Sea Ice and Climate. Part II: Model Climate Stability to Perturbations of the Hydrological Cycle

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ABSTRACT

The sensitivity of a simple climate model to variations in a global hydrological cycle is studied. The model consists of a zonally averaged single basin, two-hemisphere ocean model coupled to an atmospheric energy balance model and a thermodynamic sea ice model. Land processes are reduced to considerations of the oceanic catchment basins, which serve to amplify or decrease the oceanic freshwater forcing. The model typically admits both symmetric and asymmetric equilibria. For sufficiently strong global freshwater forcing, the symmetric circulation is replaced by asymmetric states. This is accomplished by a supercritical pitchfork bifurcation.

It is shown that sea ice enhances climate sensitivity in that larger changes of the hydrological cycle are required to induce instability if sea ice is neglected. This behavior comes from the strong damping of SST anomalies under ice, which is essential for suppressing deep ocean temperature anomalies. Thus, ocean temperature plays an important role in climate stability. This is due to the interactive atmosphere, a feature absent from ocean-only models.

The roots of transition from symmetric to asymmetric states are in the salinity balance. Differences in ocean temperature anomalies result in stronger anomalous overturning in the model with sea ice. Salt transport differences are proportional to those in overturning anomalies. The anomalous heat transports are similar due to compensating differences in the surface–abyss steady-state temperature contrasts, arising from sea ice phase transition.

In steady state, the water column under ice is homogeneous and energetically passive, consistent with oceanic insulation by sea ice. Thus, the downwelling in the region covered by sea ice is controlled kinematically by the basic flow. The width of this region is restricted by the sea ice extent. In contrast, the narrow high-latitude open ocean sinking is most dependent on the relative importance of temperature advection versus vertical diffusion in the global oceanic heat budget. Sea ice albedo and brine rejection effects have negligible influence.

Comparisons with highly truncated box model studies suggest that high horizontal resolution is essential to accurately determine model climate stability.

1. Introduction

There is evidence that the earth’s climate can be in at least two possible equilibrium states. Paleorecords indicate that modern warm conditions characteristic of the last 10 000 yr or so did not persist throughout the earth’s history; instead, the global temperature of the past exhibited significant oscillations on the centennial and longer timescales (e.g., Broecker et al. 1985). The transitions between the two states appear to be triggered by the changes of the solar insolation due to variations of the earth’s orbit (Hays et al. 1976). However, the mechanics of such global climatic shifts are unclear; in particular, the inferred amplitudes of the changes in climate require an amplification of the otherwise very small insolation induced heat budget anomalies. A plausible hypothesis is that climate transitions are conditioned by the interaction of the oceanic thermohaline circulation in the meridional–vertical plane with the perturbed hydrological cycle of the earth. The hydrological cycle is one of the most delicate processes on the earth for a number of reasons, including the sharp dependence of the atmospheric moisture capacity on temperature and the extreme sensitivity of the system to the details of the freshwater redistribution by rivers on land. Numerous modeling results indicate major changes in the global oceanic circulation caused by only slight modification to the oceanic freshwater forcing. The glacial thermohaline circulation (THC) had a structure very different from that seen today (e.g., Duplész et al. 1988), affecting the oceanic poleward heat transport and, therefore, climate.

Thus, it would appear that the ocean THC plays a major role in both setting the characteristics of climate at a given time, and determining the transitions between various climatic regimes. But the ocean in turn is potentially sensitive to local processes and feedbacks, many of which remain poorly studied. In this paper we ask how sea ice affects the stability of modern (symmetric; see below) circulation. Technically, we explore the model
response to variations of the hydrological cycle (cf. the above discussion of climate transitions) and compare the results from such an analysis for our most complete model and the one where sea ice is neglected.

Evidence for the active role of the ocean in climate transitions is also found in glacial paleorecords. Of a particular interest is the presence in these records (e.g., Broecker et al. 1985) of a series of events whose duration is estimated to be from several hundred to a thousand years. The best documented oscillation of this type is a cold anomaly just before the onset of modern, interglacial conditions, the so-called Younger Dryas event. Such phenomena as Younger Dryas are relatively short and cannot be explained exclusively by slow variations in the solar forcing (Broecker et al. 1985; Fairbanks 1989). The ocean is thought to be a major ingredient in this variability (Broecker 1997) possibly through, again, the interaction of the oceanic thermohaline circulation with the hydrological cycle (Broecker et al. 1989; Fairbanks 1989).

A specific feature of the oceanic thermohaline circulation is sinking of the water in narrow polar regions and much wider upwelling zones. This sets a natural division of the ocean into the main thermocline, where the vertical density gradients are strong, and a nearly homogeneous deep ocean. The properties of the deep water are thus determined by local surface processes in polar regions, which may in turn affect the earth’s climate globally. Such an important physical phenomenon is sea ice. In addition to affecting oceanic properties, sea ice directly influences polar atmospheric temperatures. This may be of a global significance due to the very efficient atmospheric transport of heat. It is worthwhile noting that major changes in the atmospheric temperature during glacial periods appear to be concentrated at the poles.

In Kravtsov and Dewar (1998, manuscript submitted to J. Climate, hereafter KD98), we argued that a minimal model for the study of sea ice effects on global climate is a combination of a one-basin, 2D oceanic thermohaline circulation model, 1D atmospheric energy balance model, and a thermodynamic sea ice model. As mentioned above, the model has been used to illustrate a paradigm of long-term climatic behaviors being caused by interactions of the global thermohaline circulation with various parts of the climate subsystem. An alternative view, centrally involving internal land ice dynamics conditioned by quasiperiodic solar variability, is presented in Ghil (1994). Within our paradigm, the choice of the particular physics present is still somewhat arbitrary and guided by the semi-intuitive reasoning. The climate system is regarded as the heat and salt redistributor. This redistribution can occur in both the ocean and the atmosphere. Two-dimensional thermohaline circulation models are very simple and proven tunable to reproduce oceanic zonally averaged temperature, salinity and circulation fields with a surprising qualitative realism. Fine horizontal resolution is a must for an appropriate coupling with the thermodynamic sea ice model, the simplest type of sea ice model available. The sea ice is, thus, characterized by the thickness distribution and by the extent, allowing dynamic treatment of many feedbacks (for example, sea ice–albedo feedback). Atmospheric heat and moisture transports are parameterized as temperature and specific humidity diffusion, respectively. Given such a limited set of parameterized processes, we argue that a self-contained picture of the paleoclimatic history can be deduced, which does not depend crucially on the specific choice of parameterizations or the particular choice of the parameters used in the model.

Symmetry breaking in zonally averaged ocean-only models was addressed before. The essence of the advective mechanism underlying the symmetric mode instability was demonstrated by Marotzke (1990). He considered an eight-box (two level) model of the pole-to-pole Atlantic Ocean (four boxes in each hemisphere) and showed that symmetric two-cell steady structure with polar sinking and equatorial upwelling is unconditionally unstable. The structure of the unstable eigenmode was computed analytically. Symmetry breaking occurred by means of cross-equatorial salinity transport interacting with the basic fields. If, for example, a positive northward, cross-equatorial transport anomaly is given, a negative southern surface salinity anomaly results, which is then amplified by an increased low-salinity deep southern equatorial water upwelling. By the analogous mechanism, a positive northern surface salinity anomaly arises simultaneously. The salinity anomalies spread over the entire Northern and Southern Hemispheres and pole-to-pole salinity contrast grows. The inclusion of horizontal diffusion in the model results in a conditional stability; namely, the destabilizing transport perturbation must exceed a given value.

Although Marotzke (1990) sheds light on symmetry breaking in zonally averaged models, the effects of higher model horizontal and vertical resolution remain unclear. Also, in Marotzke (1990), temperatures were prescribed. A more numerically sophisticated study of the same phenomenon was carried out by Vellinga (1996) using a higher-resolution 2D ocean model with a nearly (but not exactly) prescribed SST. Two different momentum formulations were employed. Numerical continuation was used to compute bifurcations and the structures of the host unstable eigenmodes.

The linearly perturbed equations similar to those used by Vellinga are

\[ \frac{\partial T^*}{\partial t} = -\nabla \cdot (T^* \mathbf{u}) - \nabla \cdot (\mathbf{T} \mathbf{u}') + \mathcal{L}_T T^*, \quad (1) \]

\[ \frac{\partial S^*}{\partial t} = -\nabla \cdot (S^* \mathbf{u}) - \nabla \cdot (\mathbf{S} \mathbf{u}') + \mathcal{L}_S S^*. \quad (2) \]

Here primes denote perturbation fields (T is the temperature, S is the salinity, u is the velocity field) and tildes the mean state variables. Further, L is a linear diffusion operator.
Vellinga argued the exact momentum balance formulation and convection (modeled by homogenization convective adjustment) were unessential for the symmetric state instability. As in Marotzke (1990), the mechanics of transition were argued to lie in the anomalous salt budget (2). Given a salinity perturbation with a structure of the most unstable eigenfunction, the evolution of the anomalous salinity fields depended critically on two major global advective feedbacks. The first feedback [referred to as the direct salinity feedback; first term on the right-hand side of (2)] is the stirring of anomalous salinity field by the mean state lateral advection, which tends to erode the spatial structure of a salinity anomaly, thereby disrupting coherent growth and stabilizing the mean. This is in a sense analogous to a diffusive effect, but the “mixing” (stirring) is performed by the main thermally driven overturning. The second feedback is the destabilizing indirect salinity feedback [second term on the right-hand side of (2)], that is, the anomalous advection of the mean salinity gradient. The most unstable salinity eigenfunction was found to be antisymmetric about the equator (and virtually vertically homogeneous, thus excluding convection from importance). Also, horizontal salt fluxes and their divergences \( \partial (S' \delta)/\partial x \) and \( \partial (S' y')/\partial x \) were shown to generally dominate the vertical components \( \partial (S' w)/\partial z \) and \( \partial (S' y')/\partial z \). As the overturning is proportional to meridional density (salinity) gradient, \( y' \) is symmetric about equator. Then \( S' y' \) is also symmetric as a product of two symmetric functions and \( \partial (S' y')/\partial x \) is antisymmetric. The sense of this term is such that it leads to an increase of the salinity anomaly in each hemisphere (cf. Marotzke 1990).

