THE MEASUREMENT OF VERTICAL TRANSFER OF HEAT AND WATER VAPOR BY EDDIES IN THE LOWER ATMOSPHERE

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ABSTRACT

Though the necessity of measuring the vertical flux of heat and water vapor, brought about by eddy movement in the lower atmosphere, has long been recognized, no direct method of determining these important quantities has previously appeared. The apparatus described here provides a continuous record, over a five-minute interval, of the detailed structure of temperature, vapor pressure, and total wind speed and its vertical component, of the air passing a fixed point. It is possible to derive the required fluxes from such records, and the method of analysis is discussed in some detail. Specimen records and measurements of the fluxes are presented.

1. Introduction

Though the following paragraphs may be modified to apply to the eddy transfer of other properties in the lower atmosphere, for definiteness the discussion will be restricted to heat flux. Suppose that the temperature of the air passing a fixed point is \( T \); that its density is \( \rho \), and that the upward vertical component of its velocity is \( w \). Then, at that instant, heat is being transferred upwards at the point in question at a rate \( c_p \rho w T \) per unit area, \( c_p \) being the specific heat of air at constant pressure.

If now we consider a period of time long enough to allow the passage of many eddies, some moving up, some down, the average rate at which heat has been transferred upwards at the point is \( n^{-1} \Sigma c_p \rho w T \), where \( n \) is the number of observations. Alternatively, this may be written as \( c_p \bar{\rho}w \bar{T} \), where the bar denotes the mean value of the quantity beneath it over the time considered (\( c_p \) is constant).

The temperature of the air passing the point will fluctuate, and the fluctuations may be considered as taking place above some arbitrary level of temperature \( T_0 \). Thus, the instantaneous temperature \( T \) may be written as \( T = T_0 + T' \). \( T' \) will be referred to as the temperature fluctuation.

The expression for the heat flux now becomes \( c_p \rho w \bar{T} \), which may be written

\[
\frac{c_p}{\rho w} (\bar{T} + T').
\]

The instrumental problem is to devise an apparatus to measure the quantities inside the parentheses.

Before going on to describe the apparatus that has been developed to achieve this, it would be well to consider what is meant when reference is made here to an “eddy.” Attempts have been made to represent eddies, mathematically and mechanically, as if an eddy were a discrete sample of air. But the point-to-point variations of atmospheric properties are always more or less gradual, whether the property be temperature, water-vapor content, velocity, etc. The term “eddy” will be used as an economy of expression rather than an indication that a distinct mass of air is thereby envisaged. The method of measurement of flux of heat and water vapor to be described is independent of the nature of eddies, and it would, perhaps, be better to speak of the variations of the different properties as “fluctuations.”

It seems likely that, ranging downwards from disturbances of synoptic size, there is no hiatus in the range of linear dimensions of eddies before the molecular scale is reached. Now, it is well known and easily demonstrated that molecular transfer is wholly inadequate to effect the observed changes of temperature and water-vapor structure that occur in the lower atmosphere. But at what point in the scale a limit can be set to the size of eddy that contributes materially to the flux of heat and water vapor (and other properties) cannot be decided a priori. This requires a detailed examination of eddy structure.

The recording of a fluctuating quantity always results in amplitude and phase distortion, more or less pronounced according to the period of the fluctuations and the characteristics of the recording apparatus. For the correct interpretation of such records, it is essential to know the degree of distortion introduced. It is also, of course, highly desirable to minimize this distortion, so that the range of frequencies of fluctuations that are adequately registered may be extended as far as possible.

Therefore, in the following sections, considerable attention is given to the amount of distortion introduced by the apparatus described, so that one may be fully aware of its limitations in recording the detailed atmospheric structure. In designing the apparatus, it has been assumed, with some justification from previ-
ous experience, that the fluctuations which are mainly responsible for the heat and water-vapor flux have a period of the order of a few seconds. Support for this assumption is given by an experiment described in an appendix to this paper.

2. The apparatus

Measurement of vertical-wind fluctuations.—The accurate measurement of a fluctuating vertical wind presents serious difficulties, particularly if mechanical methods are used. For example, from the behavior of a vane it is difficult to discriminate between horizontal and vertical fluctuations in wind velocity.

