Climate Simulations of African Easterly Waves

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ABSTRACT

The three-dimensional structure and propagation characteristics of African easterly waves for the 1986–94 period are studied with June–August European Centre for Medium-Range Weather Forecasts (ECMWF) operational analyses, National Centers for Environmental Prediction (NCEP) reanalyses, and Canadian Centre for Climate Modelling and Analysis GCM output. Specific consideration is given to the differences in the time-mean circulation, synoptic-scale variance, covariance, and principal oscillation patterns. The African easterly waves derived from the ECMWF and NCEP analyses are very similar, with both providing a reasonably realistic depiction of African easterly waves—given the agreement with one another, earlier station data studies, and theory. Where significant differences exist between the results from the two data assimilation systems they expectably do so over the tropical Atlantic, and in fields that are not directly observed (such as vertical velocity).

The situation with the GCM is not as favorable but there are some encouraging areas of agreement—despite the GCM’s relatively coarse resolution and absence of observed data to constrain it. Selected points of agreement and disagreement between the GCM and the analyses include the following. (i) The GCM African easterly wave energetics are comparable with the analyses in terms of the sign and magnitude of the energy transfers from the time-mean state to the waves. (ii) The northern track of easterly waves seen in the analyses terminates prematurely in the GCM at the African coast. (iii) The southerly track of moist easterly waves seen in the analyses near the seasonal rainband is absent in the GCM. Possible reasons for the deficiencies are discussed.

The sensitivity of the GCM-simulated African easterly waves to CO$_2$ doubling is investigated. Together with significant mean warming and moistening over the northern Sahara, the level of simulated African easterly wave activity increases with CO$_2$ doubling.

1. Introduction

Studies of summertime synoptic-scale disturbances in the Africa–Atlantic sector date back to at least the 1930s when using station data, Piersig (1944) and Regula (1936) identified westward-propagating cyclonic features with periods of a few days and wavelengths of several thousands of kilometers. From those early days and up to and including the 1974 Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment (GATE) much was learned about these so-called African easterly waves. In fact, from the studies of Carlson (1969a, 1969b), Burpee (1972, 1974), Reed et al. (1977), and Norquist et al. (1977), to name a few, the following picture of the quintessential African easterly wave emerges:

- Eastward-tilting lower-tropospheric synoptic-scale feature originating at 15°–30°E, which propagates westward from Africa into the Atlantic along two east–west-oriented tracks located at about 15°–20°N and 5°–10°N. Cyclonic vortices that compose by the northern track typify desert thermal lows with warm, dry air overlying the surface troughs while their southern counterparts (near the seasonal mean rainfall maximum) are associated with moist convection. Strongly implicated in the origin of these disturbances is the African easterly jet (which is a time-mean easterly zonal jet located along about 15°N at approximately 600 mb), while latent heat release in organized convection has been suggested (e.g., Kwon and Mak 1990) to maintain the disturbances beyond their region of generation.

Station data and observations taken within limited-area networks during intensive field experiments such as GATE have been extremely valuable but are not the only sources of information available to researchers. Global datasets, analyzed and archived at the European Centre for Medium-Range Weather Forecasts (ECMWF) have since been used to add to the understanding of African easterly waves—as, for example, by Reed et al. (1988) for August–September 1985 analyses or Lau and Lau (1990) for June–August 1980–

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87 analyses. On the modeling side, simple numerical models (e.g., Rennick 1976; Simmons 1977; Mass 1979; Thorncroft and Hoskins 1994; Thorncroft 1994, 1995) have helped elucidate the origin and maintenance of African easterly waves. GCM studies include Miyakoda and Sirutis (1977) using the Geophysical Fluid Dynamics Laboratory GCM, Esteque et al. (1983) using the National Aeronautics and Space Administration (NASA) Goddard Laboratory for Atmospheric Sciences GCM, and Druyan and Hall (1994) using the NASA/Goddard Institute for Space Studies GCM.

The first goal of this study is to provide a systematic comparison of the African easterly waves in two major data assimilation systems, namely, those of the ECMWF and the National Centers for Environmental Prediction (NCEP, formerly known as the National Meteorological Center). The results from Reed et al. (1988, hereafter RKH) and Lau and Lau (1990, hereafter LLA) suggest that the ECMWF analysis–forecast system provides a reasonably realistic description of African easterly waves. However, without detailed comparison with other data assimilation systems there remain open questions regarding the influence of spurious forecast model dependent features, especially so given (i) the paucity of observed data in the region and (ii) a data assimilation system that has continually and quite dramatically changed over the years. The NCEP reanalyses are particularly well suited for such a comparison since by design the assimilation system is held fixed—thereby eliminating any perceived “jumps” in the analyses.

The second goal of this study is to characterize the African easterly waves in the Canadian Centre for Climate Modelling and Analysis (CCCma) GCM2, which is a moderate resolution atmospheric Global Climate Model (T32 with 10 vertical levels) with a sophisticated package of physical parameterizations. Atmospheric GCMs such as the CCCma GCM2 have many uses including making equilibrium climate predictions and as “drivers” for high-resolution regional climate models (RCMs). For either purpose it is essential that the phenomenon being investigated, say African easterly waves as here, be evaluated for the GCM’s present climate (if, e.g., future climate predictions are the aim) and/or at the model’s usual resolution (if high-resolution regional information is sought). To this end this study compares the CCCma GCM2 African easterly waves to those derived from the ECMWF and NCEP data assimilation systems. With the information so gained, the changes in the GCM-simulated African easterly waves under a CO2 doubling are then analyzed. Fyfe and Laprise (1998, unpublished manuscript) consider changes in the simulated African easterly waves with much increased horizontal resolution.

Section 2a describes the ECMWF and NCEP data assimilation systems, as well as the CCCma GCM2. Section 2b describes the methods of analysis. Section 3 contains the main diagnostics results in terms of the comparison of means, bandpass-filtered variances and covariances, and propagation patterns. Sections 4–6 consider various supplementary issues such as African easterly wave generation, GCM2 biases, and wave activity changes with CO2 doubling. Section 7 summarizes the main findings.

