The thermohaline circulation of the ocean is compared to the hydrothermal circulation of the atmosphere. The oceanic thermohaline circulation is expressed in potential temperature–absolute salinity space and comprises a tropical cell, a conveyor belt cell, and a polar cell, whereas the atmospheric hydrothermal circulation is expressed in potential temperature–specific humidity space and unifies the tropical Hadley and Walker cells as well as the midlatitude eddies into a single, global circulation. The oceanic thermohaline streamfunction makes it possible to analyze and quantify the entire World Ocean conversion rate between cold–warm and fresh–saline waters in one single representation. Its atmospheric analog, the hydrothermal streamfunction, instead captures the conversion rate between cold–warm and dry–humid air in one single representation. It is shown that the ocean thermohaline and the atmospheric hydrothermal cells are connected by the exchange of heat and freshwater through the sea surface. The two circulations are compared on the same diagram by scaling the axes such that the latent heat energy required to move an air parcel on the moisture axis is equivalent to that needed to move a water parcel on the salinity axis. Such a comparison leads the authors to propose that the Clausius–Clapeyron relationship guides both the moist branch of the atmospheric hydrothermal circulation and the warming branches of the tropical and conveyor belt cells of the oceanic thermohaline circulation.

1. Introduction

The oceanic thermohaline circulation and the global atmospheric circulation are generally analyzed as two separate systems although they influence each other through the surface of the ocean. In the present work both are represented in thermodynamic coordinates and linked to each other. We analyze and visualize how the ocean and atmosphere are closely acting together as a number of overturning cells, expressing the mixing of air and water masses. To do so we use two recently introduced streamfunctions, one for the ocean and one for the atmosphere. The oceanic thermohaline streamfunction (Zika et al. 2012; Dőös et al. 2012) integrates the entire World Ocean circulation in potential temperature–salinity (θ–S) space. This streamfunction thus makes it possible to analyze and quantify the entire World Ocean conversion rate between cold–warm and fresh–saline waters in one single
representation. In this representation, most of the water mass conversion can be attributed to the exchange of heat and freshwater through the surface of the ocean, where the freshwater flux is the evaporation $E$ minus precipitation $P$ and river runoff $R$: $E - P - R$. The oceanic streamfunction allows us to define the thermohaline circulation as the transport across isotherms and isohalines. While this thermohaline circulation mainly takes place at the surface and mixed layer of the ocean, there are some conversions that are due to mixing in the deep ocean. This mixing can be thought of being part of the abyssal circulation. Note here that this deep ocean circulation can still be strong in terms of mass transport even if there are no diathermal, diahaline, or dipycnal mixing. In this case, the water masses are driven by the wind stress or by mass conversions at the surface of the ocean and then advected along the isotherms, isohalines, or isopycnals.

The thermohaline streamfunction captures the global thermohaline circulation in two main cells: one corresponding to the conveyor belt and the other to the shallow tropical circulation. There is also a weak polar cell corresponding to a combination of water mass conversion in the Southern Ocean near Antarctica and to a mixing of the North Atlantic Deep Water with the Antarctic Bottom Water in the abyssal ocean.

The analog for the atmosphere is the hydrothermal streamfunction (Kjellsson et al. 2014). This streamfunction is an integral of the entire atmospheric circulation in potential temperature–specific humidity $(\theta-q)$ space and captures the conversion rate between cold–warm and dry–humid air in one single representation. The hydrothermal streamfunction shows mainly one strong cell that summarizes the entire atmospheric overturning circulation in coordinates of specific humidity and potential temperature. The streamfunction describes a cycle with three branches: (i) a convective branch where latent heat is converted into sensible heat along moist adiabats, (ii) a cooling branch where dry air loses energy owing to radiative damping, and (iii) a moistening branch where cold, dry air is heated and moistened following the Clausius–Clapeyron relationship.

The thermohaline and hydrothermal streamfunctions have recently and respectively been used to study oceanic paleoclimates (Ballarotta et al. 2014), atmospheric natural variability (Kjellsson et al. 2014), and atmospheric impacts of anthropogenic forcing (Laliberté et al. 2015; Kjellsson 2015). All these studies showed important changes in the two streamfunctions due to externally forced displacements of the atmospheric and oceanic circulation in temperature, salinity, and humidity space.

In the present study, we combine and superimpose the “overturning” analysis of the ocean on the one for the atmosphere. For this we rely on the close correspondence between the temperatures of the ocean and the air at the sea surface, as seen in Fig. 1a.

