TWO SIMPLE AND ACCURATE APPROXIMATIONS FOR WET-BULB TEMPERATURE IN MOIST CONDITIONS, WITH FORECASTING APPLICATIONS

JOHN A. KNOX, DAVID S. NEVIUS, and PAMELA N. KNOX

Can you approximate the wet-bulb temperature accurately for typical temperature and moisture conditions, using weighted and unweighted averages of dry-bulb and dewpoint temperatures? How can this help students and forecasters?

The wet-bulb temperature is a mainstay of moist thermodynamics. It is defined (e.g., American Meteorological Society 2013) physically in an isobaric, or constant-pressure, context through the familiar experiment of whirling around a sling

psychrometer—a mercury thermometer with a wetted “footie” on its bulb. It is defined adiabatically using Normand’s rule (e.g., Wallace and Hobbs 2006, 86–87; see Fig. 1), by locating the lifting condensation level (LCL, where the temperature and the dewpoint are equal) of a parcel of air on a thermodynamic chart and then following the moist adiabat from the LCL back down to the surface. The temperature obtained at the surface is the wet-bulb temperature.¹

These three temperatures—dry bulb, dewpoint, and wet bulb—can be related conceptually and graphically. But can they be related *mathematically* in a simple way?

Petty (2008, p. 207) has stated that “there is no simple mathematical formula for dew-points as a function of the wet-bulb and dry-bulb temperatures.” Bohren and Albrecht (1998, p. 284) indicated that a

AFFILIATIONS: KNOX—Department of Geography, The University of Georgia, Athens, Georgia; NEVIUS—Department of Mathematical Sciences, University of Wisconsin–Milwaukee, Milwaukee, Wisconsin; KNOX—Department of Crop and Soil Sciences, The University of Georgia, Athens, Georgia

CORRESPONDING AUTHOR: Dr. John A. Knox,
johnknox@uga.edu

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¹ Technically, the wet-bulb temperature found by Normand’s rule is not quite identical numerically to the wet-bulb temperature as defined isobarically, but these differences are generally negligible for meteorological applications.

short computer program is necessary to calculate dewpoint iteratively as a function of the dry- and wet-bulb temperatures; a similar iterative computer program, based on concepts in Iribarne and Godson (1981), can calculate the wet-bulb temperature as a function of temperature and mixing ratio (J. Snider 2017, personal communication). An example of an iterative method of obtaining the wet-bulb temperature

from the dry-bulb temperature, relative humidity, and atmospheric pressure was recently published by Al-Ismaili and Al-Azri (2016). Tejeda Martínez (1994) developed a cubic equation approximation relating wet-bulb temperature to the dry-bulb temperature and relative humidity. More recently, Stull (2011) created an empirical formula for wet-bulb temperature using gene-expression methods:

$$T_{w\text{Stull}} = T_{\text{atan}} \left[0.151977(\text{RH}\% + 8.313659)^{1/2} \right] + \text{atan}(T + \text{RH}\%) - \text{atan}(\text{RH}\% - 1.676331) + 0.00391838(\text{RH}\%)^{3/2} \text{atan}(0.023101\text{RH}\%) - 4.686035, \quad (1)$$

in which the temperatures are in degrees Celsius and RH% is the relative humidity in percent. Sadeghi et al. (2013) developed a second-order polynomial fit solution to wet-bulb temperature that, unlike Stull's method, is appropriate for high elevations and a wide range of relative humidities at subfreezing temperatures.

Of these options for calculating the wet-bulb temperature discussed above, none of them fully qualifies as "simple" in the popular, nontechnical sense of the word. While this may not matter in an age of supercomputing power in people's pockets, it is relevant to our teaching and understanding of moist thermodynamics, and for applications of the wet-bulb temperature by nonspecialists. The question is, have simpler approximations for the wet-bulb temperature been largely overlooked? In this article we answer this question in the affirmative and draw attention to and analyze two such approximations.