2. Datasets and methods

a. Datasets

Our starting point is an archive of June–August, twice-daily (0000 UTC and 1200 UTC) ECMWF Tropical Ocean Global Atmosphere (TOGA) analyses, NCEP reanalyses, and CCCma GCM2 output for the 1986–94 period. All fields were transferred onto 48 (lat) × 96 (long) global Gaussian grids and North Africa–Atlantic subgrids extracted (see Fig. 1). The primary data types considered in this investigation are zonal and meridional components of the horizontal velocity (u and v, respectively), vertical velocity (ω), relative vorticity (ξ), geopotential (Φ), temperature (T), and specific humidity (q)—at eight standard pressure levels (100, 200, 300, 400, 500, 700, 850, and 1000 mb).

Detailed descriptions of the ECMWF and NCEP analysis/forecasting systems can be found in ECMWF (1994) and Kalnay et al. (1996), respectively. The ECMWF assimilation system consists of a multivariate optimal interpolation analysis while the NCEP system uses a 3D variational algorithm termed “spectral statistical interpolation (SSI).” Both systems use a high-resolution numerical model that produces a first guess forecast for subsequent analyses. Many changes to the ECMWF system occurred during the 1986–94 period. For example, the ECMWF forecast model horizontal and vertical resolution changed from T106 (1.125° or 125 km) and 19 levels in 1987 to T213 (0.5625° or 60 km) and 31 levels in 1991. The NCEP reanalysis forecast model is frozen with horizontal and vertical resolution of T62 (1.875° or 210 km) and 19 levels. Both forecast models have comprehensive physical parameterization schemes—with the ECMWF package changing over
the period (as chronicled in ECMWF 1994) and the NCEP package remaining frozen. Finally, note that the ECMWF–TOGA and NCEP fields are uninitialized.

The CCCma GCM2 is a much coarser model than either of the above forecast models, with T32 horizontal resolution (3.75° or 420 km) and 10 vertical levels. Of course, no observed data is assimilated into the GCM. All spectral transforms and physical parameterizations (clouds, convection, land surface processes, etc.) are computed on a 48 lat × 96 long global Gaussian grid (hence the analysis grid chosen for this study). The model and its climate are described in full detail in McFarlane et al. (1992).

Finally, note that the GCM and the ECMWF and NCEP forecast models were forced with observed SST and sea-ice extent.

b. Methods

1) MEANS, VARIANCES, AND COVARIANCES

Let \( \xi_{ik} \) denote the \( k \)th observation \((i = 1, \ldots, I)\) of variable \( \xi \) in the \( j \)th month \((j = 1, 2, 3)\) of the \( k \)th year \((k = 1, \ldots, 9)\). The time mean and variance in the \( j \)th month of the \( k \)th year are

\[
\bar{\xi}_{j} = \frac{1}{I} \sum_{i=1}^{I} \xi_{ik} \quad \text{and} \quad \bar{\xi}^{2} = \frac{1}{(I - 1)} \sum_{i=1}^{I} (\xi_{ik} - \bar{\xi})^{2},
\]

respectively. The covariance between variables \( \xi \) and \( \chi \) in the \( j \)th month of the \( k \)th year is

\[
\bar{\xi} \chi = \frac{1}{(I - 1)} \sum_{i=1}^{I} (\xi_{ik} - \bar{\xi})(\chi_{ik} - \bar{\chi}).
\]

For the most part, but not exclusively, the focus here will be on the monthly and yearly averaged means, variances, and covariances denoted as \( \bar{\xi}, \bar{\xi}^{2}, \) and \( \bar{\xi} \chi \), respectively.

2) PROPAGATION PATTERNS

Described here is a method of obtaining propagating patterns from variable \( \xi \). As discussed in von Storch et al. (1988, 1995) principal oscillation patterns (POPs) are the eigenvectors \( \mathbf{P}_{j} \) of the matrix \( \mathbf{A} = C_{j} \cdot C_{j}^{-1} \), where \( C_{j} \) and \( C_{j}^{-1} \) denote the lag-1 and lag-0 correlation matrices of \( \xi \). Complex POPs have complex eigenvalues defining an e-folding time \( \tau \) and period \( \Gamma \). If the real and imaginary parts (\( \mathbf{P}_{j} \) and \( \mathbf{P}_{j}' \), respectively) of a complex POP are in quadrature, then it is interpreted as propagating, such that \( \ldots \rightarrow \mathbf{P}_{j} \rightarrow \mathbf{P}_{j} \rightarrow -\mathbf{P}_{j} \rightarrow -\mathbf{P}_{j} \rightarrow \mathbf{P}_{j} \rightarrow \ldots \). After one cycle taking time \( \Gamma \) a propagating POP decays by a factor of \( \exp(-\Gamma/\tau) \). All of the POPs in this study were found to be complex with real and imaginary parts in quadrature—and hence propagating.

For propagating POP, \( \mathbf{P}_{j} \) in primary variable \( \xi \) the POP signal can be described in terms of a secondary variable using the concept of an associated correlation pattern (ACP). If the secondary variable is \( \chi \) then its ACP, denoted by \( \mathbf{P}_{\chi} \), minimizes

\[
\int_{0}^{T} \left[ \chi(x, y, p, t) - \frac{z_{\chi}(t)}{\sigma_{\chi}} \mathbf{P}_{\chi}^{T} \right]^{2} \, dt,
\]

where \( z_{\chi}(t) \) and \( \sigma_{\chi} \) are the coefficients of the POP (obtained as the dot product between \( \xi \) and the adjoint vector of \( \mathbf{P}_{\chi} \)) divided by their standard deviations \( \sigma_{\xi} \) and \( \sigma_{\chi} \) (\( T \) is the time series length). The ACP evolves as \( \ldots \rightarrow \mathbf{P}_{\chi} \rightarrow \mathbf{P}_{\chi} \rightarrow -\mathbf{P}_{\chi} \rightarrow -\mathbf{P}_{\chi} \rightarrow \mathbf{P}_{\chi} \rightarrow \ldots \). In this study the primary variable (i.e., the one for which POPs are derived) is the meridional wind on the 850-mb pressure surface, \( v_{850} \). The secondary variables (i.e., the ones for which the ACPs are derived) include all those enumerated in section 2a.