It seemed that the most satisfactory way of measuring wind fluctuations was by means of sensitive hot-wire anemometers, which do not suffer from this disability and, furthermore, have an important advantage over mechanical methods, as will be discussed later.

The techniques of hot-wire anemometry are well described by Owe (1949), and only a brief summary of the underlying principles is necessary here. King (1914), developing the theoretical treatment of Boussinesq (1901), established that the rate of heat loss $H$ per unit length from a heated wire of diameter $d$, maintained at a temperature $T$ above that of the fluid which flows normally across it with velocity $V$, could be represented by

$$H = KT + (2\pi K S d)^{1/3} V T,$$  \hspace{1cm} (1)

where $K$ is the thermal conductivity of the fluid, $S$ its specific heat at constant volume and $\rho$ its density.

King showed that this relationship should hold down to a velocity of 0.25 ft/sec for a wire with diameter 0.001 in when $T$ is 1000°C, below which wind speed the free convection currents around the heated wire distort the air flow. For lower values of $T$, this effect is less important, and, for a value of $T$ of about 500°C, that commonly used in hot-wire anemometry, it is unimportant in the measurement of atmospheric turbulence.

Because of its thermal inertia, a hot wire is not able to follow fluctuations of velocity with exactitude, and this defect becomes more pronounced the shorter the period of the fluctuations. It is possible, using (1), to calculate the amplitude and phase distortion introduced by this lag into the recording of sinusoidal fluctuations of wind velocity of any period for any wind speed. Table 1 shows the results of such calculations for a platinum wire of diameter 0.001 in.1

It is seen that the distortion only becomes noticeable for fluctuations of less than one-second period, and then only in the lightest winds. It is to be noted that the distortions in the table refer only to the hot wire itself; the instrument recording the performance of the wire introduces further distortion.

1 Similar computations were made by Dryden and Keuthe, N. A. C. A. Tech. Rep. 320, 1929.

![Fig. 1. General view of apparatus.](image)

From (1), if the wire, of resistance $R$ ohms per unit length, is heated by a current $i$, in the steady state

$$\frac{\varphi}{R} = T[K + (2\pi K S d)^{1/3} (V T)^{1/3}].$$  \hspace{1cm} (2)

This expression shows that an electrically heated wire can be used to measure the speed of air flow in two ways: (a) by maintaining the wire at a constant temperature (and therefore constant resistance), in which case the current $i$ provides a measure of the air flow, and (b) by keeping the current constant, in which case the temperature (and resistance) will vary with the air flow in accordance with (2).

The expression “air flow” is used deliberately here for, though the hot-wire instrument is customarily

<table>
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<tr>
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<th>Phase lag (deg)</th>
<th>Wind speed (m/sec)</th>
</tr>
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<tbody>
<tr>
<td>0.5</td>
<td>0.955 0.97 0.99</td>
<td>17 15 9</td>
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<tr>
<td>1.0</td>
<td>0.99 0.995 0.998</td>
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used to measure air speed, (1) and (2) show that in reality it measures the product of speed and density; in the present investigation, this proves to be a particularly valuable property.

When the air flow is steady, the hot wire may be used, in either of the above ways, as an absolute instrument. But when the flow fluctuates, it is not practicable to keep either the current or the temperature constant, and the instrument must be calibrated against another anemometer.

The vertical component of the air flow cannot be measured with a single hot wire, for such a wire is not able to distinguish between variations in the air flow and of the angle at which the wind strikes the wire. To measure this component it is necessary to know (a) the total air flow and (b) its angle of inclination, and two hot-wire circuits are necessary for this.

The hot wires that are used are shown in figs. 1 and 2. The horizontal arm is supported on ball bearings on the vertical pillar, and carries a vertical light vane of wedge shape which swivels the head carrying the hot wires, so that the single, horizontal platinum wire of diameter 0.001 in is always held broadside to the wind. Therefore, this wire is always affected in the same way by a wind of given strength, no matter what its azimuth or inclination; thus, its behavior provides a measure of the total air flow. The wire forms one arm of a Wheatstone bridge net-work, and the galvanometer current provides a measure of the air flow when the wire has been calibrated. The circuit for this wire is shown in fig. 3.

