VISUALIZING VAPOR PRESSURE

A Mechanical Demonstration of Liquid–Vapor Phase Equilibrium

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The exponential temperature dependence of the equilibrium vapor pressure of water is illustrated through a simple mechanical analogy involving bouncing balls on a vibrating plate.

The atmosphere would be a much simpler system if it did not contain water vapor—but it would also be far less interesting to atmospheric scientists and meteorologists, not to mention less hospitable to life. The presence of a trace substance—water—that exists as a gas, a liquid, and a solid under the range of typical atmospheric conditions, changes everything: water vapor is a primary greenhouse (infrared active) gas, the latent heat release in cloud convection fundamentally changes the temperature structure of the atmosphere, and of course the presence of clouds and precipitation themselves profoundly alter the radiative

balance of Earth (Durran and Frierson 2013; Stevens and Bony 2013). The simplest radiative equilibrium models do a surprisingly poor job of capturing Earth's mean temperature when water vapor and clouds are not included at least in some simplified way. Of course the hydrological cycle depends crucially on the evaporation of water from the warm oceans of Earth and the eventual condensation of water vapor to generate precipitation; and one of the key pathways for precipitation formation depends on the differential equilibrium vapor pressure of liquid and solid phases of water. It is therefore central to the teaching of atmospheric physics to correctly convey the concept of phase equilibrium. Indeed, the Clausius–Clapeyron equation that expresses the temperature dependence of equilibrium water vapor pressure on temperature could be considered one of a handful of equations that an undergraduate meteorology student should really internalize, right up there with the hydrostatic equation and equations for geostrophic balance. It is our experience, though, that the concept of equilibrium vapor pressure and its temperature dependence is deceptively subtle and challenges even the brightest of students [and sometimes their teachers; see Bohren and Albrecht (1998) for a discussion of some of the more infamous pitfalls]. In this essay we introduce a demonstration experiment that provides a vivid, mechanical analogy for the concepts of evaporation,

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condensation, and the temperature dependence of equilibrium vapor pressure. The demonstration is useful for conveying a conceptual understanding of these concepts but also can illustrate the essential concepts underlying the functional form of the most common expression for equilibrium vapor pressure.

At a given temperature, the rates of molecular exchange across the liquid–vapor interface must differ for a net transfer of matter to occur between the phases. We speak therefore of net evaporation (when the flux of molecules leaving the liquid exceeds the rate molecules reenter the liquid) and net condensation (when the condensation flux exceeds the evaporation flux). It is important to distinguish between the practical usage of the terms (evaporation and condensation) and the physical processes of molecular exchange across the liquid–vapor interface, which occur simultaneously. When the rate that molecules escape from a liquid exactly equals the rate of return, no net transfer of matter occurs, and the system is said to be in equilibrium, a dynamic steady state. Knowing the precise point of equilibrium is of course key to distinguishing the conditions for net evaporation and net condensation. The equilibrium concentration of vapor in contact with the liquid surface at a particular temperature is unique to the substance in question. Here, we focus on water and use the term “vapor pressure” as the measure of water vapor concentration (through the ideal gas equation) that maintains phase equilibrium. We wish to explore the nature of this equilibrium state and how the vapor pressure varies with temperature.

The equilibrium state is intimately linked to the process by which molecules leave the liquid state. Molecules in the liquid surface escape only if they acquire kinetic energies large enough to break the bonds holding them to neighbors. The observed vapor densities therefore tell us something about the magnitudes of the cohesive bonding energies. Whereas a weakly bonded condensate (e.g., a light alcohol) evaporates readily under normal conditions and exhibits a large vapor pressure, a strongly bonded liquid (e.g., mercury) evaporates only slowly and yields relatively low concentrations of molecules in the gas phase even at elevated temperatures. Water is a substance whose molecules are bound to each other with intermediate strength through hydrogen bonding.