3) TIME FILTERING

Figure 2 shows \( v_{850} \) power spectra at several points along \( \phi = 16.7^{\circ}N \). Most ECMWF and NCEP \( v_{850} \) power exists near the African coast around \( \lambda = 15^{\circ}W \) with a dominant period near 4 days—the period usually associated with African easterly waves. On the other hand, most GCM power is situated further inland around \( \lambda = 0^{\circ} \) where there is a relatively large peak centered near 7 days and a somewhat smaller peak near 4–5 days. It is clear that the GCM is excessively energetic over North Africa at synoptic, and longer, periods.

In this study all variances, covariances, and propagation patterns are computed on time-filtered data. The filter window \( G(f) \), that is,

\[
G(f) = \begin{cases} 
0 & \text{if } f < f_{1} \\
\frac{1}{2} \left[ 1 - \cos \left( \frac{\pi}{f_{2} - f_{1}} (f - f_{1}) \right) \right] & \text{if } f_{1} \leq f \leq f_{2} \\
1 & \text{if } f_{2} \leq f \leq f_{3} \\
\frac{1}{2} \left[ 1 + \cos \left( \frac{\pi}{f_{4} - f_{3}} (f - f_{3}) \right) \right] & \text{if } f_{3} \leq f \leq f_{4} \\
0 & \text{if } f_{4} < f 
\end{cases}
\]

(4)

is defined in terms of four parameters \( f = [f_{1}, f_{2}, f_{3}, f_{4}] \). Disturbances with periods in the 3–5-day range, normally associated with African easterly waves, are isolated in this study by setting \( f = [2, 3, 5, 8] \) days for all variance and covariance calculations (see the shaded curve in Fig. 3). The POP calculations are based on the wider filter window \( f = [2, 3, 10, 15] \) days (see
Power Spectra of $v$ at $p = 850$ mb and $\phi = 16.7^\circ$N

![Power Spectra](image)

**Fig. 2.** Power spectra of the meridional wind $v$ at $p = 850$ mb along $\phi = 16.7^\circ$N (where $f$ is the frequency and $P$ is the power). Units are in days for the abscissa and $10^{-1} \text{ m}^2 \text{s}^{-1} \text{ day}^{-1}$ for the ordinate. The period (in days) at peak $|P(f)|$ is indicated in the top left of each panel. The 4-day period is indicated with a dashed line in each panel.

**Fig. 3.** Spectral filter windows [see Eq. (4)]. The shaded curve is the filter window used for all variance and covariance computations. The solid curve is the filter window used for all propagation pattern calculations. The cosine tails are used to suppress the generation of artificial secondary maxima due to "overshooting."

1. Use a 3–5-day filter for the GCM variance and covariance excludes a significant amount of power seen to exist near but outside this range in the GCM (e.g., at 7 days).

The rationale for excluding the longer period power from the GCM variance and covariance calculations (but not the POP calculations) is based upon a detailed POP analysis, which shows two distinct modes of GCM synoptic variability—one with a period near 5 days, which is most clearly associated with African easterly waves, and another with a period near 7 days, which is most clearly not associated with African easterly waves (as will become clearer in section 3d).

3. **Main diagnostic results**

Many of the computations undertaken in this section are similar to those in RKH. RKH studied means, filtered variances, and covariances obtained from ECMWF August–September 1985 analyses. In RKH it
was concluded that based on agreement with earlier station data studies (e.g., Burpee 1972, 1974; Reed et al. 1977; Albignat and Reed 1980), the ECMWF operational analysis/forecast system gave a realistic portrayal of African easterly waves. RKH identified westward propagating synoptic disturbances that over Africa are confined to levels below 600 mb and have eastward-tilting-with-height wave axes that lean against the vertical shear of the African easterly jet (implying baroclinic growth). Below the African easterly jet core, anomalously warm dry air rises ahead (west) of wave trough axes and anomalously cool moist air sinks to the rear (east). Over the Atlantic the low-level wave structure changes drastically. As stated, all of these features were found consistent with earlier station data—an observation which led RHK to remark that "archived gridded data have much potential value for documenting wave behavior over large areas and extended time periods and for improving the diagnosis of basic physical mechanisms." Taking this remark to heart, this study proceeds to an enhanced diagnosis of ECMWF and NCEP analyses over a much longer period. In addition, diagnoses of CCCma GCM2 output for the same period are presented.

a. Means

The 850-mb time-mean velocity (Fig. 4a) is qualitatively similar in all datasets, in that each has a southwesterly monsoon flow over West Africa north of the equator and south of 20°N, a northeasterly harmattan over the Sahara, and northeasterly trade winds over the Atlantic. The GCM monsoon flow at 850 mb is stronger and the trade winds weaker than in the analyses. Both the ECMWF and NCEP latitude–pressure $\mathbf{\overline{u}}$ plots (Fig. 4b) show a near-surface westerly monsoon flow (topped at 850 mb), an African easterly jet (centered at 15°N and 600 mb), and a tropical easterly jet (centered at 5°N and 200 mb). The GCM near-surface westerly monsoon flow is too strong, too deep, and the overlying vertical wind shear too weak. Also, whereas the analyses exhibit well-defined and separated African and tropical easterly jets, the GCM has a single easterly jet structure with unrealistic speeds aloft. Given these errors in the GCM $\pi$ structure and the central role that $\pi$ plays in theories for the generation of African easterly waves, one immediately wonders about the extent to which the GCM supports such disturbances, if at all.

As an aside, it is noted that the CCCma GCM2 tendency for the easterly flow to be too pronounced in the upper levels in the Tropics was noted in McFarlane et al. (1992). This error persists, but to somewhat of a lesser extent, in CCCma GCM3, which is the current CCCma developmental atmospheric model, which features higher vertical and horizontal resolution and a better treatment of tropical deep convection. The reason for the easterly bias in CCCma GCM2 and GCM3 is unknown at this time.