![Circuit diagram of total-airflow hot wire. H.W. 0.001 in platinum wire, cold resistance 11 ohms; R₁ (= R₂): manganin wire, 100 ohms; R₃: 0.04 in platinum wire, 16 ohms; R₄: 200 ohms rheostat; R₅: 20,000 ohms; R₆: critical damping resistance of G, mirror galvanometer; B: battery; A: ammeter.](image)
reduce considerably this disability. Fig. 6 shows a calibration curve for determining the angle of inclination of the wind. (The ordinate here is the sine of the angle of inclination of the wind. This is used rather than the angle itself because, since \( pw \) = air flow \( \times \) sine of its inclination, the analysis is thereby simplified.) It will be seen that the determination of the angle of inclination requires that the total wind speed be known; it is hoped that, in the future development of the instrument, it will be possible to bring the curves of fig. 6 into practical coincidence, so that the output of the galvanometer will measure the angle of the wind independent of its speed.

The "ageing" effect of hot wires, referred to by King (1914), which causes a progressive change in the calibration of a hot wire, particularly when the wire is very fine, becomes less pronounced when the wire is worked at moderate temperatures, say 500°C. This is about the temperature used in the present arrangement. The change of calibration with time is also reduced by annealing the wires, by heating them to red heat for some time before use. But even with these precautions, it is necessary to calibrate the wires fre-

![Fig. 4. Circuit diagram of inclination-of-airflow hot wires.](image)

The second hot-wire system consists of two equal platinum wires of diameter 0.001 in, equally and oppositely inclined to the horizontal, and mounted on the head so that they are in the same vertical plane, parallel to the vane, and hence are always in the same vertical plane as the wind. These wires form the opposite arms of the second Wheatstone bridge, shown in fig. 4. Winds inclined to the horizontal affect the wires differently, and the resultant galvanometer current provides a measure of the inclination of the wind when a calibration of the system has been made. This angle, combined with the total air flow provided by the horizontal wire, gives a measure of the vertical component of the air flow, i.e., \( pw \) in the expression given above for the heat flux.

The current in each circuit is adjusted by means of the rheostat until the current in each case, as indicated by the ammeter, is 0.2 amp in still air.

*Calibration of the hot wires.*—The two hot-wire systems are calibrated in a wind tunnel against a vane-type air meter as follows: the head carrying the hot wire is removed from its support and mounted in the wind tunnel on a horizontal rod fixed normal to the air flow, so that the horizontal wire is also normal to the flow. The inclined wires can be set at any inclination, in the line of the wind, as measured by a protractor fixed to the rod. Thus, different inclinations of the actual wind can be simulated. The bridge networks used for the calibration are exactly the same as those used for the operation of the hot wires, except for the substitution during calibration of resistances equal to the long leads to the instrument when used for recording actual winds.

A typical calibration curve for air flow is shown in fig. 5. The decrease of sensitivity of hot wires with increasing wind is well shown here, but it is possible, by modifying the resistances in the bridge arms, to

![Fig. 5. Typical calibration curve of total-wind hot-wire circuit.](image)
sequently. In the present investigation, measurements of the actual wind have not been made more than two days before or after a calibration of the hot wires.

Correction of zero drift caused by varying air temperatures.—Equation (1) relates the loss of heat from the wire to the excess of its temperature over that of the air. The short-period fluctuations of air temperature that accompany similar fluctuations in the wind are much too small to be of any consequence on a wire heated to some 50°C, but the variations of temperature between one day and another, or between day and night, might easily exceed 20°C. Therefore, it is necessary to provide compensation for the influence of such changes on the calibration of the hot wire. This is accomplished by making the arm of the Wheatstone bridge opposite to the hot wire of thick platinum wire which, having the same temperature coefficient of resistance as the hot wire itself, is equally affected by changes in air temperature. Such compensation is not necessary for the second hot-wire system, each wire in this being equally affected by temperature changes.

Measurement of temperature fluctuations.—Thermocouples of 50 SWG copper and constantan wire, soldered to form a junction whose effective diameter is less than 0.002 in, are used for the measurement of the fluctuations of air temperature. They are attached to the head carrying the hot wire, but sufficiently away and to windward from the hot wires to avoid thermal influence from them. The reference junctions are buried at about one-foot depth in the ground where, for the period during which recordings are made, the temperature is virtually constant. It is not necessary to know the temperature of the reference junction, as the description of the analysis of the records, to be given presently, will show. To increase the sensitivity, two pairs of junctions in series are used. The position of the exposed junctions on the head of the instrument is indicated in fig. 2.

