ON THE THEORY OF CROSS-VALLEY WINDS ARISING FROM DIFFERENTIAL HEATING OF THE SLOPES

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ABSTRACT

By means of equations of horizontal motion and astronomical considerations, expressions are derived for a cross-valley wind caused by differential solar heating on opposite slopes. Friction, inertia, the Coriolis force, the time of the day and year, the latitude and orientation of the valley, and the inclinations of the slopes are independent variables in these expressions. Absolute wind speeds cannot be determined by this theory, but directions and variations in speed may be studied. Several examples illustrating effects of the independent variables on the cross-valley wind are discussed, and a comparison of observation with theory is made for a specific valley.

1. Introduction

The subject of thermally produced winds in a valley, which includes *slope* and *valley* winds, has received considerable attention from various investigators (for example, Defant [1] and Wagner [11]). Closely allied to the up-slope wind during the daytime is the wind, hereafter termed the cross-valley wind, produced by the difference between the temperatures of air on opposite slopes. This wind has been observed near the centers of valleys and has been discussed by Ekhardt [2] and Hewson and Longley [6].

The purposes of the present study are to investigate those physical factors upon which the cross-valley wind is dependent, and to obtain relationships between the wind and these factors. To these ends, it is first necessary that the basic mechanism responsible for this wind be understood.

The intensity of direct solar radiation received by a mountain slope at a given instant depends upon the angle of incidence of the sun's rays to the slope. Opposite slopes of a valley will experience different intensities simultaneously, except infrequently when the position of the sun and the nature of the valley permit the angles of incidence to be equal on both slopes. The difference in intensity causes unequal heating of the air adjacent to opposite slopes, with the consequent development of a pressure gradient across the valley. The pressure-gradient force generates a wind directed toward the slope having the higher temperature.

This wind is a function of the sun's position in the sky (which depends upon the time of the day and the year, and the latitude of the valley), the slopes of the valley walls, and the orientation of the valley in the plane of the horizon. These factors and their effects on the cross-valley wind will be examined in this paper.

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2. Dynamic theory of the cross-valley wind

Analysis by means of the equations of motion, containing terms describing inertia, is known as the dynamic theory. It has been used in the study of tides by Page [8], the sea breeze by Haurwitz [5], and air drainage by Fleagle [3]. This theory is employed in the present study and, because the cross-valley wind is similar in cause to the sea breeze, there will be certain resemblances between its application here and Haurwitz's study.

It will be assumed that the air at the center of a valley is acted upon by the pressure-gradient force due to unequal temperatures of the air at opposite sides of the valley, the Coriolis force, and friction. The latter force is assumed to be proportional to the wind speed and acting in the opposite direction. This is a crude representation, but it is not too unsatisfactory for the low wind speeds that will be encountered. Compressibility, variations in the vertical direction, and effects introduced by valley and slope winds will not be considered in the present model. Under these assumptions, the equations of horizontal motion may be written

$$du/dt - fv + ku = F(t) \quad (1)$$

and

$$dv/dt + fu + kv = 0, \quad (2)$$

where u and v are the components of the wind in the x direction (chosen perpendicular to the valley) and the y direction (chosen parallel to the valley), respectively, $f = 2\omega \sin \phi$ is the Coriolis parameter, k is a friction coefficient, t is time, and $F(t)$ represents the time-dependent pressure-gradient force acting in the x direction only. The presence of the terms du/dt and dv/dt indicates that the wind is not adjusted to balanced forces at every instant and, hence, the inertia of the air is recognized.

Introduction of the complex wind $V = u + iv$ sim-

plifies further operations. Thus, if (2) is multiplied by i and added to (1),

$$dV/dt + (if + k)V = F(t). \quad (3)$$

Before (3) can be integrated, an explicit representation for $F(t)$ is necessary. This will now be derived.

