Weather Derivatives and Weather Insurance: Concept, Application, and Analysis

Lixin Zeng
Risk Analysis and Technologies, E. W. Blanch Company, Minneapolis, Minnesota

ABSTRACT

The concept and applications of weather derivatives and weather insurance are introduced. Proper analysis of these financial instruments requires both statistical knowledge and thorough understanding of the physical weather and climate process. A simple but practical analysis approach is developed based on historical data and seasonal forecast. It is then applied to a single and a portfolio of weather derivative contracts. Business implications of the results from the analysis are discussed.

1. Introduction

It is almost impossible to name a type of business that is insulated from the effects of the weather. During the 1997–98 El Niño, the strongest episode in history, costly floods occurred more frequently than normal in the southern United States. However, the same climate pattern is associated with suppressed hurricane activity over the Atlantic basin. Hence, the same climate pattern had completely different impacts to insurance companies depending on the geographical distribution of their businesses. The winter of 1998/99, accompanied by La Niña, was abnormally warm for many parts of the United States. This adversely influenced revenues and expenses of many energy companies, while it saved consumers a significant amount of heating costs.

People have long used futures contracts of agricultural commodities to hedge weather-related risks. However, there is such a broad array of weather risks that the traditional methods cannot cover them all. As a result, a much more versatile financial instrument, namely, weather derivatives or weather insurance, has emerged in recent years (Jovin 1998). Consequently, financial and risk management professionals are facing the challenge of analyzing and managing these new instruments (e.g., Kaminski 1998; Stix 1998).

This paper provides a background on weather derivatives (section 2) and introduces their various applications (section 3). The main goal is to promote interest and further research on this topic among atmospheric scientists, who have the state-of-the-art knowledge and access to tools that the general business community usually does not. Although a simple and practical approach for analyzing weather derivatives is proposed here (section 4), we look forward to seeing more advanced and comprehensive methods being developed by the research community. A summary concludes the paper in section 5.

2. Basic concept

A weather derivative is a contract between two parties that stipulates how payment will be exchanged between the parties depending on certain meteorological conditions during the contract period. There are three commonly used forms of weather derivatives: call, put, and swap.

A call contract involves a buyer and a seller. They first agree on a contract period and a weather index that serves as the basis of the contract (denoted W). For
example, $W$ can be the total precipitation during the contract period. At the beginning of the contract, the seller receives a premium from the buyer. In return, at the end of the contract, if $W$ is greater than a prenegotiated threshold $S$, the seller will pay the buyer an amount equal to $P = k(W - S)$, where $k$ is a pre-agreed-upon constant factor that determines the amount of payment per unit of weather index. The threshold $S$ and factor $k$ are known as the “strike” and “tick” of the contract, respectively. The payment can sometimes be structured as “binary”: a fixed amount $P_0$ will be paid if $W$ is greater than $S$ or no payment will be made otherwise.

A put is the same as a call except that the seller pays the buyers when $W$ is less than $S$. The payment, $P$, is equal to $k(S - W)$ or $P_0$ for a linear or binary payment scheme, respectively. The payment diagrams for a linear call and put contract are illustrated in Fig. 1. A call or put is essentially equivalent to an insurance policy: the buyer pays a premium and, in return, receives a commitment of compensation if a predefined condition is met.

A swap contract between parties A and B requires no up-front premium and, at the conclusion of the contract, party A makes a payment in the amount of $P = k(W - S)$ to B. In the case of a negative $P$, the payment is actually made by B to A. In fact, a swap is a combination of (i) a call sold to B by A and (ii) a put sold to A by B. The strike $S$ is selected such that the call and put command the same premium. Thus, this study will focus on the analysis of call and put contracts only.

A generic weather derivative contract can be formulated by specifying the following seven parameters:

- contract type (call or put),
- contract period (e.g., from 1 Nov 1999 to 31 Mar 2000),
- an official weather station from which the meteorological record is obtained,
- definition of weather index ($W$) underlying the contract,
- strike ($S$),
- tick ($k$) or constant payment ($P_0$) for a linear or binary payment scheme, and
- premium.

