NOTES AND CORRESPONDENCE

Testing for a Trend in a Partially Incomplete Hurricane Record

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ABSTRACT

The record of annual counts of basinwide North Atlantic hurricanes is incomplete prior to 1946. This has restricted efforts to identify a long-term trend in hurricane activity to the postwar period. In contrast, the complete record of U.S. landfalling hurricanes extends back to 1930 or earlier. Under the assumption that the proportion of basinwide hurricanes that make landfall is constant over time, it is possible to use the record of landfalling hurricanes to extend a test for trend in basinwide hurricane activity beyond the postwar period. This note describes and illustrates a method for doing this. The results suggest that there has been a significant reduction in basinwide hurricane activity over the period 1930–98.

1. Introduction

The analysis of the historical record of basinwide North Atlantic hurricane activity is limited by the unreliability of the record prior to the advent of regular aircraft reconnaissance in 1946 (Neumann et al. 1993; Elsner and Kara 1999). For example, in testing for a trend in the basinwide frequency of North Atlantic hurricanes, Landsea et al. (1999) confined the analysis to the postwar period. In contrast to the record of basinwide hurricane activity, by virtue of their conspicuousness, the record of landfalling hurricanes in the United States is believed to be complete back to 1930 or beyond. Here, a landfalling hurricane is defined as any tropical storm making at least one landfall on the United States at category 1 or higher. As shown below, the proportion of basinwide hurricanes that made landfall has remained relatively stable since 1946. Under the assumption that this stability extends back beyond 1946, this raises the possibility of using the earlier record of landfalling hurricanes to extend the test for trend in basinwide hurricane activity as well. The basic idea is that, in conjunction with the complete record of landfalling hurricanes, knowledge of this proportion provides information about the number of basinwide hurricanes in the incomplete part of the record. In this note, this idea is formalized to test for a trend in the number of basinwide North Atlantic hurricanes over the period 1930–98. The same basic idea was used informally by Fernandez-Partagas and Diaz (1996) to estimate overall North Atlantic hurricane activity for the period 1851–1900. In contrast to results for the postwar period, the analysis presented here suggests that there has been a decline in activity over the 1930–98 period.

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2. Model and method

Consider the observation period $t = 1, 2, \ldots, n$, and let the random variable Y_t be the true basinwide number of hurricanes in year t. We assume that Y_t has a Poisson distribution with mean:

$$\mu_t = \exp(\mu_0 + \mu_1 t), \tag{1}$$

where μ_0 and μ_1 are unknown. Interest centers on testing the null hypothesis H_0: $\mu_1 = 0$ that there is no trend in the mean number of basinwide hurricanes against the general alternative hypothesis H_1: $\mu_1 \neq 0$ that there is a trend of this form.

Let the random variable X_t be the number of landfalling hurricanes in year t. We will assume that the conditional distribution of X_t given $Y_t = y_t$ is binomial with parameters y_t and p, where the unknown probability p that a hurricane makes landfall is assumed to be constant over the observation period. This assumption is critical to the method described here and some evidence to support it is given in the following section.

Finally, let the random variable Z_t be the observed number of nonlandfalling hurricanes in year t. We will assume that the conditional distribution of Z_t given $Y_t = y_t$ and $X_t = x_t$ is also binomial with parameters $y_t - x_t$ and

$$q_t = \begin{cases} q & t = 1, 2, \ldots, m \\ 1 & t = m + 1, m + 2, \ldots, n, \end{cases} \tag{2}$$

where q is unknown. Under this model, the probability that a nonlandfalling hurricane is observed is q in the first m years of the record, while in the last $n - m$ years of the record, this probability rises to 1. Thus, $X_t + Z_t \le Y_t$ for $t = 1, 2, \ldots, m$, but $X_t + Z_t = Y_t$ for $t = m + 1, m + 2, \ldots, n$.

It is straightforward to show that, under the model outlined above, the joint probability mass function of X_t and Z_t is

$$f_t(x, z) = \sum_{y=x+z}^{\infty} \binom{y-x}{z} q_t^z(1-q_t)^{y-x-z} \binom{y}{x} p^x(1-p)^{y-x}\mu_t^y \times \exp(-\mu_t)/y! \tag{3}$$