3. The pressure-gradient force

At an elevation z above a valley floor, the pressure gradient $\partial p/\partial x$ across the valley is caused by the difference in temperature of air on opposite slopes. At a higher elevation z' , the pressure p' is uniform. Integration of the hydrostatic equation between z and z' yields

$$\ln p' - \ln p = -g(z' - z)/R_a \bar{T}, \quad (4)$$

where g is the acceleration due to gravity, R_a the gas constant for air, and $\bar{T}$ the mean temperature of the layer. When differentiated with respect to x , (4) becomes

$$-p^{-1}(\partial p/\partial x) = g(z' - z)(\partial \bar{T}/\partial x)/R_a \bar{T}^2, \quad (5)$$

since $(p')^{-1}(\partial p'/\partial x) = 0$. An expression for the pressure-gradient force is obtained when the equation of state is combined with (5):

$$F(t) = -\rho^{-1}(\partial p/\partial x) = g\bar{T}^{-1}(z' - z) \partial T/\partial x, \quad (6)$$

to a first approximation, where ρ and T are the density and temperature, respectively, of air at level z . The term $g\bar{T}^{-1}(z' - z)$ will be treated as constant in the present development.

It will be assumed that the temperature T of air at an unspecified short distance from a slope is determined partly by the portion of direct solar radiation I received by the slope and transmitted to the air, and partly by various other thermal effects. These other effects need not be considered here. Then

$$T = \alpha I + T', \quad (7)$$

where α is a proportionality constant and T' is the component of T due to the net heating of the air by the various other means. This equation implies that the temperature varies instantaneously with the intensity of solar radiation, but in reality there must be a lag involved. Since this lag is difficult to estimate, it will not be considered in the present simple model.

The difference between the temperatures of air adjacent to opposite slopes at the same level may be written

$$T_2 - T_1 = (\alpha_2 I_2 + T_2') - (\alpha_1 I_1 + T_1'), \quad (8)$$

where subscripts refer to each of the opposite slopes. It will be assumed that T_1 and T_2 represent air temperatures at the same short distance from slope 1 and slope 2, respectively, thus permitting α_1 and α_2 to be

equated with a minimum loss of accuracy. It is also assumed that thermal effects other than insolation are the same at both sides of the valley, so that $T_1' = T_2'$. This has the serious consequence that the effect of temperature advection from one slope to the other cannot be considered as included in T_1' and T_2' , because advection would tend to lower the air temperature at one slope and to raise it at the other. Adequate theoretical treatment of advection would be quite involved and will not be attempted here. With the above assumptions, (8) becomes

$$T_2 - T_1 = \alpha(I_2 - I_1). \quad (9)$$

Expressions for I_1 and I_2 , the intensities of solar radiation on opposite slopes, may be derived from astronomical considerations.² (See appendix 1.) These expressions may then be combined with (3), by means of (6) and (9), so that

$$\begin{aligned} dV/dt + (if + k)V &= F(t) \\ &= R[\sin \delta \cos \beta_2 + \cos \delta \sin \beta_2 \cos(\omega t - \nu + \epsilon_2) \\ &\quad - \sin \delta \cos \beta_1 - \cos \delta \sin \beta_1 \cos(\omega t - \nu + \epsilon_1)], \end{aligned} \quad (10)$$

where subscripts 1 and 2 refer to slope 1 and slope 2, respectively, δ is the declination of the sun, β the angular distance between the celestial north pole and the perpendicular to a given slope, ω the frequency of diurnal rotation, ν the hour angle of the sun at sunset, and ϵ the angle between β and the celestial meridian. The factor R is a constant involving the maximum solar radiation intensity I_n , the term $g\bar{T}^{-1}(z' - z)$ from (6), the proportionality constant α from (9), and the horizontal distance between the air parcels whose temperatures are considered. Because the latter three quantities are indeterminate, R itself cannot be evaluated. This means that the solution to be obtained for (10) cannot be used to compute the speed of the cross-valley wind, but must be restricted to qualitative investigations of this wind.