First, Wallace and Hobbs (2006, p. 84) mention without proof that the wet-bulb temperature "usually...is close to the arithmetic mean" of the dry-bulb and the dewpoint temperatures, that is,

$$T_{w_{1/2}} = \frac{1}{2}(T + T_d), \quad (2)$$

in which T_w is the wet-bulb temperature (with the subscript 1/2 indicating the coefficient), T is the dry-bulb temperature, and T_d is the dewpoint temperature (units are irrelevant as long as they are consistently applied in the equation). But the authors supply no information regarding the origin and accuracy of the arithmetic-mean rule, nor any comparison to other, more sophisticated approximations.

Second, meteorologist Jeff Haby has discussed (e.g., <http://theweatherprediction.com/habyhints/170/>) a shortcut to calculate wet-bulb temperature known as the "one-third rule." The one-third rule is simply a weighted average of dry-bulb and dewpoint temperatures:

$$T_{w_{1/3}} = T - \frac{1}{3}(T - T_d) = \frac{2}{3}T + \frac{1}{3}T_d, \quad (3)$$

in which the subscript 1/3 indicates the coefficient of the dewpoint depression that gives the rule its name; as in (2), the units are irrelevant as long as they are consistent. This rule of thumb is, like the arithmetic-mean rule, both simple and related to common thermodynamic weather variables. But what is the justification for it, and how accurate is it?

Below we examine these two approximations, including justifications for their use and calculations of

FIG. 1. Normand's rule employed on a skew T -log p chart (obtained from www.sundogpublishing.com/AtmosThermo/Resources/skewt_part_8x11.pdf) and applied to a situation in which $T \approx 0^\circ\text{C}$ and $T_d \approx -5.6^\circ\text{C}$ for a pressure of 1,000 hPa. The dry adiabat is followed upward (blue arrow) to the LCL, where the saturation mixing ratio line intersects it. Then the moist adiabat is followed downward (red arrow) to the 1,000-hPa level, obtaining $T_w \approx -2^\circ\text{C}$.

their accuracy versus the more sophisticated Stull formula in (1)—which was chosen for comparison because of its straightforward, noniterative ease of calculation and its accuracy across a wide variety of conditions. Finally, we discuss applications of the one-third rule for quick—but *not* dirty—calculations that are useful for both precipitation-type forecasts and agriculture.

ORIGIN AND JUSTIFICATION FOR THE ONE-THIRD RULE AND ARITHMETIC-MEAN RULE.

To our knowledge, no formal and exact derivations exist of either the arithmetic-mean or the one-third rule. For example, the one-third rule seems to have originated empirically, probably with operational meteorologists (J. Haby 2015, personal communication). An Internet search of “wet-bulb temperature and arithmetic mean” in January 2017 revealed only a few relevant links; one such link, to a nineteenth-century thermodynamics textbook, includes the statement (without derivation) that “(a) 53°F [11.67°C] the reading of the wet-bulb thermometer is the arithmetic mean between the dew-point and the temperature of the air...At higher temperatures the reading of the wet-bulb is lower than this mean, and at lower temperatures it is higher” (Preston 1894, p. 365). Similarly, an Internet search of “wet-bulb temperature and one-third rule” in September 2016 revealed only a handful of mentions aside from Haby’s websites, usually from energy and heating/air conditioning vendors (e.g., www.thermcoenergysystems.com/wet_bulb_calculation_air_conditioning.html).

Our justification for the possible effectiveness of the both rules follows the following line of reasoning:

From Ferrel (1886) as reproduced in Sullivan and Sanders (1974, p. 2), the formula for saturation vapor pressure E_s (hPa) can be written as

$$E_s(T_w) - E_s(T) \frac{\text{RH}\%}{100} = 6.6 \times 10^{-4} P \left(1 + 1.15 \times 10^{-3} T_w \right) (T - T_w), \quad (4)$$

in which the first constant on the right-hand side of (4) is the psychrometric constant γ ($^{\circ}\text{C}^{-1}$), P is atmospheric pressure (hPa), and the temperatures are expressed in degrees Celsius. [Loescher et al. (2009) and Sadeghi et al. (2013) indicate that γ is not constant and is smaller than values employed in Ferrel’s time; instead, $5.48 \times 10^{-4} \leq \gamma \leq 6.42 \times 10^{-4} ^{\circ}\text{C}^{-1}$ for a wide range of wet-bulb temperatures according to Sadeghi et al. (2013). We test our approximation with different values of γ below.]