The parameters above determine the amount of payment ($P$) that the seller is obliged to make to the buyer. For a linear payment scheme,

$$P_{\text{put}} = k\max(S - W, 0)$$
$$P_{\text{call}} = k\max(W - S, 0),$$

where the function $\max(x, y)$ returns the greater of values $x$ or $y$. For a binary payment scheme,

$$P_{\text{put}} = P_0 \quad \text{if } W - S < 0; \quad P_{\text{put}} = 0 \quad \text{if } W - S \geq 0$$
$$P_{\text{call}} = P_0 \quad \text{if } W - S > 0; \quad P_{\text{call}} = 0 \quad \text{if } W - S \leq 0.$$

### 3. Applications

Because the choice of $W$ is extremely flexible, weather derivatives can be structured to meet a wide variety of risk management needs. The best-known examples are related to the consumption of electricity that is significantly affected by the seasonal temperature variations. An abnormally cool summer or warm winter decreases not only the number of kilowatt hours (KWHs) consumed but also the price per KWH in the unregulated energy trading market, reducing the revenue of utility companies that sell electricity. The same risk also applies to buyers of electricity, because an abnormally cold winter or hot summer can cause both the unit price and the consumption to rise.

To manage these risks, derivative contracts based on seasonal accumulated heating degree days (HDD) and accumulated cooling degree days (CDD) are frequently used. HDD and CDD are defined as...
HDD = \sum_{i=1}^{N} \max(0, 65^\circ F - T_i) \quad \text{and}
CDD = \sum_{i=1}^{N} \max(0, T_i - 65^\circ F),

(2)

where \(N\) is the number of days over the contract period and \(T_i\) is the arithmetic average of the observed daily maximum and minimum temperatures on the \(i\)th day of the contract. HDD and CDD measure the respective need for heating and cooling in order for people to stay comfortable.

For example, a utility company can protect itself against revenue shortfalls due to a possible warm winter by buying an HDD put with a linear payment scheme [Eq. (1a)]. The contract parameters are usually determined based on the company’s experience of the relationship between the revenue fluctuations and HDD variations. On the other hand, a major user of electricity may buy an HDD call to prepare for the high utility cost due to a winter colder than normal.

Besides the widely used HDD and CDD call and put contracts, there is a growing number of new and innovative uses of weather derivatives. In the following example, a snow blower retailer, in order to generate sales, promised its customers a sizable rebate if the total snowfall for the coming winter were less than a threshold. Consequently, the company would face a substantial liability if the snow level were below the threshold. The company, however, could completely transfer the risk by buying a total snowfall put with a strike equal to the threshold in the rebate contract. In this case, a binary payment scheme [Eq. (1b)] would serve the purpose better than a linear one does because the rebate would be triggered in a binary fashion.

The sellers of weather derivatives usually include major energy companies, who use these products to hedge their own risks and make trading profits. Insurance and reinsurance companies are also becoming important sellers, as they look for alternative ways to deploy their capital. Although the relatively short history of the weather derivative market does not allow a thorough analysis of the correlation between the performance of the weather derivatives and the general financial market, it is widely perceived that the correlation is negligible. Thus, they are appealing to a wide array of investors.

4. Analyses

To structure a weather derivative contract, the buyer must determine the type and level of protection desired and establish the first six contract parameters listed in section 2. This is generally based on the buyer’s knowledge of the relationship between the variations of (i) the weather index and (ii) the income or expense of the business, as illustrated in the examples of section 2. Because this depends on specific business needs of the buyer and can vary greatly, the present study does not intend to analyze or summarize such relationships for various types of business.

The next question is, what is a reasonable price for a contract? Although the premium is ultimately determined by the supply and demand in the marketplace, the cost of the contract usually serves as a useful benchmark. The cost of a weather derivative contract consists of the payment at the conclusion of the contract \((P)\) and the overhead expenses incurred by the seller. The former is a random variable while the latter can be determined fairly accurately. Thus, the cost is a random variable itself. Consequently, the investment return (defined as the premium plus the investment income generated by the premium minus the cost) from a seller’s perspective is also a random variable.