The thermocouple junctions, though small, still have sufficient thermal capacity to prevent the faithful recording of short-period fluctuations of temperature, and will introduce an amplitude and phase distortion more or less pronounced according to the period of the fluctuations and the speed of the wind. Table 2 shows the results of calculations of the distortions introduced by such a junction (of assumed diameter 0.002 in) for various wind speeds and periods of sinusoidal temperature fluctuations.

The table shows that though, naturally, the distortion introduced by the thermojunctions is somewhat greater than in the case of the finer hot wire, it is still, for fluctuations of greater than one-second period and for winds of 1 m/sec or more, entirely allowable in the present investigation.

<table>
<thead>
<tr>
<th>Period of fluctuations (sec)</th>
<th>Ratio of amplitude response to true amplitude</th>
<th>Wind speed (m/sec)</th>
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<td>0.5</td>
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It was at first thought unnecessary to protect such small thermojunctions from the sun's radiation, but calculation shows that the effect may not be negligible. To calculate the order of magnitude of the influence of this radiation on the performance of the thermocouples, assume the intensity of solar radiation to be 1 cal cm\(^{-2}\) min\(^{-1}\) and the reflectivity of the junction to be 50 per cent. Then, in a wind of \(\frac{1}{2}\) m/sec, the sun's radiation will be sufficient to maintain the junction at a temperature about 0.2\(^{\circ}\)C above the mean air temperature, while in a wind of 2 m/sec the temperature excess will be about 0.13\(^{\circ}\)C. Thus, since the fluctuations of air temperature that occur have, in general, an amplitude of about 1\(^{\circ}\)C, it appears that the thermojunctions should be protected against the sun's radiation which, with a fluctuating wind, may cause a spurious temperature fluctuation of perhaps one tenth of the true fluctuation.

Accordingly, when recordings are made in strong sunshine, a small shield is held so that, though it is far enough away to offer no obstacle to the air flow, its shadow falls on the thermojunctions and thus shields them from the sun's radiation.

Measurement of vapor-pressure fluctuations.—An instrument has been developed to measure the fluctuations in vapor pressure of the air passing a fixed point, and is described in another paper.\(^2\) It is sufficient, therefore, to give here only a brief description of the principle of the apparatus. The psychometric element consists of two fine wet-bulb thermocouples, built into an electrical network so designed that it simulates the behavior of Regnault's psychrometric equation for small changes in the variables (vapor pressure, saturation vapor pressure at the wet-bulb temperature, and the wet-bulb depression). The output of the circuit gives a measure of the fluctuations of vapor pressure practically linear over the range of variation that occurs naturally.

The tapered capillary tubes, providing the water supply to the fine wet-bulb thermocouples projecting from their lower openings, are shown in fig. 2. The reference junction is buried in the ground with those of the dry-bulb thermocouples.

Method of recording wind, temperature and humidity fluctuations.—The outputs of the hot wire, dry-bulb thermocouple and humidity circuits are each recorded by means of mirror galvanometers. The lights from the galvanometers (four in all) are focused by means of a horizontal cylindrical lens onto a wooden drum which, frictionally driven by a synchronous electric motor, rotates on a horizontal axis parallel to the lens once in five minutes.Photographic paper is wound around the drum, and the whole apparatus housed in a dark room. There are thus provided synchronous traces giving

\(^2\) To be published in J. sci. Instrum. (Ed. note: since published, 28, 86–89, 1951.)

<table>
<thead>
<tr>
<th>Ratio of fluctuation period to free period of galvanometer</th>
<th>Ratio of recorded to true amplitude of fluctuation</th>
<th>Phase lag (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>0.98</td>
<td>16</td>
</tr>
<tr>
<td>4.0</td>
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</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>114</td>
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wind, temperature and humidity structure over a five-minute interval for each run.