The anomalous temperature field was found by Vellinga to be much smaller in amplitude (due to strong temperature relaxation) and thus not important for model stability. The temperature effect important to the overall instability structure was that the circulation is essentially thermally driven, so increasing the basic salinity gradient reduces the mean overturning, decreasing the relative importance of the stabilizing direct salinity feedback. Inclusion of an interacting atmosphere makes the temperature feedbacks more active. An extensive discussion of the air–sea feedbacks role in the interhemispheric stability can be found in Scott et al. (1999).

The comparison between Vellinga (1996) and Marotzke (1990) shows that horizontal salt diffusivity is important as a stabilizing factor; the freshwater flux in Vellinga (1996) must exceed a certain threshold for instability. Neglecting horizontal diffusion in the 2D model of Marotzke et al. (1988) resulted again in an unconditional instability of the symmetric state.

Other studies include those by Quon and Ghil (1992) and Thual and McWilliams (1992). They also point to global advective salt feedback instability mechanics, characterized by growing cross-equatorial salt transport and amplification of interhemispheric salinity differences. The most complete mathematical study and review on the issue appeared in Dijkstra and Molemaker (1997).

The above-mentioned studies are the reference point for our analysis of sea ice effects on the interhemispheric stability of the oceanic thermohaline circulation. Interactions of sea ice with the thermohaline circulation were previously considered in Yang and Neelin (1993, 1997), Zhang et al. (1995), Lohmann and Gerdes (1998), and Jayne and Marotzke (1999). Yang and Neelin have argued that ocean–ice thermal coupling is weak and obtained behavior different from that described in former three papers, where strong relaxation to sea water freezing temperature under sea ice was used. The oscillation observed in Zhang et al. (1995) seems to be due to internal oceanic dynamics; sea ice is passive. Lohmann and Gerdes (1998) concentrated on how sea ice modifies global circulation stability through interactions with the oceanic convective pattern (see also Lenderink and Haarsma 1996) in the presence of an active atmosphere. Sea ice influence on the advective feedbacks was considered in Jayne and Marotzke (1998) in a highly truncated single-hemisphere box model of the thermohaline circulation. Sea ice was shown to destabilize the circulation. These results suggest (see also KD98) that the mechanics of the global climate transitions is advective. Here we test the conclusions of Jayne and Marotzke (1999) in a higher-resolution pole-to-pole model.

This paper is organized as follows. In section 2 we repeat and nondimensionalize the oceanic model (KD98) and derive the simplified two-layer ocean model to be used in the linear stability analysis of section 4. The modifications to models’ climates under increasing hydrological cycle are discussed in section 3, where the physical equivalency of the two-layer and 2D models is also established. Conclusions are in section 5.

2. Two-layer ocean model

The necessity to meet the global salinity balance is a powerful requirement for the climate, since only slight changes in the atmosphere and land hydrological cycle may be sufficient for a complete reorganization of oceanic circulation and the associated global heat transport (Broecker 1997). Therefore, in section 3 we will discuss how the steady states of the model constructed in KD98 are modified by responding to changes in the hydrological cycle.

An enormous number of sensitivity tests and/or long integrations are necessary to compute and fully analyze such bifurcation maps, and hence prohibitive computational demands arise (mostly in the linear stability experiments discussed in section 4, but also in the vicinity of the bifurcation points, where the internal equilibration timescales are very long). Thus, we are motivated to ask if we can simplify the model further without losing its key physical properties. If we do, we have
not only gotten a more efficient model, but also eliminated unessential physics. The simplified model must have analogous major equilibria and possess similar heat redistribution characteristics to the more complete model. We argue here such a model can be obtained by sacrificing the resolution of the oceanic main thermocline vertical structure and by parameterizing vertical mixing. In the following, we first derive this model and then demonstrate its fidelity to the complete model described above by comparing the two models mean states and their changes with the hydrological cycle (section 3). The simpler model is used (in section 4) to analyze the roots of the differences in climate response that arise if sea ice is artificially suppressed.

a. Nondimensionalization

We first introduce nondimensional variables. Hereafter, dimensional variables will be marked by asterisk.

For temperature and salinity we write

\[ T = \frac{T^* - T_0}{|T|}, \quad S = \frac{S^* - S_0}{|S|}. \]

Here the temperature scale \( T_0 = |T| = 15^\circ \text{C} \) is chosen to represent the observed equator-to-pole temperature contrasts (e.g., North et al. 1981), \( S_0 = 35 \) psu is the average ocean salinity and

\[ [S] = \frac{\alpha}{\beta} [T] = 3.75 \text{ psu}, \]

where \( \alpha = 2 \times 10^{-4} \text{ }^\circ \text{C}^{-1} \) and \( \beta = 8 \times 10^{-4} \text{ psu}^{-1} \) are thermal and haline expansion coefficients. The last relation enables us to write the nondimensional equation of state as

\[ \rho = S - T. \]

Here \( \rho \) is the density of a sea water.

Now we introduce two auxiliary geometrical parameters. One of them is the dimensional volume of the model ocean

\[ V = 2\pi f_w a_o^2 D \]

(\( f_w = \frac{1}{2} \) is the oceanic fraction of the model earth’s surface, \( a_o = 6400 \) km is the radius of the earth, \( D = 4000 \) km is the depth of the ocean). The other is the nondimensional thickness, \( \Delta \), of the upper 1000 m of the model ocean: \( \Delta = \frac{a_o}{4} \).

Let us consider “Rayleigh friction,” because this frictional parameterization is more suitable for analytical treatment. As in KD98, we are only interested in model properties that do not depend on the particular frictional form; therefore, we work with the simpler model. Here, for the coefficient of proportionality \( (r) \) in the Rayleigh law, relating local meridional density (pressure) gradient and local vorticity, we use the value of \( r = 1.5 \times 10^{-3} \) s \(^{-1} \) (i.e., half the value used in most 2D model experiments; see KD98). Convenient choices for time \([t]\), streamfunction \([\psi]\), ice thickness \([h]\), interface heat flux \([H]\), interface freshwater flux \([F_w]\), and meridional heat transport \([HT]\) scales are

\[ [t] = \frac{ra_o^2}{gD} \frac{1}{\Delta^2 \alpha[T]} \sim 117 \text{ yr}, \]

\[ [\psi] = \frac{V}{[t]} = 93 \text{ Sv}, \]

\[ [h] = k \frac{[t]}{\rho_w c_p D} \sim 0.5 \text{ m}, \]

\[ [H] = \frac{\rho_o c_p D}{[t]} [T] \sim 65 \text{ W m}^{-2}, \]

\[ [F_w] = \frac{[S]}{D} \frac{D}{[t]} \sim 3.7 \text{ m yr}^{-1}, \]

\[ [HT] = \rho_o c_p [\psi][T] \sim 5.55 \text{ PW}, \]

and arise naturally from the nondimensionalization. Here \( g = 9.82 \text{ m s}^{-2} \) is the gravitational acceleration, \( k_i \sim 2.17 \text{ W m}^{-1} \text{ C}^{-1} \) is the sea ice conductivity and \( \rho_o = 1000 \text{ kg m}^{-3}, \ c_p = 4000 \text{ J (kg}^{-1} \text{ C}) \) are water density and heat capacity. Also, nondimensional horizontal and vertical diffusivities, respectively, are

\[ k_h = k_w = \frac{k_h^*}{a_o^2 [t]} \quad \text{and} \quad k_v = \frac{k_v^*}{D^2 [t]} \]

Hereafter, all results from the standard 2D model will be rescaled according to the above formulas for comparison between the models.