4. Expressions for the cross-valley wind

The differential equation (10) is linear and may be integrated by standard methods to yield

$$\begin{aligned} V &= R \left[\frac{N_2 - N_1}{if + k} \right. \\ &\quad + (N_2' - N_1') \frac{(if + k) \cos(\omega t) + \omega \sin(\omega t)}{(if + k)^2 + \omega^2} \\ &\quad + (-N_2'' + N_1'') \frac{(if + k) \sin(\omega t) - \omega \cos(\omega t)}{(if + k)^2 + \omega^2} \left. \right] \\ &\quad + (F' + iF'')e^{-(if+k)t}, \end{aligned} \quad (11)$$

² Astronomical considerations have been used to determine sunniness on slopes by Gessler [4] and Pers [9], and on horizontal surfaces by Milankovitch [7].

FIG. 1. Cross section through Columbia River Valley at 49.6°N.

where

$$\begin{aligned} N_1 &= \sin \delta \cos \beta_1, & N_2 &= \sin \delta \cos \beta_2, \\ N_1' &= \cos \delta \sin \beta_1 \cos (\epsilon_1 - \nu), \\ N_2' &= \cos \delta \sin \beta_2 \cos (\epsilon_2 - \nu), \\ N_1'' &= \cos \delta \sin \beta_1 \sin (\epsilon_1 - \nu), \\ N_2'' &= \cos \delta \sin \beta_2 \sin (\epsilon_2 - \nu), \end{aligned} \quad (12)$$

and F' and F'' are integration constants that are real. [The real and imaginary components of (11) appear in appendix 2.]

It is to be noted that (10) and (11) are meaningful only under one circumstance: the sun is shining on both slopes. For time intervals when the sun shines on one slope but not on the other, (10) and (11) prescribe *negative* solar intensity on the slope in shadow. Therefore, it is necessary to modify (10) during these intervals by omitting from it terms for solar intensity on the shadowed slope. Because each of the two slopes may become shadowed during the day, two additional solutions similar to (11) are obtained (see appendix 2). The three solutions suffice to describe the course of the cross-valley wind throughout the day, in a given valley, with the aid of two conditions.

The first condition is that the wind is zero at sunrise, $t = 0$. This may be satisfied by setting $u = v = t = 0$ in the solution that is appropriate at sunrise, and then solving for the two integration constants.

The second condition is that the wind is continuous at a critical moment when one solution ceases to be valid and a second solution becomes valid. A critical moment occurs when the sun's rays become parallel to either of the two slopes. (See appendix 1 for further details.) The condition is satisfied by equating the two solutions at this time and solving for the integration constants in the solution to be employed thereafter.

It is apparent that the integration constants in the wind equations represent inertia. The remaining terms in these equations are too complicated to be physically interpreted, as inspection of (11), (12) and appendix 2 indicates. Therefore, several examples will be presented to illustrate how variations of friction, inertia, latitude, season, inclinations of valley slopes, and valley orientation affect the theoretical cross-valley wind. The first of these examples will also afford a comparison of theory with observations.

5. Application of the theory

Effect of friction in an actual instance.—The theoretical cross-valley wind will first be compared with pilot-balloon observations made in the Columbia River Valley, approximately four miles north of the United States-Canadian border at 49.6°N. The wind was observed here at two-hourly intervals during the summers of 1938, 1939 and 1940. Averages were computed separately for cross-valley components and components parallel to the valley, and were published in graphical form [6].

The valley is oriented in a north-south direction. Fig. 1 shows an east-west cross section through it, as interpreted from a topographic map. The present discussion will be devoted to winds at 500 ft above the valley floor (2500 ft above mean sea level). At this elevation, the west slope (hereafter referred to as slope 1) has an inclination of approximately 20 deg, while the east slope (hereafter referred to as slope 2) has an inclination of approximately 9 deg.

Let July 15 represent a typical summer day on which the wind observations were made. Then, from data presented thus far and equations developed, the theoretical cross-valley wind may be computed if a value is selected for the friction coefficient k . Haurwitz [5] and Schmidt [10] have adopted the k values 0.8×10^{-4} and $1.5 \times 10^{-4} \text{ sec}^{-1}$, respectively, in their studies of the sea breeze. The same values will be used here.

