

## A Technique to Measure Trends in the Frequency of Discrete Random Events

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### ABSTRACT

Time series of extreme meteorological and hydrological events frequently present problems with the use of traditional parametric statistical techniques. These difficulties arise from the frequent use of count data, the presence of zero values, data with nonnormal distributions, and/or truncated data. This paper presents a parametric method to evaluate temporal trends in extreme events that overcomes these problems. The technique includes the testing of the arrival structure of extreme event data for the Poisson distribution, then prepares and tests time series of interarrival times for trend analysis through linear regression. Nor'easters along the east coast of the United States and heavy rainfall events at Covington, Louisiana, are examined.

### 1. Introduction

Analyzing temporal changes in extreme (or any variable consisting of discrete random) events is becoming increasingly important as we face a future of global environmental change. Time series of extreme events, however, frequently present problems when attempting analyses using parametric techniques. When common statistical tests are used to test for trends in discrete events, underlying assumptions are often violated because of the use of enumerated (or count) data, the presence of zero values, nonnormality of data, and/or use of truncated data, where little is known about the distribution of the underlying dataset. As a result, these studies frequently implement less powerful nonparametric statistical methods (i.e., Rohli and Keim 1994; Changnon and Changnon 1992) or compare frequency-magnitude relationships for varying time samples (Keim and Muller 1992; Huff and Changnon 1987; Walsh et al. 1982), for example, the former half of a time series versus the latter half. However, the advantage of using parametric methods when possible is that these techniques are generally more powerful than their nonparametric counterparts, which leads to fewer type II errors in hypothesis testing.

This paper clarifies and simplifies a method introduced by Cox and Lewis (1966) to analyze temporal trends in the rate of occurrence of extreme events using

parametric methods. This parametric technique will be applied to datasets that initially violate each of the assumptions mentioned earlier. The technique relies on the assumption that the data counts (arrivals per time interval) are Poisson distributed, which then allows for the parametric analysis of the interarrival times between successive events (which are transformed to generate a distribution that more closely conforms to the normal).

To demonstrate, time series of the strongest East Coast nor'easters and extreme rainfall events at Covington, Louisiana, are analyzed for temporal trends. These data were selected because Davis and Dolan (1993) and Keim and Muller (1993), respectively, discuss how recent decades deviated from the long-term patterns of events in the noted regions. Analysis of these data also address important questions regarding changing extremes in a changing climate (see, e.g., Karl et al. 1995), including the potential increase in the intensity and duration of Atlantic coastal storms and of heavy rainfalls in an area long plagued with such problems. Such changes in short-duration event variability may serve as an indicator of long-term change, even in instances when seasonal or annual means remain mostly unchanged.

### 2. Poisson as a counting process

Nonparametric tests normally lack the power of parametric tests because with the former the population distribution governing the process is typically not known, and thus the sampling distribution is unknown. These tests are normally based upon order statistics (i.e., an

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ordering of the data from smallest to largest or vice versa), which are then assigned exceedence probabilities based upon empirical plotting formulas—for example, the Weibull, Gringorten, or Hazen (Stedinger et al. 1993). Thus, estimates of rejection probabilities for null hypotheses (power) are not attempted, or if so, are highly uncertain.

Time series of climatological (and other natural or anthropogenic) extreme phenomena, however, become counting processes once a truncation level is imposed. A counting process is simply a stochastic process by which a running count of the number of events occurring in a prescribed time increment (ranging from fractions of seconds to millennia) is observed (Ross 1993)—for example, the number of class 5 hurricanes per decade, or the number of river basin floods over bankfull stage in a water year (which extends from 1 October to 30 September for hydrologic purposes). Perhaps the most important and useful counting process is one in which the arrival rate of events follows a Poisson distribution. Representation of serial data in this manner allows for the use of many parametric analysis procedures. Applications of the Poisson distribution are varied—including analyses of heart disease mortality (Lovett et al. 1986) and radiation exposure (Darby et al. 1985)—and have been particularly useful in the earth sciences for representing the arrival structure of heavy rainstorms and floods (Shane and Lynn 1964; Todorovic and Yevjevich 1969; Todorovic and Zelenhasic 1970; Ashkar and Rousselle 1987).

