Destabilization of the Thermohaline Circulation by Atmospheric Transports: An Analytic Solution

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(Manuscript received 28 October 1996, in final form 14 July 1997)

ABSTRACT

The four-box coupled atmosphere–ocean model of Marotzke is solved analytically, by introducing the approximation that the effect of oceanic heat advection on ocean temperatures is small (but not negligible) compared to the effect of surface heat fluxes. The solutions are written in a form that displays how the stability of the thermohaline circulation depends on the relationship between atmospheric meridional transports of heat and moisture and the meridional temperature gradient. In the model, these relationships are assumed to be power laws with different exponents allowed for the dependence of the transports of heat and moisture on the gradient. The approximate analytic solutions are in good agreement with Marotzke's exact numerical solutions, but show more generally how the destabilization of the thermohaline circulation depends on the sensitivity of the atmospheric transports to the meridional temperature gradient. The solutions are also used to calculate how the stability of the thermohaline circulation is changed if model errors are ''corrected'' by using conventional flux adjustments. Errors like those common in GCMs destabilize the model's thermohaline circulation, even if conventional flux adjustments are used. However, the resulting errors in the magnitude of the critical perturbations necessary to destabilize the thermohaline circulation can be corrected by modifying transport efficiencies instead.

1. Introduction

The question of the stability of the North Atlantic thermohaline circulation (THC) has attracted considerable interest in recent years, in part because of paleoclimatic evidence that it has been quite variable in the past (e.g., Sarnthein et al. 1994) and in part because projections of global warming show it slowing down or even collapsing (Manabe and Stouffer 1994). However, our understanding of the processes that control the THC is still very primitive, as evidenced by the fact that no coupled atmosphere–ocean general circulation model has yet been able to simulate the THC realistically, unless ''flux adjustments'' are used (Gates et al. 1996). Consequently it is still very useful to use simple process models to try to understand which processes are important in controlling the THC, and how their representations affect the sensitivity of the THC to climate changes.

This note represents the latest in a series of studies using simple box models of the atmosphere and the ocean to examine how the behavior of the THC is affected by coupling the atmosphere and ocean's budgets of heat and moisture (Nakamura et al. 1994; Marotzke and Stone 1995; Marotzke 1996; Ahmad et al. 1998). These studies indicated that the THC was destabilized by the feedbacks associated with the meridional transports of heat and moisture in the atmosphere, and this result has now been verified by experiments using atmospheric general circulation models (Saravanen and McWilliams 1995; Schiller et al. 1997). Here we focus on how the stability of the THC depends on the relationship between the atmospheric meridional transports of heat and moisture and the meridional temperature gradient. The box model results show that the stability does depend on this relationship, and can be sensitive to it (Nakamura et al. 1994; Marotzke 1996). However, theoretical, modeling, and empirical studies give ambiguous answers for what this relationship should be. Specifically, theoretical and modeling studies of heat transports by baroclinic eddies, which are the dominant meridional transport mechanism in the atmosphere (Oort and Peixoto 1983), indicate that the heat transport should be proportional to the meridional gradient raised to some power, but the appropriate power may be anywhere from two to five (e.g., Held 1978; Zhou and Stone 1993; Held and Larichev 1996); whereas an empirical analysis, based on seasonal changes, yields an exponent that ranges from about 1.7 in midlatitudes to about 3.4 in the subtropics (Stone and Miller 1980).

The model that we use here is identical to the model of Marotzke (1996), which is a simplification of the Nakamura et al. (1994) model. The main novelty of the

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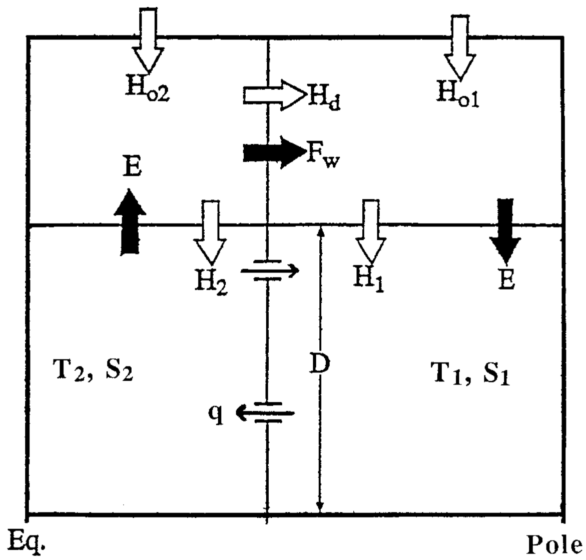

FIG. 1. Schematic diagram of the four-box model.

present study is that it solves the Marotzke model analytically, whereas Marotzke relied on numerical solutions. Thus many of the results are not new, but the analytic solutions do display the results in more general form, and the analytic solutions do, we believe, have considerable pedagogical value. Also we calculate how the stability of the thermohaline circulation is changed when model errors are ''corrected'' by using conventional flux adjustments, and what kind of corrections would be needed to preserve the model's stability characteristics. Section 2 describes the model; section 3 describes and implements the approximations that make the model equations tractable; section 4 solves the equations and analyzes their stability properties; section 5 explores the effects of ''flux adjustments'' on the model's sensitivity; and section 6 summarizes our results.

