \text{res} = [Q_e'] - [Q_i]\equiv \langle \partial / \partial t \rangle [m']' + [\nabla \cdot \nabla m']' + [\omega m/\partial p]'$, where $Q_e'$ and $Q_i'$ are apparent heat and moisture sink defined by Yanai et al. (1973).

To seek the MJO signals, we define time series as area averaged intraseasonally filtered OLR over Indian Ocean (IO) region (10°S–10°N, 80°–100°E). Then, the MJO signals are obtained by regrressing the intraseasonally filtered variables onto the normalized time series. Figure 9 shows the filtered column MSE, column DSE, column latent heat, and column MSE tendencies. It shows that in the deep convection region (around 90°E), the latent heat dominates the MSE (Figs. 9a,c.e). This is because the convection-induced waves and the associated temperature perturbations will propagate and disperse away from the convection region. To the east of the convection center, the DSE variation (Fig. 9b) resulted from the propagation of Kelvin wave response has comparable magnitude as the latent heat variation (Fig. 9b), indicating that in these regions the DSE is as important as the latent heat. Figure 9d shows that positive MSE tendencies exist to the east of the MJO deep convection while negative MSE exist to the west, signifying the eastward propagation of MJO.

To explore how the MSE amplitude changes, we multiply $[m']'$ to both sides of Eq. (28):

$$\frac{\partial}{\partial t}[m']^2 = H_1 + H_2 + H_3,$$

where $\langle \partial / \partial t \rangle [m']^2 / 2$ represents the time change of MSE amplitude. $H_1 = -\langle \omega m / \partial p \rangle'[m']'$, $H_2 = -\langle \nabla \cdot \nabla m \rangle'[m']'$, and $H_3 = \langle [Q_e'] + F_r + \text{res} \rangle [m']'$. Note that H2 corresponds to G3 in Eq. (26) of the theoretical model and part of H3 correspond to G2. H1 corresponds to G1 if the low-level vertical MSE advection dominates and D1 if the middle-level vertical MSE advection dominates, and this will be elaborated later.

Figure 10 shows the spatial distributions of each term in Eq. (29). In equatorial belt (10°S–10°N), the MSE amplitude decays in the deep convection region over IO while intensifies outside the deep convection region; the positive (negative) correlation between the column MSE and the MSE advection terms contributes to the intensification (attenuation) of MSE amplitude outside (in) the deep convection region (Figs. 10b,c); the radiation tends to intensify the MSE amplitude in the deep convection region (Fig. 10d).

To illustrate how the DSE fluctuation (and thus the temperature fluctuation) contributes to the intensification of MSE amplitude, the terms H1, H2, and H3 are split up into contributions
from DSE and latent heat. For example, $H_1$ is split up into

$$\left[ \frac{\partial}{\partial p} \frac{\partial\theta}{\partial m} \right]' [\alpha T + g \zeta']$$

and

$$\left[ \frac{\partial}{\partial p} \frac{\partial\theta}{\partial m} \right]' [L, q]',$$

representing the DSE and latent heat contributions, respectively. Figure 11 shows the domain averages of each term in Eq. (29), as well as the separate DSE and latent heat contributions. In the equatorial Indian Ocean–western Pacific (IO-WP) region ($10^\circ$S–$10^\circ$N, $40^\circ$–$160^\circ$E), the overall MSE amplitude is intensifying (Fig. 11a, first column). This overall intensification comes from the DSE contribution, which is explained by the positive correlation between the MSE advection terms (including both the horizontal and vertical advection) and the DSE (Fig. 11a, first column).
second and third column). Thus, it supports the theoretical results that the temperature feedback could amplify the MJO.

As shown by Fig. 10a, the spatial distribution of the MSE amplitude change is inhomogeneous. To see how different processes contribute to the MSE amplification, the domain in Fig. 11a is split into IO region (10°S–10°N, 40°–110°E) and Maritime Continent–western Pacific (MC-WP) region (10°S–10°N, 110°–160°E). In the MC-WP region, the MSE amplitude intensifies (Fig. 11b, first column). This intensification is attributed to H1 and H2, in which the DSE contribution plays a significant positive role (Fig. 11b, second and third column). Note that H1 in Fig. 11b is mainly contributed by the low-level vertical MSE advection and is thereby analogous to G1 in Eq. (26). To see this, H1 is split into two parts. One part is the column MSE times the vertical MSE advection integrated from 1000 to 700 hPa, and the other is the column MSE times the vertical MSE advection integrated from 700 to 200 hPa. The 1000–700 hPa integration represents the vertical MSE advection associated with the BL moisture convergence and the related shallow convection, which are shown to play significant roles in MJO dynamics (Chen and Wang 2018b). Figure 12 shows the split contribution of H1. It indicates that the positive H1 in the MC-WP region is mainly attributed to the vertical MSE advection associated with the BL moisture convergence, which is analogous to the effect of G1, therefore supporting the theoretical analysis. Contrasting to the theoretical analysis, H2 also has significant positive contribution to the MSE amplification, suggesting that the theoretical model has underestimated the effect of horizontal ME advection. On the other hand, H3 tends to attenuate the MSE amplitude. This negative contribution of H3 is caused by the negative value of $\left[\mathcal{Q}_{r}^{1} + F_{1} + \text{res}\right]$ (Fig. 10d), which might be attributed to the negative surface fluxes (Benedict and Randall 2007; DeMott et al. 2015).