With a nonhomogeneous Poisson process, arrival rates are not required to remain constant through time. This representation introduces the possibility of disaggregating the data series by climatic seasons or synoptic mechanisms. This type of disaggregation was performed by Todorovic and Rousselle (1971), Cruise and Arora (1990), and Faiers et al. (1994) on hydrological and climatological data, but other types of data can be disaggregated based on criteria appropriate to the data type. Partitioning of data series may be necessary or advantageous in many instances. For example, it is possible to have an unpartitioned series of events (e.g., floods at one gauging location) that exhibits no overall trend, but partitioning of these data may show a positive trend in one condition (e.g., snowmelt floods), which is counteracted in a series by a negative trend in another (e.g., events produced by heavy rainfall). In this instance, important deductions can be gained about the deterministic factors causing the trends. In addition, if the components of the partitioned series arise from different population distributions, then the unpartitioned data series consists of mixed distributions (Waylen and Woo 1982; Hirschboeck 1987; Cruise and Arora 1990; Faiers et al. 1994). Rasmussen and Rosbjerg (1991) and Ekanayake and Cruise (1994) demonstrated that quantiles (frequency–magnitude relationships) estimated from unpartitioned series in these cases can demonstrate significant negative bias.

### 3. Characteristics of the Poisson process

In the Poisson distribution, there is equality of mean and variance represented by the parameter  $\lambda$ . If the arrival rate remains constant with time, then the process is stationary or homogeneous and the number of realizations depends only on the length of the time increments and is directly proportional to this length (Ross 1993). However, arrival rates are not always temporally constant and the nonstationary or nonhomogeneous Poisson process results. In these cases, it is possible to partition or disaggregate the series into its individual components because of the property of regeneration of the process under addition—that is, the mean and variance of the combined process is the sum of the means and variances of the individual components, which are also Poisson processes (Karlin and Taylor 1975). Another very important property of the Poisson process is that the interarrival times between realizations of successive events are known to be exponentially distributed (Ross 1993).

The Poisson formulation has been extensively employed in the earth sciences, particularly in hydrology as the representation for an important class of stochastic flood models (Todorovic and Rousselle 1971; North 1980; Nachtnebel and Konecny 1987; Cruise and Arora 1990; Rasmussen and Rosbjerg 1991; Ekanayake and Cruise 1994). In these models, the Poisson formulation is used to facilitate the statistical analysis of the magnitudes of the observed variable (discharge) using the tools of extreme value statistics—that is, the estimation of the probabilities of extreme events from fixed or variable sample sizes. The employment of the Poisson formulation for the sample size distribution within the extreme value context allows for the computation of discharge–frequency relationships based on either partial duration series (e.g., Todorovic and Rousselle 1971; North 1981; Cruise and Arora 1990) or annual maximum data series (Gumbel 1941; Rasmussen and Rosbjerg 1991). The distribution of the event magnitudes has often been assumed to be exponential (e.g., Todorovic and Rousselle 1971); however, recent work has focused on the use of the Pareto (Rasmussen and Rosbjerg 1991) and Weibull (Ekanayake and Cruise 1994) distributions as the base distribution in the model.

Representation of storm (or flood) data series in this manner has several other advantages related to the properties of the Poisson distribution, which are not necessarily employed in the context of extreme value statistics or in the statistical analyses of the magnitudes of the variable. Since both the rate of realizations of the process and the time intervals between realizations follow known distributions, parametric statistical methods can then be employed in the analyses of these variables. These analyses include the use of powerful hypothesis tests as well as manipulations of the known probability distributions. For example, a simple test to determine if a given data series meets the Poisson criterion can be

TABLE 1. Results of Poisson tests on annual means and variances of Class IV and V nor'easters along the east coast of the United States.

Nor'easter	N	Mean	$\sigma^2$	R	Reject region	Decision R/A
Entire dataset	50	0.92	0.65	0.71	<0.75	Reject
Dataset-1 event	50	0.90	0.66	0.73	<0.75	Reject
Dataset-2 events	50	0.88	0.68	0.77	<0.75	Accept

based on the equality of mean and variance (which is subsequently demonstrated). One may also wish to estimate the time until the  $n$ th realization of the process. The distribution of these times, known as the waiting-time distribution, is usually considered to reduce to the two-parameter gamma distribution, since it consists of the sum of exponentially distributed random variables (Ross 1993). Then by taking logarithms of these variables one can transform the data into a series that approaches the normal distribution (Chow 1954).