2. Model description

The ocean–atmosphere system is represented in the model by two ocean boxes and two atmospheric boxes (see Fig. 1). Box 1 represents the high-latitude ocean and box 2 the low-latitude ocean. The ocean boxes are well mixed and have the same volume, V, and depth, D. The two atmospheric boxes are assumed to have an atmospheric surface temperature identical to the temperature of the underlying ocean box. It is assumed that the atmosphere's heat and moisture capacities are negligible compared to the ocean's.

Temperature and salinity in each ocean box are denoted by T_i and S_i, respectively ($i = 1, 2$). The conservation equations for boxes 1 and 2 are

$$\dot{T}_1 = H_1 + |q|(T_2 - T_1), \tag{1}$$

$$\dot{T}_2 = H_2 - |q|(T_2 - T_1), \tag{2}$$

$$\dot{S}_1 = -H_s + |q|(S_2 - S_1), \tag{3}$$

$$\dot{S}_2 = H_s - |q|(S_2 - S_1), \tag{4}$$

where H_1 and H_2 are the ocean's heat gain through the surface. Here H_s is the virtual surface salinity flux out of the high-latitude ocean, which is related to the surface freshwater flux into the high-latitude ocean, E, by

$$H_s = S_0\frac{E}{D}, \tag{5}$$

where S_0 is a constant reference salinity. Since moisture is conserved, H_s is also the virtual salinity flux into the low-latitude ocean. The volume flux q due to the thermohaline circulation is parameterized by

$$q = k[\alpha(T_2 - T_1) - \beta(S_2 - S_1)], \tag{6}$$

with α and β being, respectively, the thermal and haline expansion coefficients, and k is a constant parameter (Stommel 1961).

The heat fluxes at the atmosphere–ocean boundary, H_1 and H_2, are determined as the residual of the net radiative heating at the top of the atmospheric boxes, H_{01} and H_{02}, and the northward heat transport between atmospheric boxes, H_d. The salinity flux H_s is proportional to the freshwater flux F_w between boxes in the atmosphere. The meridional heat and freshwater fluxes, H_d and F_w, are derived from parameterizations based on baroclinic stability theory (Branscome 1983; Nakamura et al. 1994). Here we follow Marotzke (1996) and include only the salient feature of these parameterizations, which is their dependence on the meridional temperature gradient. We also adopt the strategy of Nakamura et al. (1994) and Marotzke (1996) and write these parameterizations in the form

$$H_d = \tilde{\chi}_n(T_2 - T_1)^n, \tag{7}$$

and

$$F_w = \tilde{\gamma}_m(T_2 - T_1)^m, \tag{8}$$

where the coefficients $\tilde{\chi}_n$ and $\tilde{\gamma}_m$ are chosen so that models with different m and n all have the same high-latitude sinking equilibrium. If this equilibrium is characterized by $\overline{T} = T_2 - T_1$, then $\tilde{\chi}_n$ and $\tilde{\gamma}_m$ for different choices of n and m are chosen so that H_d and F_w are unchanged, for example, $\tilde{\chi}_1\overline{T} = \tilde{\chi}_n\overline{T}^n$, $\tilde{\gamma}_1\overline{T} = \tilde{\gamma}_m\overline{T}^m$, etc.

We use the same parameterization of radiation at the top of the atmosphere as Nakamura et al. (1994); that is,

$$H_{01} = A_1 - BT_1, \tag{9}$$

and

$$H_{02} = A_2 - BT_2. \tag{10}$$

Here A_1 and A_2 are net incoming radiation at high and low latitudes, respectively, for a surface temperature of 0°C. Thus A_1 is negative and A_2 positive. Here BT_1 and BT_2 are longwave fluxes at high and low latitudes, respectively, caused by deviations of surface temperatures from 0°C.

Let F_i be the ocean box areas, and F_{0i} the areas of the overlying atmospheric boxes ($i = 1, 2$). We assume for simplicity that $F_1 = F_2$, $F_{01} = F_{02}$, and denote by ε the ratio,

$$\varepsilon = \frac{F_1}{F_{01}} = \frac{F_2}{F_{02}}.$$

Now using the parameterizations given by Eqs. (7), (9), and (10), we can calculate the surface fluxes H_1 and H_2 as residuals of the atmospheric heat balance:

$$H_1 = \frac{1}{\varepsilon c\rho_0 D}[A_1 - BT_1 + \chi_n(T_2 - T_1)^n], \quad (11)$$

and

$$H_2 = \frac{1}{\varepsilon c\rho_0 D}[A_2 - BT_2 - \chi_n(T_2 - T_1)^n], \quad (12)$$

where $\chi_n = \tilde{\chi}_n/F_{01}$ and $c\rho_0 D$ is the heat capacity of a unit water column. In calculating H_1 and H_2 we have assumed that all the heat imbalance in the atmosphere is taken up by the ocean boxes, that is, any land surfaces underlying the atmosphere are in equilibrium with zero net surface heat flux.