Detailed explanations of molecular phenomena are important in the sciences and engineering, particularly at the university level. Analogies can be valuable aids to instruction, in part because they highlight key features of the underlying physics, and in part because they help students visualize scales not sensed by humans. Mathematical analogies abound in technical fields

because they capture essential features of the physics and offer opportunities for detailed quantitative analysis. Maxwell’s kinetic theory of gases, for instance, while still serving as a cornerstone of theoretical physics, gently introduces new students to the molecular world and to the rigors of statistical mechanics. Physical analogies, while not capable of the precision provided by mathematical models, serve to bring the invisible microscopic world directly to students through demonstrations and experimentation. Physical analogies have been used, for instance, by Prentis (2000) to illustrate how mechanical systems with “motorized molecules” can be described well by Boltzmann statistics when using the ergodic hypothesis of statistical mechanics. Their two-level “Boltzmann machine” utilizing multiple balls is of particular relevance to our study of vaporization, which is modeled with a vibrating plate containing a depression and a countable number of metal balls. However, whereas Prentis varied the potential energy between the levels while keeping the mechanical equivalent of temperature fixed, we suggest that evaporation is best modeled as a system in which the potential (i.e., “bond”) energy is fixed while the degree of mechanical agitation (“temperature”) is varied. In both studies, the important thermodynamic and statistical–mechanical condition of temperature uniformity throughout the system has been maintained.

This paper describes a mechanical system developed for use in the teaching of atmospheric physics. We use it as a way of introducing students to the growth and evaporation of cloud drops, for which a clear understanding of equilibria between phases is imperative. We attempt here to draw parallels between the classical Clausius–Clapeyron equation (which describes the dependence of vapor pressure on temperature) and the escape of balls from a gravitational potential energy well. We first review essential theoretical principles before describing the physical system and the data derived from it. Water is the substance of most atmospheric relevance and the focus of the discussion here, but the principles are general and apply to any volatile liquid or solid. This demonstration helps answer such questions as “What causes vapor pressure to increase with temperature?” and “What physical principles determine the form of the curve describing equilibrium vapor pressure versus temperature?”

THE BOLTZMANN FACTOR: VAPOR PRESSURE AND A MECHANICAL ANALOG.

A molecular interpretation via the Boltzmann equation (see the sidebar for an overview) provides a compelling perspective for interpreting the temperature

dependence of water vapor pressure. Conceptually, it is a struggle between water molecules in the liquid state effectively trapped in a depression of potential energy (“potential well”) of magnitude l_v and the random thermal energy $k_B T$ that occasionally allows molecules to escape to the vapor. To first approximation, l_v is the binding energy per molecule. [The correction due to expansion that is needed to convert binding energy to enthalpy, e.g., shown by Baierlein (1999), section 12.3, is neglected here for reasons of pedagogical simplicity.] Binding energy is essentially the potential energy, due to Coulomb or other intermolecular forces, that must be overcome to extract a water molecule from the condensed phase. Indeed, the latent heat of evaporation of water ($L_v \approx 45 \text{ kJ mol}^{-1}$) when converted to molecular units is $l_v = 0.47 \text{ eV}$. This value is strikingly close to twice the energy of a hydrogen bond ($\epsilon_H = 0.24 \text{ eV}$) between two water molecules in the liquid phase. The ratio $l_v/\epsilon_H \approx 1.9$ provides a rough estimate of the average number of hydrogen bonds per molecule in liquid water that must be broken for a water molecule to escape the bound, liquid state and enter the unbound, gas phase. How does this binding energy compare to the thermal energy in the Boltzmann factor? At room temperature $k_B T \approx 0.025 \text{ eV}$, so we note that $l_v \gg k_B T$, which shows via the Boltzmann factor that the fraction of molecules escaping the liquid (i.e., the fraction of molecules having energy equal to or greater than l_v) is small.