Using simulations integrated with the Earth system model EC-Earth, we have obtained a single picture for the coupled ocean–atmosphere overturning circulation that can be thought of as a hydrothermohaline streamfunction. This picture highlights how the atmospheric

![Fig. 1. Surface concentration of matching air temperature near the sea surface (SSTA) with (left) sea surface temperature (SSTO) and (right) specific humidity $q$. The green dotted line in (left) is for identical SSTA and SSTO. The green dotted line in (right) is the Clausius–Clapeyron relationship, representing the highest possible humidity for a given temperature. The red dotted line in (left) is the straight line for the best fit between SSTA and SSTO and in (right) the 81% relative humidity. The blue contours show the normalized sea surface area integrated over the entire global ocean.](Unauthenticated | Downloaded 01/02/23 05:48 AM UTC)
hydrothermal cell and the oceanic thermohaline conveyor belt cell are closely linked to each other along the line of the Clausius–Clapeyron relationship (Fig. 1, right).

In the present study we follow the standard practice (Czaja and Marshall 2006; Pauluis et al. 2008, 2010; Dóos and Nilsson 2011) of using potential temperature as a vertical coordinate in the atmosphere. However, instead of using altitude as the x-axis coordinate, we use salinity and specific humidity when constructing the thermohaline and the hydrothermal streamfunctions. This approach follows previous studies (Nilsson and Körnich 2008; Ferreira and Marshall 2015) that have also related the specific humidity coordinates for the atmosphere with salinity coordinates for the ocean but only for the meridional overturning circulation.

2. Model, data, and methods

a. The EC-Earth model

The simulation analyzed in the present study was performed with version 2.2 of the Earth system model EC-Earth. In this version, EC-Earth includes an atmosphere–land surface module coupled to an ocean–sea ice module (Hazeleger et al. 2010, 2012).

The atmospheric component is based on the Integrated Forecasting System (IFS) cycle 31r1 of the European Centre for Medium-Range Weather Forecasts (ECMWF) but includes some improvements from later cycles such as a new convection scheme (Bechtold et al. 2008) and the land surface scheme HTESSEL (Balsamo et al. 2009). IFS is a spectral model with triangular truncation. In this study, the spectral T159 truncation is used. It is roughly equivalent to a 125-km (or \( \sim 1.125^\circ \)) resolution. In the vertical, 62 levels of a terrain-following hybrid coordinate are used. The lowest model level is at a height of 30 m above the ground, and the highest level is at 5 hPa. The use of a semi-Lagrangian advection scheme makes it possible to use a time step of 1 h. Results from the atmospheric component of EC-Earth (i.e., IFS) are discussed by Hazeleger et al. (2012).

The ocean component is the Nucleus for European Modelling of the Ocean (NEMO) version 2 (Madec 2008). NEMO uses the so-called ORCA1 configuration. This configuration consists of a tripolar grid with poles over northern North America, Siberia, and Antarctica at a resolution of about 1°. A higher resolution of \( \frac{1}{2}^\circ \) is applied close to the equator, and owing to the definition of the poles, the resolution around the North Pole is slightly higher than 1°. We define 42 depth levels together with a partial step representation of the bottom topography. NEMO is built on the Boussinesq approximation, is volume conserving, and has a real freshwater flux imposed as a surface boundary condition. This version 2 of NEMO used a linear free surface formulation and the UNESCO nonlinear equation of state by Jackett and Mc Dougall (1995). The effects of subgrid-scale processes (mainly mesoscale eddies) are represented by an isopycnal mixing–advection parameterization as proposed by Gent and McWilliams (1990), while the vertical mixing is parameterized according to a local turbulent kinetic energy (TKE) closure scheme (Blanke and Delecluse 1993). A bottom boundary layer scheme, similar to that of Beckmann and Döscher (1997), is used to improve the representation of dense water spreading. The Louvain-la-Neuve Sea Ice Model, version 2 (LIM2; Fichefet and Maqueda 1997; Bouillon et al. 2009), is included in NEMO, with dynamics based on Hibler (1979) and thermodynamics based on Semtner (1976).