In calculating the surface salinity flux, H_s, we allow for moisture runoff from land to ocean. Thus the net moisture flux into the high-latitude ocean is

$$E = \frac{1}{\varepsilon_w}\frac{F_w}{F_{01}},$$

where ε_w is the ratio of the ocean area to the catchment area of the ocean basin ($\varepsilon \le \varepsilon_w \le 1$). The virtual salinity flux out of the high-latitude ocean (and also into the low-latitude ocean, because of moisture conservation) is then

$$H_s = \frac{1}{\varepsilon_w}\frac{S_0}{F_{01}D}F_w. \quad (13)$$

Substituting from Eq. (8) we obtain

$$H_s = \frac{S_0}{\varepsilon_w D}\overline{\gamma}_m(T_2 - T_1)^m, \quad (14)$$

where $\overline{\gamma}_m = \tilde{\gamma}_m/F_{01}$.

From Eqs. (1) to (4), using Eqs. (11), (12), and (14), we obtain the following model equations:

$$\dot{T}_1 = \lambda(T_{01} - T_1) + \mu_n(T_2 - T_1)^n + |q|(T_2 - T_1), \quad (15)$$

$$\dot{T}_2 = \lambda(T_{02} - T_2) - \mu_n(T_2 - T_1)^n - |q|(T_2 - T_1), \quad (16)$$

$$\dot{S}_1 = -\gamma_m(T_2 - T_1)^m + |q|(S_2 - S_1), \quad (17)$$

$$\dot{S}_2 = \gamma_m(T_2 - T_1)^m - |q|(S_2 - S_1), \quad (18)$$

where $\lambda = B/\varepsilon c\rho_0 D$, $\mu_n = \chi_n/\varepsilon c\rho_0 D$, $T_{01} = A_1/B$, $T_{02} = A_2/B$, and $\gamma_m = S_0\overline{\gamma}_m/\varepsilon_w D$. Here q is determined by Eq. (6).

These equations can be rewritten in the following way: an equation for the ‘‘global’’ mean temperature $T_m = (T_1 + T_2)/2$,

$$\dot{T}_m = \lambda\left(\frac{T_{01} + T_{02}}{2}\right) - \lambda T_m, \quad (19)$$

and two equations for the temperature and salinity differences, $T = T_2 - T_1$, $S = S_2 - S_1$,

$$\dot{T} = \lambda(T_{02} - T_{01} - T) - 2\mu_n T^n - 2|q|T, \quad (20)$$

$$\dot{S} = 2\gamma_m T^m - 2|q|S, \quad (21)$$

where

$$q = k(\alpha T - \beta S). \quad (22)$$

Since $S_2 + S_1 = \text{const}$, the system of Eqs. (15)–(18) is equivalent to the system of Eqs. (19)–(21). Note that Eq. (19) is decoupled from Eqs. (20) and (21). Therefore the stability of the model can be deduced from Eqs. (20)–(22).

3. Simplification of the model equations

To analyze the properties of the model described above we take advantage of the property noted by Marotzke and Stone (1995)—that is, that the timescales of the temperature and salinity are essentially different. This difference of timescales makes it possible to introduce a small parameter and to use an expansion of the solution in powers of this parameter.

We rewrite Eqs. (20) and (21) in nondimensional variables. We define

$$x = \frac{T - \overline{T}}{\overline{T}}, \qquad y = \frac{\beta S}{\alpha\overline{T}}, \quad \text{and} \quad t' = k\alpha\overline{T}t, \quad (23)$$

where $\overline{T}$ is now explicitly defined as the solution of Eqs. (20)–(22) corresponding to the high-latitude sinking equilibrium. Because of our definitions of $\tilde{\chi}_n$ and $\tilde{\gamma}_m$, $\overline{T}$ does not depend on n or m. Thus $\overline{T}$ satisfies a cubic equation, but we do not need an explicit expression for that solution; by definition in this equilibrium $x = \overline{x} = 0$. The corresponding solution for the salinity of the high-latitude sinking equilibrium can be written nondimensionally as

$$y_1 = \overline{y} = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4h}, \quad (24)$$

where

$$h = \frac{\beta\gamma_1}{\alpha^2 k\overline{T}}. \quad (25)$$

If we now rewrite Eq. (20) in terms of the dimensionless variables (23), substituting for q from Eq. (22), we find

$$\dot{x} = -\frac{\lambda}{k\alpha\overline{T}}x - \frac{2\mu_1}{k\alpha\overline{T}}[(1+x)^n - 1]$$
$$-2[(1+x)|1+x-y| + \overline{y} - 1], \quad (26)$$