Over Africa the ECMWF and NCEP 850-mb $\mathbf{\overline{T}}$ and $\overline{q}$ plots (Fig. 4a) show regions of warm dry air north of about 15°N and cool moist air to the south (with the ECMWF analyses being generally warmer and wetter). The GCM Sahara is cooler and moister than the analyses and the meridional temperature gradient to the south is much less pronounced (consistent with the weak vertical wind shear mentioned earlier). The Saharan cold bias was noted in McFarlane et al. (1992) and is a topic to be taken up again in section 7b when a closer look is taken at the simulated surface sensible and latent heat fluxes. The ECMWF and NCEP 850-mb $\overline{Q}$ plots (Fig. 4a) show mean ascent along east–west-oriented axes located at about 10°N and 20°N (see also the latitude–pressure cross sections of Fig. 4b). The northern belt of ascending motion is likely due to surface sensible heating over the Sahara while the southern belt is most likely due to latent heat release in the seasonal rainbelt (more on this in section 7b). The GCM shows no such dual structure in the vertical velocity field.

b. Variances

We remind the reader that all data in this and the next subsection have been 3–5-day bandpass filtered in order to highlight synoptic-scale disturbances. The ECMWF and NCEP 850-mb $\mathbf{\overline{u}}^2$ fields (Fig. 5a) have maximum variance near the African coast at about (20°N, 20°W). Over the Atlantic, the maximum $\mathbf{\overline{u}}^2$ in the analyses elevates from 850 mb to 700 mb (not shown), which is indicative of significant change in the observed low-level wave structure as the waves transit the continent onto the ocean. The GCM 850-mb $\mathbf{\overline{u}}^2$ field (Fig. 5a) shows maximum variance further inland at about (20°N, 0°). Evidently, and despite the poorly represented African easterly jet, the GCM does support appreciable low-level synoptic-scale activity. Whether the GCM disturbances are physically and energetically plausible is a question to be answered in section 3c. Take note that the upper-level $\mathbf{\overline{u}}^2$ maxima around 200 mb seen in all the datasets (see Fig. 5b) are most likely associated with middle-latitude systems and not tropical waves.

We now describe the synoptic-scale variance in the other variables. Figure 5a shows that there is appreciable $\mathbf{\overline{T}}^2$ over the Sahara in all the datasets. The magnitude of the ECMWF and NCEP $\mathbf{\overline{T}}^2$ fields is also large downstream from the coastal time-mean temperature gradient. Interestingly, and for reasons unknown, the NCEP $\mathbf{\overline{T}}^2$ field is especially large over the Atlantic at about 850 mb and (20°N, 35°W). Note that the ECMWF and NCEP $\overline{q}^2$ and $\overline{\omega}^2$ fields also show substantial differences, which is perhaps not surprising given the "data poor" variables in question. The GCM $\overline{q}^2$ field (Fig. 5a) is exceedingly large over the Sahara and the western Atlantic. The vertical cross-section fields of Fig. 5b show that the maximum GCM $\overline{q}^2$ resides too southward (at about 13°N instead of 17°N) and too high (at about 500 mb instead of 700 mb) compared to the analyses.
Means at $p = 850$ mb

**ECMWF**

**NCEP**

**GCM**

**Fig. 4.** Time means at (a) $p = 850$ mb and (b) $\lambda = 7.5^\circ$W. Zonal wind $\pi$ (and velocity), temperature $\bar{T}$, specific humidity $\bar{q}$, and vertical velocity $\bar{\omega}$. In (a) interval for $\bar{T}$ is 2.5°C. In (b) intervals for $\bar{T} < 0$ and $\bar{T} > 0$ (shaded) are 10°C and 5°C, respectively. Intervals for $\pi$, $\bar{q}$, and $\bar{\omega}$ are 2 m s$^{-1}$, $2 \times 10^{-3}$ g g$^{-1}$, and $2 \times 10^{-3}$ Pa s$^{-1}$, respectively. Zero contours are dashed.

Regarding intraseasonal variations (not shown), June, July, and August rank as third, first and second in terms of the magnitude of $\bar{v}'$, $\bar{T}'$, $\bar{q}'$, and $\bar{\omega}'$ in all the datasets. While beyond the scope of this study it would be interesting to know where the month of September ranks—given that September is the month when African easterly waves are known to contribute the most toward Atlantic hurricane activity (Landsea 1993).

c. **Covariances**

Here the energetics of the ECMWF, NCEP, and CCCma GCM2 African easterly waves are inferred by considering various bandpass-filtered covariances (see Fig. 6). Specifically computed are $\bar{\omega}'\bar{T}'$, which measures the conversion of eddy available potential energy to/from eddy kinetic energy; $\bar{v}'\bar{T}'$, which when multiplied by the time-mean meridional temperature gradient measures the conversion of zonal available potential energy to/from eddy available potential energy; $\bar{u}'\bar{v}'$, which when multiplied by the time-mean zonal meridional zonal wind gradient measures the conversion of zonal kinetic energy to/from eddy kinetic energy; and $\bar{v}'\bar{q}'$, the meridional moisture transport.

In all the datasets, $\bar{\omega}'\bar{T}'$ is primarily negative, which...
Means at $\lambda = 7.5^\circ W$

ECMWF

NCEP

GCM

Fig. 4. (Continued)
Fig. 5. Bandpass-filtered variances at (a) $p = 850$ mb and (b) $\lambda = 7.5^\circ$W. Meridional wind $\vec{\gamma}^2$, temperature $T^2$, specific humidity $\vec{q}^2$, and vertical velocity $\vec{\omega}^2$ variances. Contour interval for $\vec{\gamma}^2$, $T^2$, $\vec{q}^2$, and $\vec{\omega}^2$ is 1.5 m$^2$s$^{-2}$, 0.3$^\circ$C$^2$, 5 x $10^{-3}$ g$^2$g$^{-2}$, and 10$^{-3}$ Pa$^2$s$^{-2}$, respectively.

implies a conversion of eddy available potential to eddy kinetic energy, or alternatively, anomalous upward motion ($\vec{\omega} < 0$) correlated with warm anomalies. In all the datasets, $\vec{\omega}T^2$ is centered at about 850 mb and (18$^\circ$N, 5$^\circ$W). The GCM $\vec{\omega}T^2$ field is seen to terminate prematurely at the African coast at about 18$^\circ$W, and there is small spurious $\vec{\omega}T^2$ at around 300 mb (not shown). Despite these errors it is encouraging to see how comparable to the analyses the GCM $\vec{\omega}T^2$ field actually is. The GCM $\vec{\gamma}T^2$ field, while of the right sign, has a larger magnitude, is situated too far inland, and unrealistically terminates at the coast. In all the datasets $\vec{\gamma}T^2$ is primarily negative, which implies a conversion of zonal available potential energy to eddy available potential energy, or alternatively, anomalous southward flow ($T' < 0$) correlated with warm anomalies in a region of northward increasing time-mean temperature. In all the datasets $\vec{v}T^2$ is maximum at about 850 mb and 20$^\circ$N. For the Sahara at least, the picture common to all datasets is that of low-level synoptic disturbances transporting warm anomalies upward and southward against northward-increasing time-mean temperatures— in such a way as to help maintain themselves, energetically speaking.