With \( w = \partial_x \psi \) and \( v = -\partial_y \psi, \ \psi \) is the streamfunction; the nondimensional equations governing ocean dynamics are

\[ \frac{\partial^2 \psi}{\partial z^2} = -\frac{1}{\Delta^2} (1 - x^2) \frac{\partial \rho}{\partial x}, \]

\[ \frac{\partial T}{\partial t} + \frac{\partial}{\partial x} (\nu T) = k_h \frac{\partial}{\partial x} (1 - x^2) \frac{\partial T}{\partial x} + \frac{\partial F_r}{\partial z}, \]

\[ \frac{\partial S}{\partial t} + \frac{\partial}{\partial x} (\nu S) = k_h \frac{\partial}{\partial x} (1 - x^2) \frac{\partial S}{\partial x} + \frac{\partial F_s}{\partial z}, \]

where

\[ F_r = k_v \frac{\partial T}{\partial z} - w T, \]

\[ F_s = k_v \frac{\partial S}{\partial z} - w S \]

are the downward components of oceanic heat and salt fluxes and \( x = \sin \lambda, \lambda \) is the latitude. The corresponding heat and freshwater boundary conditions at the surface of the ocean are

\[ F_r = H_{w0} = H_r, \]

\[ F_s = -F_w \]

in ice-free regions, whereas under sea ice
The heat flux experienced by the ocean surface is denoted as $H_{\infty}$, which coincides with the atmospheric surface heat loss/gain $F_{s}$ in the open ocean regions. The two fluxes are not necessarily equal in the regions covered by sea ice except when the whole system is equilibrated. The boundary conditions on salinity (10) and (12) are analogous to their dimensional prototypes (see KD98) and $\Delta_i = ([h]S_i)/(D[S]) \sim 1.2 \times 10^{-3}$. The heat flux boundary conditions (9) and (11) require some explanation. Note that we here use relaxation boundary conditions for ice-ocean coupling, as opposed to the full model where the ocean mixed layer temperature was held fixed at $T_{m} \sim T_{s}$, where $T_{m}$ is the SST. Note that in ice-free regions $T_{m} = T_{s}$, $T_{s}$ being the temperature of the surface underlying the model atmosphere, i.e., either SST or ice surface temperature) and $H_{\infty} = H_{s}$ by definition. This means that $H_{\infty}$ can everywhere be expressed as a function of $T_{m}$. The coefficient $\lambda_i$ in (11) is $\lambda = \lambda^* [T]/[H] \sim 41$. Add to the above equations a convective adjustment scheme for statically unstable density profiles and the oceanic model is complete. Aside from the thermal boundary conditions, the model is thus far identical to the complete model.

The nondimensionalization of the atmospheric and sea ice models is straightforward (see KD98 for a complete formulation of these models). An important parameter in the atmospheric model is the quantity $A$, multiplying the divergence of the atmospheric freshwater transport (KD98); the latter is parameterized as a linear specific humidity flux. Physically, it reflects the size of the atmospheric freshwater catchment basin: the more water carried to the high latitudes by the atmosphere and precipitated there on land enters the ocean, the stronger is the global oceanic freshwater forcing. Technically, we neglect the land freshwater storage and assume that the meridional transport of moisture by rivers is proportional locally to the atmospheric freshwater flux and change the coefficient of proportionality (involving $A$) to model variations in the hydrological cycle. Within such a simple closure for what in reality is a very complicated process, the model climate sensitivity to the hydrological cycle strength can be examined fully.

**b. Reduction of 2D equations**

The structure of the modeled oceanic fields (KD98) as well as observations of the real ocean suggest a natural division of the ocean into two layers. In the upper layer of approximately 1000-m depth, both horizontal and vertical density gradients are fairly large. This layer represents the main oceanic thermocline. Below the main thermocline, there is a larger volume of a comparatively homogeneous water. The imaginary interface between the two layers is nearly horizontal. Define the following averaging operations:

$$Y_u = \frac{1}{\Delta} \int_{-\Delta}^{0} Y \, dz, \quad Y_d = \frac{1}{1 - \Delta} \int_{-1}^{-\Delta} Y \, dz.$$  

The approximate equations governing the evolution of such averaged oceanic quantities are derived in Kravtsov (1998). Here we only list the final forms of the oceanic temperature and salinity equations. The atmospheric and sea ice equations are unchanged relative to those coupled to the more complete 2D ocean model. Figure 1 illustrates the setup of the problem. In addition to subscripts $u$ and $d$, we will also use “+” and “−” to denote $(z$ dependent) quantities in the upper and deep layers of the model ocean, respectively. The two-layer model is completely described by five one-dimensional time-dependent functions $T_u$, $T_d$, $S_u$, $S_d$, and $\psi_o$. The latter variable measures the northward mass transport in the upper oceanic layer. By mass continuity, this transport is compensated by the return flow in the bottom layer. The two-layer model momentum equation is

$$\psi_{0} = \psi - \frac{1}{2} \left( 1 - \Delta \right) \frac{1}{\Delta} \left( 1 - x^2 \right) \left[ \frac{\partial p}{\partial x} + \frac{(1 - \Delta)}{\Delta} \frac{\partial p}{\partial x} \right]$$

(13)
including vertical diffusion effects not explicitly present in the complete system. The vertical structure of the 2D model equations represent a Galerkin-type truncation of the full 2D model where possible to show correspondence between the models.

3. Model climate dependence on hydrological cycle

a. Mean states

To show how quantitatively similar the 2D and two-layer model climates are, we plot distributions of oceanic characteristics for these two versions of the model including sea ice for the $A_e = 1.9$ asymmetric state (Fig. 2; solid lines are used to plot the 2D model values, dashed lines are used for the two-layer model). This state has been fully described in KD98. We will see this is the only linearly stable steady state for this value of $A_e$. In Fig. 2, we use the number of grid points between equator ($n = 0$) and the actual position as a horizontal coordinate. The horizontal grid used in the model is nonuniform, with more grid points in the polar regions to resolve sea ice (see KD98 for more detail). Using grid point number as a horizontal coordinate on the plots allows one to see polar structures more clearly. Contrast two-layer and 2D models, we see from Fig. 2 that in the two-layer model, the upper layer equator-to-pole temperature contrast is slightly smaller. The salinity distribution in the two-layer model is more vertically homogeneous, but the pole-to-pole salinity difference is larger. The difference in the degree of asymmetry can also be seen in the streamfunction plots. The atmospheric structures are almost identical. We will see that the two-layer model qualitative behavior in response to the increasing hydrological cycle is essentially that of the 2D model. This implies that the two-layer model possesses the same main physical properties as the 2D model and supports our idea that the simpler model is physically equivalent to the complete 2D model. The differences between the 2D and two-layer models may be attributed to details of the frictional law formulation and imperfections of the parameterizations. All this results in a slightly different partition between the atmospheric and oceanic branches of the heat redistribution (see below).

In the subsequent discussion, we will mainly concentrate on the two-layer model bifurcation properties. However, these properties will be compared to those of the full 2D model where possible to show correspondence between the models.

b. Response to a variable hydrological cycle

One of the ad hoc parameterizations employed above is the hydrological cycle scheme, that is, complex land and atmospheric moisture pathways have been described by a single parameter $A_e$. This parameter controls the global freshwater cycle strength. Therefore, it is important to explore changes in model steady-state structure and stability for a wide range of $A_e$. Being conceptually simple sensitivity tests, such experiments...
can also be motivated from a physical point of view as paleoclimate simulations. It is generally believed that, for example, the most recent glacial period was characterized by an intensified hydrological cycle. Also, one of the theories of the Younger Dryas event (Broecker et al. 1989) involves the deflection of the meltwater pathway from the Mississippi to the St. Lawrence River. Finally, major changes in the hydrological cycle are expected in response to global warming. We will pay special attention to the ways by which sea ice inclusion modifies model behavior in these experiments.

The two-layer (hereafter 2L) model can quickly be integrated to true numerical equilibrium. In order to take full advantage of its computational efficiency, we first perform a series of experiments with a slowly changing $A_t$. Another reason for doing so is for a comparison with analogous 2D model tests (see KD98); in addition to the 2L model results, all figures in this section contain the corresponding 2D model plots. We have tried several values for $A_t$; rate of change, including $3 \times 10^{-3} \text{ yr}^{-1}$ and $6 \times 10^{-2} \text{ yr}^{-1}$. Additional tests are then performed with fixed $A_t$.

1) CROSS-EQUATORIAL MASS FLUX

In Fig. 3, we plot the $A_t$ dependence of the cross-equatorial mass flux (Fig. 3a is for the 2L model; Fig. 3b is for the 2D model). Circles mark the full (with ice; hereafter FM) model results, and crosses are used for the no ice (hereafter NI) model values. Larger bold markers stand for true numerical equilibria. Note that there are no bold markers in this (Fig. 3b) and all subsequent 2D model plots. This is due to the excessive computational expenses needed to compute equilibrium solutions in the vicinity of the bifurcation point (see below).

At $A_t = 0$, the only possible equilibrium for each model is represented by a pair of thermally driven symmetric cells, as there is no haline forcing to maintain salinity contrasts in the ocean. Naturally, no water exchange exists between the hemispheres.

For sufficiently large $A_t$, the symmetric circulation becomes unstable and gives way to an asymmetric mode. The character of this standard pitchfork bifurcation is supercritical, that is, the nearby asymmetric circulation is stable immediately upon transition and the degree of asymmetry grows with the “distance” from...
the bifurcation point. This is supported in the 2L model by the experiments with the use of fixed \( A_1 \)s. Notice that if the \( A_1 \) value is not fixed but is slowly increasing, symmetric equilibria seem to exist well beyond the bifurcation point. The reason for this is that the linear instability growth rate and initial asymmetric perturbations (caused by round-off computer errors) are small in the vicinity of the bifurcation. Very long integrations (associated with prohibitive computational expenses for the 2D model) are needed to obtain a true equilibrium here. In contrast, if experiments are started from an asymmetric state and slowly decreasing \( A_1 \) is imposed, the bifurcation point is diagnosed fairly accurately. The similarity in this return branch structure between 2L and 2D models points to similar character of models’ nonlinear stability.

