SHORTER CONTRIBUTIONS

EVALUATION OF ADVECTIVE EDDY TERM OVER NEW ENGLAND FOR JANUARY 1945

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In a recent paper on atmospheric heat-sources for monthly periods (Aubert and Winston, 1951), computations showed that the air was being heated in its mean horizontal motion in the layer, sea level to 10,000 ft, over a large area of the eastern United States, southeastern Canada and the western Atlantic, during January 1945. The maximum "horizontal heating" in this region was located over New England, where a value of 52 units[1] was calculated. In an attempt to determine the source of this heating, a direct evaluation of the contribution of the heat released by condensation in that area was made and was found to be about 23 units. This showed that about half of the observed "horizontal heating" could be accounted for by this process alone. However, the remaining 29 units were difficult to explain, when such processes as heating from the underlying surface (presumably negligible over this snow-covered land area) and radiation (generally negative) were considered. The writers concluded that heating due to advective eddy terms (or mixing) was probably the only major factor which could account for the remaining heating; however, no evaluation of the eddy term for this area was made in that study. On the other hand, it was the opinion of Dr. H. Wexler (verbally communicated) that this remaining "horizontal heating" was mainly associated with mean downward motion and dynamic warming of the descending air. In an attempt to throw more light on this interesting problem, a calculation of the advective eddy term has been made for the region and period in question.

To clarify the ensuing presentation of results, it is best to write the heating equation for mean monthly periods. The average rate at which heat is added to air parcels moving through a square-centimeter column of the atmosphere, between sea level and 10,000 ft, is expressed as

$$\frac{\overline{dQ}}{dt} = \frac{c_p\,\overline{\Delta p}}{g}\left[\frac{\overline{\partial T}}{\partial t} + \overline{V_2}\cdot\overline{\nabla_2 T} + \overline{V_2'\cdot\nabla_2 T'} + \overline{\left(\frac{p}{10^6}\right)^{\kappa} w\frac{\partial\theta}{\partial z}}\right], \quad (1)$$

where the bar represents an average over a month; the primed quantities are deviations from monthly means; Δp is the absolute pressure difference between sea level and 10,000 ft; V_2, T, p, w and $\partial\theta/\partial z$ are mean values, for the column, of horizontal velocity, temperature, pressure, vertical velocity and derivative of potential temperature with respect to height, respectively; the other symbols have their usual meaning. This equation can be derived readily from (3), (4) and (5) of the previous paper. $\overline{\partial T/\partial t}$ is generally of a smaller order of magnitude, relative to $\overline{V_2}\cdot\overline{\nabla_2 T}$; nevertheless, it was taken into account and merged with the value of $\overline{V_2}\cdot\overline{\nabla_2 T}$. Then, representing the vertical advection term by $\bar{Z}$ and $c_p\,\overline{\Delta p}/g$ by K, (1) may be written as

$$\overline{dQ/dt} = K(\overline{V_2}\cdot\overline{\nabla_2 T} + \overline{V_2'\cdot\nabla_2 T'} + \bar{Z}). \quad (2)$$

The mean heating may also be expressed in terms of individual heating processes affecting the layer of air:

$$\overline{dQ/dt} = \bar{H} + \bar{C} + \bar{R} + \overline{T_r}, \quad (3)$$

where $\bar{H}$ is the mean heating due to exchange of sensible heat with the underlying surface, $\bar{C}$ the mean heating resulting from condensation, $\bar{R}$ the mean radiational heating, and $\overline{T_r}$ the mean heating due to transformations of kinetic and turbulent energy to heat energy. Combining (2) and (3), we obtain

$$K\overline{V_2}\cdot\overline{\nabla_2 T} = -K\overline{V_2'\cdot\nabla_2 T'} - K\bar{Z} + \bar{H} + \bar{C} + \bar{R} + \overline{T_r}, \quad (4)$$

as an expression for the heating of the air in its monthly mean motion, or the "horizontal heating." Thus, $K\overline{V_2}\cdot\overline{\nabla_2 T}$ may be viewed as the net effect of the six terms on the right of (4).

The eddy term, $-K\overline{V_2'\cdot\nabla_2 T'}$, has now been computed for January 1945 at the point 45°N, 70°W in the layer, sea level to 10,000 ft. This was accomplished by measuring V_2, and the temperature gradient in the

TABLE 1. Values of terms in (4) for layer, sea level to 10,000 ft, at 45°N, 70°W for January 1945. Units are 10 cal cm^{-2} day^{-1}.

$K\overline{V_2}\cdot\overline{\nabla_2 T}$	$-K\overline{V_2'\cdot\nabla_2 T'}$	$-K\bar{Z}$	$\bar{H}$	$\bar{C}$	$\bar{R}$	$\overline{T_r}$
52	33	?*	0**	23	−8**	?***

* Measurements of part of this term ($\bar{w}\,\overline{\partial\theta/\partial z}$), using questionable gradient-wind divergence method, showed value very close to zero. Vertical eddy-term value unknown.