This same molecular interpretation of evaporation also explains the tendency for the rate of evaporation and hence vapor pressure to increase with temperature. Whereas classical thermodynamics shows this tendency to stem from the relative changes of entropy and volume across the phase boundary, it is equally valid to see the increase as arising from the need for the molecules to overcome an energy barrier: the enthalpy of phase change l_v . Higher temperatures lead to larger molecular kinetic energies and higher probabilities of molecules in the liquid surface being able to break the bonds holding them to liquid-phase neighbors. This larger evaporation flux must be balanced, in equilibrium, by a larger impingement flux, which arises in turn from a larger concentration of vapor over the liquid surface. Such a kinetic viewpoint is consistent with chemical kinetics and Boltzmann statistics, as seen by taking the logarithm of Eq. (SB5):

$$\ln \left(\frac{p}{p_0} \right) = \ln A - \frac{l_v}{k_B T}. \quad (1)$$

Figure 1 shows the same data for water as from Fig. SB1, now plotted as the ratio of vapor pressures

versus the reciprocal of temperature. The slope of the line allows one to calculate the latent heat of evaporation (approximately the energy needed to break two hydrogen bonds in the liquid). Such macroscopic measurements thus tell us how strongly the molecules must be joined to neighbors. So, at its most basic level, evaporation is simply a result of random thermal agitation of molecules in liquid water resulting in a lucky few near-surface molecules getting jostled in just the right way to gain enough kinetic energy to exceed the binding energy and so escape from the liquid and enter the vapor phase. Simply because molecules in a liquid are embedded in a potential well (roughly equal in magnitude to the latent heat of evaporation l_v), the vapor pressure must increase with increasing temperature, again in accord with the Boltzmann factor.

This recognition that molecules in a liquid effectively reside in a potential energy well imposed by the electrical forces holding them to neighbors serves as the inspiration for our mechanical demonstration. Evaporation is here made visible by replacing the molecules with small metal balls and the electric potential well with a gravitational potential well. The mechanical demonstration consists of a horizontally oriented metal plate with a central depression (the “well”) that serves to represent the liquid state as a “mean field” of the electrical binding energy. Forced vibrations of the plate serve as the analog of thermal energy and cause a few of the balls in the well to jump up to the flat upper level (see Fig. 2). The number of particles in the “gas” phase (level 2) compared with that in the “liquid” phase (level 1) can then be written as $N_2/N_1 = \exp(-\epsilon_0/k_B T)$, where ϵ_0 is the gravitational potential energy difference. Now we also need to recognize that temperature per se has little meaning in a mechanical analogy. We therefore replace thermal energy $k_B T$ with a mean mechanical kinetic energy E_{kin} . Normalization of the number densities to a reference state allows us to develop an equation that is similar in form to the integrated Clausius–Clapeyron equation. When we designate the reference condition as $E_{\text{kin},0}$ (analogous to $k_B T_0$ in a thermal system), the normalized ratio of densities in the “vapor” state (level 2) is expressed as

$$\left. \frac{N_2(E_{\text{kin}})}{N_2(E_{\text{kin},0})} \right|_{\text{mechanical model}} = \frac{\exp\left(-\frac{\epsilon_0}{E_{\text{kin}}}\right)}{\exp\left(-\frac{\epsilon_0}{E_{\text{kin},0}}\right)}. \quad (2)$$

Equation (2) is the relationship used to analyze the data from our mechanical analogy. This ratio of two Boltzmann factors is strikingly similar to the normalized concentration ratio one obtains from the thermodynamic treatment [Eq. (SB5)]:

CLASSICAL THERMODYNAMICS AND MOLECULAR VIEWS OF THE TEMPERATURE DEPENDENCE OF VAPOR PRESSURE

The term “vapor pressure” as used here is a partial pressure of the gaseous component of interest in equilibrium with its pure liquid. Vapor escapes from a unit surface of liquid with a rate that increases with increasing temperature, a fact reflected in the higher vapor pressures and larger vapor densities at higher temperatures. The Clausius–Clapeyron equation, derived from the principles of classical thermodynamics, relates an increase in vapor pressure p with temperature to the difference in entropy relative to the difference in specific volume between vapor and liquid. Ultimately, this can be written approximately as

$$\frac{d \ln p}{dT} = \frac{L_v}{RT^2}, \quad (\text{SB1})$$

where L_v is the molar latent heat and R is the gas constant. Integration of Eq. (SB1) over a modest range such that variations in L_v can be ignored gives a dependence on temperature that can be written as an Arrhenius relationship:

$$\frac{p(T)}{p_0} = A \exp\left(-\frac{L_v}{RT}\right). \quad (\text{SB2})$$

Here p_0 (≈ 611 Pa for water) is the vapor pressure at the ice point $T_0 = 273.15$ K, and $A = \exp(L_v/RT_0)$.

As seen in Fig. SB1, the vapor pressure of water increases exponentially with temperature, in accord with Eq. (SB2), which suggests that water molecules escape with increasing frequency as the temperature is raised.

Equation (SB2) is obtained from classical thermodynamics, as commonly developed in atmospheric physics texts

(e.g., Wallace and Hobbs 2006, p. 98; Lamb and Verlinde 2011, p. 134). A molecular perspective interprets the exponential expression as a Boltzmann factor, a concept already familiar to atmospheric scientists in the guise of the hydrostatic balance equation for an isothermal atmosphere:

$$p_{\text{tot}} = p_{\text{sfc}} \exp\left(-\frac{mgz}{k_B T}\right), \quad (\text{SB3})$$

where p_{tot} is the total pressure (subscript “total” to differentiate from vapor pressure), p_{sfc} is the surface pressure (at height $z = 0$), m is the mass of a gas molecule, g is the gravitational acceleration, and k_B is the Boltzmann constant (the molecular gas constant). The hydrostatic balance equation is typically derived by equating the pressure difference across a horizontal slab of air of thickness dz to the weight of the slab per unit area. One readily sees that the numerator of the exponential term (mgz) is nothing more than the gravitational potential energy of the gas molecules above the surface.

This result can be generalized to any potential field, not just that due to gravity (Feynman et al. 1963, section 40-2). For example, we can envision some generic force field of magnitude F acting on each of the molecules in population n and directed along coordinate x . We thus find a balance between the difference in pressure across a slab oriented perpendicular to x and the force per unit area acting on the slab. Assuming the ideal gas law at constant T , and noting that $-Fdx = dU$ is just the change in potential energy by virtue of

the work done in moving the molecules through distance dx against the force, integration yields the result

$$\frac{n}{n_0} = \exp\left(-\frac{U}{k_B T}\right). \quad (\text{SB4})$$

This equation, one form of Boltzmann’s law, states that at constant temperature the concentration n of molecules at elevated potential energy U is reduced over the concentration in the base state by the Boltzmann factor, $\exp(-U/k_B T)$. This expression lies at the foundation of the atomistic view of physics (statistical mechanics).

We thus gain a compelling perspective for interpreting vapor pressure resulting from the integrated Clausius–Clapeyron equation [Eq. (SB2)]. Expressing Eq. (SB2) in molecular terms, we see that the pressure ratio becomes

$$\frac{p}{p_0} = A \exp\left(-\frac{l_v}{k_B T}\right), \quad (\text{SB5})$$

where l_v is the latent heat per molecule. We can interpret the latent heat l_v as the potential energy of molecules in the vapor phase relative to those in the liquid. We have emphasized the common roots of Eqs. (SB3) and (SB5) but should also recognize that we typically consider variations in z (in the numerator) for hydrostatic balance and variations in T (in the denominator) for vapor pressure; thus, vapor pressure p must vary in the opposite sense that molecular concentration n does in the atmosphere, as the independent variable changes.