The ECMWF and NCEP $\vec{u}\vec{v}$ latitude–pressure cross sections (see Fig. 6b) show an eddy momentum flux
Variances at $\lambda = 7.5^\circ W$

**ECMWF**

**NCEP**

**GCM**

\[ \nu'^2 \]
\[ T'^2 \]
\[ q'^2 \]
\[ \omega'^2 \]

**Latitude**

**Latitude**

**Latitude**

**Fig. 5. (Continued)**
Fig. 6. Bandpass-filtered covariances at (a) $p = 850$ mb and (b) $\lambda = 7.5^\circ$W. Vertical velocity and temperature $\omega T'$, meridional wind and temperature $\overline{\nu T'}$, zonal wind and meridional wind $\overline{u \nu'}$, and vertical velocity and specific humidity $\overline{\omega q'}$ covariance. Contour interval for $\omega T'$ (negative values shaded), $\overline{\nu T'}$ (negative values shaded), $\overline{u \nu'}$ (positive values shaded), and $\overline{\omega q'}$ (positive values shaded) is $0.5 \times 10^{-2}$ over Africa around 700-mb, which is clearly against the meridional gradient of the time-mean zonal wind (see Fig. 4b), which implies a conversion of zonal kinetic energy to eddy kinetic energy. This is especially so on the equatorward flank of the African easterly jet (note that the bold dashed curve in Fig. 6b roughly identifies the axis of the African easterly jet). The GCM $u \nu'$ latitude–pressure cross section, while looking quite different from the analyses, reveals essentially the correct correlation with the meridional gradient of the time-mean zonal wind. Specifically, there is a large northward flux of westerly momentum on the equatorward flank of the GCM’s African easterly jet (see Fig. 4b), which implies, as in the analyses, a conversion of zonal kinetic energy to eddy kinetic energy. It follows that vertical and horizontal time-mean wind shears are important to African easterly wave development in the analyses as well as the GCM.

Generally speaking, the $\overline{\nu q'}$ fields are large and positive where the corresponding $q'$ fields are also large. Over the Sahara the interpretation of the $\overline{\nu q'}$ fields is roughly the same for all the datasets, that is, low-level transport of dry desert air southward (or conversely, moist monsoonal air northward). In the case of the ECMWF analyses and the GCM, the largest meridional moisture transports occur over the Sahara while in the
Covariances at \( \lambda = 7.5^\circ W \)

**ECMWF**

**NCEP**

**GCM**

![Diagrams showing covariance plots for ECMWF, NCEP, and GCM at \( \lambda = 7.5^\circ W \).](image)

*Fig. 6. (Continued) Pa s\(^{-1}\) °C, 0.5 m\(^2\) s\(^{-2}\) °C, 0.5 m\(^2\) s\(^{-2}\), and \(4 \times 10^{-4}\) m s\(^{-1}\) g g\(^{-1}\), respectively. The dashed curves on the \(u'v'\) plots show the axis of each model’s African easterly jet. Zero contours omitted.*


**Φ** Associated Correlation Patterns (imaginary) at $p = 850$ mb

<table>
<thead>
<tr>
<th>ECMWF</th>
<th>NCEP</th>
<th>GCM</th>
</tr>
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<td>[4.6, 11.6, 15.4]</td>
<td>[5.7, 13.0, 12.6]</td>
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<td>[6.4, 14.2, 15.0]</td>
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<tr>
<td>[16.7, 2.7, 10.6]</td>
<td>[12.9, 2.7, 9.9]</td>
<td>[7.9, 5.3, 6.7]</td>
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Fig. 7. POP decomposition for each dataset. Geopotential, $Φ$, associated correlation patterns (imaginary) at $p = 850$ mb. Indicated above each panel is the POP period $Γ$ (days), $e$-folding time $τ$ (days), and explained variance $EV$ (%) in the following order ($Γ$, $τ$, $EV$). Contour interval for $Φ$ is $10$ m$^2$ s$^{-2}$. Zero contours omitted. Dark shading for negative values and light shading for positive values.

NCEP analyses the largest transports occur off the African coast.

The next subsection goes beyond traditional variance and covariance statistics and considers actual patterns of propagation.

d. Propagation patterns

The computations undertaken in this section are similar to LLA. LLA employed an extended empirical orthogonal function (hereafter EEOF) technique on ECMWF operational analyses for the 1980–87 period. Using their technique LLA identified, for example, two propagation tracks over central and western Africa each consisting of disturbances with periods near 5 days and wavelengths of about 3200 km. The northern track along the southern Sahara was seen to involve the ascent of warm dry air over and ahead of surface troughs while the southern track near the climatological rainfall maximum was associated with moist convective systems. In this study a similar but somewhat more advanced empirical method is applied to ECMWF analyses for the more recent 1986–94 period, as well as to the corresponding NCEP and CCCma GCM2 datasets.

As described in section 2c the detailed structure of synoptic activity is obtained by applying a POP analysis to the primary variable $v_{850}$. The reader is reminded that in this particular analysis all the data has been 3–10-day bandpass-filtered—with the relatively wide filter window allowing for the identification of propagating disturbances other than African easterly waves (e.g., those associated with the near 7-day peak in the GCM spectra). Also, as is usual with POP analyses of this sort, the dimension of the underlying matrix-eigenvalue problem is reduced (and at the same time unwanted spatial noise suppressed) by projecting the data onto a subspace spanned by the first six EOFs. Extensive experimentation with different numbers of EOFs suggests the six EOFs are optimal in the sense of retaining signal and removing noise.
Figure 7 summarizes the POP decomposition for each of the datasets. The first POP in all the datasets is identifiable with African easterly waves—both in terms of spatial structure and period. The large e-folding times and explained variances for each of the first POPs is indicative of robust propagation patterns (von Storch et al. 1985, 1988). The fact that the first POP in the GCM has a longer period ($T = 5.7$ days) than the first POPs in the analyses ($T = 4.5$ and 4.6 days) is consistent with the spectra presented in Fig. 2. The second POPs in all the datasets have relatively long periods ($T = 5.5$, 5.8, and 6.4 days for the ECMWF, NCEP, and GCM datasets, respectively), long zonal wavelengths, and all extend into the extratropics. Based on this, and further analysis of the vertical structure of these modes (not shown), it is believed that these second POPs may be related to the 5–6-day waves identified in Reed et al. (1987) as “the signature of low-level perturbations that form in connection with upper tropospheric developments in or near the mid-ocean trough.” The third POPs have the longest periods and zonal wavelengths and explain the least amount of the total variance in each case.