In the asymmetric state, the dimensional interhemispheric mass exchange rate quickly reaches the values of 10–20 Sv [1 Sverdrup (Sv) = \( 10^6 \) m\(^3\) s\(^{-1}\)]. A striking difference between the NI and FM models for both 2L and 2D cases is the location of the bifurcation point. It is situated much to the right in the NI model (this effect is more pronounced in the 2D model). Also, to the right of the bifurcation point, the cross-equatorial transport increases faster with \( A_1 \) in the FM model; that is, similar changes in this transport correspond here to smaller increments of \( A_1 \) with respect to the bifurcation location (this will also appear in all other plots shown below, especially in heat transport). Otherwise, the NI and FM graphs look very similar.

2) OCEANIC HEAT CONTENT

Another difference between the NI and FM models can be seen in oceanic heat content. We plot its 2L model values in Fig. 4a in terms of globally averaged oceanic temperature (analogous 2D model results are visualized in Fig. 4b). The much higher heat content (warmer tem-
temperature) of the 2L FM model ocean is due to the fact that deep water temperature cannot be significantly colder than sea water freezing point, whereas no such restriction exists in NI model (to be discussed in much more detail in section 4).

An immediate effect of increased poleward freshwater transport by atmosphere and rivers (i.e., increased $A_t$) is to set up equator-to-pole salinity differences, which slows down thermally driven cells. Decreased heat exchange between low and high latitude tends to produce larger temperature contrasts in the meridional direction. Thus, lower polar temperatures result, which are then advected into the massive deep ocean. This explains negative temperature trend seen in Fig. 4a. Note that the NI model trend is somewhat steeper than that in the FM model, again because of deep water temperature capping effect in the latter.

The structure of the diagrams in the vicinity of the bifurcation point is similar. However, for larger values of $A_t (>3.5)$ more complicated behavior and multiple equilibria can be seen in the NI 2L model results. Similar states also exist in the FM model, but are less noticeable. These multiple equilibria are characterized by different degrees of asymmetry. Changes in the sign of the temperature trend, and other properties of the NI model in this region are likely to be associated with the effects of pole-to-pole circulation dynamics (Wang et al. 1999a, b). Although interesting, such phenomena are beyond the scope of our study. Therefore, we will mainly discuss the 2L model bifurcation diagrams in the region $A_t < 3.5$; also this choice is motivated because physically justified $A_t$ values for $f_w = \frac{1}{3}$ lie within the interval (1, 3) (see KD98).

The results from the 2D model integrations (Fig. 4b) are in qualitative agreement with those from the 2L model, that is, much lower deep water temperatures for the NI model and similar $A_t$ dependences in the strongly asymmetric regimes are seen. Quantitative differences do arise. In particular, the trends in the symmetric mode global temperature are much smaller and even of different sign for the FM model. However, we will see that globally averaged properties are not important in determining the model stability characteristics, but rather those associated with equator-to-pole (or deep ocean versus upper ocean) differences are the important ones. Such properties will turn out to be very similar.

3) GLOBAL ATMOSPHERIC TEMPERATURE

In Figs. 5a,b, the dependence of globally averaged atmospheric temperature on $A_t$ is visualized (2L model results are plotted as circles). In the NI model case, atmospheric temperature does not change. The reason for such a behavior is the efficient atmospheric heat transport. Small changes in atmospheric temperature are then enough to produce compensating responses to external or internal perturbations in the system. The lower atmospheric temperature of the FM model states in Fig. 5a as well as the slight drop ($\sim 0.02 \times 15^\circ C = 0.3^\circ C$; for the 2D model this value is twice as small) of temperature at transition to the asymmetric mode are essentially due to ice. Note that despite the concentration of the sea ice in a relatively small area at the poles, its effect on the atmospheric climate is global. This is again in part due to the high atmospheric diffusion. The small amplitude of the modeled atmospheric temperature changes is most probably associated with the neglect of land ice (cf. KD98; Ganapolski et al. 1998).

4) STRATIFICATION

Although volume-averaged variables (such as, for example, global oceanic temperature) may also point to some of the roots for the differences in models’ behavior, better indicators are represented by quantities characterizing equator-to-pole (or upper versus deep ocean)
differences. In particular, it is useful to form and plot the differences in integral characteristics between top and bottom layers of the ocean.

These dependences are presented in Fig. 6 for temperatures, Fig. 7 for salinities and Fig. 8 for densities [panels marked (a) show 2L model results, panels marked (b) show 2D model results]. In such a global sense, the NI model is characterized by a more stable density profile due to a lower deep ocean temperature (see above). There is difference of approximately 1°C (2°C for the 2D model) in the upper deep ocean temperature contrast between the NI and FM models. Note, however, that the two models share the same salinity contrasts in their symmetric realizations, but the solution for the FM model symmetric state becomes unstable and deviates from this (common) branch at lower $A_t$ value than the NI model solution.

The tendencies in the main model fields outlined above give a picture of the two models behavior and their differences for the changing hydrological cycle, although the information contained in the plots is not complete and the mechanisms determining these differences have yet to be explained.

5) MASS AND HEAT TRANSPORT

Finally, we should mention associated tendencies in some important derivative fields. Hemispheric extremes (absolute values) of streamfunction plots for the 2L model are shown in Figs. 9a,b and for the 2D model in Figs. 9c,d. For the symmetric mode, these extremes have the same value. Transition to an asymmetric state is characterized mainly by a significant drop of overturning strength in one of the hemispheres and only very small increase in another. Notice again the almost identical quantitative tendencies in the symmetric mode regimes of both the NI and FM models (aside from the difference in the bifurcation point location). The NI
model overturning is slightly weaker than that in the FM model (for both 2L and 2D cases), but the trends due to increasing $A_t$ look very similar.

Another important measure of modeled climates is net poleward heat transport. In Figs. 10, 11, and 12, we plot absolute values of total heat transport and its oceanic and atmospheric components at the points where the net heat flux at the top of the atmosphere is zero. The 2L model values are given in panels (a) and (b), 2D model results are displayed in panels (c) and (d) of Figs. 10, 11, and 12. Everything said above about the overturning extrema is directly applicable to the heat transport as well. It can be seen in Figs. 10 and 11 that the changes in net heat transport are

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**Fig. 8.** Upper deep ocean density difference versus $A_t$. A $\bigcirc$ is full model; a $+$ is no ice model. Bold markers are used to denote true numerical equilibria. (a) Two-layer model; (b) 2D model.

**Fig. 9.** Extrema of streamfunction versus $A_t$. Here, $\bigcirc$ is weak overturning hemisphere values; $+$ shows strong overturning hemisphere values. Bold markers are used to denote true numerical equilibria. (a) Full two-layer model; (b) no ice two-layer model; (c) full 2D model; (d) no ice 2D model.
dominated by those in the oceanic heat transport, while the atmospheric transport changes are smaller in amplitude and of different sense than the oceanic heat transport differences. This is consistent with the nearly diagnostic nature of the model atmosphere. Therefore, we should expect the physical explanation for the model behaviors to lie within the ocean model dynamical properties. The major quantitative difference between 2L and 2D models is in the partition of ocean and atmosphere in global heat transport (cf. panels (a) and (c), (b) and (d), respectively, of Figs. 10 and 11). The oceanic heat transport is generally weaker and the atmospheric heat transport is stronger in the 2D model, as compared to those in 2L model. This is the dynamical reason for all other quantitative contrasts between the 2D and 2L models. Different heat transport partitioning in the 2L model is due to crude parameterization of vertical mixing subgrid processes and a slightly different momentum law, which result in the impossibility of perfect 2L model tuning. However, the degree of correspondence between the modeled climates is satisfactory for our purposes of qualitative analysis of sea ice effects.

To complete our discussion of the FM model behavior with respect to an increasing hydrological cycle, we plot the dependencies of equilibrium ice edge positions and ice volumes for each hemisphere (Fig. 13). The ice edge position here is again measured as the number of the grid points between the equator and the actual ice edge position (see above). Significant changes in the ice volume at transition are only seen in one hemisphere. The differences between the 2D (Figs. 13a,b) and 2L (Figs. 13c,d) models are mainly in the ice edge position; the ice volume plots are very similar.

c. Summary

In summary, the FM and NI bifurcation diagrams look very similar. The similarity is almost perfect in the behavior of the mean salinity field for small $A_t$. Here the models are characterized by the same (linear) dependence of horizontally averaged upper deep layer salinity difference on $A_t$ (Fig. 7). The major quantitative difference between the diagrams is in the much stronger thermal stratification of the NI model caused by a lower value of the deep ocean temperature. The sea ice local effect of capping the surface water temperature has a global influence, since polar water properties determine those of the deep ocean. This is why stratification in the FM model under zero haline forcing is less statically stable than that of the NI

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**Fig. 10.** Oceanic heat transport versus $A_t$. Here, ◆ is weak overturning hemisphere values; + is strong overturning hemisphere values. Bold markers are used to denote true numerical equilibria. (a) Full two-layer model; (b) no ice two-layer model; (c) full 2D model; (d) no ice 2D model.
Fig. 11. Atmospheric heat transport versus $A_t$. Here, $\bigcirc$ is weak overturning hemisphere values; $+$ is strong overturning hemisphere values. Bold markers are used to denote true numerical equilibria. (a) Full two-layer model; (b) no ice two-layer model; (c) full 2D model; (d) no ice 2D model.

model. This is due to sea ice phase transition (see section 4). The thermal structure of the symmetric states changes relatively little with increasing haline forcing.