#### 4. Data

The most powerful nor'easters (in terms of coastal impact) that rate either class IV or V on the Dolan–Davis storm class scale (Dolan and Davis 1992) provide one dataset for analysis. These data extend from July 1942 through June 1992. Because nor'easters are unlikely to occur in summer, a year is defined as extending from 1 July to 30 June. The other is a partitioned dataset of two-day heavy rainfall events from 1899 to 1990 over a 76.2-mm threshold recorded at Covington, Louisiana (Keim 1996). Data can also be truncated on other criteria such as exceedences of two or three standard deviations (Ratcliffe et al. 1978) or by partial duration series, among others. With the Covington storm data, trends are investigated after partitioning the data into three synoptic weather types; fronts, tropical disturbances, and air masses. Use of this classification scheme for storms is well documented in the literature (i.e., Keim and Faiers 1996; Matsumoto 1989). Note that the nor'easter dataset could be partitioned by synoptic conditions as defined by Davis et al. (1993), but the small numbers per class would render uninterpretable results for most classes.

The Poisson process rests upon two underlying assumptions; the number of events occurring in time is independent of its previous history, and no more than one event can occur in a short interval of time—that is, as the length of the time interval decreases the number of events decreases proportionately (Karlin and Taylor 1975; Ashkar and Rouselle 1983b). In this case, the Poisson model represents the number of randomly occurring rare storm events of fixed duration over a specified threshold. The definition of “rare,” as used above, is somewhat arbitrary, but experience has shown that the fixed threshold should be set so that no more than two to three exceedences occur per unit interval (in this

TABLE 2. Results of Poisson tests on annual means and variances of heavy rainfall events partitioned by weather type at the 75-mm threshold at Covington, Louisiana.

Weather type	N	Mean	$\sigma^2$	R	Reject region	Decision R/A
Frontal	92	2.59	3.02	1.17	>1.19	Accept
Tropical dis	92	0.42	0.42	1.00	<0.82	Accept
Air mass	92	0.32	0.42	1.31	>1.19	Reject

case, year) on the average (Cunnane 1979; Ashkar and Rouselle 1983b; Cruise and Arora 1990). The average annual occurrence rate for class IV and V nor'easters is 0.92 and for rainfall events by individual synoptic weather types at Covington ranges from 2.59 (frontal) to 0.33 (air mass) (Tables 1 and 2). Since frequencies of rare events often take the form of a Poisson process, it is plausible to test these partitioned data for the Poisson assumption.

#### 5. Testing for the Poisson process

As noted earlier, a characteristic of the Poisson distribution is that the mean and variance of the counts, represented by the parameter  $\lambda$ , are equal. As a consequence, this ratio can be used to test data for a Poisson distribution. The test relies on the fact that the ratio of the sample variance and sample mean ( $R$ ) can be converted into a  $\chi^2$  random variable by means of  $R(N - 1)$ , noting the relationship between  $R$  and the  $\chi^2$  distributed Fisher dispersion statistic,  $d$  (Cunnane 1979). Using guidelines proposed by Cruise and Arora (1990) and Ashkar and Rouselle (1987), the  $R$  values of the annual frequency datasets are tested against a critical  $R$  value ( $R_c$ ), which is obtained from  $[\chi^2_{N-1, \alpha} / (N - 1)]$ , where the one-tailed  $\alpha$  level used is 0.10 and  $N$  represents the number of years in the sample. An  $\alpha$  of 0.10 (rather than the customary 0.05) makes it easier to reject the null hypothesis, which is that the sample mean and variance are equal. Hence, there is greater confidence that the data are Poisson distributed. If  $R < R_{c, 1-\alpha}$  or  $R > R_{c, \alpha}$ , depending on whether  $R$  is less than or greater than 1, then the Poisson hypothesis is rejected.