while Eq. (21) becomes

$$\dot{y} = 2h(1+x)^m - 2|1+x-y|y. \quad (27)$$

We anticipate that $x \ll 1$ for all the solutions of Eqs. (26) and (27), so long as the temperature solution is affected only weakly by the heat advection by the thermohaline circulation. Thus we expand Eq. (26) in powers of x, using the perturbation method, and find that the leading term for the equilibrium solution is

$$\overline{x} \cong -2\delta\begin{Bmatrix} \dfrac{\overline{y} - y}{1 + 2\delta(2 - y)}, & \text{if } y < 1 \\ \dfrac{y + \overline{y} - 2}{1 + 2\delta(y - 2)}, & \text{if } y > 1 \end{Bmatrix}, \quad (28)$$

where

$$\delta = \frac{k\alpha\overline{T}}{\lambda + 2\mu_1 n}. \quad (29)$$

We note that $t_a = (k\alpha\overline{T})^{-1}$ is the advective timescale associated with the thermohaline circulation, and $t_r = (\lambda + 2\mu_1 n)^{-1}$ is the temperature relaxation timescale associated with atmospheric processes. To make numerical estimates we will adopt the same input parameter values as used by Marotzke (1996) in his case 1. Thus we adopt $k = 2 \times 10^{-8}$ s^{-1}, $\alpha = 1.8 \times 10^{-4}$ K^{-1}, $\overline{T} = 23.8$°C (the exact equilibrium solution with high-latitude sinking as calculated numerically by Marotzke 1996), $\lambda = 1.7 \times 10^{-10}$ s^{-1}, and $\mu_1 = 1.3 \times 10^{-10}$ s^{-1}. If we also assume $n \geq 2$, then we calculate $t_a = 370$ yr, $t_r \leq 46$ yr, and $\delta = t_r/t_a \leq 0.12$. Thus the solutions given by Eq. (28) are indeed such that $x \ll 1$. Since $\delta \ll 1$ we can further simplify Eq. (28) to

$$\overline{x} \cong -2\delta(|1 - y| + \overline{y} - 1). \quad (30)$$

If we now approximate the right-hand side of Eq. (26) by just retaining terms up to order δ when $\delta \ll 1$, it becomes

$$\dot{x} = -\frac{x}{\delta} - 2(|1 - y| + \overline{y} - 1). \quad (31)$$

To estimate h, we use the same parameter values as above, and also $\beta = 0.8 \times 10^{-3}$ (psu)$^{-1}$ (Marotzke 1996). We then calculate $h = 0.20$. Now comparing Eq. (31) with Eq. (27), keeping our estimate of h in mind, we see that the evolution of the temperature field is much more rapid than the evolution of the salinity field. This is because oceanic temperature anomalies are damped by surface heat fluxes much more strongly than oceanic salinity anomalies are damped by surface moisture fluxes. Furthermore we see that perturbations in x are rapidly damped to the quasi-equilibrium values given by Eq. (30) with a timescale of order δ. Consequently we can determine the stability of the system by substituting these quasi-equilibrium solutions into Eq. (27) and examining the (slow) evolution of the salinity field. The resulting equation for the salinity, retaining terms up to order δ, is

$$\dot{y} = 2\{h[1 - 2m\delta(|1 - y| + \overline{y} - 1)]$$
$$- |(1 - y)y - 2\delta|1 - y|y - 2\delta(\overline{y} - 1)y|\}. \quad (32)$$

4. Analytic solution and stability properties

The right-hand side of Eq. (32) is quadratic in y, but with coefficients when $y < 1$ that differ from those when $y > 1$. Thus there are potentially four steady solutions for y, but one is spurious. We designate the three remaining equilibrium solutions by $y = y_i$, $i = 1, 2, 3$. Two are thermally dominated with $y < 1$ and $q > 0$. The first is of course $y_1 = \overline{y}$, and the second, to order δ, is

$$y_2 = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4h} + \delta(1 + \sqrt{1 - 4h} - 2mh). \quad (33)$$

Note that $y_1 < y_2 < 1$. Also, if $h > 1/4$—that is, the salinity flux is sufficiently large—then the solutions y_1 and y_2 are no longer real—that is, no longer physically realizable. The third solution is salinity dominated, with $y > 1$ and $q < 0$. This solution is

$$y_3 = \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4h} + 0(\delta). \quad (34)$$

In terms of these solutions, Eq. (32) can be written as

$$\dot{y} = 2\varphi_\delta(y)$$
$$\equiv 2\begin{Bmatrix} (1 - 2\delta)(y - y_1)(y - y_2), & \text{if } y < 1 \\ (1 + 2\delta)(y_3 - y)(y_4 + y), & \text{if } y > 1 \end{Bmatrix}, \quad (35)$$

where

$$y_4 = \frac{1}{2}[\sqrt{1 + 4h} - 1] + 0(\delta). \quad (36)$$

Note that $y_4 > 0$ and $y_3 > 1$.

