North Atlantic Power Dissipation Index (PDI) and Accumulated Cyclone Energy (ACE): Statistical Modeling and Sensitivity to Sea Surface Temperature Changes

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ABSTRACT

This study focuses on the statistical modeling of the power dissipation index (PDI) and accumulated cyclone energy (ACE) for the North Atlantic basin over the period 1949–2008, which are metrics routinely used to assess tropical storm activity, and their sensitivity to sea surface temperature (SST) changes. To describe the variability exhibited by the data, four different statistical distributions are considered (gamma, Gumbel, lognormal, and Weibull), and tropical Atlantic and tropical mean SSTs are used as predictors. Model selection, both in terms of significant covariates and their functional relation to the parameters of the statistical distribution, is performed using two penalty criteria. Two different SST datasets are considered [the Met Office’s Global Sea Ice and Sea Surface Temperature dataset (HadISSTv1) and NOAA’s extended reconstructed SST dataset (ERSSTv3b)] to examine the sensitivity of the results to the input data.

The statistical models presented in this study are able to well describe the variability in the observations according to several goodness-of-fit diagnostics. Both tropical Atlantic and tropical mean SSTs are significant predictors, independently of the SST input data, penalty criterion, and tropical storm activity metric. The application of these models to centennial reconstructions and seasonal forecasting is illustrated.

The sensitivity of North Atlantic tropical cyclone frequency, duration, and intensity is examined for both uniform and nonuniform SST changes. Under uniform SST warming, these results indicate that there is a modest sensitivity of intensity, and a decrease in tropical storm and hurricane frequencies. On the other hand, increases in tropical Atlantic SST relative to the tropical mean SST suggest an increase in the intensity and frequency of North Atlantic tropical storms and hurricanes.

1. Introduction

By convolving intensity, duration and frequency, the seasonally integrated power dissipation index (PDI; Emanuel 2005, 2007) and the accumulated cyclone energy (ACE; e.g., Bell et al. 2000; Camargo and Sobel 2005; Bell and Chelliah 2006) are concise metrics used to summarize the activity of a tropical storm season. Both of these measures are computed taking into account the lifetime of storms and the maximum sustained wind speed. The main difference between PDI and ACE is that the former is computed using the velocities cubed, while the latter the velocities squared. These metrics have been used in different studies examining past tropical storm activity as well as possible changes in climate warming scenarios.

Emanuel (2005) found a strong correlation between the North Atlantic PDI to tropical Atlantic sea surface temperature (SST; $r^2 = 0.65$). Swanson (2008) showed how comparable results could be obtained using the relative SST (difference between tropical Atlantic and tropical mean SSTs). Vecchi et al. (2008) explored the implications of Swanson (2008) for the attribution of past and projections of future PDI changes, and also showed how describing PDI as a linear function of relative SST would provide a better level of agreement with dynamical modeling results than using tropical Atlantic SST for climate change scenarios. Klotzbach (2006) found a significant increasing linear trend in North Atlantic ACE.
over the period 1986–2005 (see also Wu et al. 2008), and a statistically significant correlation between North Atlantic SST and ACE.

In studies examining the relation between PDI and ACE and climate-related predictors, linear regression is generally used after transforming the data to account for their skewness (e.g., Saunders and Lea 2005; Vecchi et al. 2008). Mestre and Hallegatte (2009) focused on the statistical modeling of the largest storm PDI each year. Despite their wide use, detailed statistical modeling of the PDI and ACE indexes is still lacking. In particular, outstanding questions revolve around the statistical distribution of these metrics, as well as the dependence of the parameters of this distribution on climate-related indices. Statistical modeling of PDI and ACE in terms of climate-related variables can suggest relationships that could lead to an improved understanding of the physical mechanisms controlling these two indices. Once these relations are explained based on our current theory of genesis and development of North Atlantic tropical storms, they could provide a foundation for improved capabilities of seasonal forecasts of tropical storm activity and better insights into possible interannual to centennial changes in tropical storm activity in response to climate variability and change. The topic of this study is, therefore, the statistical modeling of these two metrics in terms of climate indexes, and their sensitivity to uniform and nonuniform SST changes.