The atmosphere–land surface module and the ocean–sea ice module are coupled through the Ocean Atmosphere Sea Ice Soil (OASIS) coupler, version 3 (Valcke 2006); the coupling frequency is 3 h.

b. The simulation and data

Our results are based on the last 20 yr of a historical simulation performed with EC-Earth, following the guidelines for phase 5 of the Coupled Model Intercomparison Project (CMIP5; Taylor et al. 2012). The historical simulation was initialized in 1850 from the end of a 600-yr control simulation using 1850 greenhouse gas concentrations and aerosol forcing and forced by observed greenhouse gas and aerosol concentrations until 2005. The preindustrial control simulation was the continuation of an initial spinup simulation that was run for 440 yr with present-day forcing.

The climate of the model in present day and in the PI simulation is described in greater detail by Hazeleger et al. (2012) and Sterl et al. (2012) for the ocean.

From the atmospheric data, we use 6-hourly air temperature \( T_A \), specific humidity \( q \), zonal and meridional winds \( u_A \) and \( v_A \), and surface pressure \( p_s \). From the ocean, we use monthly water potential temperature \( \theta_o \), absolute salinity \( S \), and velocities \( u_O \) and \( v_O \). At 6-hourly data output and a horizontal grid of 1.125°, the atmospheric eddies are resolved as well as their associated heat and water fluxes. The ocean eddies are, however, not resolved by the ocean model and must therefore, as previously discussed, be parameterized.

To represent present-day climate, the data used here are extracted from the 1985–2005 slice of the historical simulation.

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1 See http://old.ecmwf.int/research/ifsdocs/CY31r1/index.html.
c. Thermohaline and hydrothermal streamfunctions

Thermohaline (Zika et al. 2012; Döös et al. 2012; Groeskamp et al. 2014) and hydrothermal (Kjellsson et al. 2014) streamfunctions are obtained from the thermodynamic mass flux vectors retrieved from the thermodynamics transform formalism of Laliberté et al. (2015) along with a Helmholtz decomposition that makes it possible to analyze geophysical flows that are not necessarily at steady state. This decomposition (discussed later in this section) proves important for the ocean because it cannot be considered completely steady at the time scales we are investigating. The hydrothermal and thermohaline streamfunctions reside in specific humidity–potential temperature and salinity–potential temperature spaces, respectively. Hence, neither of the two streamfunctions depends on any spatial coordinate.

\[
\dot{S}(S, \theta_O) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \left[ \delta(S - S(x, y, z, t)) \delta(\theta_O - \theta'_O(x, y, z, t)) \right] \frac{dS}{dt} \rho \, dx \, dy \, dz \, dt \quad \text{and}
\]

\[
\dot{\theta}_O(S, \theta_O) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \left[ \delta(S - S(x, y, z, t)) \delta(\theta_O - \theta'_O(x, y, z, t)) \right] \frac{d\theta'_O}{dt} \rho \, dx \, dy \, dz \, dt,
\]

where the tendencies have the units of \( \dot{S} \) \( [\text{g kg}^{-1} \cdot \text{kg s}^{-1}] \) and \( \dot{\theta}_O \) \( [\text{K}^{-1} \cdot \text{kg s}^{-1}] \). Here \( \Omega \) is the global ocean comprising individual elements \( \rho \, dx \, dy \, dz \), where \( \rho \) is the density of seawater. Moreover, from \( t_0 \) to \( t_1 \) is a time period over which the tendencies are averaged. Primes denote values in physical space. The Dirac function \( \delta(x) \) is a function with the property \( \int_{-\infty}^{\infty} \delta[x] \, dx = 1 \) with \( \epsilon > 0 \) and \( \delta[x] = 0 \) for \( x \neq 0 \).

Under the thermodynamics transform, the binned tendencies are now mass fluxes in thermohaline space. Laliberté et al. (2015) have shown that this binning is unique in the sense that computing the binning onto any other thermodynamic variables before converting it back to thermohaline space would give the same mass fluxes. Moreover, Laliberté et al. (2015) have demonstrated that under mass conservation and for a steady state the binned mass fluxes resulting from the thermodynamic transform are nondivergent and that under these conditions we can define a unique streamfunction \( \psi_O(S, \theta_O) \), such that \( \partial \psi_O / \partial S = \theta_O \) and \( \partial \psi_O / \partial \theta_O = -S \). We demonstrate this latter property in appendix A as a reference.