By inspection of Eq. (35), we see that

$$\dot{y} > 0 \quad \text{if } y < y_1,$$
$$\dot{y} < 0 \quad \text{if } y_1 < y < y_2,$$
$$\dot{y} > 0 \quad \text{if } y_2 < y < y_3,$$

and

$$\dot{y} < 0 \quad \text{if } y_3 < y.$$

Figure 2 shows a typical plot of $\dot{y}$ as a function of y. The tendencies are such that, if $y(0) < y_2$, then $y(t) \to y_1$, as $t \to \infty$, and if $y(0) > y_2$, then $y(t) \to y_3$ as $t \to \infty$. Thus the solutions y_1 and y_3 are stable, and the solution y_2 is unstable. Furthermore, if the system starts in the state y_1, which is the analog of the thermally

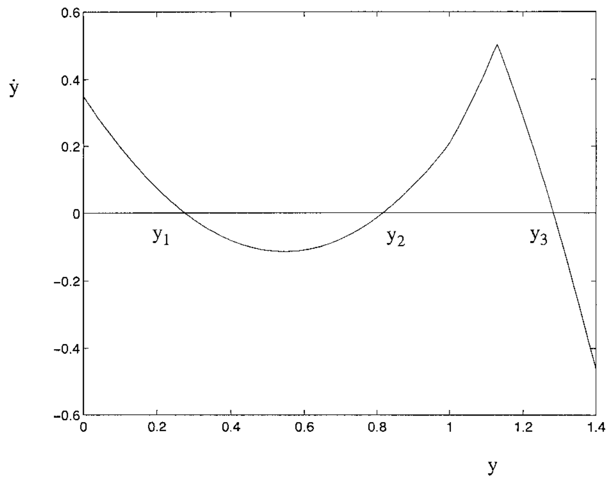

FIG. 2. Salinity tendency $\dot{y}$ versus the nondimensional salinity y for the parameter values given in the text, and $m = n = 2$. The three equilibrium solutions are indicated by y_i, $i = 1, 2, 3$.

dominated current state of the North Atlantic thermohaline circulation, then the minimum perturbation necessary to shift the system to the alternate stable equilibrium state is $\Delta y = y_2 - y_1$. This can be expressed as a minimum perturbation in salinity, since y is the nondimensional salinity parameter [see Eq. (23)]. Substituting for y_1 and y_2 from Eqs. (24) and (33) we find for this critical salinity perturbation, to order δ,

$$\Delta S_c = \frac{\alpha\overline{T}}{\beta}[\sqrt{1 - 4h} + \delta(1 + \sqrt{1 - 4h} - 2mh)]. \tag{37}$$

We can also derive the stability condition when the salinity *flux* is perturbed. We assume that the system is initially in the stable state with $y = y_1$ and the salinity flux H_s is changed to $H_s + \Delta H_s$, where $\Delta H_s =$ constant. If ΔH_s is sufficiently large, then the states y_1 and y_2 no longer exist, and the system must shift to the state y_3. What is the critical value of ΔH_s that is the minimum necessary to shift the system to this alternate stable state? Now Eq. (21) must be rewritten in the form

$$\dot{S} = 2\gamma_m T^m + 2\Delta H_s - 2|q|S, \tag{38}$$

and instead of Eq. (35) we obtain to $0(\delta)$

$$\dot{y} = 2\varphi_\Delta(y), \tag{39}$$

where

$$\varphi_\Delta(y) = \varphi_\delta(y) + h\Delta h, \tag{40}$$

and

$$\Delta h = \frac{\Delta H_s}{\gamma_1\overline{T}}. \tag{41}$$

Here Δh is the nondimensional salinity flux perturbation.

The model behavior is now determined by the roots of the equation $\varphi_\Delta(y) = 0$. If this equation no longer has real roots with $y < 1$, then only the third equilibrium state exists, and the system passes to the new equilibrium with the opposite circulation direction. Let the minimum perturbation magnitude necessary to shift the system to this equilibrium have the critical value $\Delta H_{s,c}$. It can be calculated from the condition that the discriminant of the equation $\varphi_\Delta(y) = 0$ with $y < 1$ be zero. The result is

$$\Delta H_{s,c} = \frac{k(\alpha\overline{T}^2)}{4\beta}[1 - 4h + 2\delta\sqrt{1 - 4h}(1 - 2mh)]. \tag{42}$$

Our analytic results for how large a perturbation is necessary to make the coupled system shift out of the high-latitude sinking equilibrium, Eqs. (37) and (42), allow us to deduce Marotzke's results without numerical calculations, and also allow us to extend his results to any values of n and m. Since h and $\overline{T}$ are independent of n and m, these equations show that the atmospheric feedbacks come into play only to order δ,—that is, when the meridional temperature gradient is affected by the thermohaline circulation. If the system is far from the bifurcation—that is, $h \ll 1/4$—then these feedbacks have only a small effect on the stability of the system, so long as $\delta \ll 1$. However if the system is near the bifurcation—that is, $h \cong 1/4$—then these feedbacks play a very important role in determining the stability of the system. (We note that our perturbation expansion in δ is not singular in the limit $h \to 1/4$, because the equilibrium solutions for y are still of order unity in these limits.)