For relatively small wet-bulb depressions, the first term on the left-hand side of (4) can be linearized using Taylor series as

$$E_s(T_w) \approx E_s(T) - b(T - T_w), \quad (5)$$

where b is the slope of the saturation vapor pressure curve ($\text{hPa } ^{\circ}\text{C}^{-1}$).

Also, for most meteorological applications the $1.15 \times 10^{-3} T_w$ term on the right-hand side of (4) is small and can be ignored.

With both of these linearizing assumptions, (4) can be rewritten as

$$E_s(T) - bT - E_s(T) \frac{\text{RH}\%}{100} - \gamma PT \approx -bT_w - \gamma PT_w, \quad (6)$$

or, after rearranging and dividing through, as

$$T_w \approx E_s(T) \left(\frac{\text{RH}\%}{100} - 1 \right) (b + \gamma P)^{-1} + T. \quad (7)$$

This equation is still nonlinear in T_w , because the wet-bulb temperature is implicit in the relative humidity term on the right-hand side of (7). However, Lawrence (2005) has shown that relative humidity can be approximated for moist conditions as

$$\text{RH}\% \approx 100 - 5(T - T_d), \quad (8)$$

where the temperatures are in degrees Celsius. Note that Lawrence’s approximation in (8) is not a units-consistent form; we will assume, as indicated by Lawrence, that RH% on the left-hand side of (8) is unitless. Inserting (8) into (7) eliminates the relative humidity, yielding

$$T_w \approx E_s(T) \left[\frac{1}{20} (T_d - T) \right] (b + \gamma P)^{-1} + T. \quad (9)$$

With some rearrangement, (9) can be reexpressed as

$$T_w \approx (1 - k)T + kT_d, \quad (10)$$

in which the unitless coefficient k is

$$k = E_s(T) \left[20(b + \gamma P) \right]^{-1}. \quad (11)$$

The form of (10) justifies a linear combination of dry-bulb and dewpoint temperatures as an approximation to wet-bulb temperature, for the assumptions of relatively small wet-bulb depression and relatively moist conditions.

Whether this linear combination should be a pure arithmetic mean ($k = 0.5$) or a weighted average (for the one-third rule, $k = 0.333$) is tested in Table 1, in which values of E_s , b , and P representative of near-sea level conditions for a range of dry-bulb temperatures and γ (from Sadeghi et al. 2013) are inserted into (11), obtaining a range of values for k . The values of

TABLE 1. Range of values of the coefficient k [(see (11)] at 1,000 hPa for different values of dry-bulb temperature T , saturation vapor pressure E_s [calculated using Brooker (1967)], and the slope of the Clausius–Clapeyron equation b [using Eq. (7.36b) of Miller (2015)]. Two different values of the psychrometric constant γ are used in the calculation of k : 5.68×10^{-4} and $6.42 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ (as per Sadeghi et al. 2013). Note that 1 mb = 1 hPa.

T ($^\circ\text{C}$)	E_s (mb)	b (mb $^\circ\text{C}^{-1}$)	Range of k [from (11)]
–10	2.866	0.254	0.160–0.174
–5	4.218	0.361	0.210–0.227
0	6.112	0.444	0.281–0.302
5	8.725	0.577	0.339–0.361
10	12.279	0.822	0.419–0.442
20	23.385	1.447	0.560–0.580
30	42.452	2.433	0.690–0.707
40	73.813	3.926	0.808–0.821

k in Table 1 justify the use of an average that weights temperature twice as much as dewpoint for dry-bulb temperatures between 0° and 5°C . Because $T \geq T_w$, this suggests that the one-third rule would be especially well suited for relatively moist conditions in which T_w is in the vicinity of 0°C . Based on Table 1, the arithmetic-mean rule of Wallace and Hobbs (2006) should become comparatively more accurate than the one-third rule for dry-bulb temperatures between 10° and 20°C , a result broadly consistent with Preston’s (1894) statement quoted earlier in this section. [See online supplement (<http://dx.doi.org/10.1175/BAMS-D-16-0246.2>) for an alternate derivation of the arithmetic-mean rule and an alternative estimate of its range of effectiveness.]