In the IO region, the MSE amplitude decreases (Fig. 11c, first column). This decrease is mainly associated with the latent heat contribution, while the DSE still has weak positive contribution. The H1 and H2 have strong negative contribution to the MSE amplitude change (Fig. 11c, second and third column). These negative contributions come from the deep convection zone (Fig. 10) and are associated with the negative MSE advections. Note that the negative H1 in deep convection zone (Fig. 10b) is analogous to D1 in Eq. (26). The H3 has strong positive contribution (Fig. 11c, forth column), which is mainly explained by the radiation in the convection zone (Fig. 10d). The H1, H2, and H3 in IO region are dominated by the latent heat contribution, while the DSE contribution is much weaker. This is because the latent heat variation dominates the DSE variation in IO region (Fig. 9). Interestingly, the DSE variation has negative contribution to the radiation effect (H3), contrasting to the theoretical results (Fig. 7). This is caused by the different phase relation between the DSE and radiation in the model and reanalysis data (Fig. 7 versus Figs. 9 and 10), indicating the limitation of the theoretical model.

6. Conclusions and discussion

The small Coriolis force results in weak temperature gradient in tropics, leading to the neglect of temperature feedback in the theoretical MJO studies that greatly
simplifies the problem. However, this study shows that the temperature feedback could have a significant impact on MJO. It is shown by scale analysis that the rotational effect is important (i.e., small Rossby wavenumber) for the MJO scales, so that the transient temperature variation cannot be rapidly removed by gravity wave dispersion. Consequently, the temperature feedback becomes as important as the moisture feedback.

It is further shown by the theoretical model that turning off the temperature feedback would impede the self-maintenance of the MJO over warm pool. This is because the temperature feedback can boost the energy generation. Without temperature feedback, only the latent heat can be generated, while the generation of enthalpy (and therefore APE) is not allowed. With temperature feedback, both the enthalpy and latent heat can be generated, therefore increasing the total energy generation for the maintenance of MJO. The theoretical finding that temperature feedback could boost the energy generation is further inferred from the observations.

The findings here have implications in explaining the excessive Maritime Continent “barrier effect” of MJO in the simulation. As shown by previous studies (Jiang et al. 2015; Wang and Lee 2017), the models that have excessive Maritime Continent “barrier effect” often simulate weak Kelvin wave responses when the MJO convection approaches the Maritime Continent. The weak Kelvin wave responses and the corresponding weak low-level temperature fluctuations are inefficient in amplifying the MSE [or generating APE as shown by Wang and Lee (2017)] for the growth of system, explaining why some simulated MJOs decay more quickly over the Maritime Continent. In fact, improving the simulation of Kelvin wave response by modifying the convection schemes could lead to overcoming of the “barrier effect” (Yang and Wang 2018), suggesting that the theoretical findings can be used as a guidance for improving the model simulation. However, it should be noted that results in theoretical model may not be directly applied to the numerical models, because the numerical models have more physical and mathematical complexities, and the physical mechanism in the simple theoretical model may not be directly compared to that in the numerical model.

Although the results here have revealed the importance of temperature feedback on the amplification of MJO, it does not indicate that the existence of temperature
feedback is a necessary condition of instability. This is because the results could be sensitive to the models and the associated parameterization schemes. In this study, turning off temperature feedback leads to no instability, consistent with some previous studies (Fuchs and Raymond 2017; Liu and Wang 2017; Chen and Wang 2019; Emanuel 2019), while in other studies (Sobel and Maloney 2013; Adames and Kim 2016) the instability still exists after turning off the temperature feedback. Nevertheless, this study suggests that the temperature feedback could have nonnegligible impacts on the MJO.