However, models seldom conserve mass perfectly and often are not steady. This unsteady behavior is represented by trends in salinity and ocean heat from anthropogenic forcing, natural variability, and model errors; the binned mass fluxes have a small but nonzero divergence. To calculate the thermohaline streamfunction given the binned mass fluxes in Eq. (2) [and Eq. (C1) for the atmosphere], we perform the Helmholtz decomposition of the binned mass fluxes into their rotational (i.e., nondivergent) and divergent parts, represented by a streamfunction \( \psi_O \) and a potential \( \chi_O \), respectively:

\[
\nabla^2_{s, \theta_O} \psi_O = \left[ (\Delta S)^2 \frac{\partial^2}{\partial S^2} + (\Delta \theta_O)^2 \frac{\partial^2}{\partial \theta_O^2} \right] \psi_O = (\Delta S)^2 \frac{\partial^2 \psi_O}{\partial S^2} - (\Delta \theta_O)^2 \frac{\partial^2 \psi_O}{\partial \theta_O^2} \quad \text{and}
\]

\[
\nabla^2_{s, \theta_O} \chi_O = \left[ (\Delta S)^2 \frac{\partial^2}{\partial S^2} + (\Delta \theta_O)^2 \frac{\partial^2}{\partial \theta_O^2} \right] \chi_O = \Delta S \Delta \theta_O \left( \frac{\partial \psi_O}{\partial S} \frac{\partial \psi_O}{\partial \theta_O} \right).
\]

The Helmholtz decomposition is often used to split atmospheric wind velocities at a given geopotential level in order to calculate a barotropic streamfunction (Shukla and Saha 1974). Here \( \nabla_{s, \theta_O} \) is the nondimensional nabla operator in thermohaline space:
Equation (3) is solved iteratively with a successive over relaxation (SOR) algorithm with the boundary conditions that \( c_{\theta,O} = 0 \) (since we neglect the contribution of the tendencies outside the \( \theta,O-S \) domain) and \( \nabla_S \psi_O \chi_O = 0 \) (i.e., assuming that the tendencies are constant). The tendency potential \( \chi_O \) is one order of magnitude weaker than the streamfunction \( \psi_O \). While the Helmholtz decomposition is always unique for a choice of boundary conditions, other boundary conditions could have been used here. For example, the streamfunction and the tendency potential gradient could have been set to zero on the convex hull of the binned mass fluxes. However, when the tendency potential vanishes the streamfunction becomes unique no matter where the boundaries are set (as long as all mass fluxes are included in the interior of the boundaries). Therefore, because the tendency potential is so much smaller than the streamfunction another choice of boundary conditions would have had a negligible influence on our conclusions. Note that such a Helmholtz decomposition can always be applied to any mass fluxes obtained from the thermodynamics transform.

In appendix B, we explain how \( \psi_O \) relates to the thermohaline streamfunction introduced by Zika et al. (2012) and Döös et al. (2012) as well as how this was extended by Groeskamp et al. (2014) to compute a truly diathermohaline streamfunction.

The method to obtain the thermohaline streamfunction above applies to any two thermodynamic quantities in a GCM. For the atmospheric hydrothermal streamfunction, the two tracers are specific humidity \( q \) and potential temperature \( \theta_A \) instead of the salinity \( S \) and potential temperature \( \theta,O \). As mentioned earlier, the specifics of our computation to obtain the hydrothermal streamfunction are described in appendix C. Throughout this paper, the potential temperature of air has been calculated from the dry static energy \( h_d \):

\[
\theta_A = \frac{h_d}{c_A} = T_A + \frac{gz}{c_A},
\]

where \( T_A \) is the air temperature, \( z \) is geopotential height, and \( c_A = 1004 \text{ J kg}^{-1} \text{K}^{-1} \) is the specific heat capacity of dry air evaluated at constant pressure. The units in the above calculations are expressed in kelvins. We have, however, converted temperature from kelvin to Celsius scale in all the figures to have the same units for the air and ocean temperatures. The approximation that \( c_A \theta_A \approx h_d \) is fairly accurate for most purposes and is often used when calculating the heat transport from potential temperature (Czaja and Marshall 2006; Döös and Nilsson 2011). Because \( h_d \) so closely approximates the thermodynamic variable \( \theta_A \), the binning procedure for the hydrothermal streamfunction thus has the same uniqueness properties as the thermohaline streamfunction. Both dry static energy and potential temperature are approximately conserved for dry adiabatic motion in the atmosphere. Furthermore, the moist static energy and the equivalent potential temperature are both approximately conserved for moist adiabatic motion.