2. Data

We focus on the PDI and ACE over the period 1949–2008 for the North Atlantic Basin. We have derived the time series of these two indexes from the National Oceanic and Atmospheric Administration’s (NOAA) Hurricane Database (HURDAT; Jarvinen et al. 1984; McAdie et al. 2009), which provides information on latitude, longitude, maximum wind speed, and minimum pressure of the center of circulation for recorded tropical cyclones from 1851 to the present (Fig. 1). We have used the raw HURDAT wind speeds only for the tropical–subtropical portion of the storm lifetime, not including depressions. Moreover, we have applied a correction to the pre-1970 wind speed values \( v \) (in knots) based on the following relation (Landsea 1993):

\[
\begin{align*}
  v' &= \begin{cases} 
    v - 0.14 \sin \left( \frac{v - 45}{75} \right) & \text{if } v > 45 \text{ kt} \\
    v & \text{if } v \leq 45 \text{ kt}
  \end{cases}
\end{align*}
\]

The main effect of this correction is to weaken the pre-1970 hurricanes, resulting in smaller PDI and ACE values. In addition to inhomogeneities in the wind–pressure relationship, it is likely that there are inhomogeneities in the HURDAT dataset over this long period due to storm undercount (e.g., Landsea et al. 2004; Chang and
Guo 2007; Mann et al. 2007; Chenoweth and Divine 2008; Vecchi and Knutson 2008, 2011; Villarini et al. 2011a), and different corrections have been developed (e.g., Chang and Guo 2007; Mann et al. 2007; Landsea 2007; Landsea et al. 2008; Chenoweth and Divine 2008; Vecchi and Knutson 2008; Landsea et al. 2010; Vecchi and Knutson 2011). In this study, we focus on the period from 1949 onward to limit the possible impacts of inhomogeneities in the data.

Following Swanson (2008) and Vecchi et al. (2008), we focus on tropical Atlantic (SSTAtl) and tropical mean (SST trop) SSTs as possible covariates to describe PDI and ACE data. We choose SSTAtl because of the expected local effects of SST on tropical storm development in the North Atlantic (e.g., Emanuel 2005; Mann and Emanuel 2006; Vecchi and Soden 2007; Swanson 2008; Knutson et al. 2008; Zhao et al. 2009; Villarini et al. 2010b). We include SST trop because several studies in the literature point to the impacts of tropical mean SST on wind shear (Latif et al. 2007), upper-tropospheric temperature (Sobel et al. 2002), and other quantities of thermodynamic instability (e.g., Shen et al. 2000; Tang and Neelin 2004; Vecchi and Soden 2007; Ramsay and Sobel 2011), which affect North Atlantic tropical storm activity. Moreover, high-resolution atmospheric modeling studies found that tropical Atlantic SST relative to tropical mean SST is important in describing the response of tropical storm activity to different climate change scenarios (e.g., Knutson et al. 2008; Vecchi et al. 2008; Zhao et al. 2009, 2010; Villarini et al. 2011b).

Two different input datasets are considered—the Met Office’s Global Sea Ice and Sea Surface Temperature dataset (HadISSTv1; Rayner et al. 2003) and NOAA’s extended reconstructed SST dataset (ERSSTv3b; Smith et al. 2008)—and averaged over the period June–November. As shown in Villarini et al. (2010b), there are differences between these two datasets, which tend to be larger for tropical Atlantic than tropical mean SSTs. These discrepancies are likely due to different corrections for data inhomogeneity (e.g., the “bucket to intake” adjustment), differences in the use of the satellite record, as well as differences to infill missing SST values. The use of two datasets provides information about the sensitivity of our results to uncertainties in SST reconstructions. The tropical Atlantic SST anomalies (SSTAtl) are computed over 10°–25°N and 80°–20°W, while the tropical mean SST (SST trop) is found over the global tropics (30°S–30°N).

Note that PDI is used as an approximation of the overall power dissipation (PD; Bister and Emanuel 1998), which represents the total energy dissipated by the tropical storms. The calculation of PD is based on two-dimensional wind fields, and PDI represents an approximation of PD, in which the maximum wind speed is considered to be a perfect proxy for storm structure. This approximation introduces biases that complicate the interpretation of the PDI results in terms of PD (Maue et al. 2008).

3. Generalized Additive Model in Location, Scale, and Shape (GAMLSS)

The statistical modeling of PDI and ACE (the former normalized by a factor of $10^{11}$ and the latter by $10^{9}$) is performed by using the Generalized Additive Model in Location, Scale, and Shape (GAMLSS), proposed and developed by Rigby and Stasinopoulos (2005). The advantage of GAMLSS with respect to other models, such as the Generalized Linear Model, Generalized Additive Model, and Generalized Linear Mixed Model, is that we are not restricted in using distributions from the exponential family (e.g., Gaussian, exponential) but we can fit using a distribution from a more general set of distribution functions (e.g., highly skewed and/or kurtotic continuous and discrete distributions). This statistical framework was already successfully used to describe other hydrometeorological variables (e.g., Villarini et al. 2009a,b, 2010a).