It is necessary to consider the distortion, both of amplitude and phase, of the fluctuations of the various quantities, introduced by this method of recording. Each galvanometer is critically damped, and thus spurious fluctuations are avoided. The varying resistance of the hot wires, when measuring wind in the open, does not affect materially the damping of their galvanometers, which are sensitive enough to require a series resistance of some thousands of ohms, as shown in the circuit diagrams. The shunt resistances of the hot-wire galvanometers, because of these high series resistors, are little more than their critical damping resistance. Table 3 shows the amplitude and phase distortion introduced into the recording of fluctuations (assumed sinusoidal) of various periods, expressed in terms of the free period of the galvanometer on open circuit, by a critically damped galvanometer.

The table shows that the distortion is serious for fluctuations of the same period as the galvanometer, and that an instrument with a free period something less than one quarter that of the fluctuations is necessary for adequate recording. It will, however, become evident, when the method of analyzing the records is described later, that phase distortion is not material when the individual quantities suffer equal distortion of this sort. As remarked previously, a lower limit to the period of fluctuations that contribute materially to the flux of heat, etc., in the lower atmosphere cannot be set arbitrarily. The galvanometers used in the present investigation have been the lowest-period type available, each with a free period of about two seconds. How far galvanometers of this period are able to give a true record of the fluctuations responsible for heat transport is examined in an appendix to this paper.

3. The observations

Observations have been made at an open, level grassland site, near Melbourne, by mounting the apparatus shown in fig. 1 on a theodolite stand. Thus, the measurements refer to a level about 4\(\frac{1}{2}\) ft above the ground. Up to the present, observations have been confined to occasions of clear days and clear nights. It is proposed later to extend the observations to
greater heights and to different types of weather, e.g., occasions of advective warming and cooling of air.

Simultaneous measurements of temperature and humidity are made at two levels (2 and $\frac{1}{2}$ m above the ground) and near the apparatus. These quantities are measured with electrically-driven Assman psychrometers. It is thus possible to relate the flux of heat and water vapor through the level concerned to the gradients of temperature and humidity, respectively. The instrument is levelled by reference to two spirit levels built into its base; this is done to provide free movement of the vane. The period of a run is five minutes.

Two examples of the traces so obtained are shown in figs. 7 and 8, the former referring to a clear day and the latter to a clear night.

Because fluctuations are, generally, more pronounced during the day than during the night, it has been found necessary to use different sensitivities in the recording of temperature and vapor pressures, and the scales of the fluctuations are shown in the figures.

4. Analysis of the traces

Synchronous points on the traces referring to air flow, its inclination, temperature and vapor pressure, respectively, are read off the photographic record at 300 points equally spaced over the five-minute run, i.e., at intervals of 1 sec. An accurately ruled transparent grid is used for this purpose. By reference to the calibration curves, the total air flow and the sine of its angle of inclination are determined for each of the points. Multiplication of these quantities yields the value of $\rho w$ at each point, i.e., at each instant of time.

Heat flux.—Let us refer again to the expression for the heat flux, $c_p \rho w \bar{T}$ per unit area. As stated earlier, this may be written

$$c_p \rho w (T_0 + T'),$$

where $T_0$ is some arbitrary temperature level. Alternatively, another temperature level $T_0 + T_1$ may be chosen, and the heat flux is

$$c_p \rho w (T_0 + T_1 + T' - T_1).$$

Evidently

$$c_p \rho w (T_0 + T') = c_p \rho w (T_0 + T_1 + T' - T_1),$$

i.e.,

$$c_p \rho w (T_0 + T') + c_p \rho w (T_1) = c_p \rho w (T_0 + T_1 + \rho w (T' - T_1)).$$

(3)

$T_0$ and $T_1$ being constant.

The term $\rho w$ represents the net rate at which air mass is being transferred vertically at the point in question. A net vertical mass transfer, i.e., a mean flow...
that is not horizontal, may be due to either or both of two causes: (a) the ground itself, the transfer near which is being considered, may not be horizontal and the air flow, being quasi-parallel to the ground, will have a vertical component, or (b) there may be a steady mean circulation of air in the vertical.

In either case, there results a vertical transfer of mass at the rate \( \dot{\rho w} \) and, from (3), this will effect what may be called a mass transfer of heat at the rate \( c_p \dot{\rho w} T \) or \( c_p \dot{\rho w}(T' - T) \), according to the level of the reference temperature. At the same time, there will be an eddy transfer of heat represented by \( c_p (\rho w)'^{T'} \) or \( c_p [\rho w(T' - T)] \). But that this separation of the total heat flux into the two components is not complete is evident from the fact that the magnitude of either so expressed depends upon the temperature chosen for reference, \( T_o \) or \( T_o + T_1 \). Since it is desired to measure the heat transfer brought about by genuine eddy motion, we must seek to achieve a complete separation.