The hydrothermal and thermohaline streamfunctions shown in Fig. 2 are both clearly stronger compared to the corresponding potentials in Fig. 3. Note that we have opted for both streamfunctions to use the mass transport
definition so that 1 Sv = 10⁹ kg s⁻¹ rather than the traditional oceanographic definition in volume transport. These figures show that the divergence is negligible in the atmospheric case but not entirely for the ocean. This residual divergence in the ocean is a consequence of the hydrological cycle (quantified at the ocean surface by E = P − R) that adds and remove mass in the form of freshwater from the ocean at different temperatures and salinities. The unbalanced contribution due to climate change and model drift is, however, negligible compared to E = P − R.

3. Comparison with previous results

The atmospheric hydrothermal streamfunction ψₐ(q, θₐ) and the oceanic thermohaline streamfunction ψₒ(S, θₒ) computed from the EC-Earth model integration are presented in Fig. 2. This figure shows results similar to Kjellsson et al. (2014), Zika et al. (2012), and Döös et al. (2012), where the streamfunctions were introduced. Next, we describe them separately as was done in these previous studies and then together as the different branches of the cells interact by freshwater and heat exchange between the ocean and atmosphere.

a. The atmospheric hydrothermal circulation

The atmospheric hydrothermal streamfunction comprises one strong triangular-shaped, counterclockwise circulation and beneath this cell two very weak clockwise cells. This triangular cell has a maximum amplitude of ~450 Sv and comprises most of the global atmosphere with specific humidities q from nearly 0 up to about 16 g kg⁻¹ and potential temperature from −20°C to 70°C. There are three distinct branches of the triangular cell: 1) The “moistening branch” corresponds to the surface air flowing from the poles toward the equator. As the air gets warmer it increases its specific humidity so that the flow follows the Clausius–Clapeyron relationship. 2) Once at the intertropical convergence zone (ITCZ) the air ascends and loses its moisture by precipitation and is henceforth referred to as the “precipitating branch.” The moist static energy is conserved during this adiabatic ascent with a conversion from latent heat to dry static energy. 3) Finally, the “radiative cooling branch” is the dry but warm air in the upper tropical troposphere that cools by radiation as it travels poleward and closes the loop.

In addition, there are two weak clockwise cells under the dominant counterclockwise hydrothermal cell. These two cells were shown by Kjellsson et al. (2014) to be associated with local correlations between the wind and time-varying isotherms/isohumes. Both cells are still present when using monthly mean data but disappear in annual-mean data. They will need further investigation to be fully understood, but we postulate that one might be associated with the seasonal variation of the surface air in midlatitudes and that the other might be associated with the seasonal migration of the ITCZ in the tropics.

b. The oceanic thermohaline circulation

The oceanic thermohaline streamfunction has a three-cell structure (Fig. 2b). The counterclockwise “tropical cell” at high temperature and salinity is the sum of the subtropical cells (STCs). These cells are generally visualized as the shallow meridional overturning circulation...
in both hemispheres for the three oceans. There is, however, also a zonal component that contributes to this cell. The tropical Pacific is the major contributor to this zonal component.

The clockwise “conveyor belt cell” spans considerable temperature and salinity ranges and corresponds to the interocean exchange of water masses described as a conveyor belt by Broecker et al. (1991). This cell is clearly seen as a strong thermohaline circulation in Fig. 2b. The very weak counterclockwise “polar cell” transforms water masses near freezing temperatures and high salinities. There are indications that most of the transformations within this cell are associated with the seasonal variations in surface salinity and temperature as sea ice forms or melts in the Arctic and Southern Oceans (Martin et al. 2013; Pemberton et al. 2015; Kjellsson et al. 2015). These seasonal variations also influence the formation of Antarctic Bottom Water (AABW). This AABW is subsequently also mixed with the more saline and slightly warmer North Atlantic Deep Water (NADW) in the abyssal ocean. This part of the cell takes place at higher temperatures and is likely to be hidden by the much stronger conveyor belt cell.