The product of the first three terms in this summation represents the conditional binomial probability that $Z_t = z$ given $X_t = x$ and $Y_t = y$. The product of the next three terms represents the conditional binomial probability that $X_t = x$ given $Y_t = y$. The product of the final three terms represents the Poisson probability that $Y_t = y$. The lower bound of the summation reflects the fact that, under the model considered here, the total number of hurricanes cannot be less than the observed number (i.e., $Y_t \ge X_t + Z_t$). Note that, for $t > m$, (3) reduces to

$$f_t(x, z) = \binom{x+z}{x} p^x(1-p)^z\mu_t^{x+z} \exp(-\mu_t)/(x+z)! \tag{4}$$

The likelihood function for the full set of observations is given by

$$L(\mu_0, \mu_1, p, q) = \prod_{t=1}^{n} f_t(x_t, z_t), \tag{5}$$

where x_t and z_t are the observed values of X_t and Z_t, respectively. The maximum likelihood (ML) estimates $\hat{\mu}_0$, $\hat{\mu}_1$, $\hat{p}$, and $\hat{q}$ of the unknown parameters μ_0, μ_1, p, and q are found by maximizing the likelihood or its logarithm. This maximization must be done numerically. A natural statistic for testing H_0 is the likelihood ratio (LR) statistic:

$$\Lambda = 2\{\log L(\hat{\mu}_0, \hat{\mu}_1, \hat{p}, \hat{q}) - \log L[\hat{\mu}_0(0), 0, \hat{p}(0), \hat{q}(0)]\}, \tag{6}$$

where generally $\hat{\mu}_0(\mu)$, $\hat{p}(\mu)$, and $\hat{q}(\mu)$ are the ML estimates of μ_0, p, and q under the restricted model with $\mu_1 = \mu$. Under H_0, the LR statistic Λ has an approximate chi-squared distribution with 1 degree of freedom (Silvey 1975).

If H_0 is rejected, then it may be of interest to construct a confidence interval for the trend parameter μ_1. An approximate $1 - \alpha$ confidence interval for μ_1 is given by the set of values of μ satisfying

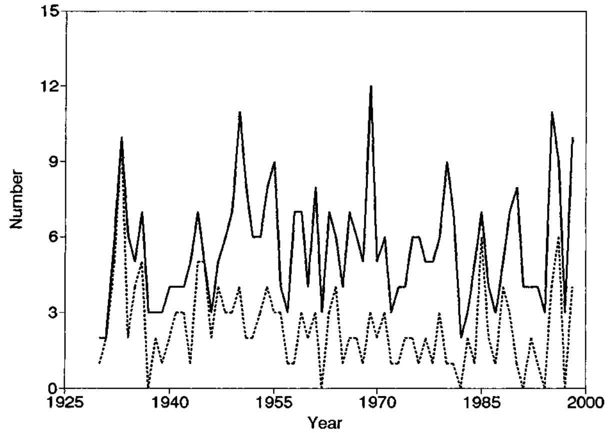

Fig. 1. Time series of total observed (solid) and landfalling (dashed) hurricane counts, 1930–98. The former time series is incomplete over the period 1930–45.

$$2[\log L(\hat{\mu}_0, \hat{\mu}_1, \hat{p}, \hat{q}) - \log L(\hat{\mu}_0(\mu), \mu, \hat{p}(\mu), \hat{q}(\mu)] < \chi_1^2(\alpha), \tag{7}$$

where $\chi_1^2(\alpha)$ is the upper α-quantile of the chi-squared distribution with 1 degree of freedom. Confidence intervals for the other parameters can be constructed in the same way.

3. Results

Figure 1 shows the time series of the total number $X_t + Z_t$ of recorded hurricanes and the number X_t of landfalling hurricanes for the $n = 69$-yr period 1930–98. We will treat the former time series as incomplete over the $m = 16$-yr period 1930–45 and as complete thereafter. Over the period 1930–45, a total of 76 hurricanes were recorded, for an annual average of 4.75. Of these, 51 were landfalling, for an annual average of 3.19. The proportion of hurricanes in the record that were landfalling over this period was 0.67. In contrast, over the period 1946–98, a total of 310 hurricanes were recorded, for an annual average of 5.85. Of these, 116 were landfalling, for an annual average of 2.19. The proportion of hurricanes that were landfalling over this period was 0.37.

Before applying the test described in the previous section, for the purposes of comparison, we tested H_0 using the last $n - m = 53$ yr of the record of basinwide hurricane counts. The approximate significance level was 0.57, so H_0 could not be rejected by conventional notions of significance. This is consistent with earlier analyses (Landsea et al. 1999; Elsner and Kara 1999). Let $\hat{Y}_t$ be the fitted value of Y_t for $t = m + 1, m + 2, \ldots, n$. As a check of the Poisson assumption, we calculated the Poisson deviance:

$$D_P = 2 \sum_{t=m+1}^{n} Y_t \log(Y_t/\hat{Y}_t) \tag{8}$$

for the fitted model. Under the Poisson assumption, D_P

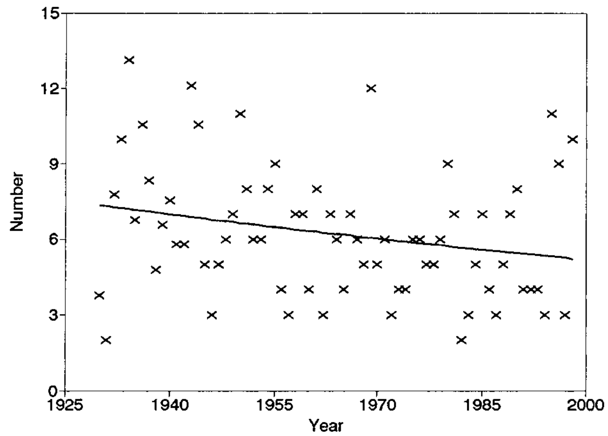

FIG. 2. Estimated trend in mean annual basinwide hurricane counts, 1930–98. Also shown is a reconstruction of the observed counts for 1930–45 and the actual observed counts for 1946–98.