This can be done by introducing the quantity \( (\rho w)' \) defined as \( (\rho w)' = \rho w - \dot{\rho w} \), i.e., the quantity \( (\rho w)' \) is the fluctuation in \( \rho w \) referred to the mean flow. It is determined, for each individual point, by measuring the value of \( \rho w \) for all the points, and then measuring the 

departure of \( \rho w \) for the point from this mean.

\[
(\rho w)' = \rho w - \dot{\rho w}.
\]  

(4)

The expression for the heat flux due to fluctuations about the mean flow, the true eddy flux, may thus be written as

\[
c_p (\rho w)'^{T'} = c_p (\rho w)'^{T'} = c_p (\rho w)'^{(T' - T)} = c_p (\rho w)'^{(T' - T)}.
\]

in consequence of (4).

In eliminating the mass transfer of heat in this way, we have at the same time provided justification for referring the temperature fluctuations to any convenient level, for the eddy flux is represented either by \( c_p (\rho w)'^{T'} \) or \( c_p (\rho w)'^{(T' - T)} \).

A point-by-point multiplication of the quantity \( (\rho w)' \) by the synchronous value of \( T' \) is made, and the mean product, multiplied by \( c_p \), yields the quantity \( c_p (\rho w)'^{T} \), the heat flux due to eddy motion.

It will be recognized from the foregoing that the determination of \( \dot{\rho w} \), and the measurement of fluctuations of \( \rho w \) from this mean, is an essential and unavoidable step in the analysis if a distinction is to be made between mass and eddy transfer of heat. It cannot be circumvented by any method of combining directly temperature and wind fluctuations, which would provide a heat flux composed inseparably of the two parts, eddy and mass transfer.

It is a particular merit of the hot-wire method of measuring air flow that, in providing a simple product of the velocity and density of the air, it makes easy the elimination from the analysis of the heat transfer brought about by the mass movement of air through the level of reference.

Some consideration must be given at this stage to the accuracy with which the method described here measures the eddy flux of heat.

Let \( (\rho w)' \) be the true component of vertical eddy momentum, \( (\rho w)' \) its measured value. Then the true value of the eddy heat flux is \( c_p (\rho w)'^{(T_0 + T)} \) or \( c_p (\rho w)'^{T'} \), for \( (\rho w)' \) = 0 by definition. The measured value of the eddy heat flux is \( c_p (\rho w)'^{(T_0 + T)} \). But \( (\rho w)' \) is made zero by measurement in analysis of the traces. Hence the measured heat flux is \( c_p (\rho w)'^{(T_0 + T')} \). Thus the error in measurement is \( c_p [(\rho w)'^{T'} \equiv (\rho w)'^{(T_0 + T')}] \).

It is seen that the accuracy of measurement of the eddy heat flux is comparable with that of \( (\rho w)' \) itself; indeed, the latter error sets an upper limit to that for the heat flux, and the more random are the errors in the measurement of \( (\rho w)' \), the less will be the error in the measurement of \( (\rho w)'^{(T_0 + T')} \). So far as the error in measurement of \( (\rho w)' \) is concerned, it is estimated to be less than 5 per cent.

It remains to consider the effect of inaccuracy in zeroing \( (\rho w)' \) in the analysis. Suppose that the estimated zero of \( (\rho w)' \) is, in fact, a distance \( \epsilon \) from its true zero on the scale of the record (due to reading the traces to the nearest millimeter). Then, when we assume that we measure \( (\rho w)'^{(T_0 + T')}_0 \), in fact we measure

\[
[(\rho w)'^{(T_0 + T')}_0 - \epsilon](T_0 + T') = (\rho w)'^{(T_0 + T')} + (\rho w)'^{(T_0 + T')}_0 - \epsilon(T_0 + T') = \epsilon(T_0 + T') + (\rho w)'^{(T_0 + T')}_0 - \epsilon(T_0 + T')
\]