FIG. SB1. The dependence of the equilibrium vapor pressure ratio of water on temperature, as calculated from the Magnus equation (Pruppacher and Klett 1997, p. 854).

$$\left. \frac{n(T)}{n(T_0)} \right|_{\text{real}} = \frac{T_0}{T} \frac{\exp\left(-\frac{l_v}{k_B T}\right)}{\exp\left(-\frac{l_v}{k_B T_0}\right)}, \quad (3)$$

when the ideal gas law is used to convert pressure into concentration. (The factor T_0/T , arising from the expansion of the real vapor with temperature, can be ignored here.)

MECHANICAL SYSTEM, DATA COLLECTION, AND ANALYSIS. The fundamental concept behind this demonstration, replacement of the mean electrostatic binding potential in a real liquid by a gravitational potential energy well, is realized by milling a depression into the center of a flat aluminum disk (refer again to Fig. 2). Small spherical balls of copper or aluminum, the analogs of molecules, were able to bounce and migrate anywhere within the confines of the outer wall in response to the plate vibrations. Particles (balls in the mechanical system or molecules in a real liquid) naturally seek the lowest gravitational potential energy in the absence of other forces, but they may be ejected from the potential well if they happen to acquire sufficient kinetic energy (from the plate or from neighboring particles). The circular symmetry of the mechanical system (Fig. 2, bottom) offers the perspective, when viewed from above, similar to that of a drop of water in air.

The mechanical system was devised with a few considerations in mind. The plate (15 cm in diameter) was constructed of aluminum (6 mm thick) to minimize its mass and provide a hard, robust surface upon which the metal balls could bounce readily. Also, the rigidity of the plate served to prevent unwanted mechanical resonances. It was found that machining the surface with a very gentle slope ($\sim 1/4^\circ$) toward the center helped compensate for the friction that naturally arises in a mechanical system. The central depression (2.5 cm in diameter) was cut 3 mm deep and placed in the center of the disk, mainly for symmetry. The outer wall was constructed of parchment paper and attached to the outer edge of the plate to keep the balls confined to the plate. The entire plate was spray-painted black to give visual contrast to the shiny balls. The bottom of the aluminum disk was outfitted with an adapter that permitted attachment to a mechanical wave generator, the frequency and amplitude of which were driven by a function generator. Further details on constructing the apparatus are provided in the online supplement (<http://dx.doi.org/10.1175/BAMS-D-15-00173.2>).

The amplitude of plate oscillations, needed to estimate the kinetic energy imposed on the particles,

FIG. 1. An alternative way of showing the dependence of the equilibrium vapor pressure ratio of water on temperature: Arrhenius plot of the vapor pressure normalized to the ice point $T_0 = 273.15$ K (at which $p_0 = 611$ Pa).

was determined optically. As illustrated in Fig. 2 (top), a (green) light from a laser pointer was focused and made to reflect first off a plane mirror attached to the bottom of the vibrating plate, then off a spherical mirror (focal length ~ 20 cm) that was mounted near the opposite side of the plate. This technique allowed the variations in the light path arising from the small (<1 mm) vertical excursions of the plate to be greatly amplified (~ 240 times) when projected onto the room wall about 6 m from the apparatus. A set of calibration experiments was performed in which the peak-to-peak range of the projected light was measured for each voltage setting of the function generator. The resulting regression equation was used, in conjunction with the measured magnification factor, to calculate the amplitude A of the plate oscillations from the measured voltages with a precision of a few percent for each experiment.