Figure 8 shows the imaginary part of each ACP based on the first POP of the respective datasets bandpass-filtered $\Psi_{\Lambda C}$. The real part of each ACP (not shown) is essentially the imaginary part shifted westward one-quarter of a wavelength $L$ which leads to the view of the ACPs as propagating westward with phase speed $c = L/T$ [where $c = (3000 \text{ km})/(4.5 \text{ days}) \approx 8 \text{ m s}^{-1}$ for the analyses and $c = (3000 \text{ km})/(5.7 \text{ days}) \approx 6 \text{ m s}^{-1}$ for the GCM]. Referring to Fig. 8a, note that the ECMWF and NCEP $\Phi$ ACPs are very similar while the GCM $\Phi$ ACP originates and terminates more toward the east. Also notice the unrealistic southwest to northeast tilt of the wave trough over Africa (as indicated by the dashed line) in the GCM. The ECMWF and NCEP $\zeta$ and $\omega$ ACPs suggest a southern propagation track at about 10$^\circ$N. Further calculation (not shown) with $\zeta$ as the primary variable (instead of $\nu$) accentuates the southern track. On the other hand, neither the GCM $\zeta$ or $\omega$ ACPs (whether based on $\zeta$ or $\nu$ as the primary variable) show evidence of a southern track. The missing southern track and the premature termination of the northern track are taken as significant shortcomings of the GCM depiction of African easterly waves.

The ECMWF and NCEP 850-mb $T$ and $q$ ACPs show some differences over the Atlantic—with the magnitude of the NCEP perturbations exceeding the ECMWF perturbations (as expected given the differences in the variance plots of Fig. 5a). Over Africa the phasing between the $\Phi$, $T$, and $q$ fields at 850 mb is essentially the same between datasets—conforming to the usual picture of desert disturbances carrying anomalously warm dry Saharan air southward ahead of surface troughs and cold moist monsoonal air northward behind. In the ECMWF and NCEP desert disturbances there is a clear correlation between vertical motion and moisture [i.e., warm (cold) anomalies ascending (descending)]. In the GCM this $\omega-q$ correlation is not as evident.

The longitude–pressure cross sections of Fig. 8b show from another view the unphysical termination of the GCM track at the African coast. Also seen in Fig. 8b is the fact that the GCM perturbations extend far too deep into the upper troposphere (seen especially in vertical velocity ACP). All $\Phi$ ACPs show disturbances that are eastward tilting with height in the lower troposphere—with the ECMWF and NCEP ACPs showing a very clear tendency to elevate from near the surface to about 700 mb as they propagate from Africa to the Atlantic.

From the ACPs of the previous subsection it is possible to compute associated covariances. Specifically, given secondary variables $\Psi$ and $\chi$ the associated covariance between the two is defined here as the real part of the product of their respective ACPs; that is, $\Psi \chi = \Psi \chi_x + \Psi \chi_y$. An analysis of the associated covariances (not shown) confirms the picture presented in Fig. 6. Namely, the ECMWF, NCEP, and GCM African easterly waves are maintained through the combination of eddy available potential energy conversion to eddy kinetic energy (i.e., $\omega T < 0$), zonal available potential energy conversion to eddy available potential energy (i.e., $\nu T \partial \nu / \partial y < 0$), and zonal kinetic energy conversion to eddy kinetic energy (i.e., $\nu T \partial \nu / \partial y < 0$). Additionally, the meridional moisture flux in the three datasets is of the same sign but differs in magnitude and location. Also, the ECMWF and NCEP $\nu q$ fields are localized near the African coast (at 850 mb) while the corresponding GCM flux is much stronger and is situated too far inland over the Sahara.

**e. Comparison with wave structures during phase III of GATE**

While we have found that the ECMWF and NCEP wave structures are very similar, it remains to compare these structures against detailed observational studies. Reed et al. (1987) compared the wave structures obtained from operational ECMWF analyses for the August–September 1985 period against the GATE structures for August–September 1974 [see Table 1 of Reed et al. (1987)]. In that study it was concluded that the baroclinic, barotropic, and convective structures seen during phase III of GATE were nearly repeated during August–September 1985. The question here then is two-fold: 1) Given that the ECMWF analysis scheme has changed dramatically since 1985 and that the density of observational data available over Africa has decreased does the Reed et al. (1987) comparison still stand up? 2) How does the NCEP reanalyses system fair in this regard? (Note that there has been no previous direct comparison of African easterly waves from the NCEP reanalyses with detailed observational studies.)
Fig. 8. Associated correlation patterns (imaginary) (a) $p = 850$ mb and (b) $\phi = 16.7^\circ$N. Geopotential $\Phi$ (and velocity), relative vorticity $\zeta$, vertical velocity $\omega$, temperature $T$, and specific humidity $q$. Intervals for $\Phi$, $\zeta$, $\omega$, $T$, and $q$ are $10$ m$^2$s$^{-2}$, $2 \times 10^{-3}$ s$^{-1}$, $5 \times 10^{-3}$ Pa s$^{-1}$, and $2 \times 10^{-4}$ g g$^{-1}$, respectively. Zero contours omitted. Dark shading for negative values and light shading for positive values.
Fig. 8. (Continued) The bold arrows in the ζ 850-mb and Φ cross-section plots indicate propagation tracks while the dashed curve in the Φ cross-section plot indicates phase line tilt (see text for discussion).
tigation and those found in Reed et al. (1977), Norquist et al. (1977), and Albignat and Reed (1980) (hereafter referred to as RNR, NRR, and AR) for phase III of the GATE. In making this comparison it must be kept in mind that the GATE description obtained from these various studies is based mainly on a composite analysis for August–September 1974 while the present is based on a variance/covariance and POP analysis for June–August 1986–94. Having said this we conclude from Table 1 that the basic picture of wave dynamics obtained from the ECMWF and NCEP analysis systems conforms quite well with the picture obtained from the GATE analyses. Specifically, from either, we see a two-component system consisting of a northerly branch of moderately shallow disturbances and a southerly branch of deeper more convective structures. The northern disturbances are located below the African easterly jet and carry warm (cold) dry (moist) air southward (northward) and upward (downward), and the wave axes are tilted eastward in a sense opposite to the vertical shear of the African easterly jet. Wave energetics computed here and from the GATE observations (Norquist et al. 1977) show that the northern wave structures are maintained through conversions of zonal to eddy available potential energy and from eddy potential energy to eddy kinetic energy. In short, the ECMWF and NCEP wave structures are in reasonable accord with the detailed GATE-based observations (despite the reduction of observational data available).