[for we have now postulated \( (\rho w)'_0 = \epsilon \), not 0],

\[
= (\rho w)'^{(T_0 + T')}_0 - \epsilon(T_0 + T').
\]

Hence, there is introduced an error \( \epsilon(T_0 + T') \) into the eddy heat flux measurement, due to the inability to zero \( (\rho w)'_0 \) exactly. But if 300 points are used to determine \( (\rho w)'_0 \), and there is a random error of measurement of 1 mm in each point, the value of \( \epsilon \) is (1/300) \( \chi \) mm, \( \approx 1/170 \) cm. Therefore, since \( (\rho w)'_0 \) and \( T' \) are represented on the traces by lengths of the order of centimeters, the error so introduced into the measurement of the eddy heat flux will be much less than 1 per cent. This, of course, is aside from the error introduced by inaccuracy in the measurement of instantaneous \( (\rho w)' \) discussed above.

It has been felt necessary to present the above discussion at some length because there is a certain subtlety in the argument which can only be clarified by a step-by-step treatment.

**Water-vapor flux.**—If the vapor pressure is \( e \) and temperature \( T \), the density of water vapor \( \rho_w \) is given by \( \rho_w = e/R_w T \), where \( R_w \) is the gas constant for water vapor. As in the case of heat flux, the vapor pressure in the air passing the reference point may be regarded as \( e = e_0 + e', e_0 \) being the vapor pressure at
some arbitrary level of reference, and $e'$ the fluctuation above that level.

Then the flux of water vapor through the level considered is $\mathcal{w}_w$, and

$$\mathcal{w}_w = \frac{w(e_0 + e')}{R_w(T_o + T')} = \frac{\rho w(e_0 + e')}{R_w p(T_o + T')}.$$

It is necessary to consider the range of variation of the quantities in this expression. $\rho w$, in general, varies from positive to negative, and $e$ varies by a considerable fraction (about 10 per cent) of its average value. On the other hand, $T_o + T' = p/R_p$, where $p$ is the atmospheric pressure and $R$ the gas constant for air, is virtually constant. Hence, the expression for the vapor flux may be written as

$$\mathcal{w}_w = \frac{\rho w}{R/R_p} \left[ \frac{(\rho w)'}{\rho w} \right] \left[ e_0 + e' \right].$$

In this expression, $(\rho w)(e_0 + e')$ represents the mass transfer of water vapor, and $(\rho w)'e'$ an eddy transfer. The middle term vanishes, since $(\rho w)' = 0$ by definition. Hence, the quantity we wish to measure, the eddy transfer, is represented by $(0.622/p)(\rho w)'e'$. In the above, no account has been taken of virtual-temperature effects in the expression $\rho(T_o + T')$, for such influence in the present considerations is quite insignificant. Nor, throughout this investigation, is any account taken of the influence of varying water-vapor content on the behavior of the hot-wire anemometer, for the same reason.

5. Results

In table 4 are presented measurements of the eddy flux of heat and water vapor on the two occasions to which the traces in figs. 7 and 8 refer. The second occasion (5 May) has been included not because it is typical, but because it illustrates a rather anomalous behavior in the flux of water vapor which has been observed on several clear nights. It will be seen that on this occasion there was an upward eddy flux of water vapor, small it is true, even though the gradient of absolute humidity was directed downwards. Furthermore, at the time of observation the grass was heavily covered with dew, and this was also the case on the other nights referred to. These observations confirm some surprising results that have been observed in the direct measurement of evaporation from pots sunk level with the ground which, during some clear nights, lose weight even though they are dew covered at the end of the night. It may be that dew, which is commonly regarded as deposition of water vapor from the air is, on occasion, entirely transpiration, some of which may be evaporated into the air. Confirmation and explanation of these results await further work.
The values of the fluxes shown in table 4 are not necessarily typical, but (except for the flux of water vapor on 5 May) they are in accord with general concepts of their respective values.

It has been argued (Priestley and Swinbank, 1947) that the coefficients of eddy transfer, as customarily defined, should vary from one transferred property to another, if the considered property influences the motion. It is therefore of some interest to compare the transfer coefficient for heat with that for water vapor, which can influence the motion only slightly, on the occasions listed in table 4. They have been estimated, for heat, from the equation

\[ \text{Heat flux} = K_{H_{2}O}(\partial T/\partial z + \Gamma), \]

in the usual notation, and for water vapor from the equation

\[ \text{Water-vapor flux} = K_{W_{2}O} \partial q/\partial z, \]

where \( q \) is the specific humidity. The values of the coefficients so calculated are shown in table 4.

