Simple Procedures for Extrapolation of Humidity Variables in the Mountainous Western United States

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ABSTRACT

A series of simple procedures are presented for extrapolating climatic averages of humidity variables from a reference location with long-term humidity measurements to nearby higher elevation locations. The extrapolation of monthly average dewpoint temperature is accomplished by using an exponential function for the height decrease of water vapor pressure. This procedure results in a lapse rate of dewpoint temperature which is to a first approximation a constant. The root-mean-square (rms) error in estimating dewpoint temperature at nine higher elevation locations averaged 1.1°C. Summer wet-bulb design temperatures are estimated using a region-wide lapse rate of 4.4°C km^{-1}. The rms error in estimating 1%, 2.5%, and 5% wet-bulb design temperatures at 12 higher elevation locations was 0.9°C. Three-hour monthly average wet-bulb temperatures and relative humidity are estimated from the 3-hour monthly average dewpoint temperature and the 3-hour monthly average air temperature at the reference location with the following procedure. An estimate of the 3-hour monthly average air temperature at the desired location is obtained using an average lapse rate calculated from the monthly mean maximum and minimum temperatures at the two locations. An estimate of 3-hour monthly average dewpoint temperature is obtained using the same procedure that was developed for monthly average dewpoint temperature. Estimates of 3-hour monthly average wet-bulb temperature and relative humidity are then calculated from the estimated air and dewpoint temperatures. A test of this procedure for two station pairs resulted in good agreement for 3-hour monthly average air and wet-bulb temperatures with rms values of 0.8°C and 0.7°C, respectively. The rms error for 3-hour monthly average relative humidity was 5%, however, with some individual errors around 10%.

1. Introduction

In the mountainous areas of the western United States, climatic conditions can change rapidly over short distances due to elevation changes and the interaction of atmospheric motions with the complex terrain. In the case of temperature and precipitation, a moderate number of cooperative observer stations, located at middle and higher elevation locations, provide data for climatic applications. For other climatic variables such as humidity, wind, and cloud cover, however, long-term climatic averages are often available only at lower elevation locations, primarily at airports. In many parts of the west, the population is increasing rapidly at both lower and middle elevation towns and cities. Consequently, the demand for climatic information is increasing. With the general lack of data at middle and higher elevations, techniques for extrapolating climatic data are highly desirable.

In this report, simple procedures for extrapolating climatic averages of humidity variables from lower to higher elevations are presented. A search of recent literature did not reveal any significant research on this topic. This work was initiated as the result of the need for wet-bulb temperatures to estimate cooling requirements at middle elevation locations. Other applications are possible, however, including estimation of potential evapotranspiration for forest and agricultural activities.

Procedures for estimating dewpoint temperatures, wet-bulb temperatures, and relative humidity are presented in sections 3, 4, and 5 respectively. These procedures assume that climatic averages of the pertinent humidity variable are available at a nearby reference location. The procedures use these averages as a basis for extrapolating to the location of interest.

The general equations for the humidity variables used in this report are presented in appendix A.

2. Data

A variety of datasets were used in the development and validation of these procedures. Average dewpoint temperatures for a variety of locations in the western United States were obtained from tabular information in the *Climatic Atlas of the United States* (1968). Wet-

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bulb design temperatures were obtained from ASHRAE (1981). More detailed data for a number of New Mexico stations and El Paso, Texas were extracted from the TD-1440 digital datasets obtained from the National Climatic Data Center. These datasets, which consist of hourly surface observations primarily at airports, were analyzed to yield 3-hour averages of the pertinent climatic variables. Hourly averages of surface observations for several Oregon stations were obtained from the Oregon Office of State Climatologist. Finally, average radiosonde data for a number of western stations were obtained from Ratner (1957).

The coordinates and elevation of those stations used in this report are given in appendix B.

3. Estimation of dewpoint temperature

Figure 1 shows a plot of the altitude dependence of average water vapor pressure at White Sands Missile Range, New Mexico, for three representative months. The height dependence of the vapor pressure is well represented here by an exponential function. This general behavior was observed at all of the locations in the west with long-term radiosonde measurements. Furthermore, the rate of decrease was observed to be quite similar from location to location. The fundamental assumption then in the procedure for estimating dewpoint temperature is that the vertical profile of the average water vapor pressure e can be characterized by an exponential function with a rate of decrease with height which is only a function of the time of year and not a function of location within the region. In mathematical terms

$$e(z) = e(z_0)\exp[-a_m(z - z_0)], \quad (1)$$

where z_0 is the elevation of the reference location, z is the elevation of the desired location, and a_m is a coefficient that is a function only of the month number m.

With this basic assumption, the step-by-step approach for calculating the average dewpoint temperature for a particular month m is quite simple. First, the average dewpoint temperature at the reference location $T_d(z_0)$ is used to calculate the average water vapor pressure $e(z_0)$ from (A.1). Second, the average water vapor pressure $e(z)$ at the desired location is calculated from (1). Finally, the average dewpoint temperature $T_d(z)$ at the desired location is calculated from $e(z)$ using (A.4).

The coefficients a_m were estimated from averages of radiosonde data found in Ratner (1957). These data include temperature, relative humidity, and the height of various pressure levels. Eleven stations in the intermountain west were used. These are: Albuquerque, New Mexico; Boise, Idaho; El Paso, Texas; Ely, Nevada; Grand Junction, Colorado; Great Falls, Mon-

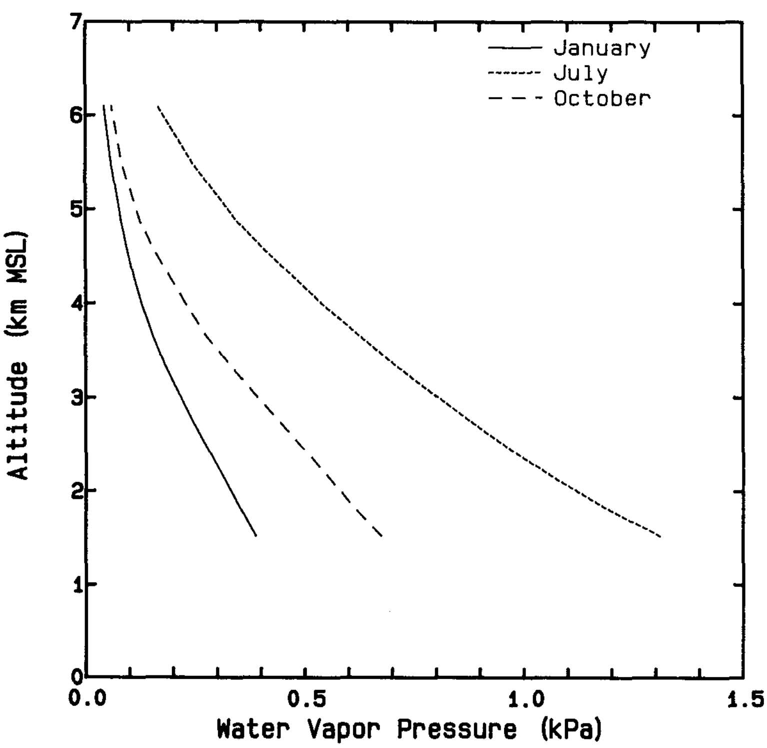

FIG. 1. Average vertical profiles of the water vapor pressure for January, July, and October at White Sands Missile Range, New Mexico. Altitude is in km above mean sea level (MSL).

tana; Lander, Wyoming; Las Vegas, Nevada; Medford, Oregon; Phoenix, Arizona; and Spokane, Washington. Using data at the pressure levels of 85, 80, 75, and 70 kPa, a_m was estimated for each station for each month using standard least squares formulae. For each month, the values of a_m were subsequently averaged for all stations. These values plus the standard deviations among the stations are shown in Table 1. There is a moderate seasonal dependence. The standard deviations are rather small, which suggests that a single set of coefficients applied to the entire region may be adequate as a first approximation.