We provide here a brief overview of GAMLSS and point the interested reader to Rigby and Stasinopoulos (2005) for a detailed discussion of the theory behind these models. Let us consider the predictand $Y$ to have a cumulative distribution function $F_Y(y_i, \theta')$, where $\theta' = (\theta'_1, \ldots, \theta'_q)$ is a vector of $q$ parameters and $y_i$ are $n$ observations. In general, $q$ is smaller than or equal to 4, because four-parameter distributions are flexible enough for most applications. We focus in this study on a semi-parametric additive model formulation to relate the predictors to the parameters of the selected distribution. Let $g_k(\cdot)$, for $k = 1, \ldots, q$, be monotonic link functions relating the parameters of the distribution to the predictors through

$$g_k(\theta_k) = x_k \beta_k + \sum_{j=1}^{J_k} h_{jk}(x_{jk}),$$

where $\theta'$ is a vector of size $n$, $\beta'_k = (\beta'_{1k}, \ldots, \beta'_{J_k})$ is a parameter vector of length $J_k$, $x_k$ is a known design matrix of order $n \times J_k$, and $h_{jk}$ is a function of the predictor $x_{jk}$. The functions $h_{jk}$ are smoothing terms allowing for a higher degree of flexibility in modeling the relation between the parameters of the distributions and the predictors. In this study, we use cubic splines as smoothing functions.

Because PDI and ACE are continuous and can only have positive values, we explore these four two-parameter
distributions: gamma, Gumbel, lognormal, and Weibull (e.g., Krishnamoorthy 2006). We model the parameters of these distributions as a linear or nonlinear (via cubic splines) function of covariates. Model selection, both in terms of predictors and their functional relation to the parameters of these distributions, is performed by penalizing more complex models with respect to the Akaike information criterion (AIC; Akaike 1974) and the Schwarz Bayesian criterion (SBC; Schwarz 1978). Because AIC and SBC do not provide information about the quality of the fit (e.g., Hipel 1981), we assess the quality of the fit by comparing the first four statistical moments of (normalized quantile) residuals against a standard normal distribution, together with their Filliben correlation coefficient (Filliben 1975), which represents the correlation coefficient between the order statistics of the residuals and those of a standard normal distribution, and by visual examination of the residuals’ plots, such as quantile–quantile (qq) plots and worm plots (van Buuren and Fredriks 2001; Stasinopoulos and Rigby 2007). The latter are detrended forms of qq plots, where the agreement between the observations and the selected distribution is represented in the form of the “worm.” A flat worm supports the choice of the selected distribution. Because of sampling uncertainties, in particular for the high and low quantiles, the points should be within the 95% confidence intervals.

All the calculations are performed in R (R Development Core Team 2008) using the freely available GAMLSS package (Stasinopoulos et al. 2007).

4. Results

a. Statistical modeling

Modeling of the PDI and ACE in terms of tropical Atlantic and tropical mean SSTs is performed using GAMLSS. Focusing first on PDI, Figs. 2 and 3 show the
results obtained using AIC and SBC as penalty criteria. Summary of the models’ fit is presented in Table 1. Independently of the penalty criterion and SST input data, both tropical Atlantic and tropical mean SSTs are always retained by the model as significant predictors (see also Villarini et al. 2010b). Moreover, the former has a positive coefficient, while the latter a negative one. This is in agreement with the results in Swanson (2008) and Vecchi et al. (2008). The magnitudes of these coefficients are larger for the tropical Atlantic, suggesting that uniform SST warming should lead to tropical storm seasons with larger PDIs. The ratio of the coefficients linking SST_{trop} and SST_{Atl} to the mean is between 0.85 and 0.92, similar to the results of Swanson (2008) using linear regression. These models describe very well the variability exhibited by the data, with alternating periods of increased and decreased activity. The fit diagnostics (Figs. 2 and 3, right; Table 1) support the choice of these models. When using ERSSTv3b data for modeling PDI, independently of the penalty criterion the gamma distribution with the logarithm of the \( \mu \) parameter linear function of both tropical Atlantic and tropical mean SSTs is selected as final model. The picture is slightly different when using HadISSTv1 data. The Weibull distribution with log(\( \mu \)) depending on both predictors by means of a cubic spline is selected when penalizing with respect to AIC. On the other hand, a gamma distribution with log(\( \mu \)) depending linearly on both predictors is selected when penalizing with respect to SBC.

The results and conclusions for the ACE are similar to those found for the PDI (Figs. 4 and 5; Table 2). Both tropical Atlantic and tropical mean SSTs are included in the final models, with the coefficient of the former (latter) having a positive (negative) sign (see also Villarini et al. 2010b). The results using ERSSTv3b data are the same independently of the penalty criterion, with the gamma distribution being the selected distribution with the log(\( \mu \)) depending linearly on both predictors. The results for the HadISSTv1 data, both in terms of the parametric distribution and functional relation of its parameters on the covariates, depend on the penalty criterion. When using AIC, the data can be described by a Weibull distribution with the \( \mu \) parameter depending on the SST predictors by means of a cubic spline (via a logarithmic link function). The gamma distribution with log(\( \mu \)) depending linearly on both predictors is selected when penalizing with respect to SBC. These models are able to describe very well the variability exhibited by the
data, as is also supported by the fit diagnostics (Figs. 4 and 5, right; Table 2). The absolute value of the ratio between the coefficients of the $\mu$ parameter for the two covariates ranges between 0.8 and 0.9, similar to what is found for PDI and by Swanson (2008). Because of the larger values of the tropical Atlantic SST coefficient, a uniform SST warming would result in increasing ACE values.