There is visual justification for both of these rules. Typical diagrams in textbooks explaining Normand’s rule [e.g., Fig. 3.11 in Wallace and Hobbs (2006)] show T_w approximately equidistant from T and T_d ; in such circumstances the arithmetic-mean rule would be a good approximation. Justification for the one-third

TABLE 2. Dry-bulb temperatures (white numbers) corresponding to the domains of superior accuracy vs $T_{w\text{Stull}}$ of $T_{w1/3}$ (blue) and $T_{w1/2}$ (magenta). Black regions indicate where both approximations are superior in accuracy to $T_{w\text{Stull}}$. Boldfaced numbers indicate dry-bulb temperatures for which $T_{w1/3}$ or $T_{w1/2}$ has zero error. Gray boxes at right are for extremely dry conditions for which no psychrometric data are available.

		$T - T_w$ ($^\circ\text{C}$)																	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
T_v ($^\circ\text{C}$)	–5	–4	–3	–2	–1	0	1												
	–4	–3	–2	–1	0	1	2												
	–3	–2	1	0	1	2	3												
	–2	–1	0	1	2	3	4	5											
	–1	0	1	2	3	4	5	6	7										
	0	1	2	3	4	5	6	7	8	9									
	1	2	3	4	5	6	7	8	9	10									
	2	3	4	5	6	7	8	9	10	11	12								
	3	4	5	6	7	8	9	10	11	12	13	14							
	4	5	6	7	8	9	10	11	12	13	14	15	16						
	5	6	7	8	9	10	11	12	13	14	15	16	17	18					
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20				
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22			
	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24		
	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33

FIG. 2. Absolute error (°C) for the arithmetic-mean rule [(2)] for dry-bulb temperatures between -5° and 45°C and a wide range of dewpoint depressions. Absolute errors between -1° and +1°C are shaded.

rule is provided by the Normand's rule diagram in Fig. 1. Note that the example chosen for this figure was for a cold-weather situation in which the surface dry-bulb temperature is 0°C. It is obvious in Fig. 1 that the wet-bulb temperature does *not* lie equidistant from T and T_d —a weighted average, with dry-bulb temperature weighted more than dewpoint as in (3), is required. And, in fact, $T_{w1/3} = -1.9^\circ\text{C}$ for this example, which is very close to the graphically obtained $T_w \approx -2^\circ\text{C}$ (and $T_w = -1.9^\circ\text{C}$ from an online calculator; Brice and Hall 2013). This is also consistent with Preston's (1894, p. 365) statement that “at lower temperatures [the wet-bulb temperature] is higher [i.e., warmer than the arithmetic mean of T and T_d].”

Arithmetic Mean Rule Absolute Error (°C)

TESTING WET-BULB TEMPERATURE APPROXIMATIONS WITH REAL DATA: METHODS. To test the accuracy of the two rules for less idealized circumstances, we compare the approximations to wet-bulb temperatures using psychrometric data at 1,000 hPa from Petty (2008), and from Brice and Hall (2013), which was used to obtain results for wet-bulb temperatures below 0°C and for mean sea level pressure (MSLP; for Stull's method). We calculate relative error for Stull's empirical formula [(1)], the arithmetic-mean rule [(2)], and the one-third rule [(3)] for a range of moisture conditions for dry-bulb temperatures between -4° and 33°C. Absolute error is plotted for (2) and (3) for temperatures between -5° and 45°C and dewpoint depressions from 1° to over 70°C.

One-Third Rule Absolute Error (°C)

FIG. 3. As in Fig. 2, but for the one-third rule [(3)].

FIG. 4. Ice accumulations on 16–17 Feb 2015 across northeastern Georgia (from www.srh.noaa.gov/ffc/?n=20150216_winterstorm). Athens is located in Clarke County, east-northeast of Atlanta (the cluster of interstates in red).