Table 1 displays the class IV and V nor'easter results in which the null hypothesis was initially rejected. In this case, the truncation level was raised and new means and variances were derived and retested until the hypothesis was not rejected. Since the number of class IV and V nor'easters is small, the single weakest nor'easter of the time series was successively eliminated until the count data were Poisson distributed, which occurred after the elimination of the third event. If a dataset of larger sample size was in use, the truncation level could have been raised gradually (eliminating more than one event at a time) until the null hypothesis was not rejected. Once a truncation level is found to be Poisson admissible, the data should remain so at any higher level of truncation (Ashkar and Rouselle 1983a).

TABLE 3. Results of Poisson tests on the Covington, Louisiana, airmass series using higher threshold levels.

	N	Mean	$\sigma^2$	R $\sigma^2/\text{mean}$	Reject region	Decision
Dataset-5 events	92	0.27	0.38	1.38	>1.19	Reject
Dataset-10 events	92	0.22	0.28	1.30	>1.19	Reject
Dataset-15 events	92	0.16	0.18	1.12	>1.19	Accept

Results from the partitioned heavy rainfall data show that the null hypotheses of equal means and variances for the counts of frontal events and tropical disturbances, respectively, were not rejected; hence they are assumed to be Poisson distributed. However, the null hypothesis was rejected for airmass events at the 76.2-mm threshold (Table 2). The airmass storms were processed in the same manner as the nor'easters, and the null hypothesis was not rejected after eliminating the smallest 15 events in the series, which corresponds to raising the truncation level to 93 mm (Table 3). If the ultimate purpose is to reassemble the storms of various weather types (frontal, tropical disturbance, and air mass) for extreme statistical analyses, then these data should be tested for the Poisson process collectively, then disaggregated. That way, each series would be truncated at the same base level.

## 6. Testing for trend

These series of enumerated events are now associated with a parametric distribution that enables more rigorous analysis than provided by nonparametric methods. Although truncated data are in use, they are Poisson distributed, hence analysis of the interarrival times (defined as the time increments between successive events) removes the enumeration ("counts" of events) statistical assumption violation in parametric testing. Using a modification of guidelines proposed by Cox and Lewis (1966), these data were tested for any gradual trend in the rate of occurrence through an analysis of processed interarrival times using standard linear regression analysis. Advantages of using linear regression on transformed interarrival time data generated by a Poisson process include the testing of both the sample mean and variance simultaneously, successive values (events) are known to be independently distributed, and the probability distribution is moderately nonnormal. Therefore, since the residuals are near normal, the first two moments of the regression hold exactly and the  $t$  tests and  $F$  tests (which test the significance of the regression coefficients and the residuals, respectively) hold to a good approximation (Cox and Lewis 1966).

This method regresses the natural log of the intervals between successive events ( $Y_i$ ), or successive groups of events (in days), on the independent variable ( $X_i$ ), which is the cumulative frequency of days (or the midpoint of  $Y_i$  with grouped data) from the occurrence of the first event. In essence, testing of the data in this manner determines whether both the mean and variance vary

with time. Since the interarrival times have high variability, it is advantageous to cluster these values into groups to reduce the variance. This in turn will increase the efficiency of the statistical testing because it provides sample estimates closer to the population parameters. Therefore, taking the natural log of these values further reduces the variance, ensures a constant known variance of  $Y_i$ , while making the data conform more closely to the normal distribution. Although the conformance to normality of transformed data depends upon the scale parameter of the original gamma distribution, the logarithms of data arising from summation processes are often assumed to be normally distributed in practice (Chow 1954).

The series of interarrival times must begin with the onset of an event (rather than from the point from which data are first available) because the length of time from some arbitrary point to the first event does not belong to the same distribution of intervals between events (Cox and Lewis 1966). The last issue to be addressed is the number of intervals to include in groups and the corresponding efficiencies of the tests on nonnormal distributions. Problems exist with the implementation of least squares estimates of the  $y$  intercept and slope rendering this method inefficient compared to all possible estimates. However, efficiency (in percent) increases as the intervals are grouped together and the larger the number of intervals ( $g$ ) placed in the group, the greater the efficiency. In this analysis, however, the number of intervals in a series ranges from 15 (airmass events) to 238 (frontal), and grouping of the airmass series would reduce the number of elements in the regression analysis to unacceptable levels. Hence, the grouping criteria used in this study is as follows:

$$\begin{aligned} \text{if } n < 20, & \quad \text{then } g = 1, \\ \text{if } 20 \leq n \leq 60, & \quad \text{then } g = 2, \\ \text{if } n > 60, & \quad \text{then } g = 4, \end{aligned}$$

where the efficiency estimate of trend based on least squares estimates of logged intervals of  $g = 1, 2,$  and  $4$  is 61%, 78%, and 88%, respectively (Cox and Lewis 1966). Percent efficiency is the ratio of the variances of the minimum variance—unbiased statistic to that of the estimated statistic. Obviously, the efficiency increases with larger cluster sizes since the variance of the sample becomes smaller as the cluster size increases.

Figure 1 provides an illustration of this method where interarrival times between powerful nor'easters are examined. Figure 1a displays the number of days between each nor'easter event. Events in succession are plotted across the  $x$  axis and the number days from the end of one event to the beginning of the next event in sequence is shown on the  $y$  axis. Since the efficiency estimate of trend increases as intervals are analyzed in clusters, and the total number of intervals is 44, these events are clustered into groups of two. Therefore, in Fig. 1b, the number of bars is reduced by half and each bar repre-

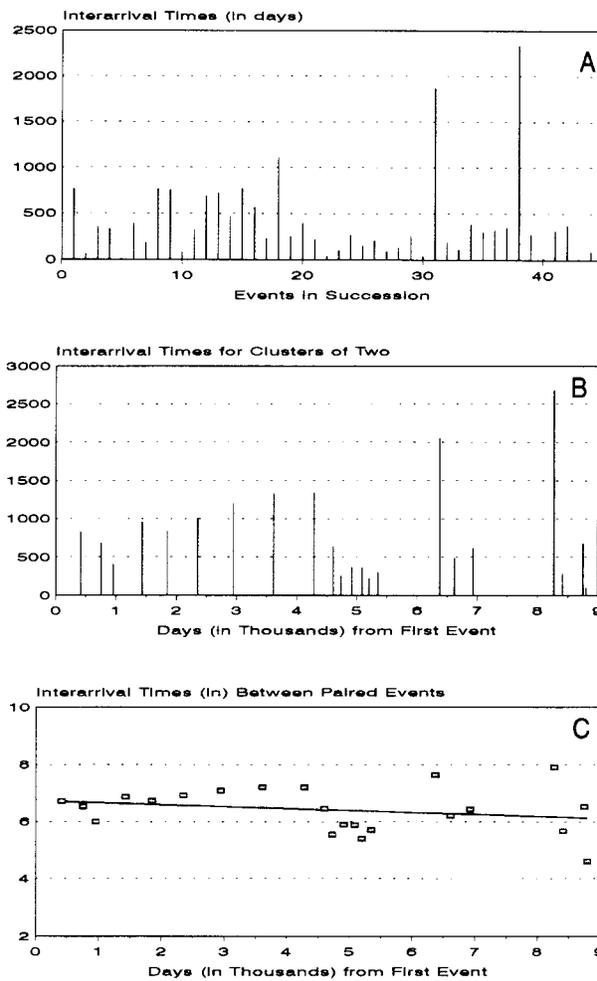


FIG. 1. Time series of class IV and V nor'easters along the east coast of the United States: (a) intervals between events, (b) intervals within clusters of two, and (c) linear regression slope.

sents the summed number of days of paired intervals (the waiting-time distribution). In other words, the first two bars in Fig. 1a are summed and are represented by the first bar in Fig. 1b, and intervals 3 and 4 in 1a are represented by the second bar in 1b, and so on. Since the 44 intervals are divisible by two, no problem is encountered regarding extraneous events. However, if the number of intervals are not divisible by the number used in the clustering procedure, the interval(s) nearest the center of the series, without interrupting a cluster in succession, is (are) omitted (Cox and Lewis 1966). The  $x$  axis in Fig. 1b no longer displays the intervals in succession, but rather time (in days) from the occurrence of the first event in the series, plotted at the midpoint of the cluster. In Fig. 1c, the graph is again modified where the number of days in each cluster is transformed into its natural log ( $\ln$ ) and is then plotted on the  $y$  axis. Rather than bars, the data are plotted in point form along with the linear least squares fit.