### b. Sensitivity to SST changes

We can couple the information from the PDI and ACE gamma models presented in this study with the statistical models describing the frequency of tropical storms and hurricanes to examine the sensitivity of North Atlantic hurricane frequency, duration, and intensity to uniform and nonuniform SST changes.

Let us start with uniform warming. We can describe the basin-wide tropical storm and hurricane count $f_A$ with a Poisson regression model, in which the logarithm of the rate of occurrence is a linear function of $SST_{At}$ and $SST_{trop}$ (Villarini et al. 2010b; Vecchi et al. 2011; Villarini et al. 2012). The expected value is

\[
E[\phi] = \bar{\phi} = \Phi \exp(\alpha_{\phi} T_A + \beta_{\phi} T_T), \quad (3)
\]

where to simplify the notation we indicate the tropical Atlantic SST and tropical mean SST with $T_A$ and $T_T$, respectively. Taking the logarithmic differential of Eq. (3),

\[
\frac{d\bar{\phi}}{d\phi} = \alpha_{\phi} dT_A + \beta_{\phi} dT_T. \quad (4)
\]

The fractional sensitivity of frequency to uniform warming is

\[
\frac{d\bar{\phi}}{\bar{\phi}} \bigg|_{dT_A=dT_T=dT} = (\alpha_{\phi} + \beta_{\phi}) dT = \gamma_{\phi} dT. \quad (5)
\]

The various factors can be computed from the statistical modeling results of Villarini et al. (2010b), Vecchi et al. (2011), or Villarini et al. (2012)—depending on whether we want to focus on hurricanes or tropical storms, and whether we want to train the statistical model on observations or dynamical models. Notice that $\alpha_{\phi}$ is positive, while $\beta_{\phi}$ is negative. If $\gamma_{\phi} > (<) 0$, then frequency has a positive (negative) sensitivity to uniform warming. The published results indicate that the frequency sensitivity is negative, except for the raw HURDAT data, which does not correct for likely storm undercount in the earliest part of the record. Taking the results of Villarini et al. (2010b), we get $\gamma_{\phi} \sim -3\%$ to $-7\% \; ^{\circ}\text{C}^{-1}$ for tropical storms. For hurricane frequency, we get $\gamma_{\phi} \sim -12\%$ to $-22\% \; ^{\circ}\text{C}^{-1}$ from the results in Villarini et al. (2012), a range that includes the value of $-13\% \; ^{\circ}\text{C}^{-1}$ that was estimated by Vecchi et al. (2011) from the sensitivity of the HiRAM-C180 dynamical model (Zhao et al. 2009).

Let us take the definition of PDI and ACE (to simplify the notation, we will indicate them with $P$ and $A$, respectively):

\[
P = \sum_{s=1}^{\phi} \sum_{\tau=1}^{d(s)} u^3(s, \tau)
\]

\[
A = \sum_{s=1}^{\phi} \sum_{\tau=1}^{d(s)} u^2(s, \tau), \quad (6)
\]

where $d$ is the duration of each storm and $u$ is the wind speed at each time interval.

Based on the results described in section 4a, we can write the expected values of PDI and ACE as

\[
E[P] = \bar{P} = k \exp(\alpha_P T_A + \beta_P T_T) \quad \text{and} \quad (7)
\]

\[
E[A] = \bar{A} = l \exp(\alpha_A T_A + \beta_A T_T). \quad (8)
\]
By taking the logarithmic differential of Eqs. (7) and (8), we get

$$\frac{d \mathcal{P}}{\mathcal{P}} = \alpha_p dT_A + \beta_p dT_T$$  \hspace{1cm} (9)

and

$$\frac{d \mathcal{A}}{\mathcal{A}} = \alpha_A dT_A + \beta_A dT_T.$$  \hspace{1cm} (10)

To move forward, let us assume that the expected value of PDI and ACE can be approximated as the product of a scaling factor (different between PDI and ACE), the expected frequency, an expected duration scale, and the cube of an expected wind speed scale:

$$\mathcal{P} \approx K \bar{\phi} \bar{\delta}^3$$ \hspace{1cm} (11)

and

$$\mathcal{A} \approx L \bar{\phi} \bar{\delta}^2.$$ \hspace{1cm} (12)