This method was tested on nine pairs of stations which are located in close proximity, are separated in elevation by a minimum of 500 m and have long-term climatic averages of dewpoint temperature (see Table 2 for list of station pairs). In two cases (Mt. Shasta and Stampede Pass), two different reference locations were used to make the estimate. For Tucumcari and Las Vegas, New Mexico average dewpoint temperatures were calculated from the TD-1440 hourly dataset obtained from the National Climatic Data center in Asheville, North Carolina. Dewpoints were obtained for the other stations from tabular information (NOAA 1968).

The results of this test are shown in Table 2. The general agreement between measurements and estimates is good. For almost all individual months, the absolute error is less than 2.0°C. The root-mean-square (RMS) error is less than 1.7°C for all station pairs except one—Stampede Pass estimated from Yakima. In this one case, the method appears to fail, although this failure is instructive. Stampede Pass, located in the Cascade Mountains, is in an area of rapidly changing climatic conditions from a moist, rainy regime on the west side of the mountains to a dry, rain shadow regime on the east side. The high mean annual precipitation at Stampede Pass (2313 mm) is a strong indication that conditions there are more appropriate to the wet west side, a supposition that is verified by the excellent agreement with dewpoint temperatures estimated from Seattle located on the west side. This indicates that care must be taken when applying this method to stations located in regions where rapid spatial changes in climate are present.

TABLE 1. Average values and standard deviation of coefficients a_m.

Month	a_m (km^{-1})	Standard deviation
January	0.41	0.05
February	0.42	0.08
March	0.40	0.07
April	0.39	0.05
May	0.38	0.07
June	0.36	0.04
July	0.33	0.05
August	0.33	0.05
September	0.36	0.07
October	0.37	0.06
November	0.40	0.05
December	0.40	0.06

In one other case, Baker estimated from Pendleton, the agreement is rather poor during the summer months (June, July, August). The reason for the poor behavior is not known. The dewpoint temperatures at Baker appear to be anomalously high. In fact, the method successfully estimated dewpoint temperatures for Burns using the same reference station, Pendleton, even though Burns is located about twice as far from Pendleton as Baker.

Figure 2 shows a plot of estimated values versus the measured values of dewpoint temperature for all pairs except Stampede Pass–Yakima. This plot graphically illustrates the good agreement over a wide range of dewpoint temperatures.

Overall this method appears to be successful in spite of several simplifying assumptions. These include:

1) One set of the coefficients a_m is applied to the entire region.

2) Local effects are ignored. In much of the west, higher elevation locations receive more rainfall and have greater amounts of vegetation. Therefore, evaporation and transpiration will be greater and might be expected to cause increases in the near surface moisture content. As a result, the rate of decrease of $e(z)$ with height might be different following the terrain than the rate of decrease in the free atmosphere upon which the values of a_m are based. However, this effect does not appear to be a major factor.

The analysis in this section can be used to estimate a lapse rate for dewpoint temperature. Combining Eqs. (1) and (A.1) yields

$$\begin{aligned} e(z) &= A \exp\{BT_d(z_0)/[C + T_d(z_0)]\} \\ &\quad \times \exp[-a_m(z - z_0)] \\ &= A \exp\{-a_m(z - z_0) + BT_d(z_0)/ \\ &\quad [C + T_d(z_0)]\}. \quad (2) \end{aligned}$$

Equation (A.4) can be applied to estimate $T_d(z)$:

$$T_d(z) = C \ln[e(z)/A]/\{B - \ln[e(z)/A]\}. \quad (3)$$

Substituting (2) into (3) gives

$$\begin{aligned} T_d(z) = \{&-Ca_m(z - z_0) + CBT_d(z_0)/[C + T_d(z_0)]\}/ \\ &\{B + a_m(z - z_0) - BT_d(z_0)/[C + T_d(z_0)]\}. \quad (4) \end{aligned}$$

Multiplying the numerator and denominator of (4) by $[C + T_d(z_0)]/BC$ gives

$$\begin{aligned} T_d(z) = \{&-(a_m/B)(z - z_0)[C + T_d(z_0)] + T_d(z_0)\}/ \\ &\{1 + a_m(z - z_0)[C + T_d(z_0)]/BC\}. \quad (5) \end{aligned}$$

An approximate relationship can be derived by noting that

$$a_m(z - z_0)[C + T_d(z_0)]/BC \ll 1$$

TABLE 2. Comparison of measured and estimated dewpoint temperatures (°C).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	RMS error
Flagstaff estimated from Phoenix (elevation difference = 1972 m)													
Measured	−10.0	−8.9	−8.3	−6.7	−5.6	−3.9	6.1	6.1	1.7	−3.9	−6.7	−9.4	
Estimated	−9.1	−9.3	−8.9	−7.7	−6.9	−3.5	5.7	6.7	2.3	−2.6	−7.4	−9.0	0.8
Mt. Shasta estimated from Red Bluff (elevation difference = 887 m)													
Measured	−3.9	−2.2	−3.3	−1.1	1.7	4.4	5.0	3.3	3.3	1.1	1.1	−1.1	
Estimated	−3.8	−2.8	−2.6	−0.9	1.4	2.6	4.1	4.1	1.6	0.4	−1.6	−2.7	1.3
Mt. Shasta estimated from Medford (elevation difference = 587 m)													
Measured	−3.9	−2.2	−3.3	−1.1	1.7	4.4	5.0	3.3	3.3	1.1	1.1	−1.1	
Estimated	−3.2	−2.2	−1.5	0.1	2.4	4.7	6.6	6.6	4.2	3.0	−0.5	−2.1	1.5
Butte estimated from Missoula (elevation difference = 709 m)													
Measured	−14.4	−11.1	−8.9	−5.0	0.0	3.3	3.9	3.9	0.6	−2.8	−7.8	−10.6	
Estimated	−13.0	−9.9	−8.7	−5.4	−0.9	2.4	3.9	2.8	0.3	−2.5	−7.1	−10.3	0.8
Baker estimated from Pendleton (elevation difference = 572 m)													
Measured	−9.4	−5.0	−2.8	−1.1	2.8	5.6	6.7	5.6	2.8	0.0	−3.5	−5.0	
Estimated	−7.5	−4.3	−4.2	−1.9	1.4	2.6	3.4	3.4	2.1	0.9	−2.6	−4.7	1.7
Burns estimated from Pendleton (elevation difference = 807 m)													
Measured	−7.2	−5.0	−4.4	−2.8	0.6	2.8	3.3	2.8	0.0	−1.1	−3.3	−7.2	
Estimated	−8.7	−5.6	−5.4	−3.2	0.2	1.4	2.3	2.3	0.9	−0.3	−3.8	−6.0	0.9
Stampede Pass estimated from Yakima (elevation difference = 886 m)													
Measured	−7.8	−4.4	−4.4	−2.2	1.1	3.9	5.6	6.7	5.0	1.7	−1.7	−3.9	
Estimated	−10.7	−8.2	−7.4	−5.2	−1.8	1.5	2.5	3.6	1.1	−1.8	−5.8	−8.0	3.3
Stampede Pass estimated from Seattle (elevation difference = 1009 m)													
Measured	−7.8	−4.4	−4.4	−2.2	1.1	3.9	5.6	6.7	5.0	1.7	−1.7	−3.9	
Estimated	−4.9	−4.6	−4.3	−2.1	0.3	3.1	5.8	6.3	4.3	1.9	−2.2	−3.9	0.9
Las Vegas estimated from Tucumcari (elevation difference = 854 m)													
Measured	−9.4	−8.3	−7.8	−5.0	−0.6	4.4	10.0	9.4	5.6	−1.1	−6.7	−8.9	
Estimated	−11.7	−10.4	−9.4	−5.6	0.4	5.4	9.6	9.1	5.0	−1.6	−7.3	−10.5	1.2