Davis and Dolan (1993) report atypical frequencies

TABLE 4. Results of linear regression analysis of intervals;  $g$  = number of intervals in group,  $n$  = cases included in regression,  $r$  = Pearson's correlation coefficient,  $p$  = significance of slope.

Weather type	Intervals	$g$	$n$	$r$	$p$
Nor'easters	44	2	22	-0.23	0.30
Frontal	237	4	59	-0.16	0.22
Tropical dis	38	2	19	-0.47	0.04
Air mass	14	1	14	0.59	0.03

of these powerful nor'easters during the latter portion of this time series. The increase in frequency may be related to recent circulation shifts leading to increased cold air advection into the southeastern United States (Davis et al. 1993). Cold air advection would strengthen the East Coast baroclinic zone, thereby increasing cyclogenesis in the region. This is highlighted in Fig. 1a with the relatively short interarrival times between events, with the exception of intervals 31 and 38, which are the longest of the series. As a result of this interarrival time variability, the slope depicted in Fig. 1c was insignificant (Table 4).

Each of the heavy rainfall series by synoptic weather type were processed in the above manner and the linear regression of the log transformed intervals are shown in Fig. 2. Specifics of the sample sizes, groups, and significance levels are provided in Table 4. The sign of the Pearson  $r$  represents the association between  $\lambda$  and time. A positive slope signifies that the time between events is increasing and that events are becoming less frequent. In contrast, a negative association indicates that the intervals between events are becoming shorter and that the frequency of events is temporally increasing. Hence, heavy rainfall events induced by tropical disturbances at Covington have a significantly increasing pattern of frequencies (and variance). In contrast, events produced by airmass storms are significantly decreasing in frequency and variance. Considering the opposing directions of these trends, in addition to the relatively large sample size of frontal events with no trend, it is not surprising that Keim (1997) found no overall trend (using nonparametric techniques) in all events at this site collectively.

If trends are discovered, one should always attempt to relate the statistics to the underlying physics behind the observed patterns, which should be the ultimate purpose behind statistical analyses of earth science data. As such, the deterministic causes of the results at Covington may be related to shifting intensity and position of the Atlantic subtropical high. For example, Coleman (1988) and Henderson and Vega (1996) documented increasing aridity in the extreme southeastern United States because of strengthening of the Atlantic subtropical high in recent years. Furthermore, Keim (1997) reported that temporal and spatial fluctuations of heavy rainfalls across the southeastern United States are consistent with documented shifts in the Atlantic high pressure complex. Since most of the airmass storm activity

