NOTES AND CORRESPONDENCE

Comments on "Calculation of Average, Uncertainty Range, and Reliability of Regional Climate Changes from AOGCM Simulations via the 'Reliability Ensemble Averaging' (REA) Method"

Doug Nychka and Claudia Tebaldi

National Center for Atmospheric Research, Boulder, Colorado

1. A statistical interpretation of "reliability ensemble averaging"

In their article "Calculation of average, uncertainty range and reliability of regional climate changes from AOGCM simulations via the 'reliability ensemble averaging' (REA) method," Giorgi and Mearns (2002, hereafter GM02) propose a new summary measure of climate change derived from nine different AOGCM models. This summary measure consists of a weighted average of the individual models' responses, and the weights, taking into account two indicators of model reliability:

- the bias between present-day climate as simulated by the model and observations,
- the "location" of each model's future climate change response within the ensemble of the nine members.

In this note we want to show that by using the latter criterion, in its specific form and algorithmic solution, GM02 are in fact solving a least L_p regression problem, with $0 \leq p < 2$, and in so doing are choosing a robust estimate for the climate change measure.

Here the term "robust" is used in a strictly statistical sense. Robust measures are the statistical answer to the problem of estimating the center of a distribution in the presence of "heavy tails," that is to say, in the likely presence of outliers among the observations. This is exactly what the REA method of GM02 set out to tackle, and we show how their heuristic solution can be recast in a rigorous statistical framework.

When the authors apply an iterative procedure in order to obtain the final weighted mean, they are applying an algorithm known in the statistical literature as iter-

atively reweighted least squares (Green 1984). In GM02's notation, they define $\widetilde{\Delta T}$ as the summary measure of the temperature change (i.e., the goal of the estimation) and ΔT_i as the ith AOGCM response. At the kth step of the iteration the algorithm finds the value $\widetilde{\Delta T}_{(k)}$ that minimizes

$$\sum_i R_i[\widetilde{\Delta T}_{(k-1)}] \times (\Delta T_i - \widetilde{\Delta T})^2, \tag{1}$$

where the weights are defined as

$$R_i[\widetilde{\Delta T}_{(k-1)}] = C_i |\Delta T_i - \widetilde{\Delta T}_{(k-1)}|^{-1/n} \tag{2}$$

with $n \geq 1$. Notice that we gather the part of the weight that does not depend on $\widetilde{\Delta T}$ into the constant C_i. The exact formulation, again following GM02's notation, is

$$C_i = \left(\frac{\epsilon^{(m+n)/n}}{|B_i|} \right)^{1/m},$$

where ϵ is a measure of natural variability, B_i is the bias of the ith AOGCM in reproducing present-day climate, and m, like n in (2), are exponents chosen in order to differently weigh the two components of the weight.

It is well known that this algorithm converges to the unique solution of the least L_p regression problem:

$$\min_{\widetilde{\Delta T}} \sum_i C_i (\Delta T_i - \widetilde{\Delta T})^p \tag{3}$$

(see, e.g., Lange 1999, chapter 12). Here $p = 2 - 1/n$.

Because of the particular choice of $n = m = 1$, in GM02, $p = 1$ and the nature of the optimization problem allows for a closed form solution: least L_1 regression is optimized by the median value of the observations, in this case a weighted median, because of the component C_i in (2) and (3).

In summary, GM02's choice is motivated by a sen-

Corresponding author address: Claudia Tebaldi, National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307.
E-mail: tebaldi@ucar.edu

sible attempt at treating some of the differing outputs of the nine AOGCMs as statistical outliers with respect to a central value. This verdict is reached through a completely data-based procedure. From a strictly statistical perspective, however, their choice is equivalent to the optimal solution of a well-posed problem of robust estimation.

REFERENCES

Giorgi, F., and L. O. Mearns, 2002: Calculation of average, uncertainty range, and reliability of regional climate changes from AOGCM simulations via the ''reliability ensemble averaging'' (REA) method. *J. Climate,* **15,** 1141–1158.

Green, P. J., 1984: Iteratively reweighted least squares for maximum likelihood estimation, and some robust and resistant alternatives. *J. Roy. Stat. Soc.,* **46B,** 149–192.

Lange, K., 1999: *Numerical Analysis for Statisticians.* Springer-Verlag, 356 pp.