and

$$C + T_d(z_0) \approx C,$$

which is adequate for the usual conditions encountered in much of the west where

$$-15°\text{C} < T_d < 15°\text{C}.$$

Therefore,

$$T_d(z) = -a_m C(z - z_0)/B + T_d(z_0) \qquad (6)$$

or

$$dT_d/dz = -\Gamma_d = -a_m C/B, \qquad (7)$$

which implies that the lapse rate of the average dewpoint temperature is to a first approximation a constant. Using values of a_m from Table 1, Γ_d varies from 4.5°C km^{-1} in July and August to 5.7°C km^{-1} in February.

4. Estimation of wet-bulb temperature

The estimation of wet-bulb temperature is more complex because of its dependence on both atmospheric water vapor content and air temperature. The wet-bulb temperature exhibits a more pronounced diurnal dependence than dewpoint temperature. Of most interest in this study are two measures of wet-bulb temperature: 3-hour averages on a monthly basis and summer design wet-bulb temperatures. Summer design wet-bulb temperatures are values that are exceeded a certain percentage of the available hours and are used to determine requirements for air-conditioning capacity. For instance, the 1% wet-bulb design temperature will be exceeded during 1% of the hours during the summer. In this case, summer is considered to be the months of June, July, August, and September.

a. 3-hour monthly average wet-bulb temperatures

For three-hour monthly averages the simplest possible approach was taken. The average dewpoint temperature and air temperature were used to calculate an average wet-bulb temperature as given by (A.7). This calculated value was used as the average wet-bulb temperature. This approach assumes that any problems associated with the nonlinear relationship between these variables are insignificant. This approach was

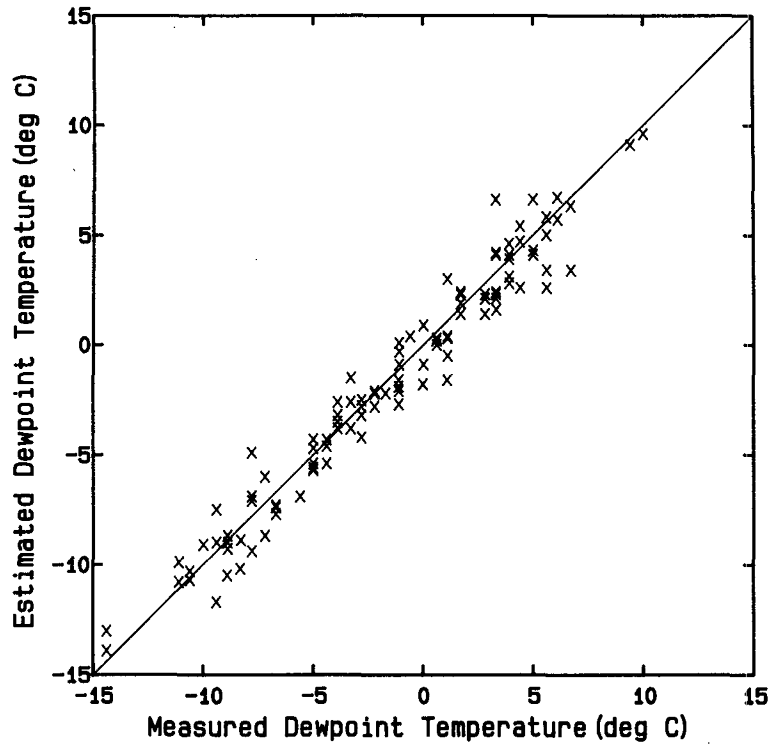

FIG. 2. Estimated vs measured values of the monthly average dewpoint temperature for all station pairs except Stampede Pass–Yakima.

tested by using 3-hour averages of air, wet-bulb, and dewpoint temperatures calculated from the TD-1440 hourly dataset for El Paso, Texas and 13 stations in New Mexico. Figure 3 shows a scatter plot of the errors in the estimated wet-bulb temperature as a function of the measured wet-bulb temperature. There is a slight tendency for overestimation of wet-bulb temperature. In nearly all cases, however, errors are less than 1°C. There is some dependence on time of day with highest values occurring in the afternoon. The RMS error for all stations and all months varies from 0.2°C for nighttime hours to 0.6°C for midafternoon.

The application of this approach requires some method for estimating 3-hour monthly average air temperatures at the desired location. This matter will be discussed further in section 6. It is informative, however, to look at the height dependence of wet-bulb temperature for the case of constant lapse rates for T and T_d. [The vertical air temperature data of Ratner (1957) are fit well by a straight line.] An approximate relationship for the lapse rate of T_w can be derived from the following approximate relationship for wet-bulb depression (Abbott and Tabony 1985):

$$T - T_d = 0.34(T - T_d) + 0.006(T^2 - T_d^2), \quad (8)$$

where all temperatures are in °C. Rearranging this equation yields

$$T_w = 0.66T + 0.34T_d + 0.006(T_d^2 - T^2). \quad (9)$$

Taking the height derivative, substituting $T(z) = T(z_0) - \Gamma(z - z_0)$ where Γ = air temperature lapse rate and $T_d(z) = T_d(z_0) - \Gamma_d(z - z_0)$, and rearranging yields

$$\begin{aligned}\Gamma_w = -dT_w/dz = {} & [0.66 - 0.012T(z_0)] \\ & + \Gamma_d[0.34 + 0.012T_d(z_0)] \\ & + 0.012(z - z_0)(\Gamma^2 - \Gamma_d^2). \quad (10)\end{aligned}$$

As a result of the third term on the right, Γ_w is not constant with height. This term can be significant when the difference in elevation and the difference in air and dewpoint temperature lapse rates are large, but it is possible to minimize this effect by calculating Γ_w at the midpoint of the elevation range of interest. For instance, in order to extrapolate T_w from an elevation of 1 km to 3 km, Γ_w would be evaluated at an elevation of 2 km. Figure 4 shows an example of the vertical profile of wet-bulb depression for typical August conditions where Γ = 8.1°C km^{-1}, Γ_d = 4.5°C km^{-1}, $T(z_0)$ = 30°C, and $T_d(z_0)$ = 5°C over the elevation range of 1–6 km. The solid curve shows the exact solution from (A.7) while the dashed curve shows an approximate linear solution calculated from (10) with $(z - z_0)$ = 2.5 km. The linear approximation differs by less than 1°C from the exact solution over the entire range. This indicates that a linear approximation to the wet-bulb temperature height profile can be an adequate first approximation when using the approach outlined above.