Taking the logarithmic differential of Eqs. (11) and (12), we get

$$\frac{d \mathcal{P}}{\mathcal{P}} = \frac{d \bar{\phi}}{\bar{\phi}} + \frac{d \bar{\delta}}{\bar{\delta}} + 3 \frac{dt}{\bar{\tau}}$$ \hspace{1cm} (13)

$$\frac{d \mathcal{A}}{\mathcal{A}} = \frac{d \bar{\phi}}{\bar{\phi}} + \frac{d \bar{\delta}}{\bar{\delta}} + 2 \frac{dt}{\bar{\tau}}.$$ \hspace{1cm} (14)

After substituting Eqs. (9) and (10) into Eq. (13) and (14), respectively, and subtracting Eq. (4) from them, we obtain

$$(\alpha_p - \alpha_\phi) dT_A + (\beta_p - \beta_\phi) dT_T = \frac{d \bar{\delta}}{\bar{\delta}} + 3 \frac{dt}{\bar{\tau}}$$ \hspace{1cm} (15)

and

$$(\alpha_A - \alpha_\phi) dT_A + (\beta_A - \beta_\phi) dT_T = \frac{d \bar{\delta}}{\bar{\delta}} + 2 \frac{dt}{\bar{\tau}}.$$ \hspace{1cm} (16)

Subtracting Eq. (16) from (15), we can find the sensitivity equation for the scale intensity:

$$\frac{dt}{\bar{\tau}} = (\alpha_p - \alpha_A) dT_A + (\beta_p - \beta_A) dT_T.$$ \hspace{1cm} (17)

We can define the sensitivity parameters of the expected intensity scale as $\alpha = (\alpha_p - \alpha_A)$ and $\beta = (\beta_p - \beta_A)$, with the proportional sensitivity parameter to uniform warming for the intensity scale being $\gamma = (\alpha_p - \alpha_A) + (\beta_p - \beta_A)$. Based on the values in Tables 1 and 2, we find a central estimate for the sensitivity of the
intensity scale to tropically uniform warming of 0% to −2% °C−1. The uncertainty on that sensitivity, however, is quite large.

We can find a sensitivity equation for the duration scale by subtracting 2 times Eq. (17) from Eq. (16):

\[
\begin{align*}
\frac{d\bar{\sigma}}{\delta} &= (3\alpha_A - \alpha_\phi - 2\alpha_p)\ dT_A + (3\beta_A - \beta_\phi - 2\beta_p)\ dT_T, \\
&= (3\alpha_A - \alpha_\phi - 2\alpha_p)\ dT_A + (3\beta_A - \beta_\phi - 2\beta_p)\ dT_T.
\end{align*}
\]  (18)

We can define the sensitivity parameters of the expected duration scale as \(\alpha_\bar{\sigma} = (3\alpha_A - \alpha_\phi - 2\alpha_p)\) and \(\beta_\bar{\sigma} = (3\beta_A - \beta_\phi - 2\beta_p)\). The proportional sensitivity parameter to uniform warming for duration scale is \(\gamma_\bar{\sigma} = (3\alpha_A - \alpha_\phi - 2\alpha_p) + (3\beta_A - \beta_\phi - 2\beta_p) = 3\gamma_A - \gamma_\phi - 2\gamma_p\). Based on the values in Tables 1 and 2, and assuming that the relevant frequency scaling is that of hurricanes, we find a central estimate for the duration-scale sensitivity to tropically uniform warming of +41 to +44% °C−1. If the relevant frequency scaling is that of tropical storms, we find a duration-scale sensitivity to uniform warming of +25% to +36% °C−1.

The above relations were valid for uniform SST warming. We can also modify the fractional sensitivity equations for frequency, as well as the duration and intensity scale, to explore the sensitivity of each to the nonuniform components of warming. The sensitivity equations are of the form

\[
\frac{d\xi}{\xi} = \alpha_\xi\ dT_A + \beta_\xi\ dT_T.
\]  (19)

By defining the nonuniform component of SST change as \(dT_{rel} = dT_A - dT_T\), the sensitivity equations can be rewritten as

\[
\frac{d\xi}{\xi} = \alpha_\xi\ dT_{rel} + (\alpha_\xi + \beta_\xi)\ dT_T.
\]  (20)

Therefore, the total fractional sensitivity is the sensitivity to uniform warming (\(\gamma_\xi = \alpha_\xi + \beta_\xi\)) described above plus the sensitivity to the nonuniform SST change, which is much larger per unit temperature change for all quantities except the duration scale. From internal variations of the climate system, the nonuniform component of the SST change tends to be much larger than the uniform component, so one could approximate the sensitivity of North Atlantic tropical storm activity based on relative SST. Meanwhile, in response to changes in the
top-of-the-atmosphere radiative forcing, the amplitude of the uniform SST changes can be substantially larger than that of the nonuniform component, yet the cyclone sensitivity is larger for the latter than the former, meaning that both the uniform and nonuniform components must be considered. We have summarized in Table 3 the observationally estimated values of the nonuniform and uniform fractional sensitivities of North Atlantic cyclone activity indices to SST change.