The dependence of the critical perturbation on m is explicitly shown in Eqs. (37) and (42), and the dependence on n comes in implicitly because δ is a function of n [see Eq. (29)]. The dependences are illustrated in Fig. 3. The difference between the curves for $m = 0$ and $m = n$ in Fig. 3 illustrates the destabilizing effect of the feedback in the moisture flux—that is, of the EMT (eddy moisture transport) feedback, as it was called in Nakamura et al. (1994). This feedback works as follows: decreasing the strength of the thermohaline circulation and its poleward heat transport causes the meridional temperature gradient and the atmospheric moisture flux to increase; this freshens the high-latitude waters and further weakens the thermohaline circulation—that is, the feedback is positive. Increasing m destabilizes the system, as illustrated in Fig. 3, because it enhances the response of the moisture transport to changes in the meridional temperature gradient. The sign of this feedback is given by the sign of the term in Eq. (32), which is proportional to both m and y. For thermally dominated states, $y < 1$, it is positive, as illustrated in Fig. 3. However, for salinity-dominated states, $y > 1$, the sign of this term changes, and the EMT feedback would be negative.

The exponent n measures the strength of the negative feedback between the atmospheric heat transport and

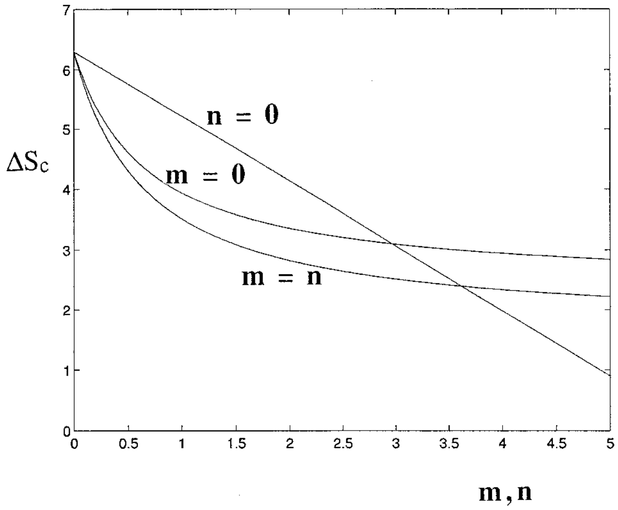

FIG. 3. The minimum perturbation in the salinity gradient, ΔS_c, necessary to shift the system from the high-latitude sinking state to the low-latitude sinking state. Here ΔS_c is shown versus n for $m = 0$ and $m = n$; and versus m for $n = 0$. The typical values of the parameters given in the text were used to calculate the curves.

the meridional temperature gradient. This negative feedback inhibits changes in the meridional temperature gradient, and thus affects the stability of the thermohaline circulation in two ways (Marotzke 1996.) First, it weakens the negative feedback between the strength of the THC and its poleward advection of heat; thus this effect destabilizes the THC. Second, it weakens the positive EMT feedback; thus this effect stabilizes the THC. The first effect dominates when m is small—that is, when the EMT feedback is weak—but the second dominates when m is large—that is, when the EMT feedback is strong. This is illustrated by comparing the curves for $n = 0$ and $n = m$ in Fig. 3. Increasing n destabilizes the system if m is small, but increasing n stabilizes it if m is large.

Qualitatively these results are the same as those found in Marotzke (1996). However our results allow us to calculate the critical values of m, which separate the two kinds of behavior as n increases. Inspection of Eqs. (37) and (42) shows that, with respect to salinity perturbations, the critical value, m_s, is given by

$$m_s = \frac{1 + \sqrt{1 - 4h}}{2h}; \qquad (43)$$

and with respect to salinity flux perturbations the critical value m_f is given by

$$m_f = \frac{1}{2h}. \qquad (44)$$

If the system is near the bifurcation, $h \cong 1/4$, then both of these critical values are close to 2. If $h < 1/4$, then m_s and $m_f > 2$; for example, if $h = 0.2$ as estimated above, then $m_s = 3.6$ and $m_f = 2.5$.

In the parameterizations used by Nakamura et al. (1994), m and n are necessarily equal because of the approximations used. In particular, the key approximations are 1) that fluctuations in relative humidity are small compared to fluctuations in specific humidity; and 2) that fluctuations in absolute temperature are small. Analyses of observations support these approximations (Gutowski et al. 1995; Stone 1984). With these approximations, the fluctuations in specific humidity are proportional to the fluctuations in temperature and the eddy flux of moisture is proportional to the eddy flux of sensible heat (Leovy 1973); thus, $m = n$.