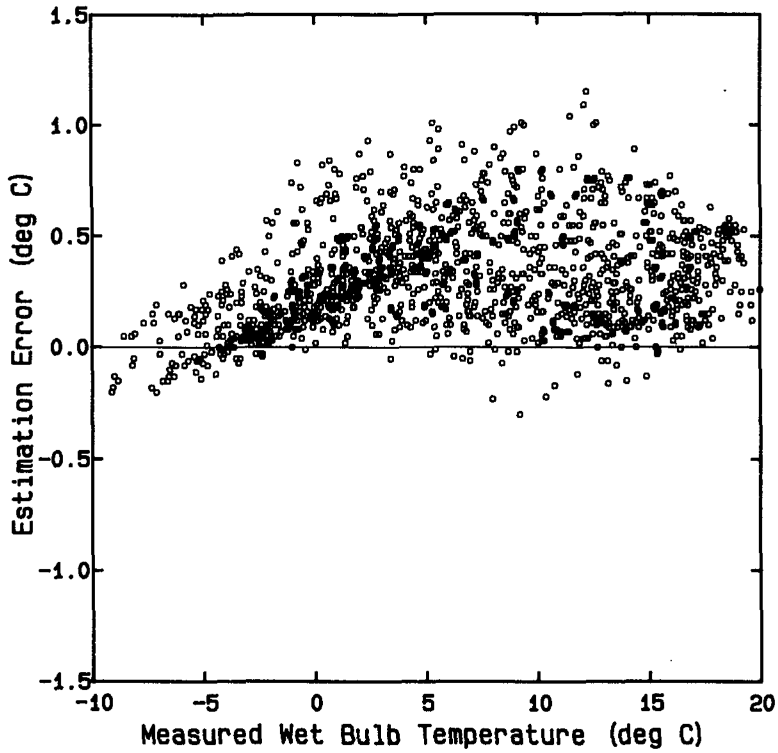

FIG. 3. Errors in the 3-hour monthly average wet-bulb temperature estimated from the measured air and dewpoint temperatures as a function of the measured wet-bulb temperature. Data for El Paso, Texas and 13 New Mexico locations are included.

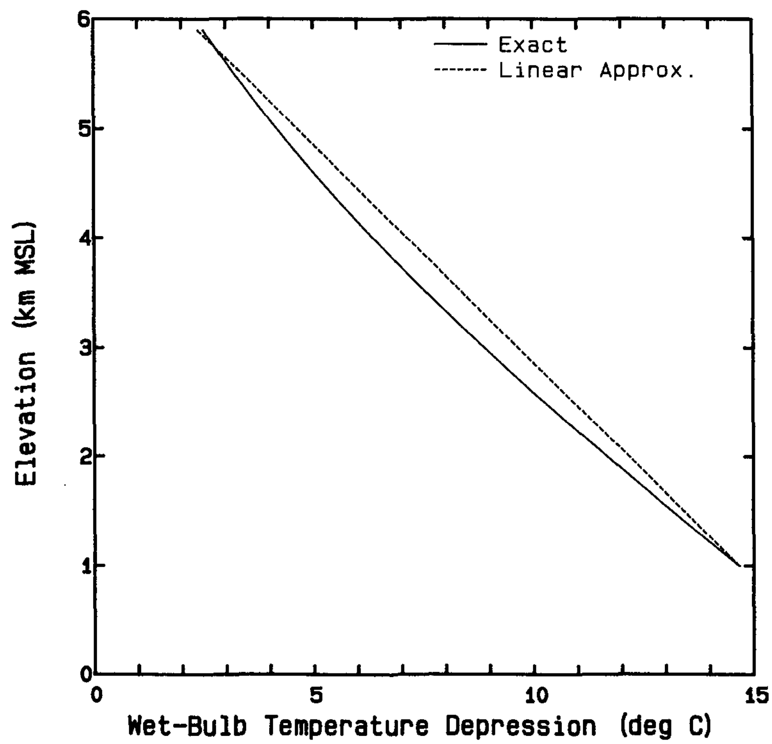

FIG. 4. Vertical profile of wet-bulb temperature depression for $\Gamma = 8.1°C\ km^{-1}$, $\Gamma_d = 4.5°C\ km^{-1}$, $T(z_0) = 30°C$, $T_d(z_0) = 5°C$. Solid line gives exact solution while dashed line gives an approximate linear solution. Elevation is in km above mean sea level (MSL).

b. Wet-bulb design temperatures

The extrapolation of wet-bulb design temperatures appears at first glance to be a complex problem. Design temperatures are calculated from the tail of the probability distribution which in turn is determined by a small number of extreme meteorological events. It is not immediately clear what the height dependence of either temperature or water vapor content might be during these infrequent events and how the resulting probability distribution of wet-bulb temperature might change with height. Such a question could probably be answered by comparing the probability distributions of wet-bulb temperatures at locations with different elevations. The following very simple procedure, however, appears to work satisfactorily avoiding the necessity for the above approach.

ASHRAE (1981) has published 1%, 2.5%, and 5% summer wet-bulb design temperatures for numerous locations. These were calculated from wet-bulb temperature measurements primarily at National Weather Service offices and at airports. Figure 5 shows a plot of the 1% design temperatures for New Mexico and Arizona plotted against elevation. It appears that a linear relationship between wet-bulb design temperatures and elevation is a very good first approximation. A least-squares fit to these data indicates a decrease of about 4.2°C km for New Mexico and 4.6°C km for Arizona. An intermediate value of 4.4°C km was tested on 12 pairs of western stations located in close proximity but with a significant elevation difference. Table 3 gives the estimated design temperatures compared with the ASHRAE (1981) values. The agreement in general is very good with errors of less than 1.0°C in most cases. The worse results were obtained for Leadville, the only station located above 2500 m, with errors of 2°–3°C; This perhaps indicates that, at the highest elevations, this simple linear decrease no longer applies.

The general success of this simple method is somewhat surprising since a single gradient value is applied to the entire western region. This suggests that, even though the climate of the west is highly diverse, the particular situations resulting in high summer wet-bulb temperatures are similar throughout the region.