RESULTS. *Relative error.* The relative error of the arithmetic-mean rule [(2)] and the one-third rule [(3)] was calculated for a range of wet-bulb depressions ($1^{\circ} \leq T - T_w \leq 18^{\circ}\text{C}$) and wet-bulb temperatures ($-5^{\circ} \leq T_w \leq 15^{\circ}\text{C}$); refer to the supplemental material (<http://dx.doi.org/10.1175/BAMS-D-16-0246.2>). As a summary of these calculations, in Table 2, the dry-bulb temperatures for which (2) and (3) are more accurate than (1) are depicted (shaded regions), including the temperatures for which (2) and (3) yield zero error (in boldface font). For near-saturation conditions, either the arithmetic-mean rule or the one-third rule—or both—is better than Stull’s approximation for temperatures from -4° to 15°C .

These results are consistent with, but exceed, expectations based on our linearized analyses in the previous two sections. The one-third rule is highly accurate—that is, superior to (1)—for relatively moist conditions [mean RH% where it is superior to (1) = 50%, in general agreement with the assumptions implicit in the use of Lawrence (2005) in the previous section] and near-freezing dry-bulb temperatures. The arithmetic-mean rule is superior to (1) for somewhat warmer and similarly moist conditions (mean RH% = 61%) several degrees either side of $T = 13^{\circ}\text{C}$.

But the arithmetic-mean rule is also highly accurate for a broader range of temperatures than our derivation suggested. In addition, the one-third rule similarly shows unexpected accuracy versus the analysis in the previous section for a narrow range of warmer and progressively drier situations (e.g., $T = 27^{\circ}\text{C}$ with RH% = 3%). These extremely simple approximations exhibit “unreasonable effectiveness” in the Wignerian phrase (Wigner 1960). While the Stull formula is definitely the superior approximation overall, it is shown that simple averages can provide similar or superior results in certain temperature/moisture circumstances.

Absolute error. A more common measure of error in

research on wet-bulb temperature approximations is absolute error. Figures 2 and 3 depict the absolute error for the arithmetic-mean rule and the one-third rule, respectively, for dry-bulb temperatures between -5° and 45°C and a wide range of dewpoint depressions.

The absolute error results for the arithmetic-mean rule are depicted in Fig. 2. Its region of maximum accuracy is in moist conditions for the full temperature range shown, as well as a corridor of drier conditions at above-freezing temperatures. The one-third rule (Fig. 3) is also useful in very moist conditions throughout the domain, and it has less than $\pm 1^{\circ}\text{C}$ error even for warm ($>25^{\circ}\text{C}$) and very dry conditions (dewpoint depressions $> 50^{\circ}\text{C}$).

A comparison of Figs. 2 and 3 with Fig. 3 in Stull (2011) is favorable, given that the absolute errors in Stull’s (1) are generally larger in magnitude than 0.5°C for relative humidities of 50%–80% in the temperature range from -5° to 5°C . For this same temperature range, the absolute errors of the one-third rule are generally within a few tenths of a degree Celsius. As we will see in the next section, the one-third rule is especially useful in applications because its domain of maximum accuracy includes the phase change for water from solid to liquid and vice versa.

FIG. 5. Meteogram for Athens (KAHN) for 16 Feb 2015 (from www.wunderground.com/history/airport/KAHN/).

APPLICATIONS OF THE ONE-THIRD RULE TO FORECASTING PRECIPITATION TYPE AND FROST PREVENTION IN AGRICULTURE. *Precipitation-type forecasts.* Knowing the surface wet-bulb temperature near 0°C is extremely useful for the forecasting of rain versus freezing rain, sleet, and/or snow. When precipitation falls into unsaturated air near the surface, the dry-bulb temperature eventually drops to the wet-bulb temperature in the absence of other thermodynamic or dynamic forcings. This convergence of the two temperatures occurs because the unsaturated air is cooled by the process of continual evaporation, akin to the classic sling psychrometer measurement of wet-bulb temperature. If the wet-bulb temperature is at or below 0°C, then the result can be a transition from liquid to freezing or frozen precipitation (depending on the depth of the cold air near the surface) as the surface temperature drops to the wet-bulb temperature. This is particularly common, for example, in the U.S. Southeast during wintertime. In the Southeast, moisture advancing over a warm front associated with a Gulf of Mexico low can precipitate into cold and very dry air entrenched

near the surface over the region (and often enhanced by cold-air damming; see Rackley and Knox 2016).