If we indeed assume $m = n$, then substituting Eq. (29) into Eq. (37) gives us

$$\Delta S_c = \frac{\alpha\overline{T}}{\beta}\left[\sqrt{1-4h} + \frac{k\alpha\overline{T}}{\lambda + 2\mu_1 n}(1 + \sqrt{1-4h} - 2nh)\right]. \qquad (37')$$

This shows that increasing n *always* destabilizes the system when $m = n$. This result is also illustrated in Fig. 3, for typical values of the parameters. Note that the error in our asymptotic solution is largest when δ is largest—that is, when $n \to 0$—and smallest when δ is smallest—that is, when $n \to \infty$. Comparing the results for $m = n$ in Fig. 3 with Marotzke's numerical solutions (note that our definition of ΔS_c is twice Marotzke's), we calculate that the error in our asymptotic solution is $\leq 7\%$ for $n \geq 2$, the range of values suggested by the studies cited in the introduction. For $n = 0$, the error becomes as large as 30%. In all cases our asymptotic solution overestimates the exact result. Similarly, when $m = n$, one can also show that increasing n always destabilizes the system with respect to a salinity *flux* perturbation.

5. Effect of flux adjustments

Finally, we can use our solution to investigate the influence of using ''flux adjustments'' in a model with errors. We will follow the strategy of Nakamura et al. (1994) but instead of their numerical approach we are again able to obtain our result in analytical form. The problem of flux adjustment in our case can be posed as follows. Consider a model that differs from the model described above in that the efficiency of the meridional transports are incorrect. Now these transports, q', H'_d, and F'_w (instead of q, H_d, and F_w), are written in the form:

$$q' = k'[\alpha(T_2 - T_1) - \beta(S_1 - S_2)], \qquad (6')$$

$$H'_d = \tilde{\chi}'_n(T_2 - T_1)^n + H^0_d, \qquad (7')$$

and

$$F'_w = \tilde{\gamma}'_m(T_2 - T_1)^m + F^0_w, \qquad (8')$$

where $k' \neq k$, $\tilde{\chi}'_n \neq \tilde{\chi}_n$, and $\tilde{\gamma}'_m \neq \tilde{\gamma}_m$; and H^0_d and F^0_w are constants chosen so that the temperature and salinity gradients in equilibrium in the model with errors are the same as in the model without errors—that is, the same as in ''observations.'' Thus the model that uses Eqs. (6′), (7′), and (8′) instead of Eqs. (6), (7), and (8), which we call the model with flux adjustment (Sausen et al. 1988), has the same equilibrium state (corresponding to the current climate) as the original model.

Using the same approach as before, we can derive the equation for the nondimensional salinity of the model with flux adjustment. Instead of Eq. (32), we obtain

$$\dot{y} = 2\begin{Bmatrix} h' + y^2 - y - 2\delta'(y - \overline{y}) \\ \quad \times (y - mh') + a, \quad \text{if } y < 1 \\ h' - y^2 + y - 2\delta'(\overline{y} + y - 2) \\ \quad \times (-y + mh') + a, \quad \text{if } y > 1 \end{Bmatrix}. \quad (32')$$

Here $a \equiv h - h' = \beta S_0 F_w^0 / k' \alpha^2 \overline{T}^2 \varepsilon_w F_{01} D$, where h' and δ' are determined by Eqs. (25) and (29) with μ_1, and γ_1 is replaced by μ_1' and γ_1', respectively, where $\mu_1' = \tilde{\chi}_1' / \varepsilon c \rho_0 D F_{01}$, and $\gamma_1' = S_0 \tilde{\gamma}_1' / \varepsilon_w D F_{01}$, and k is replaced by k'. We have assumed that $\delta' \ll 1$ as well as $\delta \ll 1$.

Critical perturbations for the model with flux adjustments, $\Delta S_c'$, can be found again as the distance between the roots y_1' and y_2' of the equation $\dot{y} = 0$ with $y < 1$. Taking into account that Eqs. (32) and (32′) have the same root, $y_1' = y_1 = \overline{y}$, since H_d^0 and F_w^0 are chosen to guarantee this, we find for the difference between the critical perturbations of the model with flux adjustment and the original model:

$$\Delta S_c' - \Delta S_c = \frac{\alpha \overline{T}}{\beta}[(\delta' - \delta)(1 + \sqrt{1 - 4h}) - 2m(\delta' h' - \delta h)]. \quad (45)$$

We note that the error in the flux-adjusted model's stability is of order δ. Thus the error is small if the system is far from the bifurcation at $h = ¼$ [cf. Eq. (37)]. However if the system is near the bifurcation, the error is potentially large—that is, of order ΔS_c itself.

If the only model ''error'' is in the atmospheric moisture flux, then $\delta' = \delta$, $h' \neq h$, and Eq. (45) shows that the model with flux adjustment is more (less) stable than the original model if the moisture flux—that is, h (or correspondingly the EMT feedback)—is underestimated (overestimated), as noted by Nakamura et al. (1994). If the only model error is in the heat flux—that is, $\delta' \neq \delta$, but $h' = h$, the effect on the stability is more complicated, because of the two competing effects of the atmospheric heat flux noted in the previous section. An overestimate of the heat flux—that is, an underestimate of δ'—reduces the stability if $1 + \sqrt{1 - 4h} > 2mh$ but increases the stability if $1 + \sqrt{1 - 4h} < 2mh$. The latter case corresponds to Nakamura et al.'s (1994) numerical calculations. The critical value of m, which separates these two behaviors, is the same as that given by Eq. (43). Last, if the only model error is in the efficiency of the oceanic transports, then $\delta' h' = \delta h$, but $\delta' \neq \delta$, and an overestimate (underestimate) of the ocean transports—that is, an overestimate (underestimate) of δ—increases (reduces) the stability.