5. Estimation of relative humidity

The extrapolation of relative humidity is complicated by its highly nonlinear dependence on temperature. As a result, it is possible that a simple average of measured relative humidity values could be significantly different from a calculated value of relative humidity computed from an average air temperature and average dewpoint temperature. As a first approxima-

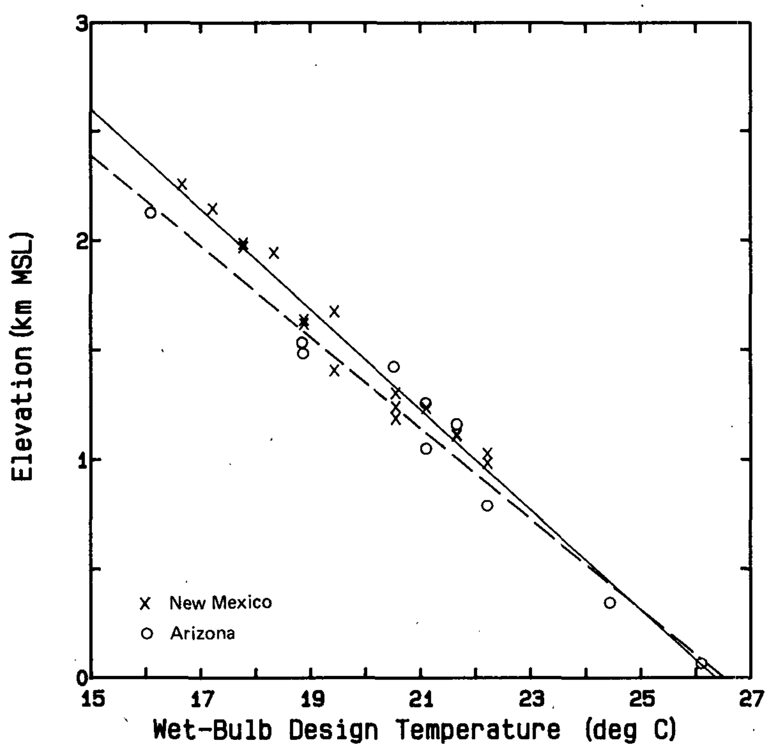

FIG. 5. 1% wet-bulb design temperatures vs elevation for New Mexico ("X") and Arizona ("O") stations. Solid line gives a least-squares linear fit to the New Mexico data while the dashed line is a fit to the Arizona data. Elevation is in km above mean sea level (MSL).

TABLE 3. Comparison of measured and estimated 1%, 2.5%, and 5% wet-bulb design temperatures (°C). Elevation difference (m) is given following the reference station name.

Estimated station	Reference station	1%		2.5%		5%	
		Measured	Estimated	Measured	Estimated	Measured	Estimated
Flagstaff	Phoenix (1972)	16.1	16.6	15.6	16.1	15.0	16.1
Prescott	Phoenix (1198)	18.9	19.2	18.3	18.7	17.8	18.7
Winslow	Phoenix (1148)	18.9	19.4	18.3	18.9	17.8	18.9
Leadville	Denver (1401)	13.3	11.3	12.8	10.7	12.2	10.1
Butte	Missoula (709)	15.6	15.2	14.4	14.1	13.9	13.6
Ely	Las Vegas (1334)	15.6	16.2	15.0	15.7	14.4	15.1
Los Alamos	Farmington (580)	16.7	16.9	16.1	15.8	15.6	15.2
Silver City	White Sands Missile Range (345)	18.9	18.6	17.8	18.0	17.2	17.5
Klamath Falls	Grants Pass (953)	17.2	17.4	16.1	16.3	15.6	15.8
Bend	The Dallas (978)	17.8	16.4	16.7	15.3	15.6	14.8
Cedar City	St. George (817)	18.3	17.5	17.2	16.4	16.7	15.8
Spokane	Kennewick (602)	18.3	18.5	17.8	17.4	16.7	16.8
RMS error = 0.9							

tion, however, this simple approach was used to estimate 3-hour monthly averages of relative humidity. Three-hour average values of T and T_d were used to calculate an average relative humidity (RH) with the following simple procedure. For each month, $e_s(T_d)$ and $e_s(T)$ were calculated using (A.1). Next, $r[e_s(T_d)] \equiv r$ and $r[e_s(T)] \equiv r_s(T)$ were calculated using (A.2). Finally, RH(T, T_d) was calculated using (A.5).

This method was tested using 13 New Mexico stations and El Paso, Texas where three-hour monthly averages of T, T_d, and RH were available. The probability distributions of measured hourly relative humidity values often indicates a significant asymmetry in the arid west. In this case, the median may be a better measure of the central tendency. Both the mean and median values of the relative humidity measurements were compared to the calculated averages as a function of time of day. Table 4 shows the results of this comparison for each 3-hour period averaged over all stations and over all months. This table shows both the root-mean-square (rms) error and the mean error which gives an estimate of any systematic biases in the procedure. For the mean RH, rms errors are in the range of 3%–5%. The mean error indicates the presence of a systematic bias with calculated values typically 3–4% lower than measured values. For the median RH the systematic bias as indicated by the mean error is reduced considerably. The rms errors are also reduced significantly. Figure 6 shows a scatter plot of the errors in estimating the median relative humidity as a function of the relative humidity. While the absolute value of most errors is less than 3%, some larger errors exist particularly at higher relative humidity values. Therefore, this procedure may be an adequate first approximation for estimating median values of relative humidity in most cases but there may be situations in which significant errors result. As in the case of wet-bulb temperatures, the application of this procedure requires an estimate of the diurnal variation of temperature. This will be discussed in section 6.

TABLE 4. Results of estimating mean and median relative humidity.

Time (hours)	Mean RH: Mean error (%)	Mean RH: Rms error (%)	Median RH: Mean error (%)	Median RH: Rms error (%)
0100–0300	−3.7	4.0	−2.0	3.5
0400–0600	−3.7	4.0	−3.0	4.2
0700–0900	−3.9	4.3	−2.4	3.7
1000–1200	−3.8	4.1	−0.1	1.7
1300–1500	−3.7	3.9	0.9	1.5
1600–1800	−4.1	4.4	1.1	1.7
1900–2100	−4.3	4.5	0.0	2.3
2200–2400	−3.9	4.1	−1.2	3.0

An approximate relationship for the vertical profile of relative humidity can be derived for the case of a constant air temperature lapse rate. With the approximation that $(P - e) = P$, (A.2) can be combined with (A.5) to give

$$\mathrm{RH}(z) = 100\%\ e(z)/e_s[T(z)]. \quad (11)$$

Substituting (1) for $e(z)$, (A.1) for $e_s[T(z)]$, and $T(z) = T(z_0) - \Gamma(z - z_0)$, Eq. (11) becomes

$$\mathrm{RH}(z) = 100\%\ e(z_0)A^{-1} \times \exp\{-a_m(z - z_0) - B[T(z_0) - \Gamma(z - z_0)]/[C + T(z_0) - \Gamma(z - z_0)]\}. \quad (12)$$

Noting that $\mathrm{RH}(z_0) = 100\%\ e(z_0)A^{-1}\exp\{-BT(z_0)/[C + T(z_0)]\}$, Eq. (12) can be written as

$$\mathrm{RH}(z) = \mathrm{RH}(z_0)\exp\{-a_m(z - z_0) - B[T(z_0) - \Gamma(z - z_0)]/[C + T(z_0) - \Gamma(z - z_0)] + BT(z_0)/[C + T(z_0)]\}$$