** Rough estimates of these quantities, considering snow-covered ground in regard to $\bar{H}$ and frequent cloudiness extending through 10,000-ft level in regard to $\bar{R}$.

*** Magnitude unknown.

[1] Units are 10 cal cm^2 day^{-1} or 10 ly/day.

direction of V_2, twice daily throughout January 1945. The values of the eddy term and previously measured quantities of (4) are presented in table 1.

It is readily apparent that the eddy term is quite large and that, when added to the heat of condensation, it comes close to balancing the mean horizontal heating term on the left side. Thus, as anticipated, the eddy term helps explain some of the sizeable heating of the air in its mean horizontal motion. Physically, this apparently means that large-scale eddies (cyclones and anticyclones), which are smoothed out in the monthly mean circulation, were bringing heat into the New England area during the month.

There are still several unknowns in table 1, notably the vertical term and the energy-transformation term. They may be fairly large in magnitude. However, the large value of the eddy term, which comes as close to providing a balance as one might expect, indicates that the vertical-motion term does not have to be invoked to provide a balance in (4).

The results have an additional interpretation in regard to computation of the average monthly heating $\overline{dQ/dt}$ of air passing over the New England area in January 1945. Evaluation of the eddy term means that three of the four terms in (1) have now been calculated, yielding a more refined estimate of $\overline{dQ/dt}$. Inclusion of the eddy term in (1) results in a net value of $\overline{dQ/dt}$ of 19 units, or 33 units less than was estimated by evaluating only the first two terms on the right of (1). Thus, if the air is followed in its daily motion over New England (*i.e.*, as observed on twice-daily charts), it is found that the air is actually heated, on the average, to a considerably smaller degree than one would expect from the mean monthly motion. This indicates that advective eddy terms can be of marked importance in modifying estimates of average "horizontal heating" made by use of mean maps. However, whether one considers these advective eddy terms as corrections to be applied to measurements from mean maps or as effects of large-scale turbulence associated with mean monthly motion, it appears that further study of their magnitudes in other situations would be worthwhile.

REFERENCE

Aubert, E. J., and J. S. Winston, 1951: A study of atmospheric heat-sources in the northern hemisphere for monthly periods. *J. Meteor.*, **8**, 111–125.

SEVERE TURBULENCE RESULTING FROM EXCESSIVE WIND-SHEAR IN TROPICAL CYCLONES

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Through works by Solberg (1939), Arakawa (1941a; 1941b), Kleinschmidt (1941), Van Mieghem (1946a; 1946b; 1948) and others, the question of the dynamic stability of zonal or circular flow has been introduced into meteorology. In the case of stability the absolute vorticity must be positive, while in the case of instability the absolute vorticity must be negative. The critical shear, for concentric circular flow around the origin of the coordinate system, is given by

$$\partial v_\theta/\partial r = -(v_\theta/r) - 2\omega \sin \phi, \qquad (1)$$

where r is the radius of curvature of the flow, v_θ the speed at any point of the path, ω the angular speed of the earth's rotation, and ϕ the latitude. If the shear is algebraically less than the value given by (1), dynamic instability is present; if the shear is greater, the motion is dynamically stable.

Arakawa (1941a; 1941b) has given a criterion to determine whether turbulent motion in the field of circular flow will increase or decrease. The criterion that turbulence should increase, even though the flow is dynamically stable, can be written

$$\partial v_\theta/\partial r > (2v_\theta/r) + \omega \sin \phi. \qquad (2)$$

A new derivation of (2) will be presented below. For clarity, it is desirable first to consider the derivation of (1). The rate of rotation of the horizon about the vertical at any latitude ϕ is $\omega \sin \phi$, where we assume that the variation of latitude in the field of circular flow can be neglected. The absolute velocity of the air is $v_\theta + r\omega \sin \phi$. The absolute angular momentum Ω is composed of the relative angular momentum rv_θ of the flow and the angular momentum of the earth $r^2\omega \sin \phi$. Thus,

$$\Omega = r(v_\theta + r\omega \sin \phi). \qquad (3)$$

It is well known that Ω is conserved when an individual parcel travels cross-stream in the field of circular flow, where the isobars are concentric circles. When a parcel of air is displaced from a position $r - dr$ to r with conservation of its angular momentum, the speed v_θ^* of the parcel in its new position is given by

$$r\{v_\theta^*(r) + r\omega \sin \phi\} = \{r - dr\} \{v_\theta(r - dr) + [r - dr]\omega \sin \phi\}.$$

The wind speed of the surroundings is $v_\theta(r)$. So the difference $v_\theta' = v_\theta^*(r) - v_\theta(r)$ between the wind speed of the displaced air-parcel and that of the surround-