On 16–17 February 2015, one such event occurred, in which a rain event turned to unforecasted ice and snow across northeastern Georgia. In Athens, Georgia, the short-term forecast called for a cold rain event to occur through the afternoon and evening hours. However, by the next morning, parts of northeastern Georgia received more than 1.25 cm (0.5 in.) of ice, including around 0.85 cm (0.33 in.) of ice on the north side of Athens (Fig. 4). A continuous supply of cold unsaturated air from the east allowed the surface temperature to converge with the dewpoint temperature and drop below freezing as precipitation fell on the afternoon of 16 February. This is illustrated in the meteogram from the Athens–Ben Epps Airport (KAHN) in Fig. 5.

In Table 3, we show the calculation of wet-bulb temperature via the one-third and arithmetic-mean rules, illustrating both the accuracy and the simplicity of the one-third rule for these situations. As precipitation began to fall around noon local time (1651 UTC), the one-third rule estimated the wet-bulb

TABLE 3. Observations at Athens (KAHN) at the onset of precipitation [light rain (–RA); 1651 UTC 16 Feb 2015] and shortly after the onset of freezing precipitation [light freezing rain (–FZRA); 2213 UTC], with the wet-bulb temperature estimated by the Stull [(1)], arithmetic-mean rule [(2)], and one-third rule [(3)] approximations, and as reported by the National Weather Service. (Source for observational data: <http://mrcc.isws.illinois.edu/CLIMATE/>.)

Time (UTC)	T (°C)	T_d (°C)	Present weather	$T_{w\text{Stull}}$ (°C)	$T_{w1/2}$ (°C)	$T_{w1/3}$ (°C)	$T_{w\text{NWS}}$ (°C)
1651	3	–9	–RA	–1.6	–3.0	–1.0	–1
2213	–1	–2	–FZRA	–1.8	–1.5	–1.3	–1

temperature at –1°C, the same as the National Weather Service wet-bulb reading. As precipitation fell all afternoon, by 2213 UTC the dry-bulb temperature dropped to near the wet-bulb temperature (which held nearly constant near –1°C), where it remained throughout the evening during the freezing rain event (see Fig. 5).

As expected from the results of the previous sections, the one-third rule (seventh column of Table 3) performs very well under these conditions of near-freezing dry-bulb temperatures and relatively moist conditions. In contrast, the arithmetic-mean rule gives an absolute error of about 2°C for the wet-bulb temperature at 1651 UTC and is of less utility as a result. Stull’s method [(1)] yields errors of several tenths of a degree Celsius, also as expected based on our previous discussion.

Protection from frost damage in agriculture. Many types of fruit crops, such as strawberries and blueberries, are susceptible to frost damage in late winter and early spring. The damage occurs when the water in the plant cells freezes, expands, and ruptures the cells. Depending on what stage the plant is in, the critical temperature for frost damage is from approximately –12° to –2°C (10°–28°F; Demchak 2007). A technique for preventing below-freezing temperatures from affecting the plants on relatively calm, clear nights when radiative cooling is dominant is to irrigate them using overhead irrigation systems (Williamson et al. 2004). The water on the outside of the plants freezes and releases latent heat, preventing subfreezing temperatures from affecting the water in the plant cells as long as a thin layer of water is present on the bloom or on the ice (Fisher and Shortt 2006). This relates to the wet-bulb temperature “because [the wet-bulb temperature] essentially is what the plant temperature will be once the irrigation is started and evaporative cooling has taken place” (Demchak 2007, p. 15).