At some value of $A_t$, each model experiences a supercritical pitchfork bifurcation and asymmetric circulations arise. These $A_t$ values are different for the two models, the NI model being destabilized at a much stronger hydrological cycle amplitude. This in a sense points to a decreased nonlinear stability of the FM model. Indeed, let us conduct the following thought experiment. Let us suppose that our models are in steady symmetric states for some sufficiently small strength of a hydrological cycle. In KD98, we have associated symmetric modes with the present climate. Important climate characteristics, such as overturning strength, meridional heat transports, SST, and atmospheric temperature distributions are very similar in the NI and FM models. Now consider finite-amplitude, permanent (positive) perturbations to the hydrological cycle. They may, for example, be associated with a global warming or changes in the ocean’s catchment basin structure. Our results indicate that FM model climate will become unstable for a smaller perturbation compared to NI model climate. Note also that near the bifurcation the degree of asymmetry upon transition grows faster with $A_t$ for the FM model (smaller changes in $A_t$ are sufficient to induce similar changes in model characteristics), as can be seen in Fig. 2, but most clearly in Figs. 10 and 12. This also indicates a decrease in stability (cf. Wang et al. 1999b).

A good quantitative and qualitative comparison between the results from the 2D and 2L models is seen in both the mean states and bifurcation diagrams. The quantitative differences in 2D and 2L model behaviors arise from the different partition of the ocean and atmosphere in the steady-state net poleward heat transport. The impossibility of perfect 2L model tuning is mainly due to simplified parameterization of subgrid vertical mixing processes as well as a slightly different momentum formulation. These differences are immaterial for our purposes of elucidating sea ice climatic influences.

Experiments with the 2D model employing the Rayleigh friction law (not shown) confirm the robustness of our results; that is, the NI and FM model salinity $A_t$ dependences are very close, the NI model deep layer temperature is much lower than that in the FM model, and, finally, the symmetric state supercritical pitchfork bifurcation occurs at a much stronger hydrological cycle if sea ice is suppressed.
4. Linear stability analysis

We have seen that inclusion of the sea ice in the modeled climate system leads to significant changes in its nonlinear (finite amplitude) response to hydrological cycle perturbations. In summary, sea ice appears to increase climate sensitivity. The roots of such behavior can be understood by analyzing the linear stability properties of the model. Based on the results of section 3, we will use the more efficient 2L model for this experimentation. The similarity of the 2D and 2L models in both their mean states and bifurcation diagrams suggests that examining the 2L model linear stability properties is useful for understanding the more complete 2D model stability.

a. Numerical technique

We start with introducing the symbolic form of oceanic temperature and salt perturbation equations. In a standard way, model fields are represented as

\[
T = \tilde{T} + T', \quad S = \tilde{S} + S', \quad u = \tilde{u} + u', \ldots,
\]

(22)

where \(\tilde{T}, \tilde{S}, \tilde{u} \equiv (v, w)\) denote mean oceanic temperature, salinity, and vector velocity fields of the model steady state, the stability of which is explored. Primed variables denote perturbations to this state. A similar decomposition is applied with respect to the atmospheric and sea ice equations. Substituting (22) in the oceanic heat and salt equations and neglecting quadratic terms, we get

\[
\frac{\partial T'}{\partial t} = -\nabla \cdot (T' \tilde{u}) - \nabla \cdot (\tilde{T} u') + L_d T' = \mathbf{A}_1 \left( \begin{array}{c} T' \\ S' \end{array} \right),
\]

(23)

\[
\frac{\partial S'}{\partial t} = -\nabla \cdot (S' \tilde{u}) - \nabla \cdot (\tilde{S} u') + L_d S' = \mathbf{A}_2 \left( \begin{array}{c} T' \\ S' \end{array} \right),
\]

(24)

where \(L_d\) is a linear diffusive operator. The operators \(\mathbf{A}_1\) and \(\mathbf{A}_2\) on the right-hand sides of the above equations depend only on steady-state quantities. Note that \(\tilde{u}\) and \(u'\) depend on \(\tilde{T}\), \(\tilde{S}\) and \(T'\), \(S'\) diagnostically, which is implicit in the last equalities of (23) and (24). Both sea ice and atmospheric equations are also nearly diagnostic, meaning that the associated feedbacks only modify the stability of the model through an influence on oceanic equations, but do not directly determine it. We will discuss this point in more detail below.

The right-hand sides of (23), (24) do not depend on \(t\). Therefore, the general solution of this system can be represented by a sum of particular solutions that depend exponentially on time:

![Fig. 12. Total heat transport versus \(A_t\). Here \(\bigcirc\) is weak overturning hemisphere values; \(+\) is strong overturning hemisphere values. Bold markers are used to denote true numerical equilibria. (a) Full two-layer model; (b) no ice two-layer model; (c) full 2D model; (d) no ice 2D model.](image-url)
Substitution of (25) in (23), (24) gives
\[ AX = \lambda X, \]
where
\[ A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \]
and \( X \) is a vector containing spatial structure of the perturbation temperature and salinity, that is, it is the so-called linear eigenfunction of the equations corresponding to an eigenvalue \( \lambda \). Also, \( X \) and \( \lambda \) are in general complex. Below, talking about the structure of eigenfunctions, we will assume the real part of \( X \exp(\lambda t) \) is considered. Equation (26) is solved with proper boundary conditions. The steady solution of the problem is unstable if there exists an eigenvalue \( \lambda \) with a positive real part.

Given \( A_1 \), we use the “breeding” technique to find the most unstable (least stable) eigenfunction and its eigenvalue (Toth and Kalnay 1991). By definition, this eigenvalue \( \lambda \) is the most positive in the unstable regime or the least negative if the flow is stable. Isolating the most unstable eigenmode permits us to analyze the mechanisms of model linear stability.

A remarkable difference between NI and FM models is the location of the bifurcation, namely, the NI model is destabilized at higher values of hydrological cycle strength parameter \( A_t \) (see section 3). To explain this effect we will need to consider the NI and FM model eigenmodes in detail.

b. Comparison between no ice and full model: Global feedbacks

The differences in model behavior are most clearly demonstrated if we compare the two models for \( A_t \) values just beyond their respective bifurcation points. Rather arbitrarily, we pick the FM model state corresponding to \( A_t = 1.75 \) and NI model state corresponding to \( A_t = 2.6 \) for such comparison. The analysis shows that the NI model state here is slightly less unstable (perturbation grows slower) than the FM model state, meaning that the contributions of stabilizing and destabilizing feedbacks of the models are slightly different. This difference will not affect our conclusions.
1) CONTROL MEAN STATES FOR THE STABILITY ANALYSIS

Before considering the properties of the eigenfunctions and anomalous balances, it is useful to see how different the (unstable) reference steady solutions are. These states are plotted in Fig. 14. The distributions of upper (solid line) and deep (dashed line) oceanic layer temperature appear in Fig. 14a, SST/IST (solid line) and atmospheric temperature (dashed line) in Fig. 14b, upper (solid line) and deep (dashed line) layer salinity divided by $A_t$ in Fig. 14c, and model streamfunction in Fig. 14d. Bold lines are used to plot the FM model values. It is immediately obvious that the two states are very similar. The correspondence in salinity is due to the normalization of the salinity field by $A_t$ (see Fig. 7). Aside from differences in the salinity field (steeper salinity gradients in the NI model), another significant difference is in the value of abyssal water temperature. It is close to sea water freezing point in the FM model and is much lower in NI model. Surface ocean structure as well as atmospheric structure are similar in the two models. The overturning magnitude in the FM model slightly exceeds that of the NI model. In general, the biggest differences between the states are seen at the poles, whereas equatorial fields look very similar.

If the climates look so similar, why is the much larger mean field salinity gradient characteristic of a stronger hydrological cycle needed to destabilize the NI model? Also, what is the physical role of the difference in deep water temperatures? To answer these questions we consider anomalous fields and analyze the feedbacks responsible for the model behavior.

2) ANOMALOUS FIELDS AND BUDGETS IN NON-POLAR REGIONS

As suggested by previous studies (see section 1), the source of the symmetric mode instability is localized in near-equatorial region. Therefore, we here first concentrate our attention on the balances in the “central” part of the basin, which does not include the regions of strong polar downwelling. These sinking regions are fairly narrow, so in fact the central portion covers most of the basin.
(i) Active role of temperature in the instability

First note that the FM model overturning magnitude is stronger than that in the NI model (Fig. 14d). This is generally true for all values of $A_t$ where symmetric modes exist, meaning that the stabilizing, direct salinity feedback (see section 1) is stronger in the FM model. Thus, it does not explain the destabilization of the FM model at smaller values of hydrological cycle strength.

Ocean anomaly fields from the most unstable eigenfunction structure of each model are plotted in Fig. 15. Figure 15b shows an asymmetric, nearly vertically homogeneous salinity perturbation profile. This is analogous to a result in Vellinga (1996). Note how similar the anomalous salinity fields are in the NI andFM models. Subject to such virtually identical salinity perturbations, the two models nonetheless possess anomalous overturning fields of very different strength (Fig. 15d). The FM model anomalous overturning is much stronger and here lies the explanation for the differences in the models' behavior: The stronger FM model overturning perturbation associated with similar salinity perturbations increases the importance of the destabilizing indirect salinity feedback. Therefore, a smaller mean salinity gradient is needed for the influence of the destabilizing indirect feedback to become dominant and overcome the action of stabilizing effects.