4. African easterly wave generation

It has been seen that the GCM supports African easterly waves despite a rather unrealistic looking African easterly jet (see Fig. 4b). Given existing theoretical and observational evidence that African easterly waves originate as dynamical instabilities on the African easterly jet, it would be prudent to determine the stability characteristics of the simulated zonal winds. If the simulated African easterly jet was founded to be stable, then the origin and nature of the simulated African easterly waves would certainly be in doubt. To determine the stability of the time-mean zonal winds we compute the time-mean Ertel’s potential vorticity on isentropic surfaces; that is,

$$\Pi = -\left(\frac{\partial x}{\partial y}\right) + f \frac{\partial \theta}{\partial p},$$  \hspace{1cm} (5)$$

where \(\theta\) is potential temperature, \(f\) is the Coriolis parameter, and \(g\) is the gravitational constant (note that the \(\partial x/\partial x\) term normally appearing in the square brackets has been dropped). Charney and Stern (1962) have shown that the vanishing of the meridional gradient of \(\Pi\) along an isentropic surface is a necessary condition for instability of \(\Pi\).

Figure 9a shows \(\partial \Pi/\partial y\) on the 315-K isentropic surface (which is generally around the 700-mb pressure surface) for the month of August. Also shown is \(\zeta_{2}\) on the 850-mb surface for August. All the models, including the GCM, show a region of reversed \(\Pi\) gradient collocated with the region of large \(\zeta_{2}\). Figure 9b shows longitude-vertical cross sections of \(\partial \Pi/\partial y\) and \(\zeta_{2}\). Interestingly, the GCM region of negative \(\partial \Pi/\partial y\) stops abruptly at the African coast, which may help explain the unphysical termination of the GCM propagation patterns at the coast. Also revealing is how in the case of the ECMWF and NCEP models, the region of negative \(\partial \Pi/\partial y\) elevates above the surface over the Atlantic—as do the corresponding propagation patterns (and \(\zeta_{2}\) for that matter). Take note that the other summer months (not shown) show similar results.

5. The GCM Saharan cold bias

The question addressed here is why does the GCM have a mean cold bias over the Sahara? Figure 10a

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<table>
<thead>
<tr>
<th>Baroclinic features</th>
<th>GATE*</th>
<th>ECMWF and NCEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm (T), northerly (u)</td>
<td>850 mb, 15°–20°N</td>
<td>1000–600 mb, 10°–25°N</td>
</tr>
<tr>
<td>Warm (T), upward (\omega)</td>
<td>850 mb, 15°–20°N</td>
<td>1000–600 mb, 10°–25°N</td>
</tr>
<tr>
<td>(u/\omega) with negative maximum in regions of (\partial^2 T/\partial y &gt; 0)</td>
<td>850 mb, 16°N</td>
<td>850 mb, 18°N</td>
</tr>
<tr>
<td>(u/\omega) negative maximum</td>
<td>850 mb, 16°N</td>
<td>850 mb, 18°N</td>
</tr>
<tr>
<td>Eastward tilt of wave axis with height</td>
<td>850–700 mb, 0°–45° phase tilt</td>
<td>850–700 mb, 25° phase tilt</td>
</tr>
</tbody>
</table>

Barotropic features

<table>
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<tr>
<th>(u/v) maximum in region of (\partial^2 \theta/\partial z &gt; 0)</th>
<th>11°N, 650 mb</th>
<th>10°N, 700 mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRR: Fig. 2</td>
<td>Figs. 6b</td>
<td></td>
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</table>

* RNR: Reed et al. (1977); NRR: Norquist et al. (1977); AR: Albignat et al. (1980).
August Means

<table>
<thead>
<tr>
<th>ECMWF</th>
<th>NCEP</th>
<th>GCM</th>
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August Means at $\phi = 16.7^\circ N$

<table>
<thead>
<tr>
<th>ECMWF</th>
<th>NCEP</th>
<th>GCM</th>
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</table>

Fig. 9. August 850-mb vorticity variance $\zeta^2$ and 315-K time-mean meridional EPV gradient $\partial \Pi / \partial y$: (a) $p = 850$ mb for $\zeta^2$ and $\theta = 315$ K for $\partial \Pi / \partial y$; (b) $\phi = 16.7^\circ N$. Interval for $\zeta^2$ is $1.75 \times 10^{-11}$ s$^{-1}$. In (a) the interval for $\partial \Pi / \partial y$ is $5 \times 10^{-11}$ m$^{-1}$ s$^{-1}$. In (b) the intervals for $\partial \Pi / \partial y < 0$ and $\partial \Pi / \partial y > 0$ are $5 \times 10^{-11}$ m$^{-1}$ s$^{-1}$ and $25 \times 10^{-11}$ m$^{-1}$ s$^{-1}$, respectively. Zero contours are dashed.
Means Near the Surface

(a) NCEP  

(b) GCM

Fig. 10. Screen temperature $T_s$, precipitation $P$ (and superimposed 10-m winds), surface sensible heat flux $Q_s$, and surface latent heat flux $Q_L$. (a) Time means and (b) bandpass-filtered variances. In (a) the contour intervals for $T_s$, $P$, $Q_s$, and $Q_L$ are 2.5°C, 2×10^{-3} Kg m^{-2} s^{-1}, 25 W m^{-2}$, and 25 W m^{-2}, respectively.