However, the timing and amount of irrigation can be problematic. If too much irrigation is used,

then the weight load of the ice damages the plants and defeats the purpose of preventing crop damage. This can happen with air that is so exceptionally cold and dry that no amount of irrigation will prevent damage. So knowing when to irrigate, or not to irrigate, for maximum benefit is a complicated and potentially costly decision: “more

art than science,” with significant financial benefits to be gained from better precision (Borisova et al. 2015).

One aspect of better precision not discussed by Borisova et al. (2015) is improvement in the temperature guidelines for when to begin and end irrigation. Fisher and Shortt (2006) provide a table of dry-bulb and dewpoint temperatures for this purpose, but they say that “if wet bulb temperatures are available, these can be used directly to determine when irrigation should begin, and when the system can be shut off.” The implication is that dry-bulb and dewpoint temperatures are more readily available than wet-bulb temperatures; this is echoed in other agricultural extension documents discussing irrigation and frost prevention. However, a main result of this article is that, given temperature and dewpoint, the wet-bulb temperature for conditions relevant to frost prevention is *easily and accurately*

TABLE 4. (left) Dewpoint temperature and (middle) dry-bulb thresholds for irrigation of blueberries and strawberries to prevent frost damage, adapted from Longstroth (2012). (right) Wet-bulb temperature calculated from the other two columns, using the one-third rule [(3)].

T_d (°F)	T to start irrigation (°F)	$T_{w1/3}$ (°F)
26	34	31.3
25	35	31.7
24	35	31.3
23	36	31.7
22	36	31.3
21	37	31.7
20	37	31.3
19	38	31.7
18	38	31.3
17	38	31.0
16	39	31.3
15	39	31.0

calculated by the one-third rule. There is therefore no need for guesswork using rules of thumb for temperature and dewpoint; the wet-bulb temperature can be obtained from them virtually instantaneously via (3).

We illustrate this skip-the-middleman approach in Table 4, adapted from Longstroth (2012). It is apparent that Longstroth's table (the first two columns in our Table 4) can be collapsed to a simple maxim: "Begin irrigation when wet-bulb temperatures go below 32°F." With (3), moreover, the wet-bulb temperature can be obtained easily from dry-bulb and dewpoint temperatures. Using (3) to give farmers an easily accessed wet-bulb temperature might be an improvement upon current practices, for example, "growers typically initiate their irrigation systems between air temperatures of 31°F–35°F" (Borisova et al. 2015, p. 3) with no reference to moisture conditions.

In addition, the vexing and costly problem of irrigating in vain in too-dry conditions should also be clarified with reference to the wet-bulb temperature, rather than dewpoint. The salient issue is that the air is so cold *and* so dry that the *wet-bulb* temperature is below the critical temperature for the plant at its particular stage—no amount of irrigation will warm the plants enough to avoid crop damage. Here again, the simplicity and accuracy of the one-third rule should provide a guide to agricultural interests that is more succinct and informative than consulting tables of dewpoints or dry-bulb temperatures.

CONCLUSIONS. This work sheds new light on two very simple and surprisingly accurate approximations for the wet-bulb temperature, both of which employ only dry-bulb and dewpoint temperatures. These approximations are justified from basic thermodynamics principles, and perform well in moist conditions in the vicinity of freezing (the one-third rule) and for somewhat warmer conditions (the arithmetic-mean rule). The one-third rule is particularly useful for quick and accurate estimates of the wet-bulb temperature relevant to both precipitation-type forecasting in winter weather situations and frost prevention in agricultural applications.

These approximations do not supplant other, more sophisticated approximations for wet-bulb temperature that are accurate over broader ranges of temperature and moisture conditions. But their simplicity, and accuracy in some commonly encountered situations, makes them highly useful for both education and outreach. Because of the utter simplicity of these approximations, we encourage thermodynamics instructors and textbook authors to include discussion of the one-third rule and the arithmetic-mean rule in their

classes and books. Because of its particular utility for agricultural interests, we encourage extension agents to popularize the one-third rule.

Future work to benefit agriculture includes the development of an application to allow instantaneous calculation of wet-bulb temperatures via (3) for use by farmers and other agricultural interests.

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