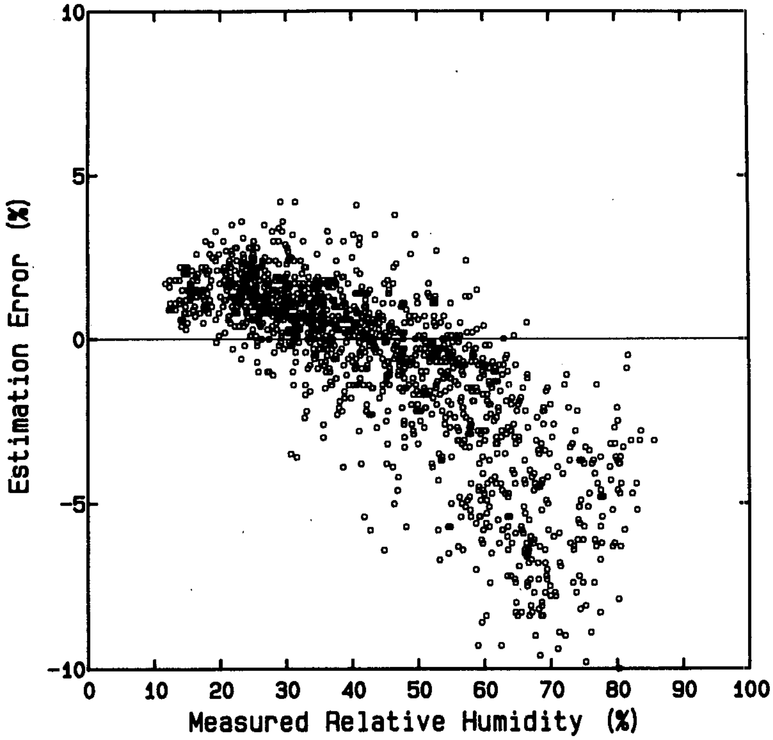

FIG. 6. Errors in the 3-hour monthly median relative humidity estimated from the measured air and dewpoint temperatures as a function of the measured median relative humidity. Data for El Paso, TX and 13 New Mexico stations are included.

$$= \mathrm{RH}(z_0)\exp\{-a_m(z-z_0) + BC\Gamma(z-z_0)[C+T(z_0)]^{-1} \times [C+T(z_0)-\Gamma(z-z_0)]^{-1}\}. \quad (13)$$

The relative humidity lapse rate, Γ_{RH}, will be defined as

$$\Gamma_{\mathrm{RH}} = -a_m + BC\Gamma/ \{[C+T(z_0)][C+T(z_0)-\Gamma(z-z_0)]\}. \quad (14)$$

TABLE 5. Typical values of Γ_{RH}.

Month	$T(z_0)$ (°C)	Γ (°C/km)	Γ_{RH} (km^{-1})
January	0	4.4	−0.09
February	2	5.9	0
March	7	7.1	0.09
April	15	7.8	0.11
May	23	8.1	0.11
June	27	8.2	0.12
July	30	8.1	0.14
August	27	8.1	0.15
September	23	7.7	0.11
October	15	6.8	0.07
November	7	5.5	−0.02
December	2	4.7	−0.07

Equation (13) can then be simply written as

$$\mathrm{RH}(z) = \mathrm{RH}(z_0)\exp[\Gamma_{\mathrm{RH}}(z-z_0)]. \quad (15)$$

Relative humidity therefore varies exponentially with height for a constant temperature lapse rate and a vapor pressure profile following (1). Γ_{RH} may be positive or negative depending on the values of the variables in (15). Table 5 gives values of Γ_{RH} based on the values of a_m in Table 1, typical values for $T(z_0)$, and for $z = z_0$. Also given in Table 5 are values of Γ derived from the data of Ratner (1957). With these values, the average relative humidity is expected to decrease slightly with height during the winter months and increase with height during the rest of the year.

6. Application

The series of simple procedures presented above for estimating humidity variables in the western United States assume that long-term average humidity data are available at a nearby reference location. Humidity variables may then be calculated at the desired location on a monthly basis. The estimation of average water vapor pressure and dewpoint temperature, as outlined in section 2, is straightforward. The lapse rate of dewpoint temperature was shown to be approximately constant with some variation with time of year. This constant lapse rate could be used as a first approximation of dewpoint temperature. The estimation of wet-bulb design temperatures is also simple, assuming a single lapse rate of 4.4°C km^{-1}.

The estimation of average wet-bulb temperature and median relative humidity is complicated by their dependence on temperature. As a result, these variables exhibit a considerable diurnal variation and measurements or estimates of the diurnal dependence of temperature at the desired location are required for calculation. Many locations have long-term average values of daily maximum and minimum temperatures, although more detailed temperature data are often unavailable. One possible solution is the use of techniques which have been developed for estimating the diurnal variation of temperature in mountainous terrain (McCutchan 1979; Bradshaw and Salazar 1985).

A somewhat simpler solution was used here that involves estimating 3-hour monthly average temperatures at the desired location from the 3-hour monthly average temperatures at the reference location. The approach is quite simple. The monthly mean maximum and minimum temperatures at the reference and desired locations can be used to estimate a temperature lapse rate. This lapse rate can then be used to estimate the 3-hour monthly average temperature at the desired location, i.e.,

$$T(t, z) = T(t, z_0) - \Gamma(t)(z - z_0), \qquad (16)$$

where

$$\Gamma(t) = 0.5(\Gamma_n + \Gamma_x) + 0.5(\Gamma_n - \Gamma_x)\cos[2\pi(t - 5)/24] \qquad (17)$$

with

$\Gamma_x = -[T_x(z) - T_x(z_0)]/(z - z_0)$,
$\Gamma_n = -[T_n(z) - T_n(z_0)]/(z - z_0)$,
T_x mean monthly maximum temperature,
T_n mean monthly minimum temperature, and
t time of day in hours.

The time of day dependence for $\Gamma(t)$ accounts for situations in which the diurnal temperature amplitude is different at the desired location relative to the reference location. The 3-hour monthly average dewpoint temperatures at the desired location can be calculated from the 3-hour average dewpoint temperatures at the reference location using the method of section 3. Although not large, T_d does exhibit some diurnal variation, typically about 1°–3°C. This step allows this diurnal variation to be accounted for. This approach was tested on two locations: Las Vegas, Nevada using Tucumcari, New Mexico as the reference and Burns, Oregon using Pendleton, Oregon as the reference. In the case of Burns, only mean values (not medians) of RH were available.

Figure 7 is a scatter plot of the errors in estimating

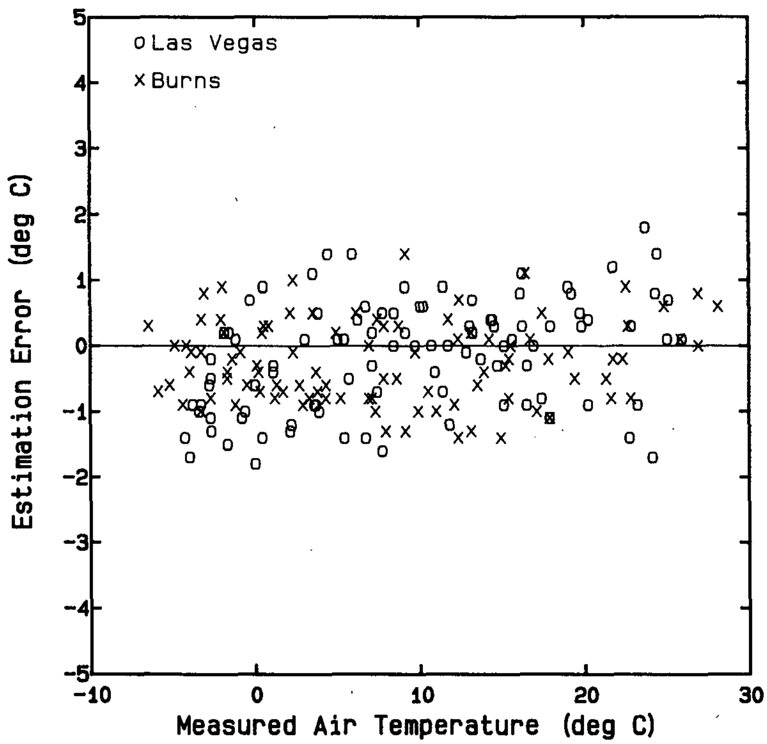

FIG. 7. Errors in the estimated 3-hour monthly average air temperature vs the measured air temperature for two station pairs: Las Vegas, Nevada estimated from Tucumcari, New Mexico and Burns, Oregon estimated from Pendleton, Oregon.