From the seller’s perspective, a contract must sell at a price above the expected value of the cost so that it can be profitable in the long term (i.e., with a positive expected return). The seller also demands an increased level of expected return for an increasing level of risk that can be measured by the standard deviation of the return. However, a detailed discussion of investment risk and return is beyond the scope of this paper. Interested readers are referred to, for example, Rudd and Clasing (1988). From the buyer’s point of view, the difference between the premium and the expected cost of the contract represents the price of transferring the weather-related risk. The buyer must ensure this cost is not excessive compared to the weather risk itself.

The key to analyzing the probabilistic characteristics of the contract cost is to analyze the probability density function (PDF) of \(P, f_p(p)\). We first analyze the PDF of \(W, f_w(w)\), since \(P\) is determined by \(W\) [Eqs. (1a) or (1b)].

a. Probabilistic characteristics of \(W\)

Commonly adopted ways to obtain \(f_w(w)\) include (a) fitting a probability model to the historical data,
which are treated as independent samples, and (b) fitting a time series model, such as the autoregression moving average model, to the historical data. These approaches are based on well-defined statistical assumptions and have been successfully applied to many areas of social and natural sciences.

However, the historical weather records that these models rely on are not necessarily independent. In addition, there may exist trends or interannual fluctuations that are caused by physical phenomena but cannot be adequately modeled by pure statistical models. This is because these models do not take advantage of the fact that $W$ is controlled by the atmosphere–ocean circulation, whose physics and dynamics can be described by physically based models. Consequently, the effectiveness of the pure statistical models may have reached a relative plateau compared to the more persistent development and performance increases of dynamic models (Barnston et al. 1999). As the accuracy of seasonal prediction produced by these models is improving continuously (Livezey et al. 1997), $f_w(w)$ would be ideally calculated by such a dynamic model that reflects the variability of the current state and trend of the atmosphere–ocean system. However, a dynamic model that directly and explicitly calculates the desired PDF has not yet become available to the general business community. Thus, it is currently impossible to quantify the degree of improvement (e.g., reduced prediction uncertainty) by using a climate model–based PDF. However, because such a model would explicitly include the physical processes of the atmosphere–ocean system, it is believed to be a better tool for stochastic weather and climate prediction.

In this study, we propose a quick approach to predict $f_w(w)$ before the ideal solution suggested above becomes reality. It is designed for $W$ being HDD, CDD, total precipitation, or accumulated snowfall, which are the weather indices used in the majority of weather derivatives in the market. This approach combines the information in historical data and the official seasonal forecast of temperature or precipitation for the United States. The former is archived at the National Climate Data Center. The typical data length is 50 years. Instrument changes and relocations (vertically or horizontally) may have introduced bias to the data and can be corrected using approaches described in, for example, Quayle et al. (1991). However, it is beyond the scope of this study to correct such possible biases. Seasonal forecasts of temperature and precipitation are produced by the National Weather Service’s Climate Prediction Center (Barnston et al. 1999). Figure 2 (top panel) shows a CDD $f_w(w)$ predicted by this approach. In this figure, $f_w(w)$ is represented in a nonparametric fashion by a normalized histogram. The details of the approach are described in the appendix.

The cumulative probability distribution function (CPDF) of $W$ is displayed in Fig. 2b. Although it contains the same information as the PDF does, it allows the buyer of a weather derivative to explicitly examine the probability of $W$ reaching different values, which would have different levels of impacts on its business. This will also help the buyer determine if the premium for the contract is reasonable.

b. The premium and the probabilistic characteristics of $P$

Because the relationship between $W$ and $P$ [Eqs. (1a) and (1b)] is generally not determined by an invertible function, there is no simple analytic derivation of $f_P(p)$ from $f_w(w)$. We opt for a simulation approach, which generates a large number of random samples of $W$ from $f_w(w)$. Each of the samples, denoted $w_*$, is used to calculate a sample payment $p_i$ based on Eqs. (1a) or (1b). The normalized histogram $p_i$ represents $f_P(p)$ in a nonparametric way. Figure 3a shows the $f_P(p)$ of a call contract with a strike of 570 degree days and a tick of $5,000 based on the data in Fig. 2.
The sample mean of \( p_j \) denoted \( \mu_i \) is an unbiased estimate of the expected value of \( P \), that is, the average amount that the seller would pay if the contract were executed for an infinite number of times. It can be used as a benchmark for the premium (discussed above). In addition, the CPDF of \( P \) (Fig. 3b) directly demonstrates the probabilities that the seller experiences different levels of losses (i.e., the seller’s downside risk). This information can be factored into the premium calculation. For example, the seller may have a policy stating that the probability of losing $100,000 or more must be below 10%. However, Fig. 3b shows that this probability is approximately 20%. As a result, the seller must purchase a separate call contract with a higher strike to reduce the downside risk. The extra cost of doing so will be considered when the premium of the current contract is negotiated.