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APPENDIX A

Numerical Schemes

For the simulation with temperature feedback (section 4a), the full dimensional equations [i.e. Eqs. (1)–(7)] are solved numerically. The numerical scheme exactly follows Wang and Chen (2017). The model is solved in an aquaplanet channel between 40°S and 40°N on a spherical coordinate by converting Eqs. (1)–(7) to the spherical coordinate. The differences between using β-channel model and spherical coordinate are very small. A periodic zonal boundary condition is used, while the meridional boundary conditions are specified as that the fluxes of mass, momentum, and heat normal to the boundaries vanish. The numerical method introduced by Lin and Rood (1997) is adopted for solving the model equations. The time step is 10 min, and the spatial resolution is 4° longitude × 2° latitude.

For the simulation without temperature feedback (section 4b), we seek the quasi-steady circulation (including winds and geopotential) responses. The moisture equation is solved in C grid with time step of 10 min. Within each time step of moisture equation, the momentum equations (including the BL momentum equations) and thermodynamic equation are first integrated for 15 days, with time step of 30 min. The quasi-steady circulation responses are then obtained by averaging the results of last 5 days. The method of Lin and Rood (1997) is used for integration of momentum and thermodynamic equations. The quasi-steady winds are then used for the integration of moisture equation. The initial perturbation is a dipole precipitation anomaly centered at equator with meridional width of 20°. One part of the initial dipole precipitation is centered at 80°E with zonal width of 40°, and the other part is centered at 130°E with zonal width of 40°.

APPENDIX B

Alternative Precipitation Scheme

To consider the effect of temperature in the precipitation parameterization, alternative Betts–Miller type parameterization Pr = \[ q - \alpha \Delta p_C/(g\ell_C) T \]/\(T\), following Frierson et al. (2004), is tested. Here, \(\Delta p_C/g\ell_C\) represents column-integrated enthalpy and \(\Delta p_C/(g\ell_C)\sim 1.6\). This parameterization is equivalent to what is used in Ahmed et al. (2021). In the trio-interaction model, this parameterization becomes Pr =
Alternative B-M scheme

\[ \frac{q}{\alpha} + \frac{\Phi}{\tau} \quad (\text{Wang and Chen 2017}), \quad F = \frac{2P_c C_p}{L_c R_g}. \]

Figures B1 and B2 show the results in the simulations with temperature feedback. Figure B1 shows that including \( T \) in the precipitation parameterization could slow down the propagation speed of the MJO mode and reduce the growth rate (reflected in the growth of amplitude). The reduction of growth rate can be explained as that including temperature in the precipitation parameterization amounts to increasing the thermal damping in the thermodynamic equation, i.e., adding \(-\alpha \Phi' / \tau\) to the right-hand side of Eq. (3). Figure B2 shows that including \( T \) in precipitation does not qualitatively change horizontal structures of the MJO mode, compared to Fig. 4. Given that the effect of the temperature feedback on MJO maintenance depends on the horizontal structures, it suggests that including \( T \) in precipitation will not affect the conclusions of this study.
APPENDIX C

Sensitivity to the Convective Adjustment Time Scale

One of the important parameters of the trio-interaction model is the convective adjustment time scale, i.e., $\tau = \frac{\tau_0}{\eta(\varphi)}$. To see the model sensitivity to $\tau$, Figs. C1 and C2 show the model sensitivity (with temperature feedback) to $\tau_0$ and $\eta(\varphi)$, respectively. Figure C1 shows that shorter convective time scale corresponds to larger growth rate (reflected in the growth of amplitude), whereas Figure C2 shows various distribution of mean precipitation $\eta(\varphi)$.
which is consistent with the results of Wang and Chen (2017). Figure C2 shows that wider distribution of mean precipitation leads to larger growth rate. This is because wider distribution of mean precipitation corresponds to shorter convective time scale over equatorial region.

APPENDIX D

Monopole Initial Condition for Simulation without Temperature Feedback

To see the model sensitivity to initial condition, the monopole initial perturbation is tested. The initial monopole precipitation perturbation is centered at equator with meridional width of 20° and zonal width of 40°. Figure D1 shows that with monopole initial precipitation perturbation, the simulated MJO mode propagates faster and lasts longer than the simulation with dipole initial condition (compared to Fig. 5b). Figure D2 shows that for the monopole initial condition, the Kelvin wave component is stronger. This is because the Kelvin wave component tends to be muted by dipole heating. The stronger Kelvin wave response can cause more zonally elongated precipitation, further enhancing the Kelvin wave component at later time. With stronger Kelvin wave component, the MJO mode lasts longer due to stronger BL feedback. The MJO mode also propagates faster for stronger Kelvin wave component, consistent with Chen and Wang (2020).