the 3-hour monthly average air temperature as a function of the air temperature. Absolute values of the errors are less than 2.0°C in all cases. The RMS error for all values is 0.8°C. Figure 8 shows a plot of errors in estimating the 3-hour monthly average wet-bulb temperature as a function of the wet-bulb temperature. Despite a slight tendency to underestimate T_w, absolute values of the errors are again less than 2.0°C in all cases. The rms error for all values is 0.7°C. Finally, Fig. 9 is a scatter plot of errors in estimating the 3-hour monthly relative humidity as a function of relative humidity. The agreement between estimates and measurements is rather poor, with errors exceeding 10% in a few cases. There appears to be an offset in the estimated values for Burns, which may be due to using the mean rather than the median. The overall agreement, however, is no better for Las Vegas where medians are used. The rms error for all values is 5%.

The rather poor results for RH may be due in part to the limitations in estimating relative humidity suggested by the results of Fig. 6. A closer examination of individual values indicated a second source of error: errors in the estimation of 3-hour monthly average dewpoint temperature. In some cases (e.g., winter at Las Vegas), the diurnal variation of T_d was significantly different at the desired location compared to the reference location. Figure 10 is a scatter plot of errors in estimating the 3-hour monthly average dewpoint temperature as a function of the measured dewpoint temperature. Absolute values of errors frequently exceed 1.5°C. This contributes significantly to the errors in estimating relative humidity.

In conclusion, the methods for estimating average monthly dewpoint temperatures, summer design wet-bulb temperatures, and 3-hour monthly average wet-bulb temperatures were quite accurate. The method for estimating monthly 3-hour average relative humidity was less successful, however, with errors of greater than 5% rather common.

The locations used for validation of these procedures represent a range of topographic settings from valley floors to valley sidewalls to mesas. The topographic relief varies from mild to moderate. It is expected then that these procedures should be generally applicable throughout the west, but it is possible that, in cases of extreme topographic relief where nocturnal microclimatic gradients are large, estimation errors will be greater for nighttime hours than the errors experienced in this study. This possibility is partially mitigated by the necessity of having measurements of mean maximum and minimum temperatures. These will partially incorporate the effects of these nocturnal gradients. More study would be required to determine whether extreme topographic relief results in greater errors.

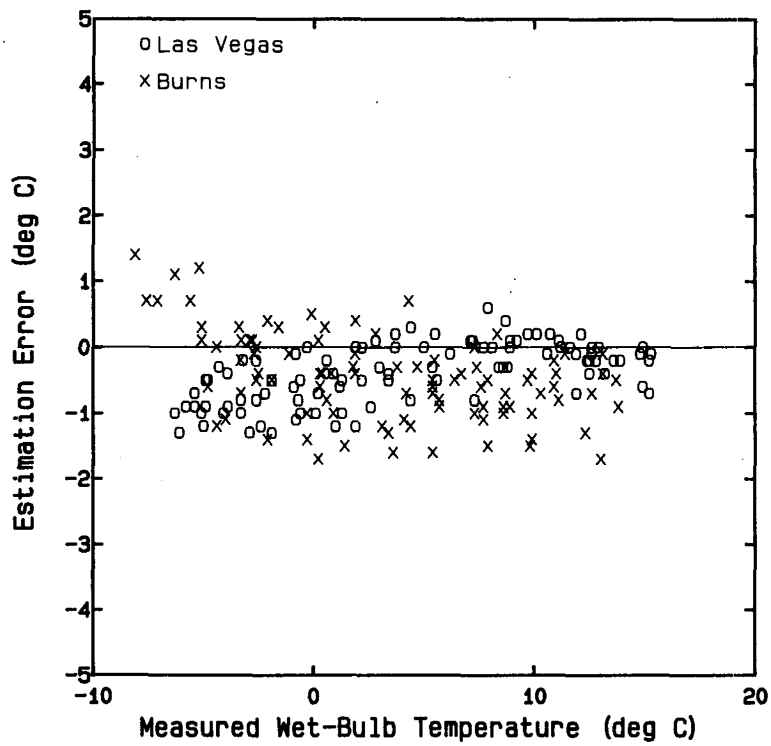

FIG. 8. Errors in the estimated 3-hour average wet-bulb temperature vs the measured wet-bulb temperature for two station pairs: Las Vegas, Nevada estimated from Tucumcari, New Mexico and Burns, Oregon estimated from Pendleton, Oregon.

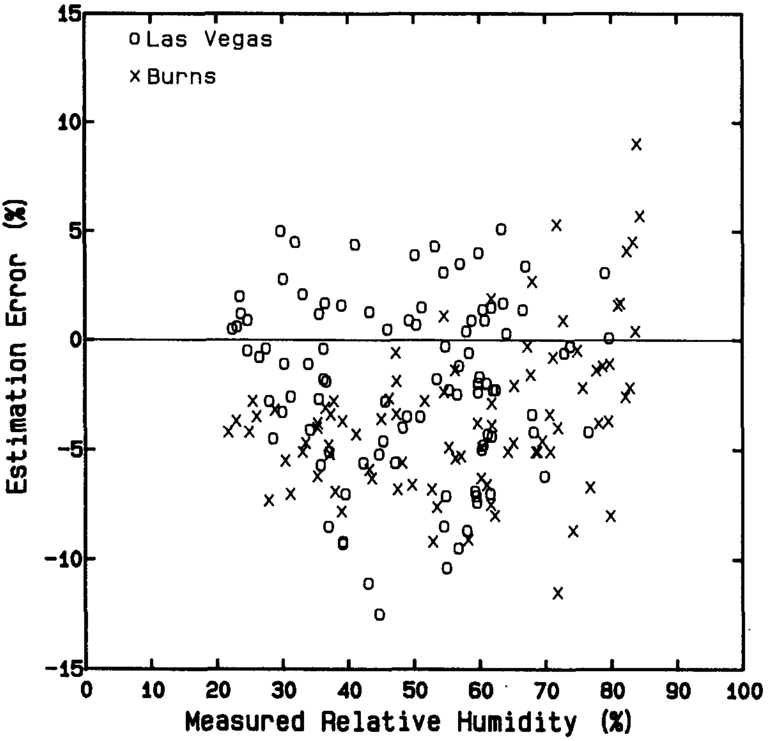

FIG. 9. Errors in the estimated 3-hour average relative humidity vs the measured relative humidity for two station pairs: Las Vegas, Nevada estimated from Tucumcari, New Mexico and Burns, Oregon estimated from Pendleton, Oregon. For Las Vegas, estimated values were compared with the measured median values while for Burns, estimated values were compared with the measured mean values.