has an approximate chi-squared distribution with $n - m - 2$ degrees of freedom (McCullagh and Nelder 1989). In this case, the value of D_P was 47.3 with 51 degrees of freedom for an approximate significance level of 0.65, so that the Poisson assumption is reasonable.

Turning to the methods described above, the ML estimates under the full model were $\hat{\mu}_0 = 2.00$, $\hat{\mu}_1 = -0.005$, $\hat{p} = 0.39$, $\hat{q} = 0.36$, while the ML estimates under the restricted model were $\hat{\mu}_0(0) = 1.80$, $\hat{p}(0) = 0.40$, $\hat{q}(0) = 0.43$. The value of the LR statistic Λ was 2.90 with an approximate significance level of 0.09, which is at least marginally significant. The approximate 0.90 confidence interval for μ_1 constructed as outlined in the previous section is $(-0.009, -0.001)$.

In Fig. 2, the estimate of $\mu(t)$ under the full model is shown along with a rough reconstruction of basinwide hurricane counts given by

$$\tilde{Y}_t = \begin{cases} X_t + (Z_t/\hat{q}) & t = 1, 2, \ldots, m \\ X_t + Z_t & t = m + 1, m + 2, \ldots, n. \end{cases} \quad (9)$$

The estimate of $\mu(t)$ declines from around 7.4 hurricanes in 1930 to around 5.4 hurricanes in 1998. It is important to emphasize that the goal of the analysis presented here is to test for a trend in annual mean hurricane activity and not to reconstruct the incomplete part of the hurricane record. The reconstructed values are shown in Fig. 2 solely as a graphical aid. A rough standard error for $\tilde{Y}_t$ for $t \leq m$ is $[(\tilde{Y}_t - X_t)(1 - \hat{q})/\hat{q}]^{1/2}$. The average of this standard error for the reconstructed counts in Fig. 2 is around 2.6.

As noted, a key assumption underlying the methods described in this note is that the probability p that a hurricane makes landfall is constant over the entire observation period. Due to the incompleteness of the early part of the record, it is not possible to test this assumption. However, it is possible to test it over the period for which the records are complete. Under the assumption that the landfall probability p is constant and conditional on the observed values of Y_{m+1}, Y_{m+2}, . . . , Y_n, the binomial deviance

$$D_B = 2 \sum_{t=m+1}^{n} X_t \log(X_t/\hat{X}_t) + Z_t \log(Z_t/\hat{Z}_t) \quad (10)$$

has an approximate chi-squared distribution with $n - m - 1$ degrees of freedom, where $\hat{X}_t = \hat{p}_c Y_t$, $\hat{Z}_t = (1 - \hat{p}_c)Y_t$, and

$$\hat{p}_c = \sum_{t=m+1}^{n} X_t \bigg/ \sum_{t=m+1}^{n} Y_t \quad (11)$$

is the ML estimate of p based on the complete part of the record (McCullagh and Nelder 1989). For the data used here, $\hat{p}_c = 0.37$ and $D_B = 43.33$ with approximate significance level 0.80, suggesting that the assumption of constant p is reasonable for the period 1946–98.

4. Discussion

In considering the results of the previous section, it is worth revisiting the underlying logic of the method described in section 2. To begin with, we assumed that the landfall probability p is constant. This assumption appears to be reasonable for the period 1946–98. It then follows from the fact that the annual counts of landfalling hurricanes are greater on average during 1930–45 than during 1946–98 that the true annual counts of basinwide hurricanes were also greater on average during the earlier period. We have stressed the importance to this result of the assumption that p is constant. A second assumption is that the probability q of observing a nonlandfalling hurricane is also constant during this earlier period. It would be possible to relax this assumption. For example, in analyzing a partially incomplete record of tropical storms in the Australian region, Solow (1989) assumed that the sighting probability increased monotonically to 1 according to a simple parametric model. As there is no clear trend in the ratio $X_t/(X_t + Z_t)$ between 1930 and 1945, as would be expected if there were a trend in q, this additional complication does not seem justified here. However, such an extension would probably be necessary to extend this analysis back beyond 1930.

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