A coordinate system will be chosen whose origin is at the center of the valley, with the x direction positive from west to east across the valley and the y axis positive from south to north parallel to the valley. Fig. 2 shows time variations in the x direction of (a) the theoretical difference between intensities of solar radiation on the west slope (slope 1) and the east slope (slope 2); (b) observed u -components of the average winds at 500 ft (solid curve) and 1500 ft (dashed curves), as interpolated from a diagram [6, p. 309], and (c) theoretical u -components of the cross-valley wind for $k = 0, 0.8 \times 10^{-4}$ and $1.5 \times 10^{-4} \text{ sec}^{-1}$. The curve for $k = 0$ is reproduced at one half the calculated size.

FIG. 2. Time variations in west-east direction, across Columbia River Valley, of (a) theoretical difference between intensities of solar radiation on west slope (slope 1) and east slope (slope 2), (b) observed u -components of average winds at 500 ft (solid curve) and 1500 ft (dashed curves), (c) theoretical u -components of cross-valley wind for $k = 0$, 0.8×10^{-4} and $1.5 \times 10^{-4} \text{ sec}^{-1}$. Curve for $k = 0$ is reproduced at one-half calculated size. Date is 15 July.

On the abscissa, local time is measured in hours. The range is from sunrise (between 0400 and 0500) to sunset (between 1900 and 2000). On the ordinate of (a), the ratio of intensity difference to maximum possible intensity is positive when the east slope experiences greater intensity than the west slope, and negative when the converse situation prevails. On the ordinate of (b), the velocity is indicated for 5 mph or less. Velocities in excess of 5 mph were omitted from the averages, since it was believed that additional causes besides solar heating were largely responsible for them [6]. The ordinate of (c) is measured in units of $10^3 R$, which have no particular significance since R is indeterminate. Extreme values on curves are indicated by small circles.

Several features are of interest in this figure:

1. The intensity of solar radiation is greater on the western slope than on the eastern slope during the morning,

and conversely during the afternoon, as would be expected.

2. At local noon, the intensity difference is not zero, but has already become positive. This is due to the fact that the inclination of the east slope is less than that of the west slope.

3. The maximum intensity difference in the afternoon is slightly greater than the maximum intensity difference in the morning, essentially for the same reason.

4. In general, both observed and theoretical u -components of the wind are directed toward the warmer of the two slopes throughout the day.

5. Near mid-day, the time at which the theoretical wind changes direction becomes later as friction increases. If friction is absent ($k = 0$), the wind, acted upon by the cross-valley pressure-gradient force, inertia and the Coriolis force, becomes westerly even while the west slope is still warmer than the east slope. With friction present ($k = 0.8 \times 10^{-4}$, $1.5 \times 10^{-4} \text{ sec}^{-1}$), the changes in direction occur after the intensity difference has changed sign. In this respect, the observed and theoretical winds show best agreement for $k = 1.5 \times 10^{-4} \text{ sec}^{-1}$.

6. Near mid-day, the observed wind changes direction approximately one hour after the intensity difference changes sign.

7. For both the theoretical and observed winds, the maximum speeds occur in the afternoon, just as the maximum intensity difference occurs in the afternoon. This may be explained, in the case of the theoretical winds, by the dependence of wind speed on solar heating. However, there may be other factors responsible for the maximum speed of the observed wind in the afternoon.

8. The observed maximum wind-speeds in morning and afternoon occur closer to noon than do the theoretical maximum wind-speeds, when friction is present. In reality, the sun begins to shine on the west slope later and ceases to shine on the east slope earlier than in theory, because of the shadow cast on one slope by the other, which is not accounted for in the theory. Near the top of the valley this shadowing effect is small, as indicated by the winds at 1500 ft (dashed curves).