Many more observations are required before definite conclusions can be drawn, but the discrepancy in the values of the coefficients listed here is interesting.

6. Further work

The method described in this paper seems to be capable of providing reliable measurements of the vertical transfer of heat and water vapor brought about by the movement of eddies in the lower atmosphere. Furthermore, it can be used for the measurement of these quantities over any sort of surface. The results quoted readily show, by comparison against incoming solar energy or outgoing long-wave radiation, that the values so measured are considerable, and that this mechanism of transfer must be of major importance in the transport of heat and water vapor in these regions.

It must be said that the present method of analysis, point by point, is laborious, and further work is being concentrated on its mechanization. The only obstacle remaining in the way of this is the separation of the calibration curves shown in fig. 6. If the response of this hot-wire circuit can be made to depend only on the angle of inclination of the wind and not on its speed, the whole analysis can be made mechanical. There has also been designed a simple addition to the instrument which, by providing a continuous record of the instantaneous azimuth of the wind, will provide measurements, in exactly the same way as for heat and water vapor, of the eddy transfer of momentum.

When this has been achieved, a program of investigation into heat, water-vapor and momentum flux at various heights, and in a variety of synoptic situations, will be carried through.

\[ \text{Fig. 9. Superimposed traces of temperature fluctuations measured with short and long period galvanometer, and of angle of wind and total wind at same site during clear weather at 1500 EST 26 July 1950.} \]
APPENDIX

It has been stressed earlier (section 3) that a lower limit to the dimensions of the eddies that contribute materially to the flux of the various properties cannot be set arbitrarily, and that the eddies must be examined quantitatively before the necessary speed of response of the recording apparatus can be decided.

The ideal way to do this would be to obtain traces of the various properties, as described above, using galvanometers all of a certain free period, and, simultaneously and at the same place, another set of traces using galvanometers of much shorter period. A comparison of the fluxes of heat and water vapor calculated from the two sets of data could then be made, and certain conclusions drawn. If the respective fluxes showed no material difference, it could be said that the longer period galvanometers were adequate for the purpose at hand. But if there were differences, this need not be true of either the longer or shorter period instruments, and further comparison against yet faster galvanometers would be necessary.

The galvanometers providing the traces shown in figs. 7 and 8 were all of two-seconds period on open circuit. Instruments of shorter period than this are not readily available, and it was not possible to carry out the comparison described above. Therefore a different comparison has been made as follows: one galvanometer of free period 0.2 sec has been used to provide a record of the temperature fluctuations at a point, and simultaneously and at the same place (i.e., at about one-inch distance) a similar set of thermo-couples has provided a record of the temperature fluctuations on a two-second galvanometer.

The circuit resistances of the two galvanometers were so arranged that they were critically damped, of the same voltage sensitivity, and with the same zero on the recording paper. As before, records of the total wind and its inclination were made synchronously, using the same galvanometers as before. Traces so obtained on a clear day (26 July 1950) are shown in fig. 9.

The temperature traces are rather striking in that they reveal that, though the shorter period galvanometer shows detail of fine structure suppressed by the other, this detail consists entirely of short-period fluctuations of small amplitude. The fluctuations of large amplitude are of much longer period, and these the slower galvanometer appears to record faithfully. Some difference in phase lag can be detected in the two traces.

The heat flux has been calculated from each temperature trace in the manner described previously. The values that result are, for the short-period galvanometer, $13 \times 10^{-4}$ cal cm$^{-2}$ sec$^{-1}$, and for the other $14 \times 10^{-4}$ cal cm$^{-2}$ sec$^{-1}$. The difference between these two values is perhaps within the error of this method of measuring heat flux and, particularly since the shorter period galvanometer trace yields the slightly smaller value, probably not significant.

This test, though not conclusive, affords good support for regarding apparatus able to record fluctuations of a few-seconds period as sufficiently fast for measuring the flux of heat, and of other properties, by eddies in the atmosphere near the ground.

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