Data of particle “evaporation” were gathered from video records of the vibrating plate with bouncing balls. The digital video camera that generated these records was mounted above the plate and yielded many complete images (as in Fig. 2, bottom) for a period of 3 min at each amplitude. The stored images were later played back and stopped every 5 s, yielding 37 frames for data collection. This time interval was chosen as a compromise between the needs for the separate realizations of the particle distributions to be independent and for the number of samples (determinations) to be statistically meaningful. A relatively few (~ 60) large (2.4-mm diameter) copper balls work well for demonstration purposes, but the data here were gathered using a greater number (300) of small

(0.79-mm diameter) aluminum balls (McMaster-Carr) to improve statistics. The data from each voltage setting (yielding amplitude category k) of the mechanical vibrator are expressed as the average number N_2 of balls observed to be out of the well (i.e., “evaporated” and on level 2), from which a normalized ratio was calculated:

$$r_k = (N_2/N_1)_k / (N_2/N_1)_0. \quad (4)$$

An “uncertainty” δN_2 about the mean, taken as the standard deviation, was propagated to the overall

uncertainty in r_k by standard methods (e.g., Taylor 1997), as outlined in the online supplement.

The mean kinetic energy E_{kin} imparted to the balls on the plate by the mechanical vibrator served as the analog of the thermal energy $k_B T$ that drives the motions of molecules in a real liquid–vapor system. Determining the mechanical kinetic energy is challenging, in part because no convenient “thermometer” is available and because some of the mechanical energy is quickly dissipated by friction, something that does not exist as such in the molecular world.

However, our purposes here are served adequately by assuming that the energy used for analysis of the data are linearly related to the hypothetical “true” energy imparted to the balls. We therefore use the square of maximum plate speed for the “relative energy” $E_{\text{rel}} = (A\omega)^2 \propto E_{\text{kin}}$, where A is the amplitude of plate motion (as described above) and $\omega = 2\pi f$ is the angular frequency of the applied voltage.

RESULTS AND DISCUSSION.

The results presented here were derived from a series of nine experiments using the $N_{\text{tot}} = 300$ aluminum balls. The conditions and experimental results are summarized in Table ES1 in the online supplement. The number ratio N_2/N_1 (where $N_1 = N_{\text{tot}} - N_2$) for each amplitude category k is shown in Fig. 3 (top) plotted against the relative energy E_{rel} of the plate. One clearly sees the qualitative trend that the balls “evaporate” and jump from the lower level (the potential energy well) to the upper level more readily as the imposed energy (the “temperature”) increases. Low energies permit only a tiny fraction of the balls in the well to gain the requisite energy needed for promotion to the higher level; large energies, by contrast, let many of the balls escape the well. To first approximation, the balls mimic the quasi-exponential pattern from the evaporation of water (Fig. 1) as the temperature increases.

The data from balls jumping out of a potential well in this mechanical

FIG. 2. The mechanical system used to demonstrate phase equilibrium. (top) Schematic diagram of the plate and optical arrangement for determining the amplitude of imposed oscillations. (bottom) Photograph of the plate from above, showing a set of copper balls and various ancillary components: the leveling base, electrical connection to the driving speaker, a “wall” to hold in the balls, and a mirror for measurement of amplitude.

system can be interpreted further with the help of Boltzmann statistics. An underlying feature of classical statistical mechanics is the fact that all energy states are equally likely, so a closed system overall tends toward the most probable distribution. If the balls in our mechanical analogy interact with the plate and with each other in random and independent ways, then the form of the dependence of the relative number densities in the upper and lower states should depend on the mean kinetic energy in the same way it does for the evaporation of true liquids [Eq. (3)]. We have put our data into an analogous form [Eq. (2)] by normalizing the number densities to a particular value (N_{20} , from amplitude category $k = 9$) and then plotting the normalized ratio [Eq. (4)] against the reciprocal of the relative energy. We see (by comparing the bottom panel of Fig. 3 with Fig. 1) that the overall pattern exhibited by the data from the mechanical analogy is, to good approximation, linear in semilogarithmic coordinates, just as it is with true molecular evaporation. It surprised us to find that such a few short segments of data from a system containing a mere 300 particles can be so well described by Boltzmann statistics.