9. As friction decreases, the theoretical wind speeds increase, as might be expected.

10. The theoretical winds are zero at sunrise (between 0400 and 0500), as prescribed by the theory. However, the observed wind is different from zero at this time and is, in fact, a continuation of a nocturnal wind as shown by additional data [6].

A comparison of the v components of the theoretical and observed winds will not be made, since the observed v component is too heavily influenced by the valley wind for the comparison to be significant. It seems probable that complete verification of the theory for other valleys would be equally difficult for the same reason.

Fig. 3 shows time hodographs of the theoretical cross-valley wind computed for k equal to 0.8×10^{-4} and $1.5 \times 10^{-4} \text{ sec}^{-1}$. North is at the top of the figure. Local time, in hours, is indicated along each curve. Wind speed is marked off in units of $10^3 R$ along the borders.

Several features in fig. 3 deserve comment:

1. The Coriolis force introduces a north-south component into the cross-valley wind. This will be illustrated in further detail later.

FIG. 3. Time hodographs of theoretical cross-valley wind in Columbia River Valley, 15 July, for (a) $k = 0.8 \times 10^{-4} \text{ sec}^{-1}$, (b) $k = 1.5 \times 10^{-4} \text{ sec}^{-1}$.

2. As friction decreases, the wind becomes stronger. This was also apparent in fig. 2.

3. As friction decreases, the north-south component of the wind increases relative to the east-west component. Obviously this is due to the fact that the Coriolis force varies with the wind speed, which in turn varies inversely with the friction force.

Effect of inertia.—If inertia were to be disregarded, theoretically the cross-valley wind would at every instant be in equilibrium with the forces acting upon it. Thus, for example, the wind would be determined in part by the *instantaneous* difference in solar-radiation intensity on opposite slopes, rather than by the *integrated* difference which has a cumulative effect on the wind in the dynamic theory.

In the equilibrium theory,³ variations of the u component of the wind would be represented by the bottom curve in fig. 2. This may be demonstrated as follows: Let $du/dt = dv/dt = 0$ in (1) and (2), so that inertia is not recognized. Simultaneous solution of the altered equations yields

$$\begin{aligned} u &= k F(t)/(k^2 + f^2), \\ v &= -f F(t)/(k^2 + f^2). \end{aligned} \quad (13)$$

Since k and f are constant with time, u varies solely with $F(t)$, whose variations are shown in the bottom curve of fig. 2. Comparison of the curves in fig. 2 then

³ The equilibrium theory has been applied to studies of tides [8], the sea breeze [5], and air drainage [3].

indicates that neglect of inertia reduces agreement between theory and observation considerably.

The equilibrium theory yields straight-line hodographs through the origin, described by

$$v = -fu/k,$$

from (13). These are not plotted, because of the obvious inadequacy of this theory.

Effect of latitude.—The following example illustrates the effect of latitude on the theoretical cross-valley wind. Assume that valleys, identical in form and orientation with that portion of the Columbia River Valley considered previously, exist at 25°N and the equator. Case I in table 1 exhibits pertinent data concerning the theoretical winds in these valleys on 15 July. In this table, it is assumed that $k = 1.5 \times 10^{-4} \text{ sec}^{-1}$. It will be noted in this case that:

1. As latitude decreases, the north and south components of the maximum winds decrease relative to the east and west components, because of decreasing influence of the Coriolis force.

2. Maximum wind speeds at 25°N are considerably larger than at the other two latitudes. This is due to the term $\omega^2 - f^2$ in the denominators of coefficients in the equations for the wind components [see (25) in appendix 2]. This term is smaller at 25° than at the other two latitudes. It is zero at 30° , resulting in maximum wind speeds at that latitude. Physically this indicates that, at 30° , the period of the diurnal variation of the cross-valley wind coincides with the period of inertia motion. In reality, excessive wind speeds would not be expected, however, because the temperature difference across the valley, upon which the wind depends, would be decreased rapidly by temperature advection across the valley.

3. Wind speeds are generally smaller at the equator than at 49.6° . An indication of why this is so is given by the fact that the intervals during which the sun heats the west slope exclusively in the morning and the east slope exclusively in the afternoon are shorter at the equator than at the higher latitude.