To complete this argument, it is also necessary to show that the differences in stability of the NI and FM models are not caused by temperature feedbacks. Note that changes in $S_u - S_d$ with $A_t$ are larger than those of $T_u - T_d$ (cf. Figs. 6 and 7). The quantity $S_u - S_d$ affects the strength of the indirect salinity feedback, consistent with the above explanation. In Fig. 6, a slightly faster trend in $T_u - T_d$ of the NI model, as compared to that of the FM model. We will see below that the indirect heat feedback [see (1)], analogous to the indirect salinity feedback, is stabilizing. Therefore, a faster increase in $T_u - T_d$ with $A_t$ in the NI model also increases its stability. We thus see that the NI model is inherently more stable than the FM model. However, analyzing the mean state tendencies is not sufficient to decide whether the differences in stability between the NI and FM models are due to salinity and not due to temperature. We must consider the anomalies. Such an analysis below will show that the roots of instability are indeed in the perturbation salinity balance and not in the perturbation temperature balance, confirming the results by Vellinga (1996).
The differences in anomalous overturning are mainly due to differing anomalous temperature fields. In principle, both upper- and bottom-layer temperature anomalies contribute to the anomalous overturning through their effect on the layers density. In Fig. 15c, the slope of the upper-layer density profile for the NI model (thin solid line) is larger than that in the FM model (bold solid line) and the inverse is true for the bottom-layer densities (dashed lines). Therefore, the larger anomalous cross-equatorial transport in the FM model (Fig. 15d) is due to bottom-layer density.

The upper-layer anomalous temperature structures for the two models are similar; thus, the difference in the deep oceanic density (Fig. 15c) is associated with that in deep layer temperature (Fig. 15a). The abyssal temperature gradient is relatively weak in the FM model compared to the upper-layer gradient. Conversely, it is quite noticeable and comparable with surface water anomalous temperature gradient in the NI model. Therefore, as meridional salinity anomaly gradients are similar, a larger depth-integrated meridional density gradient results in the FM model, meaning anomalous overturning is stronger [(cf. Eq. (13)].

Note that in the above explanation of the stability differences between models with and without ice, temperature plays an important active role. This can happen because, in a coupled model, temperature anomalies are no longer constrained by a strong relaxation. In contrast, in ocean-only models (Marotzke 1990; Vellinga 1996), we would expect no change in stability if sea ice is artificially suppressed. Scott et al. (1999) discuss the role of the air–sea feedbacks in the interhemispheric stability (see also Marotzke 1996; Rahmstorf 1996).

### (ii) Heat and salt budgets in the central region

Let us consider heat and salt budgets in some more detail. Graphic representations of the salt budget for the FM and NI models are displayed in Fig. 16.
(iii) Mean salinity budget; linear dependence on hydrological cycle

In Figs. 16a and b, the mean state balances for the FM and NI models, respectively, are illustrated. Naturally, the atmospheric freshwater forcing of the ocean (circles) is balanced by advective and diffusive oceanic transports.

To the first approximation, the atmospheric temperature profile does not change with $A_t$ (at least in equatorial regions). The polar and subpolar atmospheric changes do not cause significant corrections to the freshwater budget as well, due to the generally very low atmospheric moisture capacity in this region. Finally, the main thermally driven symmetric overturning decreases only slightly with increased hydrological cycle. In short, the shape $F(x)$ of the atmosphere–ocean freshwater flux and the structure of the symmetric circulation $\psi_0(x)$ are nearly independent of $A_t$. The general form of the steady-state salinity equation is

$$\square S = A_t F(x),$$

where operator $\square$ depends on $\psi_0$ and $x$, but not on $S$ or $A_t$. It then follows that $S/A_t$ does not depend on $A_t$, meaning that changes in mean salinity field and salinity budget are linear with respect to $A_t$.

The similarity of the mean fields in NI and FM models ensures this dependence is virtually identical in the two (cf. Fig. 7). So again, the convenient fact is that for the symmetric mode, the shapes of all salinity related steady fields remain the same, and only their amplitude increases linearly with $A_t$. This linear connection between the hydrological cycle strength and mean field salinity contrast is largely true also for the asymmetric mode.

(iv) Anomalous salinity budget; Stabilizing and destabilizing feedbacks

We now turn to anomalous salinity budgets plotted in Figs. 16c,d. First, notice that these are linear eigenfunctions that are plotted, so their amplitude has no physical meaning. It is the relative magnitude of the anomalous fields that is important. We will refer to each term in the linearized anomalous salinity and temperature budget equation as representing a certain feedback. In accordance with Vellinga (1996), the terms in the salt budget equation that tend to erase the interhemispheric salinity contrast, driving the instability, degrade the unstable eigenmodes and are, thus, stabilizing, whereas those amplifying the perturbation are destabilizing.

Not surprisingly, we see from Figs. 16c,d that the relative contributions of different feedbacks are very similar in the two models, which is related to the fact that we are comparing the states from analogous locations in the bifurcation map.

The fact that the diffusion influence is stabilizing provides us with an easy indication of a salinity feedback effect on the system. In general, if it is in the same sense as the diffusive effect, the feedback is stabilizing, and vice versa.

In Figs. 16c,d, the divergence of anomalous diffusive salt transport is plotted by dots. We thus see immediately that the direct salinity feedback [solid lines; see (2)] is stabilizing and that there are two destabilizing feedbacks. The first one is the indirect salinity feedback [dashed lines; see (2)] discussed above (Vellinga 1996). We have seen that this is the key feedback in explaining the differences between the NI and FM models. Despite very different mean salinity gradients in the two, this effect has similar relative magnitude in the budgets plotted in Figs. 16c,d, due to the different anomalous overturning strengths between the models (Fig. 15d).

The second destabilizing feedback is due to an anomalous divergence of atmospheric moisture transport (circles in Figs. 16c,d). This is the so-called EMT (eddy moisture transport–thermohaline circulation) feedback, after Nakamura et al. (1994). We see that in our models this feedback is quite important in determining overall model stability properties, so that excluding it will undoubtedly lead to an increase in model stability (cf. Saravanan and McWilliams 1995; Lohmann et al. 1996). However, the modifications to the EMT feedback caused by the sea ice inclusion are insignificant, as is seen by comparing Figs. 16c and 16d.

(v) Heat budgets

The basic-state heat budgets are qualitatively analogous to those of salinity and are again very similar in the two models. The counterintuitive influence of the temperature and salinity feedbacks on the overall climate linear and nonlinear stability was discussed by Marotzke (1996) and Scott et al. (1999). Their discussion applied to our model implies that those temperature effects that tend to reduce the cross-equatorial mass exchange (which maintains interhemispheric salinity contrasts, the main feature of the instability), are stabilizing and vice versa. The anomalous cross-equatorial flux is driven by the interhemispheric salinity differences and is opposed by the interhemispheric temperature differences (Fig. 15). Therefore, those terms in the perturbation heat budget equation that flatten the anomalous temperature profile (e.g., diffusion) are destabilizing.

The biggest contributors to the anomalous heat budget in both models (not shown) are the stabilizing indirect heat feedback [i.e., the anomalous advection of the mean temperature gradient; see (1)], and the destabilizing anomalous ocean–atmosphere heat flux, as indicated by relative sense of these terms and the destabilizing diffusive effect. The destabilizing ocean–atmosphere anomalous heat flux feedback can be related to atmospheric anomalous heat budget and is essentially the well-known atmospheric heat transport feedback, which involves the removal of SST anomalies by effective atmospheric diffusion (e.g., Marotzke and Pierce 1997). We now discuss the stabilizing feedback.
Recall that the anomalous overturning has a smaller magnitude in the NI model case (Fig. 15). How is it that the indirect heat feedback has the same relative importance in the NI and FM models (not shown)?

To answer this question, let us write an explicit expression for the indirect heat feedback in the perturbation heat budget of our 2L model. In the upwelling region it is

$$\frac{\Delta \partial T_d}{\partial t} = -\frac{\partial}{\partial x} \psi_0(T_u - T_d) + \frac{\partial}{\partial x} \psi_0(T_u - T_d) + \frac{\partial}{\partial x} T_d + \ldots$$

where primes are used to denote anomalies to model fields. The last term in the third line can be neglected as \(\frac{\partial T_d}{\partial x} \approx 0\) (see Fig. 14a).

Jayne and Marotzke (1999) argue that the differences in the destabilizing indirect heat feedback were responsible for the decreased stability of their full box model relative to their no ice model. In contrast, our results show this feedback is the same for the two models. This is because the abyssal−surface mean temperature difference is smaller due to sea ice. Thus, a smaller temperature difference coupled to the stronger FM anomalous overturning produces negligible change in the indirect heat transport effect (see the above formula). This formal distinction between the results of Jayne and Marotzke and ours is due to a different formulation of the problem.

In general, our results indicate that global oceanic and atmospheric feedbacks represented by highly truncated box models do exist and work similarly in higher-resolution models. However, in a highly truncated box model, it is not really possible to distinguish between global and local feedbacks, whereas in fact sea ice structures are concentrated at the poles and can only affect global feedbacks indirectly. One then should interpret with caution the properties of the low-resolution box models that crucially depend on the parameterization of local effects. We should also acknowledge that the model we use is by itself a highly parameterized and simplified version of the “true” dynamics. Many of the important feedbacks are simply not represented (lead formation in the ice, ice dynamics, wind-driven oceanic circulation, details of the atmospheric hydrological cycle, etc.). However, the inclusion of these effects is not possible without a significant change in the model’s level of complexity. The corresponding modifications and their effect on long-term climate remain the subject for future studies.