c. Combination of weather indices and derivative contracts

Frequently, the risk management needs cannot be totally satisfied by just one weather derivative. For example, the revenue of a large electricity supplier with a geographically diverse client base is affected by the temperatures at many locations throughout its service territory. Consequently, it needs to purchase more than one weather derivative contract to manage weather-related risks. In another example, a seller of a put can protect downside risk by buying another put with the same contract parameters except for a lower strike.

In general, most of the active participants in the market are buying and selling multiple call and put contracts simultaneously. To analyze such a portfolio of contracts, it is necessary to analyze the net payment that the portfolio can potentially make, defined as

\[
P_N = \sum_{i=1}^{n} \alpha_i P_i = F(W_1, W_2, ..., W_n),
\]

where \( n \) is the total number of contracts in the portfolio, \( P_i \) is the payment for the \( i \)th contract, \( \alpha_i \) is 1 (or \(-1\)) if the contract is sold (or bought), and \( W_i \) is the weather index underlying the contract. The second equality is based on Eqs. (1a) and (1b), showing \( P_N \) as a function of all the weather indices underlying the contracts.

Let \( W \) represent the vector \( \{W_1, W_2, ..., W_n\} \) and \( f_w(w) \) be the joint PDF of \( W \). The PDF of \( P_N, f_{PN}(p) \), can be calculated based on \( f_w(w) \). The main challenge of obtaining \( f_w(w) \) is that the individual \( W_j \)'s are not independent, because the behaviors of all weather parameters are generally correlated as they are controlled by the same global atmosphere-ocean circulation. This is particularly true for temperature and precipitation integrated over periods from a week upward (e.g., Huang and van den Dool 1993; Kozuchowski and Marciniak 1988), and more so for the former than the latter. These two are the most commonly used parameters in weather derivatives.

Ideally, similar to an individual weather index, \( f_{w_j}(w) \) would be calculated using a state-of-the-science dynamic model. However, until such a model becomes available to the business community, we use a method to estimate \( f_{w_j}(w) \) based on the correlation among the \( W_j \)'s in the historical data. This and the calculation of \( f_{PN}(p) \) are described in the appendix.

This methodology is applied to a portfolio of five contracts (Table 1), all of which are CDD calls or puts for the summer of 1999 and have the same tick of $5,000. Figure 4 shows \( f_{PN}(p) \) and the CPDF of \( P_N \) for this portfolio. This information allows the owner of the portfolio to evaluate the expected return of the portfolio and the risk associated with such a return.
TABLE 1. A sample portfolio of weather derivatives.

<table>
<thead>
<tr>
<th>City</th>
<th>Station ID</th>
<th>Contract type</th>
<th>Start date</th>
<th>End date</th>
<th>Strike (degree days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco</td>
<td>23234</td>
<td>Put</td>
<td>1 Jun 1999</td>
<td>30 Sep 1999</td>
<td>70</td>
</tr>
<tr>
<td>Tucson</td>
<td>23160</td>
<td>Put</td>
<td>1 May 1999</td>
<td>31 Oct 1999</td>
<td>2,650</td>
</tr>
<tr>
<td>Houston</td>
<td>12918</td>
<td>Call</td>
<td>1 May 1999</td>
<td>31 Oct 1999</td>
<td>2,700</td>
</tr>
<tr>
<td>Miami</td>
<td>12839</td>
<td>Put</td>
<td>1 May 1999</td>
<td>31 Oct 1999</td>
<td>2,850</td>
</tr>
<tr>
<td>Boston</td>
<td>14739</td>
<td>Call</td>
<td>1 May 1999</td>
<td>30 Sep 1999</td>
<td>850</td>
</tr>
</tbody>
</table>