Effect of inclination of the slopes.—The effect of inclination of the slopes on the theoretical cross-valley wind, in a north-south valley, is illustrated in case II, table 1. In this case it may be noted that:

1. As the slopes become steeper, the maximum speed of the cross-valley wind increases, because larger differences between solar heating of opposite slopes develop.

2. As the east slope becomes steeper while the west slope is unchanged [see II (a) and (b), table 1], the maximum wind speed in the morning occurs later. This is because the sun rises later over the east slope, so that the east wind accelerates toward the warmer west slope for a longer period during the morning. Moreover, near midday, the easterly component of the wind vanishes later as a consequence.

3. In case II (a) or (b), where opposite slopes in the same valley have different inclinations, the maximum wind speed in the afternoon surpasses the maximum speed in the morning. The reason for this behavior is different in each case. Thus in (a), where the east slope is less inclined than the west slope, the sun heats the east slope for a longer interval during the day than it heats the west slope. This

TABLE 1. Effect of various parameters on cross-valley wind.

Case	Latitude (°N)	Date	Slope 1		Slope 2		Maximum velocities		
			orientation	inclination (deg)	orientation	inclination (deg)	direction (deg E of N)	speed (10°R)	approximate local time
I. Effect of latitude	(a) 49.6	15 July	N-S (west slope)	20	N-S (east slope)	9	120	2.35	0800
							300	2.45	1815
	(b) 25						105	6.20	0830
							285	6.80	1710
	(c) 0						90	0.80	0900
							270	0.95	1700
II. Effect of inclinations of slopes	(a) 49.6	15 July	N-S (west slope)	20	N-S (east slope)	9	120	2.35	0800
							300	2.45	1815
	(b)						120	4.30	0815
							300	4.90	1810
	(c)						120	5.20	0845
							300	5.20	1730
III. Effect of season	(a) 49.6	15 July	N-S (west slope)	20	N-S (east slope)	9	120	2.35	0800
							300	2.45	1815
	(b)						114	1.20	1000
							294	1.10	1530
	(c)						17	1.70	0610
							212	5.10	1315
	(d)	22 December	E-W (south slope)	65	E-W (north slope)	45	6	2.00	1930 (sunset)
							214	4.90	1300
IV. Effect of valley orientation	(a) 49.6	15 July	N-S (west slope)	45	N-S (east slope)	45	120	5.20	0845
							300	5.20	1730
	(b)						142	4.60	0945
							322	5.00	1815
	(c)						167	5.10	1030
							347	4.30	1915

has the cumulative effect of producing the strongest winds toward the east slope in the afternoon. However, in (b) another effect must be predominant. Since the west slope is less inclined than the east slope, solar radiation is more intense on the east slope in the late afternoon than on the west slope during the first hours after sunrise. As a result, the strongest winds are again directed toward the east slope in the afternoon, even though the east slope is heated for a shorter period than the west slope.

4. In II (c), where opposite slopes of the valley have the same inclination, the maximum wind speeds in morning and afternoon are equal.

Effect of season.—An indication of the seasonal effect on the theoretical cross-valley wind in the Columbia River Valley is afforded by case III, (a) and (b), in table 1. It will be observed that:

1. The wind is generally weaker on 22 December than on 15 July. In December, the shortness of the day and the weak intensity of solar radiation on the slopes do not allow the winds to develop extensively.

2. On 15 July, as noted previously, the maximum wind speed occurs in the afternoon; but on 22 December, the maximum speed occurs in the morning. Since the sun remains low on the horizon during the daytime in December, the intensity of solar radiation becomes greater on the west slope in the morning than on the less steep, east slope in the afternoon. Hence, the wind becomes strongest when directed toward the west slope in the morning.