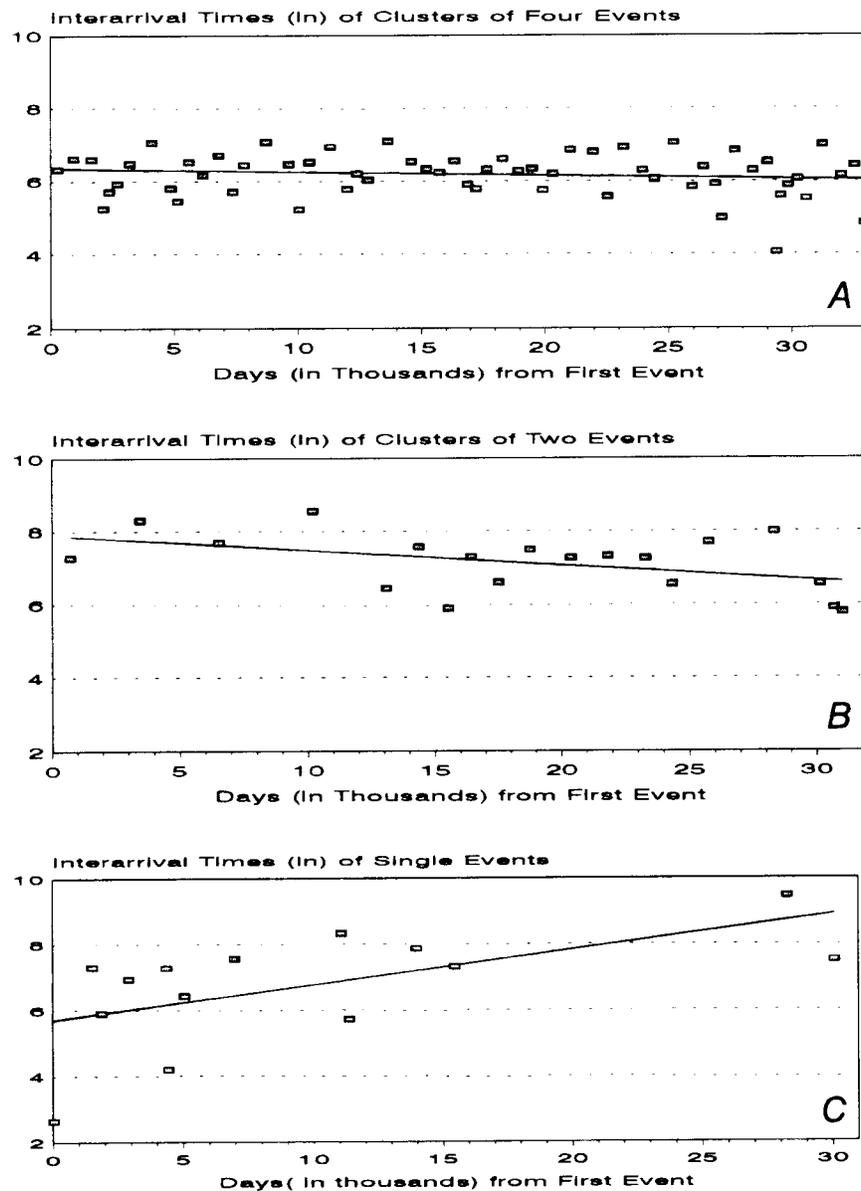


FIG. 2. Linear regression of heavy rainfalls at Covington, Louisiana, derived by (a) fronts, (b) tropical disturbances, and (c) air masses. Time is the independent variable.

takes place in summer months (which is when the subtropical high is at its maximum intensity), this phenomenon may also be having an effect as far west as Covington, thereby decreasing free convection and limiting opportunities for air mass-induced heavy rainfalls. In addition, an expanded Atlantic subtropical high may also direct more tropical storm activity (ranging from relatively weak easterly waves to hurricanes) to the central Gulf Coast, rather than along the east coast of the United States.

Since interannual variability is great within the observed data, however, inferences made beyond the periods of record are limited. Therefore, the slope com-

ponent of the regression models and their probabilities of significance are considered valid for the period of record under examination only. Extrapolating the derived slopes beyond the periods of record is not recommended since this would assume that storm frequencies would continue to increase (to infinity) or decrease (to zero) along the derived lines of best fit (Woodward and Gray 1993).

### 7. Summary and conclusions

Discrete random events can often be difficult to analyze statistically through powerful parametric testing.

However, this paper outlines a method to overcome many of the statistical assumption violations associated with parametric testing of these data types. The technique is summarized by these two important points.

- 1) Extreme events are often analyzed by means of truncation at some fixed threshold. These data can often be considered as counting processes in which the arrival rates follow a Poisson distribution. Hence, the arrival data follow a parametric distribution, with the most important parameter  $\lambda$  representing the mean and variance per unit time interval.
- 2) Once data are Poisson permissible, the interarrival times (time between events) are known to follow the exponential distribution. As a result, when these interarrival times are clustered into groups of two or more through summation, an approximately gamma-distributed waiting-time distribution is created. These data are transformed by converting the raw values into natural logarithms ( $\ln$ ) so that they will more nearly conform to the normal distribution, which allows for statistical tests associated with normal distributions. Furthermore, analysis of interarrival times eliminates zeroes in a dataset of rarely occurring events.

The advantage to using parametric methods is that the tests are generally more powerful and efficient. Therefore, this technique may prove useful in studying changing patterns of extreme events in a changing climate to address this important manifestation of global change. The example provided herein examined temporal trends in nor'easters and heavy rainfalls but can be applied to the temporal and spatial frequencies of other extreme (or any discrete random) events as well.

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