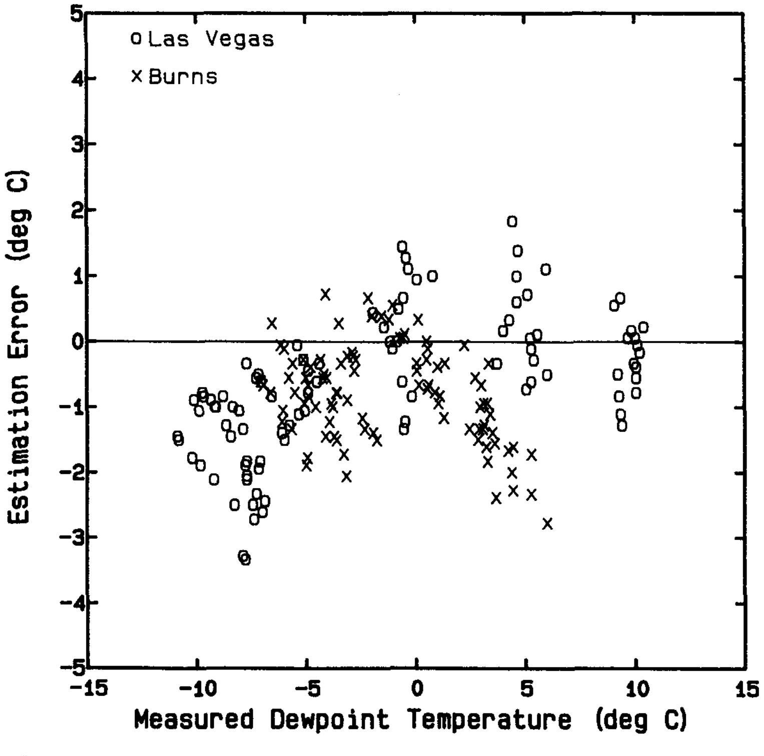

FIG. 10. Errors in the estimated 3-hour monthly average dewpoint temperature vs the measured average dewpoint temperature for two station pairs: Las Vegas, Nevada estimated from Tucumcari, New Mexico and Burns, Oregon estimated from Pendleton, Oregon.

Acknowledgments. I thank Kelly Redmond for supplying data for several Oregon stations. I also thank Kelly Redmond and Nolan Doesken for helpful comments on the manuscript.

APPENDIX A

General Equations for the Humidity Variables

The saturation water vapor pressure $e_s(T)$ in mb at temperature T is given by (Buck 1981)

$$e_s(T) = A \exp[BT/(C + T)], \quad \text{(A.1)}$$

where T is in °C and A, B, and C are constants. Over water, the values of these constants are $A = 6.1121$, $B = 17.502$, and $C = 240.97$. Over ice, the values are $A = 6.1115$, $B = 22.452$, and $C = 272.55$.

The mixing ratio r is related to the water vapor pressure e by (List 1949)

$$r = 0.62197e/(P - e) \quad \text{(A.2)}$$

or

$$e = rP/(0.62197 + r), \quad \text{(A.3)}$$

where P = pressure.

The dewpoint temperature T_d can be calculated from the water vapor pressure by (Buck 1981)

$$T_d = C \ln(e/A)/[B - \ln(e/A)]. \quad \text{(A.4)}$$

The relative humidity RH in percent at some temperature T is given by (List 1949)

$$\mathrm{RH}(T) = 100\% \; r/r_s(T), \quad \text{(A.5)}$$

where $r_s(T)$ is the saturation mixing ratio at temperature T.

The relationship between wet-bulb temperature T_w, air temperature T, and water vapor pressure e is given by (List 1949)

$$e_s(T_w) - e = 0.000660(1 + 0.00115T_w)P(T - T_w), \quad \text{(A.6)}$$

where T and T_w are in °C and P, e, and $e_s(T_w)$ are in consistent units.

The wet-bulb temperature may be calculated from the dewpoint and air temperatures by combining (A.1) for $T = T_d$ with (A.6):

$$e_s(T_w) - e_s(T_d) = 0.000660(1 + 0.00115T_w)P(T - T_w). \quad \text{(A.7)}$$

Equation (A.7) may be solved for T_w using the Newton–Raphson iterative approximation (Abbott and Tabony 1985).

APPENDIX B

Station Locations

The latitude, longitude, and elevation for the stations used in this study are given below. An asterisk "*" in front of the name indicates that the TD-1440 hourly surface observation data were used.

Name	North latitude	Longitude	Elevation (m)
*Albuquerque, NM	35°03′	106°37′	1620
Baker, OR	44°50′	117°48′	1027
Bend, OR	44°05′	121°12′	1052
Boise, ID	43°34′	116°24′	871
Burns, OR	43°35′	118°57′	1262
Butte, MT	45°58′	112°30′	1685
Cedar City, UT	37°42′	113°06′	1712
*Clayton, NM	36°27′	103°09′	1540
*Clovis, NM	34°23′	103°19′	1311
Denver, CO	39°46′	104°53′	1625
*El Paso, TX	31°48′	106°24′	1194
Ely, NV	39°17′	114°51′	1907
*Farmington, NM	36°45′	106°15′	1679
Flagstaff, AZ	35°18′	111°51′	2131
*Gallup, NM	35°31′	108°43′	1971
Grants Pass, OR	42°26′	123°20′	294
Grand Junction, CO	39°06′	108°32′	1475
Great Falls, MT	47°31′	110°10′	1077
*Holloman Air Force Base, NM	32°51′	106°05′	1241

APPENDIX B (*Continued*)

Name	North latitude	Longitude	Elevation (m)
Kennewick, WA	46°13′	119°08′	119
Klamath Falls, OR	42°09′	121°43′	1247
Lander, WY	42°48′	108°47′	1669
*Las Vegas, NM	35°39′	105°08′	2093
Las Vegas, NV	36°15′	115°02′	573
Leadville, CO	39°13′	106°18′	3026
Los Alamos, NM	35°52′	106°19′	2259
Medford, OR	42°24′	122°52′	405
Missoula, MT	46°54′	114°05′	976
Mt. Shasta, CA	41°15′	122°16′	992
Pendleton, OR	45°41′	118°51′	455
Phoenix, AZ	33°26′	112°23′	339
Prescott, AZ	34°39′	112°25′	1537
*Raton, NM	36°45′	104°30′	1944
Red Bluff, CA	40°09′	122°15′	105
*Roswell, NM	33°18′	104°32′	1110
St. George, UT	37°05′	113°35′	895
Seattle, WA	47°27′	122°18′	117
Silver City, NM	32°38′	108°10′	1637
Spokane, WA	47°37′	117°31′	721
Stampede Pass, WA	47°17′	121°20′	1216
The Dalles, OR	45°37′	121°09′	74
*Truth or Consequences, NM	33°14′	107°16′	1481
*Tucumcari, NM	35°10′	103°36′	1239
Winslow, AZ	35°01′	110°44′	1487
*White Sands Missile Range, NM	32°22′	106°29′	1292
Yakima, WA	46°34′	120°32′	330
*Zuni, NM	35°06′	108°48′	1963

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