Another example of the seasonal effect is illustrated by case III, (c) and (d), table 1, for a valley oriented

in an east-west direction. On 15 July, the sun rises north of the valley and initially shines on the south slope exclusively. Later it shines on both slopes, but before noon it ceases to shine on the south slope as it moves southward. In the afternoon, it shines on both slopes once more. Before sunset the sun ceases to shine on the north slope, but continues to shine on the south slope. On 22 December, there is sunshine on the north slope exclusively throughout the day.

Directions of maximum winds are what would generally be expected from these descriptions. It will be noted further that the maximum wind speed on 15 July is greater than on 22 December. Essentially this is explained by the greater solar heating of the north slope in July.

Effect of valley orientation.—The effect of valley orientation on the theoretical cross-valley wind is illustrated in case IV, table 1. It is evident in this case that:

1. As the slopes are rotated from a north-south direction to a northeast-southwest direction, the maximum winds rotate toward the north-south direction.

2. Simultaneously with these variations, the maximum wind speeds occur at successively later times in the morning and afternoon. This is because the sun shines longer on the west slope and less on the east slope as the valley is rotated. As a result, the easterly wind accelerates for a longer time in the morning and attains its maximum speed later. As a further result, the time at which the westerly wind attains its maximum speed is also delayed.

3. In (a), the maximum wind speeds in morning and afternoon are equal, as noted in the discussion of II (c).

4. In (b), the maximum speed is less in the morning than in the afternoon. Essentially this is because the intensity of solar radiation is weaker on the northwest slope in the morning than on the east slope in the afternoon.

5. In (c), the maximum speed is less in the afternoon than in the morning, because the sun shines all morning on the northwest slope but only for the latter part of the afternoon on the southeast slope.

The examples that have been presented are, of course, no more than a small sample from an indefinitely large number of possible situations that may be studied by means of the present theory. Obviously, it would be a lengthy undertaking to formulate general rules for the theoretical behavior of cross-valley winds under the collective influence of the physical factors considered above. Moreover, such an undertaking may not be sufficiently warranted in view of the rather simple model of the cross-valley wind dealt with here.

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APPENDIX 1

In fig. 4, N(north), E(east), S(south), and W(west) are four points that lie in the plane of the horizon at latitude ϕ . Assume that at J, the center of the plane, there is a mountain slope having a uniform inclination, z' deg, to the plane. The celestial meridian is represented by NLOQS, the celestial pole by L, the celestial equator by WPQE, the zenith by O, the position, declination and zenith distance of the sun by N' , δ and z^* , respectively, and the intersection of the perpendicular drawn to the mountain slope with the celestial hemisphere by M.

It is apparent that the great-circle arc z is equivalent to the angle of incidence of the sun's rays on the slope. Thus, when $z = 0$, the sun's rays are perpendicular to the slope; when $z = 90$ deg, the sun's rays are parallel to the slope. From examination of spherical triangle LMN', the following expression for z may be obtained:

$$\cos z = \sin \delta \cos \beta + \cos \delta \sin \beta \cos t', \quad (14)$$

since $LN' \equiv 90 - \delta$. Equation (14) may be combined with the

FIG. 4. Celestial hemisphere at latitude ϕ . Points N, E, S, W lie in plane of horizon. Celestial meridian is represented by NLOQS, celestial pole by L, celestial equator by WPQE, zenith by O, position, declination and zenith distance of sun by N' , δ , and z^* respectively, and intersection of perpendicular drawn from mountain slope (at J) with celestial meridian by M.

cosine law for solar radiation,

$$I = I_n \cos z, \quad (15)$$

where I_n is the intensity when the sun's rays are normal to the slope, to yield

$$I = I_n(\sin \delta \cos \beta + \cos \delta \sin \beta \cos t'). \quad (16)$$

In the evaluation of (16), the declination δ of the sun is available for any day of the year from a nautical almanac. The term β may be calculated from

$$\cos \beta = \cos z' \sin \phi + \sin z' \cos \phi \cos \psi, \quad (17)$$

which refers to spherical triangle LMO. Previously, ϕ and $z' \equiv MO$ were defined as the latitude and inclination of the slope, respectively. The angle ψ is equivalent to the angle between the geographic meridian and the projection in the plane of the horizon of a perpendicular drawn outward from the slope. It will be considered positive when measured west from north.