(vi) Summary

Thus, the differences between the FM and NI model stability are due to global advective feedbacks. The indirect sea ice influence on these feedbacks is accomplished through its effect on deep water temperature. The effect is twofold, both steady-state temperature and anomalies being affected. To understand the mechanics of this effect, we need to consider local polar balances.

c. Local feedbacks

1) Negligible role of the albedo and brine rejection effects

We begin the discussion with brine rejection and sea ice−albedo feedbacks. To check if the former is important, it is artificially suppressed and the linear stability experiments are repeated. The eigenvalues and the spatial structure of the eigenfunctions are nearly unchanged. Thus, brine rejection can be safely neglected. This is not unexpected given the very long climatic timescales of the effects we are modeling (e.g., Zhang et al. 1995; Jayne and Marotzke 1999). The albedo effect appears to be an important contributor in determining the ice edge position and may in principle contribute to the steeper slope of the FM bifurcation curves (e.g., Fig. 2) in the region right beyond the bifurcation point. However, in the linear stability experiments, it turns out that the sea ice edge does not move. Thus, the ice−albedo feedback cannot be significant. (Recall again that land ice is neglected in our model).

2) Sea ice and narrow sinking

Let us look at the mean structure in the polar regions more carefully (Fig. 14). In the model without sea ice, surface water is cooled during its northward transit. At the poles, the model must sink (Fig. 14d; thin line). The deep water is the coldest and the heaviest (Fig. 14a), as its temperature (density) properties are gained at the surface location where downwelling occurs. An intriguing feature of steady oceanic circulation, consistent with the observed oceanic climate, is narrow intense polar downwelling regions and wide zones of deep water upwelling at lower latitudes. Stommel (1962), using a multipipe model of convection, has shown that similar asymmetry develops and becomes progressively more pronounced as the importance of density advection increases relative to vertical diffusion (see also Rossby 1965; Winton 1995).

Contrast this with the full model. Due to sea ice phase transition, the oceanic temperature cannot be colder than sea water freezing point. In ice-covered regions, the ocean−ice heat flux, \(\lambda^* (SST^* - T^*)\), is positive, indicating oceanic heat loss through the ice pack. Therefore, \(SST^* > T^*\). The surface oceanic heat loss in the ice-covered regions is small, because sea ice conductivity is inversely proportional to the ice thickness. This leads to effective insulation of the ocean from atmosphere, allowing the latter to acquire the low temperatures required to satisfy the heat balance at the top of the atmosphere.

In steady state, the poleward heat transport into the
ice-covered ocean is small, consistent with insulation by rigid boundaries and sea ice. The water in this region therefore has a nearly homogeneous temperature, which naturally is close to sea water freezing point. Consider a hypothetical initial value experiment of our FM model from uniform oceanic temperature and salinity. As in Rossby (1965), sinking will occur at the pole and upwelling at lower latitudes, the main thermocline will develop and heat will penetrate diffusively into the deep. However, the physics of the polar region will be different due to ice. Sea ice formation will prevent cooling of polar oceanic water below freezing and all extra heat will be stored as latent heat of fusion. Sea ice thickness will increase, leading to eventual near insulation of the ocean from the atmosphere. The homogeneous water density under the ice makes this portion of the ocean energetically inactive: the motions here do not involve net heat transports and buoyancy work. These properties mean that the sinking in the thick ice-covered region is only determined by the kinematics of the flow and not by the integral dynamic and thermodynamic requirements controlling open water sinking (e.g., Rossby 1965). In particular, the width of the sinking region can in principle be as wide as or larger than the sea ice extent.

The near absence of flow under sea ice shown in our model results is essentially due to the special form of our momentum equation, relating the meridional density gradient to the vertical shear of horizontal velocity. There is nonetheless a small (factor of 1.5) difference in the area of the sinking regions between the NI and FM models in our model solutions as well.

Under the momentum formulation employed in our study, the main oceanic sinking occurs just equatorward of the ice edge (Fig. 14d; thick line). Here, due to strong atmospheric heat transports off the ice pack, the ocean surface is subject to strong heat loss. As in the NI model, this heat loss is consistent with the strong oceanic downwelling here. In contrast to the NI model, however, part of this heat loss contributes to cooling fluid that advects under ice, rather than only to balancing upward heat transport by sinking motions. Note again that insulation and phase transition effects are correlated: changes in the ice thickness caused by phase transition affect sea ice conductivity.

3) THE SEA ICE EFFECT ON DEEP TEMPERATURE ANOMALIES: LOCAL ADVECTIVE MECHANISM AND STRONG TEMPERATURE RELAXATION UNDER SEA ICE

We now consider the anomalous fields and balances in polar regions. Figure 17 is an extension beyond the central regions shown in Fig. 15 to include the poles. Eigenfunction fields are displayed. Without loss of generality we pick the “left” hemisphere of the model earth for analysis. As expected, the NI model temperature anomalies do not exhibit much structure at the poles. The deep water temperature anomalies here are determined by polar SST anomalies. Polar surface regions are thermodynamically equivalent to any other surface location in the basin, implying that temperature anomalies are free to penetrate into the deep.

In contrast, the FM model temperature anomalies vanish at the poles (Fig. 17a). Large SST anomaly gradients over the ice edge result in corresponding density gradients and a strong polar anomalous overturning cell (Fig. 17d). We stress again that the meridional spatial scale of the ice-covered region is very small, thus local effects determine the difference in NI and FM anomalous temperature fields.

The key factor in this behavior is the strong damping of SST anomalies under ice. Note that the differences in the steady-state temperature field between the NI and FM model discussed above do not crucially depend on the large value of \( \lambda^* \). In fact, a smaller \( \lambda^* \) would result in even larger differences between the deep water temperatures of the NI and FM models, as the FM model deep ocean would become warmer. [The amplitude of ice–ocean coupling, however, may indirectly affect the role of brine rejection in the overall stability of the system, cf. Yang and Neelin (1997). We have seen that in our model brine rejection effect is negligible.]

SST anomalies under ice tend to be weakened by strong negative feedback. Still, a very small-amplitude cold anomaly survives under the thinnest ice portion, cooling down the base of the ice and providing the tendency for sea ice growth. The deep water temperature anomaly is determined by the corresponding surface temperature anomaly at the location where the major sinking occurs. In our model, this location is situated just equatorward of the ice edge (and not under ice). It can be demonstrated that coupled ice–atmospheric feedbacks in the near-ice edge region contribute to amplify the SST anomaly in the sinking region. Oceanic diffusion is without question too weak to explain the observed damping of SST anomalies (Fig. 15a) equatorward of the ice edge and deep water temperature anomalies. Thus, the damping is associated with the anomalous advective overturn. This is seen in Fig. 17d. The anomalous cell has a positive sense (upwelling under ice, downwelling equatorward, consistent with the anomalous temperature, and, therefore, density distribution) and is very intense. To understand the local mechanics of damping, we consider the anomalous temperature balances in the upper and deep oceanic layers in the polar downwelling regions. It can easily be shown that in the downwelling region just equatorward of the ice, the major advective contributors to the surface and bottom layers heat balance can be expressed as

\[
\frac{\Delta T''}{\partial t} \sim -\psi_0 \frac{\partial T''}{\partial x} - \phi_0 \frac{\partial T''}{\partial x},
\]

\[
(1 - \Delta) \frac{\partial T''}{\partial t} \sim \phi_0 \frac{\partial T''}{\partial x} + \psi_0 \frac{\partial T''}{\partial x}.
\]
In deriving the above expressions we have used the fact that $T_u \sim T_d$ and $T_u' \sim T_d'$ in the downwelling region (see Figs. 14 and 15). Note that the first term on the right-hand side of the upper-layer equation is negative, meaning that this feedback also amplifies the cold temperature anomaly just off the ice edge. Therefore, this anomaly, originating at lower latitudes, gets advected into the deep ocean by the strong mean downwelling (second term on the right-hand side of the upper-layer equation). The deep ocean anomaly damping is dominated by the anomalous advection of the mean temperature by the perturbation overturning (first term in the bottom-layer equation). Therefore, the following picture of the suppression of the deep ocean temperature anomalies in the model with sea ice emerges: the damping of temperature anomalies under ice results in a strong perturbation cross-ice edge temperature gradient, driving the strong local anomalous overturning. This provides an effective anomalous stirring of the polar region and allows the strong ocean–ice surface temperature damping (modeling the sea ice phase transition) to affect the whole region in the vicinity of sea ice.

Note that if a considerable portion of the sinking occurs under sea ice (which is possible with different momentum laws), then the strong SST relaxation due to phase change damps the deep ocean temperature anomalies directly (in contrast to indirect damping through anomalous overturning discussed above). Therefore, the ultimate cause of the suppression of the deep oceanic layer temperature anomalies is strong ice–ocean coupling, independent of the form of the oceanic momentum equation.

4) **Summary**

In summary, due to sea ice phase transition, the deep water temperature is always near sea water freezing point and is much lower in the model without ice. This is because the heat, used to cool the particles below freezing in the no ice model, is stored as the latent heat of fusion if sea ice is included. Phase transitions and sea ice growth are accompanied by sharp drop in sea ice conductivity, which is inversely proportional to ice thickness, and, thus, effective insulation of the ocean from the atmosphere takes place. All of this results in the formation of a homogeneous body of water under ice in the steady state, so that the ice-covered ocean is energetically passive. The underice sinking is, therefore, purely “kinematic” and its width is only constrained by the sea ice extent. In contrast, the narrowness of the
sinking in the open ocean follows from the global energetic requirements, involving weakness of vertical diffusion relative to density advection effects.