The scatter in the data, too, provides insight into the statistics of balls bouncing on a plate. The “error” bars shown in Fig. 3 (bottom) are everywhere large compared with the standard errors of the means (white boxes), which shows the distinction that must be made between the fluctuations inherent in any statistical system and the ability to approach physically meaningful expectation values (the means) by repetitive sampling. The individual determinations (of r_k from the counting of balls on the upper level of the plate) will always vary greatly when the numbers are small. Nevertheless, we see from Fig. 3 (bottom) that the standard deviations (indicated by the error bars) follow the boundaries (shaded region) described by Poisson statistics quite well. Increasing the number of determinations will improve the estimation of the mean but not the statistical fluctuations. The only way to beat down that statistical uncertainty is to increase the number of particles in the system. Using more balls in the vibrating-plate experiment would help here; real liquid–vapor systems do this naturally because of the huge numbers of molecules involved. (Note that uncertainties in vapor pressure data arise from imprecision in the instruments making the measurements and not from the type of statistical uncertainties discussed here.)

Several caveats need to be stated about our physical system. The regular sinusoidal motion imposed on the plate by the mechanical actuator is not at

FIG. 3. Data from the vibrating plate using a total of $N_{\text{tot}} = 300$ aluminum balls. (top) Count ratio vs relative energy applied to the vibrating plate. Here N_2 is the number in the upper level and $N_1 = N_{\text{tot}} - N_2$ is the number in the well. (bottom) Arrhenius plot of the concentration ratios normalized to the maximum degree of “vaporization” from the vibrating-plate experiment. The filled circles represent the mean values of the normalized ratio from 37 trials at each energy level, while the white boxes represent the standard error of the means. The solid line is the best fit to the mean values and accounts for 99.5% of the variance. The error bars indicate the plus and minus standard deviations about the means. The shaded region identifies the theoretical range of uncertainty arising from a system obeying Poisson statistics.

all similar to the random way in which thermal energy is transferred to or from a temperature bath. Moreover, most of the particle motions arise from interactions with the plate itself and not with other particles. The fact that this mechanical system obeys Boltzmann statistics so well may arise from the chaotic nature of particle–particle collisions and even of particle–plate interactions when many impacts occur per second.

SUMMARY. The mechanical analogy of balls on a vibrating plate containing a depression serves a couple of useful purposes. The mechanical system has a behavior predicted by Boltzmann statistics and therefore also illustrates the essential physics of the Clausius–Clapeyron equation. The mere existence of a potential energy well in a system of agitated particles is sufficient to guarantee that the equilibrium “vapor” density increase with “temperature,” a measure of the energy imposed on the particles comprising the system. Escape of molecules from the “liquid” phase is simply a matter of probability; those lucky few with kinetic energy exceeding the energy depth of the potential well are likely to escape; otherwise, they remain trapped in the well. Finally, the coupling of theory (thermodynamics and statistical mechanics) with data from a macroscopic system yields valuable information about the particle-scale and thermodynamic properties of the system. Vapor pressure above a liquid increases with temperature simply by virtue of the cohesive forces holding molecules in the liquid. It is one of the beauties of science that a molecular phenomenon like the probability of escape from a binding energy in a liquid can have global consequences: the physics of balls bouncing out of a depression in a vibrating plate or of molecules escaping from a liquid underlies the “water vapor feedback,” the increase of water vapor concentration in the atmosphere as global-mean temperature increases, and the concomitant enhancement of the greenhouse effect.

For teachers, we envision that the system described here can be useful as a classroom demonstration in an atmospheric thermodynamics course or similar. To further that end, we have made an animation of the mechanical demonstration, which is available online (<http://phy.mtu.edu/vpt/>). Moving the slider along the temperature scale in the animation shows how adding energy to the system increases the likelihood of particles escaping the central potential well. Or the system can be reproduced without great expense or effort to serve as part of an undergraduate atmospheric physics laboratory. We find the quantitative illustration of Boltzmann statistics to be highly instructive, and the analysis also serves to teach several useful concepts in

error analysis. It is our hope that this demonstration serves to enlighten all of us students about the inner workings of liquid–vapor interactions.

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