The angle t' in (16) may be calculated from the following relations obtained from fig. 4:

$$t' = t^* + \epsilon, \quad (18)$$

and

$$\sin \epsilon = \sin z' \sin \psi / \sin \beta. \quad (19)$$

In (18), t^* represents the hour angle of the sun, measured positive westward from the celestial meridian. Angle $\epsilon \equiv$ angle OLM is chosen to be positive when measured westward from β , and may be computed from (19).

It will be assumed that at sunrise the time t is zero, and is therefore related to the hour angle t^* as follows:

$$t^* = \omega t - \nu, \quad (20)$$

where $\omega = 7.292 \times 10^{-5} \text{ sec}^{-1}$ is the frequency of diurnal rotation and $-\nu$ is the value of t^* at sunrise. The latter value may be computed from

$$\cos t^* = -\tan \delta \tan \phi. \quad (21)$$

Equation (21) is obtained from fig. 4. For spherical triangle LN'O,

$$\cos z^* = \cos t^* \cos \delta \cos \phi + \sin \delta \sin \phi.$$

At sunrise the zenith distance z^* of the sun is -90 deg, and $\cos z^* = 0$. Equation (21) then follows directly.

Combination of (18) and (20) in (16) yields

$$I = I_n[\sin \delta \cos \beta + \cos \delta \sin \beta \cos(\omega t - \nu + \epsilon)]. \quad (22)$$

When (22) is referred to each of the opposite slopes of a valley, then, by use of (6) and (9), an expression may be obtained for $F(t)$, the pressure-gradient force directed across the valley.

To determine the critical moment τ , let (22) refer to the slope parallel to the sun's rays. Then, because I is zero at $t = \tau$,

$$\cos(\omega \tau - \nu + \epsilon) = -\tan \delta \cot \beta.$$

Thus, τ may be computed from this relation by use of $\omega = 7.292 \times 10^{-5} \text{ sec}^{-1}$ and specified values for ν , ϵ , δ , and β .

APPENDIX 2

The real and imaginary components of (11) may be written

$$u = R[D + A \cos(\omega t) + B \sin(\omega t)] + e^{-kt}[F' \cos(ft) + F'' \sin(ft)], \quad (23)$$

$$v = R[D' + A' \cos(\omega t) + B' \sin(\omega t)] + e^{-kt}[F' \cos(ft) - F'' \sin(ft)], \quad (24)$$

where

$$A = \frac{(N_2' - N_1')k(k^2 + \omega^2 + f^2) - (-N_2'' + N_1'')\omega(k^2 + \omega^2 - f^2)}{4k^2f^2 + (k^2 + \omega^2 - f^2)^2},$$

$$B = \frac{(N_2' - N_1')\omega(k^2 + \omega^2 - f^2) + (-N_2'' + N_1'')k(k^2 + \omega^2 + f^2)}{4k^2f^2 + (k^2 + \omega^2 - f^2)^2},$$

$$D = \frac{k(N_2 - N_1)}{(k^2 + f^2)},$$

$$A' = \frac{(N_2' - N_1')f(-k^2 + \omega^2 - f^2) + 2kf\omega(-N_2'' + N_1'')}{4k^2f^2 + (k^2 + \omega^2 - f^2)^2},$$

$$B' = \frac{-2kf\omega(N_2' - N_1') + (-N_2'' + N_1'')f(-k^2 + \omega^2 - f^2)}{4k^2f^2 + (k^2 + \omega^2 - f^2)^2},$$

and

$$D' = \frac{-f(N_2 - N_1)}{(k^2 + f^2)}.$$

These equations are valid when opposite slopes are both sunlit. When one slope is in shadow, (23) and (24) are altered solely by omission of those terms in A , B , D , A' , B' , and D' , whose subscript (1 or 2) refers to the shadowed slope.

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