To interpret these results, it is worth clarifying that the intensity and duration scales do not refer to the average duration of a tropical storm (e.g., from first to last gale-force record) or the average intensity of all storms (e.g., averaging all records together equally), respectively. Rather, the duration scale is some representative length of time during which each storm is at the range of intensities that contribute to the bulk of PDI and ACE (i.e., typically, how long each storm is at the intensities of the strongest storms in the basin). Similarly, the intensity scale refers to a representative intensity of the storm records that contribute to most of the PDI and ACE values (i.e., the typical intensity of the strongest storms). Therefore, based on the apparent historical sensitivities of the frequency, PDI, and ACE, the statistical modeling results indicate that uniform warming is expected to lead to (i) a modest decrease in tropical storm and hurricane frequency in the Atlantic, (ii) little change in the typical intensity of the strongest storms, and (iii) storms being expected to spend a substantially larger aggregate time as the “strongest” storms.

The modest sensitivity of intensity to uniform warming appears to be consistent with the small sensitivity of the global-mean potential intensity (PI) from coupled global climate models (CGCMs) in twenty-first century warming scenarios (e.g., Vecchi and Soden 2007). Further, the modest implied sensitivity of the intensity of strongest storms to uniform warming suggests that the observed 1980–2006 increase in the intensity of the strongest storms in the North Atlantic was not driven by the uniform warming component of the observed SST change, but by the warming of the Atlantic relative to the tropical mean over this period. The negative frequency of hurricane activity indices to uniform warming is consistent with the atmospheric general circulation model (AGCM) results of Zhao and Held (2011).

The sensitivity to nonuniform SST changes is much more marked. These results indicate that increases in tropical Atlantic SST relative to the tropical mean SST should lead to large changes in tropical storm and hurricane frequencies and intensity scale, with a reduction in duration scale. It is harder to explain the opposite sensitivity of the duration scale to uniform and nonuniform warmings. Our results suggest that for nonuniform warming, storms become more frequent and stronger, but they spend, on average, less time as strong storms. On the other hand, for uniform warming there are fewer storms that are approximately of the same

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### Table 2. As in Table 1, but for ACE.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>ERSSTv3b</th>
<th>HadISSTv1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.64 (0.08)</td>
<td>1.73 (0.07)</td>
</tr>
<tr>
<td></td>
<td>1.64 (0.08)</td>
<td>1.63 (0.07)</td>
</tr>
<tr>
<td>Log(μ): SST_{Atl}</td>
<td>1.61 (0.32)</td>
<td>1.54 (0.29; cs)</td>
</tr>
<tr>
<td></td>
<td>1.61 (0.32)</td>
<td>1.56 (0.28)</td>
</tr>
<tr>
<td>log(μ): SST_{trop}</td>
<td>-1.43 (0.42)</td>
<td>-1.31 (0.42; cs)</td>
</tr>
<tr>
<td></td>
<td>-1.43 (0.42)</td>
<td>-1.27 (0.41)</td>
</tr>
<tr>
<td>Log(σ)</td>
<td>-0.72 (0.09)</td>
<td>0.97 (0.10)</td>
</tr>
<tr>
<td></td>
<td>-0.72 (0.09)</td>
<td>-0.73 (0.09)</td>
</tr>
<tr>
<td>Mean (residuals)</td>
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<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Variance (residuals)</td>
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<td>1.02</td>
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<td></td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>Skewness (residuals)</td>
<td>-0.08</td>
<td>0.12</td>
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<tr>
<td></td>
<td>-0.08</td>
<td>-0.15</td>
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<td>Kurtosis (residuals)</td>
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<td></td>
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<td>Filliben (residuals)</td>
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<td></td>
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<td>AIC</td>
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<td></td>
<td>282.5</td>
<td>280.9</td>
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<tr>
<td>SBC</td>
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<td>298.9</td>
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<td></td>
<td>290.9</td>
<td>289.2</td>
</tr>
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</table>