shows the NCEP and GCM time-mean screen temperature $T_s$. A cold bias of about −4°C exists in the GCM around 0° long and 15°N. McFarlane et al. (1992) suggest that the cold bias may result from a “surface sensible heat flux parameterization [that] overestimates that quantity in dry desert regions.” The argument is that in the absence of evaporation/precipitation (as in deserts) excessive surface sensible heat flux would imply colder than observed screen temperatures. Figure 10a shows the opposite to be true, in that there is a deficit rather than a surplus of surface sensible heat flux $Q_s$ in the region. The real culprits are simulated precipitation $P$ and surface latent heat flux $Q_L$, both of which are excessively large in the south Sahara. It is suggested in the RCM study of Fyfe and Laprise (1998, unpublished manuscript)
that this anomalous convection may in part be due to underresolved coastal mountains, which would otherwise block, drag, and/or deflect the moisture-laden monsoonal winds flowing from the Gulf of Guinea. On this point, note that the NCEP 10-m winds decrease markedly as they encounter the coast blowing northeastward out of the Gulf of Guinea. On the other hand, the GCM monsoonal winds are apparently undeterred at the coast.

Not surprisingly, the GCM Saharan surface cold bias goes hand-in-hand with deficiencies in the synoptic-scale variance near the surface in the area. Figure 10b shows the NCEP and GCM bandpass-filtered screen temperature, precipitation, and surface sensible and latent heat flux variances. Take note that the GCM $P'$ contour interval is 10 times the corresponding NCEP contour interval. The GCM has far too much precipitation and surface latent heat flux variance over

Variances Near the Surface

(a) NCEP

(b) GCM

Fig. 10. (Continued) In (b) the contour intervals for $T_{s}'^2$, $Q_s'^2$, and $Q_L'^2$ are 0.4°C², 75 W m⁻², and 150 W m⁻² m⁻¹, respectively. In (b) the NCEP (GCM) contour interval for $P'$ is $2 \times 10^{-10}$ ($2 \times 10^{-9}$) Kg² m⁻⁴ s⁻². Zero contours omitted.
Africa. On the other hand, over the Atlantic the GCM has too little precipitation and surface latent heat flux variance.

Finally, take note that the precipitation and surface sensible and latent heat fluxes are not included in the ECMWF–TOGA dataset and so are not available to evaluate the NCEP surface fields. Inadequacies in the NCEP surface fields are discussed in Kalnay et al. (1996).

6. African easterly waves and CO2 doubling

It is of interest to know if given a doubling of CO2 in the atmosphere if the level of African easterly wave activity changes. Figures 11a,b (right-hand sides) show the changes in selected means, variances, and covariances between two 9-yr integrations with CCCma GCM2—one integration with present levels of CO2 (i.e., 330 ppm) and the other with double the present levels of CO2 (i.e., 660 ppm). Both runs incorporate a slab ocean and thermodynamic sea-ice model, as detailed in Boer et al. (1992). As can be seen, with a doubling in CO2 there is a mean warming and moistening throughout the region and at all levels. [Note that all changes discussed here are statistically at the 95% confidence level (based on the standard t test)]. Along with these mean changes there is a modest increase in the bandpass-filtered $\nu$ variance (note...
Climate Signals and Change at $\lambda = 7.5^\circ W$

$1 \times CO_2$  $2 \times CO_2$  Change

$T$

$q$

$v'^2$

$v'T'$

Fig. 11. (Continued)
also the corresponding changes in the $v-T$ covariance). Inspecting the $1 \times CO_2$ and $2 \times CO_2$ $v^2T$ fields themselves, it can be seen that the increased wave activity does not appear to involve any major systematic latitudinal nor altitudinal changes in the low-level storm tracks. Finally, we note that while the fact that the simulated wave activity increases with CO2 doubling is an interesting and provocative one, the reader must be reminded and cautioned that the CCCma GCM2 suffers from some fairly significant biases in the region.

7. Summary and discussion

This study systematically compares the ECMWF, NCEP, and CCCma GCM2 synoptic scales over the Africa–Atlantic region for the northern summers from 1986 to 1994. The ECMWF and NCEP datasets are very similar in terms of means, variances, covariances, and propagation patterns—despite the paucity of observed data assimilated into either in this region. Where significant differences exist they expectably do so over the Atlantic and in those variables that are not directly observed (such as vertical motion). Notable differences between the analyses are found in the low-level time-mean moisture content over the Sahara and in the bandpass-filtered temperature variance over the Atlantic. The situation with the GCM is not as favorable, but even so there are some areas of agreement with the analyses despite the model’s coarse resolution and absence of observed data to constrain it. Specific points of agreement and disagreement between the GCM and the analyses include the following.

- The GCM African monsoon flow is too strong, too deep, and the overlying time-mean vertical wind shear too weak. Also, whereas the analyses exhibit well-defined and separated African and tropical easterly jets the GCM has a single easterly jet structure with unrealistic speeds aloft. The GCM Sahara is cooler and moister than in the analyses.
- The GCM synoptic low-level meridional wind variance is too large in magnitude and centered too far over the Sahara with no extension into the Atlantic. The GCM has excessive moisture variance throughout the Africa–Atlantic sector, while over Africa the vertical velocity variance extends unrealistically far into the upper troposphere.
- The GCM $\omega T^2$ and $v^2T^2$ fields have the same sign as in the analyses, but as expected from the previous bulleted comment are too large in magnitude and are displaced too far inland. The GCM $\omega v^2 T^2$ field correctly implies zonal kinetic energy conversion to eddy kinetic energy on the equatorward flank of the African easterly jet.
- The GCM African easterly wave propagation pattern terminates prematurely at the African coast.

Over the Sahara the GCM disturbances are eastward tilting with height in the lower troposphere and realistically transport anomalous warm dry desert air southward ahead of surface troughs and cold moist monsoonal air northward behind. The GCM shows no indication of a track of moist waves near the observed seasonal rainband.

As can be seen, the GCM has its failings with respect to the simulation of African easterly waves but so does it have its successes. The fact that the GCM disturbances have roughly the correct period, zonal wavelength, and energy conversions for their maintenance is quite encouraging.

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