5. Summary

The concept and applications of weather derivatives and weather insurance are introduced. They are shown to be effective risk management tools for a variety of businesses. Proper analysis of this type of financial instrument becomes increasingly important and represents an area where advanced knowledge of climate and seasonal variability can make a significant impact. This requires not only statistical methods but also deep understanding of the physical processes of the atmosphere–ocean system. It is recognized that a robust solution of analyzing weather derivatives would ideally be built upon a state-of-the-science dynamic seasonal prediction model. However, before such a solution becomes available to the general business community, a simple but practical method of analyzing weather derivative contracts is proposed and applied to several examples. In addition, the financial implications of these analyses are briefly discussed.

Acknowledgments. This work is supported by the E. W. Blanch Company. The author would like to thank the anonymous reviewers and Drs. Chet Ropelewski, Robert Livezey, and Gad Levy for helpful discussions.

Appendix: The PDF of weather indices and contract payment

a. The calculation of $f_w(w)$

A PDF can be represented by a formula (e.g., normal distribution) or approximated by a histogram (the graphic representation of a data table). We opt for the latter in this paper because it does not require an a priori assumption of the functional form of the PDF.

We first retrieve the official seasonal forecast of temperature or precipitation for the United States from the National Weather Service’s Climate Prediction Center (CPC). The forecast is based on a combination of general circulation model analyses and statistical approaches (Barnston et al. 1999). It starts with an assessment of what aspects of climate variability are most important to the current situation. This can lead to sharply focused, high-confidence predictions that were not previously feasible with pure statistical tech-
techniques that attempted to treat all factors simultaneously and indirectly.

The forecast provides the probabilities of monthly and quarterly temperature being above, near, and below the climate norm, denoted \( p_{a}, p_{n}, p_{b} \). We assume the following. 1) If \( W \) is CDD, then \( p_{a}, p_{n}, p_{b} \) are the probabilities that the CDD is above, near, and below the CDD climate norm, respectively. 2) If \( W \) is HDD, then \( p_{a}, p_{n}, p_{b} \) are the probabilities that the HDD is below, near, and above the HDD climate norm, respectively. The validity of the first assumption is examined by calculating the rank order correlation (i.e., the linear correlation of the order statistics, denoted \( R \)) between the monthly mean temperature, \( T_{m} \), and monthly HDD data for a given month over the historical data record. This calculation is performed on 250 weather stations across the United States and shows that all \( R \) values are greater than 0.9, with 99% greater than 0.95. An \( R \) value near or equal to one indicates that warmest year tends to be the year with the highest CDD, and so on. Thus, the first assumption is shown to be acceptable. The second assumption is tested by calculating \( R_{j} \) between \( T_{m} \) and monthly HDD over the same 250 stations. They are shown to be close to -1 (all \( R_{j} \) values are less than -0.95, with 95% less than -0.99), meaning that warmest year tends to be the year with the least HDD, and so on. This validates the second assumption.

The CPC seasonal forecast also provides the \( p_{a}, p_{n}, p_{b} \) values for monthly and quarterly total precipitation. When \( W \) is the total precipitation, \( p_{a}, p_{n}, p_{b} \) are naturally used as the probabilities that \( W \) is above, near, and below the climatic norm, respectively.

The procedure above produces the probabilities that \( W \) is above, near, and below the norm, denoted \( p_{AW}, p_{NW}, p_{BW} \). Next, we sort the historical record of \( W \) and divide the sorted list into three groups: 1) the highest 33%, 2) the middle 34%, and 3) the lowest 33%. Samples are taken from the three groups with replacement. The numbers of samples taken from groups 1, 2, and 3 are proportional to \( p_{AW}, p_{NW}, p_{BW} \), respectively. The total number of samples is 30 000 in the present study. The group of sampled \( W \) (e.g., summarized by the histogram Fig. 2a) reflects both historical observations and the CPC forecast of deviation from the climate norm.

b. The calculation of \( f_{w}(w) \) and \( f_{p}(p) \) for a combination of contracts

The PDF of \( W \), a vector of random variables, can be represented by an analytic function (e.g., multivariate normal distribution) or approximated by a multidimensional histogram. The former is not a feasible option because it is impossible to determine a priori the general functional form of \( f_{w}(w) \). In addition, when the dimension of \( W \) is greater than two, it is difficult to construct and visualize the latter. Instead, we characterize the random vector \( W \) with an \( M \) by \( n \) matrix \( Z \), where \( M \) is a large integer and \( n \) is the dimension of \( W \). Each row of \( Z \) is a realization of \( W \), and its \( j \)th column contains \( M \) realizations of \( W_{j} \).