### Table 3. Observationally estimated values of the nonuniform and uniform fractional sensitivities of the North Atlantic cyclone activity indices to SST changes. In each row, the top (bottom) line indicates the values for the statistical model trained on data from the HadISST.v1 (ERSST.v3b) dataset. Values in parentheses indicate the standard error. The values for tropical storm frequency are based on Villarini et al. (2010b), those for hurricane frequency are based on Villarini et al. (2012), and those for PDI and ACE are from the present study. The estimates of sensitivity for intensity and duration scale are based on Eqs. (17) and (18).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Sensitivity to uniform SST (% °C⁻¹)</th>
<th>Sensitivity to relative SST (% °C⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS frequency</td>
<td>-3 (24)</td>
<td>+102 (15)</td>
</tr>
<tr>
<td></td>
<td>-7 (26)</td>
<td>+105 (14)</td>
</tr>
<tr>
<td>Hurricane frequency</td>
<td>-12 (27)</td>
<td>+105 (16)</td>
</tr>
<tr>
<td></td>
<td>-22 (30)</td>
<td>+111 (17)</td>
</tr>
<tr>
<td>PDI</td>
<td>+29 (58)</td>
<td>+187 (33)</td>
</tr>
<tr>
<td></td>
<td>+16 (62)</td>
<td>+194 (37)</td>
</tr>
<tr>
<td>ACE</td>
<td>+29 (49)</td>
<td>+156 (28)</td>
</tr>
<tr>
<td></td>
<td>+18 (52)</td>
<td>+161 (32)</td>
</tr>
<tr>
<td>Intensity scale</td>
<td>0 (75)</td>
<td>+31 (46)</td>
</tr>
<tr>
<td></td>
<td>-2 (94)</td>
<td>+33 (49)</td>
</tr>
<tr>
<td>Duration scale</td>
<td>+41 (189)</td>
<td>-11 (108)</td>
</tr>
<tr>
<td></td>
<td>+44 (215)</td>
<td>-16 (122)</td>
</tr>
</tbody>
</table>
maximum intensity but, on average, they remain strong for longer.

5. Discussion and conclusions

In this study we have focused on the power dissipation index (PDI) and accumulated cyclone energy (ACE) for North Atlantic tropical storms over the period 1949–2008. We have examined the dependence of these two metrics on tropical Atlantic and tropical mean SSTs. Statistical modeling was performed using GAMLSS. Two different penalty criteria (AIC and SBC) were selected, as well as two different SST input datasets (ERSSTv3b and HadISSTv1).

Our results indicate that both tropical Atlantic and tropical mean SSTs are significant covariates in describing the variability of PDI and ACE for North Atlantic seasonal tropical storm activity, providing additional evidence as to the importance of relative SST on tropical storm activity. For both PDI and ACE, the coefficient of tropical Atlantic SST had a positive sign, while the coefficient for tropical mean SST was negative. For both PDI and ACE the coefficient for the Atlantic SST was larger than for the tropical SST.

Given these models, and studies describing the frequency of tropical storms and hurricanes in terms of $SST_{\text{Atl}}$ and $SST_{\text{trop}}$ using a Poisson regression model (Villarini et al. 2010b; Vecchi et al. 2011; Villarini et al. 2012), we have examined the sensitivity of frequency, duration, and intensity of North Atlantic tropical cyclones to SST changes. Under uniform SST warming, these results indicate that we should expect a decrease in North Atlantic tropical storm and hurricane frequency, small changes in the typical intensity of the strongest storms, and that storms should spend a longer amount of time as the strongest storms. We have obtained a larger sensitivity to relative SST (tropical Atlantic SST minus tropical mean SST), with large increases in the tropical storm and hurricane frequencies, PDI, ACE, and intensity scale. While these results for uniform warming are consistent with findings from climate models (e.g., Vecchi and Soden, 2007; Zhao and Held, 2011), it is worth reminding that they are based on the relations obtained from statistical models and the assumptions made to obtain Eqs. (11) and (12).

In addition to modeling the adjusted records, we have also examined the sensitivity of our results to the adjustment in Eq. (1). Independently of the penalty criterion and input dataset, the parametric distributions are the same as in Tables 1 and 2; moreover, tropical Atlantic and tropical mean SSTs are always retained as important predictors, with the coefficient of the former (latter) being positive (negative). For both PDI and ACE when using ERSSTv3b data, however, the coefficient of $SST_{\text{trop}}$ is larger than that of $SST_{\text{Atl}}$, suggesting that uniform SST warming would lead to a decrease in tropical storm seasonal activity. If HadISSTv1 data are used as input, the absolute value of the $SST_{\text{Atl}}$ coefficient is slightly larger than the one for $SST_{\text{trop}}$, effectively offsetting the impacts of uniform SST warming. The sensitivity of our results to the data used for model development highlights the importance of efforts to reanalyze the HURDAT database (e.g., Landsea et al. 2004, 2008), in particular for studies trying to examine possible changes in North Atlantic tropical storm activity in a warmer climate.