The strong damping of SST anomalies under ice results in suppressing the deep ocean temperature anomalies either directly, if the sinking occurs under ice, or by creating steep small-scale cross–ice edge SST gradients, which produce intense secondary anomalous advective over-turns. This perturbation circulation damps deep temperature anomalies. These local feedbacks have a global influence, thereby affecting global feedbacks as discussed in the preceding section.

d. Random forcing experiments

None of the models discussed in this study were found to exhibit nonlinear, self-sustained variability; however, damped oscillatory modes were found. Such modes can be excited by forcing the system with a random, noisy signal. This signal is usually interpreted as “weather.”

To explore possible forced oscillatory regimes in our model, we introduce anomalies to the oceanic salinity field by a random component to the hydrological cycle \( A_t \). The value of the anomalous hydrological perturbation in each hemisphere was changed randomly every time step \( \Delta t \) (several days) and the maximum amplitude of this change is half of the nonrandom amplitude. If, for example, the maximum ocean–atmosphere freshwater flux is \( 1 \text{ m yr}^{-1} \), the random perturbation is between \(-0.5\) and \(0.5 \text{ m yr}^{-1}\).

Using the numerically efficient 2L model allows very long integrations to be performed, ensuring correct representation of low-frequency variability. All experiments consistently show that the main contribution to the ocean spectrum is in the “advective” range (associated timescale in our model is several hundred years). The absence of any singularities in the spectrum at low frequencies argues that the oscillatory linear advective eigenmode is the main feature of the model response to white noise perturbations. These oscillations are analogous to linear oscillations found by Mysak et al. (1993) and discussed by Griffies and Tziperman (1995). Sea ice is not an essential ingredient in these oscillations as indicated by the presence of the latter in the no ice model (cf. Yang and Neelin 1997; Jayne and Marotzke 1999).

5. Summary and discussion

The transition from the symmetric to asymmetric climate at a sufficiently strong hydrological cycle (controlled by \( A_t \)) is characterized by a supercritical pitchfork bifurcation. The main effect detected in simulations is the instability of symmetric circulation at much lower value of \( A_t \) in the model including sea ice. An analysis of mean fields at sufficiently small \( A_t \) reveals two major properties of no ice and full model steady states. First, the dependence of an equator-to-pole (equivalently, surface-to-abyss) salinity contrast on \( A_t \) is linear and virtually identical in the two models. The reason for this is the weak dependence of the atmospheric temperature profile (and, therefore, basic freshwater flux shape) and thermally driven overturning on \( A_t \). The two dependences are very close due to the general similarity of the the upper oceanic layer fields in the models. Second, the deep ocean temperature in the full model is near the sea water freezing point and it is much lower in the no ice model. This is a consequence of the sea ice phase transition.

The general structure of the oceanic thermal forcing is such that surface particles are cooled as they travel toward the pole. An excessive cooling of the surface water, which happens if sea ice is artificially suppressed in the model, is prevented by taking up extra heat to form ice. The increase in ice thickness results in the sharp drop in sea ice conductivity, which leads to the effective insulation of the underice ocean from the atmosphere in a steady state. Such sequence leads in turn to the homogenization of water under ice. Therefore, the sinking occurring under ice cannot contribute to the global oceanic heat redistribution and the structure of this sinking (its width in particular) is determined by the sea ice extent and kinematic characteristics of the flow.

The sinking region in the open ocean is narrow as a consequence of integral constraints on the global oceanic energy budget involving nonlinear density (temperature) advection and the relative weakness of vertical diffusion. Although the relative width of the sinking region is somewhat larger in our model including sea ice, our special momentum formulation, where vertical shear of horizontal velocity is connected to the horizontal density gradient, restricts the motions under ice. For more general forms of momentum law, more intense underice sinking can be realized. Major sinking in our model occurs right beyond the ice edge and is consistent with strong heat loss by the surface ocean to the atmosphere due to cold air transport off the ice pack.

Numerical linear stability analysis of the two models reveals the mechanisms responsible for differences in the behavior. Symmetry breaking roots are identified by considering the anomalous salinity budget near the transition point. The main difference between the no ice and full models is in the magnitude of anomalous overturning resulting from identical salinity perturbations. Such perturbations are accompanied in the no ice model by compensating deep water temperature anomalies of similar spatial structure. These are absent in the full model. Therefore, a smaller mean salinity gradient (hence, smaller \( A_t \); see above) is required to destabilize the model by anomalous overturning. Note the active role of temperature in the mechanism of instability. This is due to atmospheric coupling (cf. Marotzke 1990; Vellinga 1996; Marotzke 1996; Scott et al. 1999).

The damping of deep water temperature anomalies in the full model is a consequence of strong ice–ocean thermal coupling (damping of SST anomalies under sea
ice). When SST anomalies originating in lower latitudes approach the sinking region, steep cross–ice edge anomalous density gradients are induced. Strong anomalous overturning results and damps the temperature anomalies in the polar region. Were the major sinking to occur under ice, the deep ocean temperature anomalies would be damped directly by the same strong relaxation condition at the bottom of the ice. Thus, in both cases and independent of the exact form of the momentum equation, the damping of the deep ocean temperature anomalies is due to strong relaxation of SST under ice, modeling sea ice phase transition.

The anomalous upper ocean temperature balance in both models is between the destabilizing influence of atmospheric heat transport feedback and the stabilizing advection of mean temperature gradient by anomalous overturning. This by itself, however, does not lead to a decreased stability of the full model, as deep water temperature is warmer (and upper- versus bottom-layer temperature difference is smaller) providing necessary compensation.

Brine rejection effect was found negligible. Sea ice albedo effect is important in determining ice edge position and thus may play a role in the differences in models’ nonlinear stability, expressed through faster development of the degree of asymmetry in the full model. However, in the initial stages of asymmetry development it does not matter. Thus, sea ice has been found to increase climate sensitivity.

This is in agreement with the box model results of Jayne and Marotzke (1999), but different from the conclusion reached by Lohmann and Gerdes (1998), who considered a 3D one-hemisphere model of the thermohaline circulation. Wind forcing and simple bottom topography were also included in the latter. The Lohmann and Gerdes study concentrated on polar interactions between the sea ice, oceanic convection, air–sea feedbacks, and the thermohaline flow. This emphasizes a different aspect of the climate, which involves, among other things, shorter (decadal and interdecadal) timescales. In contrast, our, in a sense, much simpler study (as well as Jayne and Marotzke 1999), addresses a conceptually different problem of the mechanics behind the global climate transitions of the past, which we argue to be possibly an advective phenomenon. The two sets of interactions are not exclusive. Complete analysis of climate behavior should include all possible spatial and temporal scales of interactions within the system. The idea of the oceanic advective trigger to global climate switches forced by the slowly varying insolation should be checked using long integrations of more complete models (such as that of Lohmann and Gerdes 1998).

The damped oscillatory (advective timescale) response has also been detected in the no ice and full models [consistent with previous studies: Mysak et al. (1993), Tziperman et al. (1994), Griffies and Tziperman (1995)]. This allows us to suggest a “purely advective” scenario of such paleoclimatic phenomena as the Younger Dryas event, which differs from the convective scenarios suggested elsewhere. Here the large-scale response of the ocean to the global perturbations, not localized in polar regions, leads the global climate. Broecker et al. (1989) argued that the Younger Dryas was in part caused by the deflection of freshwater pathways from the Mississippi to the St. Lawrence River. Such an asymmetric perturbation to the basic hydrological cycle could launch a sufficiently strong advective anomaly whose thermal influence on North Atlantic climate would be amplified by advancing sea ice cover (i.e., sea ice is essential for this interpretation). The duration of such an event (several hundred years) is consistent with the advective timescale and the ultimate onset of interglacial conditions is guaranteed by the damped nature of the oscillation. The connection between Arctic and Antarctic, which are clearly out of phase in our experiments during the oscillation (warmer Atlantic is in phase with colder Antarctica due to opposite signs of ice cover changes) was recently noticed in the paleoobservations (Broecker 1997). Another interesting feature of the paleorecord is a distinct preference for centennial oscillations during glacial, although these events can also be noticed under interglacial conditions, or even in the remote past, where the global temperature contrasts and continental boundaries were quite different (Broecker 1997). This suggests that these phenomena are indeed driven by the oceanic heat–salt interactions. The changes in the probability of centennial climate oscillation for different climates may be associated with changes in the net amplitude of random atmospheric forcing. In particular, larger mean atmospheric temperature contrasts during glacial might mean that the intensity of baroclinic disturbances in the glacial atmosphere was increased, thus providing intensified anomalies to the glacial ocean. Finally, it would be interesting to compare temperature–salinity phase relations during such oscillation with paleorecords (Fairbanks 1989).

Last, contrasting our model results with analogous box model studies by Jayne and Marotzke (1999) and J. R. Scott and J. Marotzke (1997, personal communication) suggests that sufficiently high horizontal resolution is a key factor in determining climate model linear and nonlinear stability, in particular the global properties of climate transitions. In a highly truncated box model, Scott and Marotzke find a subcritical bifurcation, as opposed to our own supercritical bifurcation. This result is however ambiguous. The 3D GCM study of Klinger and Marotzke (1999) shows the behavior consistent with the low-resolution model of Scott and Marotzke in terms of the bifurcation structure. This could indicate that local character of the momentum parametrization employed in high-resolution zonally averaged models is deficient (see KD98). This is also a subject for future studies.

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REFERENCES