The above result [Eq. (45)] makes it clear that the conventional additive flux adjustment does not correct the stability of a model that has errors in its simulation of the atmospheric and oceanic meridional transports or of their efficiencies. This is a common problem in GCMs: atmospheric GCMs tend to overestimate heat transports (Stone and Risbey 1990; Gleckler et al. 1995) and oceanic GCMs tend to underestimate heat transports (Manabe and Stouffer 1988; Maier-Reimer et al. 1993). According to our results both kinds of errors tend to destabilize the thermohaline circulation, even when conventional flux adjustments are used.

By contrast, inspection of Eqs. (37) and (42) suggests a different kind of adjustment that would at least preserve the magnitude of the critical perturbations if there are errors in the transports or, equivalently, in their efficiencies. These efficiencies affect only the two nondimensional parameters δ and h. Thus an adjustment that preserved the correct values of δ and h would preserve the magnitude of the critical perturbations. This can be accomplished by adjusting transport *efficiencies*. For example, if the ocean transport, or, equivalantly, its efficiency, k, is underestimated, then an appropriate reduction of the efficiencies of the atmospheric heat and moisture transports, γ_1 and μ_1, would preserve the values of δ and h [see Eqs. (25) and (29)], and thus preserve the size of the critical perturbations. Such adjustments would be less rigorous than those suggested by Marotzke and Stone (1995), which were designed to restore the full coupled equations to their correct form. However adjusting the transport efficiencies is simpler, and the critical perturbations for destabilizing the thermohaline circulation are perhaps the most important feature that one would like coupled atmosphere–ocean models to simulate correctly.

6. Summary

We have presented an analytical solution for a simple coupled ocean–atmosphere model, and used it to investigate how the thermohaline circulation's sensitivity depends on different parameterizations of atmospheric moisture and heat transports. A detailed discussion and numerical solution of this problem has already been given by Marotzke (1996). However the analytical approach permits us to display the results in more general form and to understand better the relationship between the model behavior and its parameters.

Our analytic results show how large a perturbation is necessary to make the coupled system shift out of the equilibrium, which corresponds to the current state of the North Atlantic circulation. In particular, we related these critical perturbations to the exponents describing the sensitivity of the atmosphere's meridional fluxes of heat and moisture to the meridional temperature gradient. The results show that the atmospheric feedbacks have only a small effect on the stability of the system if the system is very stable, but these feedbacks play a very important role if the system is only weakly stable.

Overestimating (underestimating) the sensitivity of the atmospheric moisture transport to the meridional temperature gradient always destabilizes (stabilizes) the system, because of the positive EMT feedback (Nakamura et al. 1994.) However this is true only as long as the system is in the thermally dominated state corresponding to the current state of the North Atlantic circulation. For a salinity-dominated circulation, the EMT feedback would be negative.

Errors in estimates of the sensitivity of the atmospheric heat transport have two competing effects, and therefore can either stabilize or destabilize the system. Furthermore, which effect dominates depends on the strength of the EMT feedback. If the sensitivity of the heat and moisture transports are the same—that is, if the exponents in the parameterizations of the heat and moisture transports are equal, which we would argue is realistic, then overestimating (underestimating) the sensitivities always destabilizes (stabilizes) the system.

A model that uses flux adjustments to correct for errors in model parameters will generally have an error in the size of its critical pertubations. The analytical relationship between the critical perturbations of the flux-adjusted model and its parameters shows that these errors as before have only a small effect on the model stability if the model is very stable, but the effect becomes large if the model is weakly stable. Errors in the atmospheric transports or their efficiencies have the same effect on the stability of the system as errors in the sensitivities of the transport parameterizations described above. In addition, overestimating (underestimating) the ocean transport always stabilizes (destabilizes) the model. In particular, errors like those common in GCMs destabilize the model's THC, even if conventional flux adjustments are used. By contrast, adjustments in some of the model's transport efficiencies can compensate for errors in other efficiencies, and thereby preserve the magnitude of the critical perturbations necessary to destabilize the thermohaline circulation.

Acknowledgments. We are indebted to Jochem Marotzke for many useful discussions. Seventy percent ($17,000) of this research was funded jointly by the U.S. Deptartment of Energy's (DoE) National Institute of Global Environmental Change (NIGEC) through the NIGEC Northeast Regional Center at Harvard University (DoE Cooperative Agreement DE-FC03-90ER61010) and by the DoE's Program for Computer Hardware, Applied Mathematics, and Model Physics. Financial support was also contributed by the Tokyo Electric Power Company (TEPCO) through the TEPCO/MIT Environmental Research Program. Financial support does not constitute an endorsement by DoE or TEPCO of the views expressed in this article.

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