The statistical models provide a framework with which to reconstruct the PDI and ACE time series prior to 1949 using reconstructed SST time series (e.g., Fig. 6, top). These reconstructions could provide information about the North Atlantic tropical storm activity in the past, placing recent variations into a larger context. The
centennial reconstruction of PDI indicates periods of enhanced and reduced variabilities over the past 130 yr on a variety of time scales. Thus, the PDI reconstruction indicates that there have been periods before 1949 that were relatively active compared to the post-1995 era of heightened activity. Future work will explore modifying the methodology of Mann et al. (2009) using these models to build multi-centennial reconstructions of PDI and ACE.

Apart from information about possible changes in tropical storm activity from decadal to centennial climate variations and change, another application of our models is related to the seasonal forecasts of PDI and ACE (e.g., Camargo et al. 2007; Klotzbach 2007; Klotzbach and Gray 2009; Vecchi et al. 2011). For instance, the NOAA/Climate Prediction Center (CPC) uses the ACE value to classify a North Atlantic tropical storm season as being above, near, or below normal. Recently, Vecchi et al. (2011) proposed a hybrid statistical–dynamical model that can be used to forecast hurricane counts starting from September of the previous year. As an example, we have “forecasted” the PDI distribution using a 10-member June–November tropical Atlantic and tropical mean SST forecast set initialized in January. The correlation coefficient between the observations and the median of the PDI distribution over the period 1982–2009 is 0.75, with an RMSE of 1.43 m$^3$ s$^{-2}$ and an MAE of 1.06 m$^3$ s$^{-2}$ (Fig. 6, bottom). Even though we have forecasted the period used for model fitting, results obtained from leave-one-out cross validation support the predictive capability of this model (compared to the full model, the correlation coefficient is 0.49 versus 0.56, the RMSE is 1.40 versus 1.33 m$^3$ s$^{-2}$, and the MAE of 1.02 m$^3$ s$^{-2}$ versus 0.97 m$^3$ s$^{-2}$; these results are for the period 1949–2008). We have used the model obtained from the period 1949–2008 to do retrospective forecasts for 2009 and 2010. The PDI values for these 2 yr are 1.25 and 4.11 m$^3$ s$^{-2}$, respectively. For 2009, the median forecast is 1.76 m$^3$ s$^{-2}$, with the 5th and 95th percentiles being 0.43 and 5.34 m$^3$ s$^{-2}$, and the 25th and 75th percentiles being 1.01 and 2.88 m$^3$ s$^{-2}$. For 2010, the median forecast is 2.53 m$^3$ s$^{-2}$, with the 5th and 95th percentiles being 0.73 and 7.82 m$^3$ s$^{-2}$, and the 25th and 75th percentiles being 1.57 and 4.03 m$^3$ s$^{-2}$. These preliminary results are encouraging, and in a future study we will examine the applicability of our statistical models to the seasonal forecasts of PDI and ACE, in a fashion similar to that described in Vecchi et al. (2011). For the 2011 season, based on June–November tropical Atlantic and tropical mean SST forecasts initialized in January, the median PDI forecast is 2.85 m$^3$ s$^{-2}$, with the 5th and 95th percentiles being 0.81 and 8.46 m$^3$ s$^{-2}$, and the 25th and 75th percentiles being 1.76 and 4.49 m$^3$ s$^{-2}$. The median PDI forecast indicates that the 2011 season is slightly more active than the 1980–2010 average (2.60 m$^3$ s$^{-2}$), but less active relative to the 1995–2010 average (3.52 m$^3$ s$^{-2}$).

One element that requires further discussion is the fact that tropical Atlantic and tropical mean SSTs are correlated (the correlation between these two predictors is equal to 0.73 for HadISSTv1 and 0.78 for ERSSTv3b). At the onset, it is worth clarifying that, even though these values may appear large, they are not nearly as large as those in studies from other disciplines (e.g., Burnham and Anderson 2004; Stasinopoulos and Rigby 2007). As a rule of thumb, Burnham and Anderson (2002) suggested keeping all of the predictors unless the correlation coefficient is extremely high, with [0.95] as a cutoff value for dropping a covariate. To assess whether collinearity may have affected our results, we use the variance inflation factor (VIF). This is a diagnostic tool commonly used to evaluate the impacts of collinearity, by quantifying the impacts of the correlation among predictors on inflating the sampling variance of an estimated regression coefficient. For the gamma models, we compute the VIF using the vif function in the Design package (Harrell 2009) in R (R Development Core Team 2008), in which the method described in Davis et al. (1986) is implemented (see also Wax 1992). A VIF value of 10 is generally used to decide whether the collinearity is high or not (e.g., Davis et al. 1986; O’Brien 2007), and this is the cutoff value we use. Independently of the SST input data and tropical storm activity metric, the VIF values are smaller than 3, indicating that the impacts of collinearity do not significantly affect the results of this study [see also the discussion in Villarini et al. (2011a)].

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