A principal component analysis method is used to construct \( Z \). Since this type of method has been commonly used in the literature (e.g., Bretherton et al. 1992), detailed mathematical derivations are not given for brevity. We first remove the temporal mean from the historical records of \( W_{j} \), where \( j \) varies from 1 to \( n \). Next, a historical data matrix, \( W \), is constructed such that its \( j \)th column is the mean-removed historical records for \( W_{j} \). Here, \( W \) is an \( m \) by \( n \) matrix, where \( m \) is the number of historical records. The covariance matrix of \( W \) is calculated and denoted \( A \).

Then, an eigenvalue decomposition on the covariance matrix \( A \) is performed:

\[
A = U \times U^{T}, \tag{A1}
\]

where \( U \) is an orthogonal matrix and \( \times \) is a diagonal matrix whose elements are the eigenvalues. The PC of \( W \) is defined as

\[
Y = WU. \tag{A2}
\]

The covariance matrix of \( Y \) is diagonal because

\[
\frac{1}{m} Y^{T}Y = \frac{1}{m} U^{T}W^{T}WU = U^{T}UA = \Lambda. \tag{A3}
\]

This implies that the columns of \( Y \) have zero correlation with each other.

Next, each column of \( Y \) is independently sampled \( M \) times with replacement to form the corresponding column of an \( M \) by \( n \) matrix \( Y^{*} \). Since the elements in the \( j \)th column of \( Y^{*} \) are random samples of the elements of the \( j \)th column of \( Y \), the variance of the former is the same as that of the latter. In addition, because the samplings among different columns of \( Y \) are independent, the columns of \( Y^{*} \) are independent of each other. This means that the covariance matrix of \( Y^{*} \) is diagonal. Hence, the covariance matrix of \( Y^{*} \) is also \( A \). Let
\[
Z^* = Y^*U^T. \quad (A4)
\]

It can be verified that the covariance matrix of \( Z^* \) is

\[
\frac{1}{M} Z^{*\text{T}}Z^* = \frac{1}{M} (Y^*U^T)^T(Y^*U^T)
= \frac{1}{M} UY^*Y^*U^T = U^\Lambda U^T = A. \quad (A5)
\]

Because each column of \( Z^* \) is a linear combination of the columns of \( Y^* \) and the mean for each column of \( Y^* \) is zero, the mean of each column of \( Z^* \) is zero. Finally, \( Z \) is formed by adding to the \( j \)th column of \( Z^* \) the temporal mean of \( W \).

In summary, the covariance of \( Z \) is the same as that of the historical records of \( W \). In addition, the \( j \)th column of \( Z \) has the same mean as that of the historical records of \( W_j \). Moreover, the large number of rows allows \( Z \) to provide a better probabilistic representation of \( W \).

The \( z_{ij} \), the element at the \( i \)th row and \( j \)th column of \( Z \), can be used to calculate a realization of \( P_j \), the payment of the \( j \)th contract. Summation of \( P_j \) over \( j \) from 1 to \( n \) yields a realization of \( P_N \) [Eq. (3)]. Completing this calculation for all rows of \( Z \) produces the underlying data that form the histogram and CPDF of \( P_N \) (Fig. 4). The approach used here, unlike the one used for single contracts described in the previous subsection, does not incorporate the CPC forecast in the sampling frequency. Consequently, it does not take into account the possibility that the historical records can be nonstationary at relatively long timescales (e.g., Livezey and Smith 1999). Future study is needed for a method of biasing the sampling frequency according to the CPC forecasts while maintaining a proper correlation among different geographical locations.

References


Huang, J., and H. M. van den Dool, 1993: Monthly precipitation-temperature relations and temperature prediction over the United States. J. Climate, 6, 1111–1132.