4. Results

a. Heat transport across isohumes and isohalines

Using the fact that $C_{PA}\theta_A(\partial \psi / \partial \theta_A) d\theta_A$ and $C_{PO}\theta_O(\partial \psi / \partial \theta_O) d\theta_O$ are the heat transports in the $q$ and $S$ directions respectively in the potential temperature range from $\theta$ to $\theta + d\theta$, we can calculate the heat transport accomplished by a closed cell bounded by the streamline $\psi = \psi_1$. Integrating by parts, we find that the heat transport in the $q$ or $S$ direction for the atmosphere and the ocean, respectively, is given by the following:

$$H_A(q) = -C_{PA} \int_{\theta_{min}}^{\theta_{max}} \left[ \psi_A(q, \theta_A) - \psi_1 \right] d\theta_A$$ and $$H_O(S) = -C_{PO} \int_{\theta_{min}}^{\theta_{max}} \left[ \psi_O(S, \theta_O) - \psi_2 \right] d\theta_O,$$

where $\theta_{min}$ and $\theta_{max}$ are the potential temperatures on the boundary of the cell, and the specific heat capacity for dry air is $C_{PA} = 1004 \, J/(kg \cdot K)$ and for seawater $C_{PO} \approx 3992 \, J/(kg \cdot K)$. Note that these heat transport calculations are independent of reference temperature since there is no net mass transport across the isohumes and isohalines for a given streamline and can hence be equally computed using kelvins or degrees Celsius as temperature units. This heat transport across isohumes and isohalines as a function of isohumes and isohalines is presented in Fig. 4. It is, according to Eq. (6), approximately 4 times ($C_{PO}/C_{PA} \approx 4$) stronger per Sverdrup (Sv) in the ocean than in the atmosphere. It is therefore useful to rescale the ocean thermohaline streamfunction and the atmospheric hydrothermal streamfunction in such a way that they can be related energetically. Czaja and Marshall (2006) chose to present ocean and atmospheric circulations on the $C_P\theta$ axis; with this choice, a given circulation $\psi$ represents the same heat transport in both cases. However, we would expect the sea surface and the base of the atmospheric boundary layer to be at the same potential temperature (e.g., due to sensible heating) rather than the same $C_P\theta$. For this reason, we argue that potential temperature is a better choice for a mutual vertical between the atmosphere and the ocean.

Summing it up, we have rescaled the atmospheric hydrothermal streamfunction by a factor of $C_{PO}/C_{PA}$ so that the newly scaled streamfunction $\psi^*_A$ carries the same heat as $\psi_O$ for a given temperature difference. The rescaled hydrothermal streamfunction is then

$$\psi^*_A(q, \theta) = \frac{C_{PA}}{C_{PO}} \psi_A(q, \theta),$$

which will be used to compare the two streamfunctions in section 4e.

b. Freshwater transport across isotherms

Using similar arguments, we find that the freshwater transport in the potential temperature $\theta$ direction for the atmosphere and the ocean, respectively, is given by
The blue curve shows the net evaporation minus precipitation and river runoff integrated so that a positive gradient indicates
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\[ F_A(\theta) = \gamma \int_{q_{\min}(\theta)}^{q_{\max}(\theta)} \psi_A^*(q, \theta) \, dq \quad \text{and} \quad (8) \]
\[ F_O(\theta) = -\frac{1}{S_r} \int_{S_{\min}(\theta)}^{S_{\max}(\theta)} \psi_O(S, \theta) \, dS. \quad (9) \]

We have multiplied the freshwater transport for the atmosphere by the factor defined as
We have multiplied the freshwater transport for the atmosphere by the factor defined as
We have multiplied the freshwater transport for the atmosphere by the factor defined as

\[ \gamma = \frac{\text{kg} \, C_{PO}}{1000 \, \text{g} \, C_{PA}} \quad (10) \]
in order to use specific humidity units of grams per kilogram and to retain the heat-transport-adjusted atmospheric streamfunction \( \psi_A^* \). We have set the salinity and humidity boundaries of the cells to \( S_{\min}(\theta), S_{\max}(\theta), q_{\min}(\theta), \) and \( q_{\max}(\theta) \). The quantity \( S_r = 35 \, \text{g} \, \text{kg}^{-1} \), which is a constant reference salinity. This freshwater transport across isotherms as a function of temperature is presented in Fig. 5.

**c. The heat and freshwater exchanges of the hydrothermal and thermohaline circulations**

The atmospheric and oceanic circulations influence each other by heat and freshwater exchange through the ocean surface. To emphasize this mutual forcing, the heat flux and \( E - P \) for the atmosphere and the \( E - P - R \) for the ocean have been superimposed on the two streamfunctions shown in Fig. 6. The \( E - P \) fields are projected on the surface values of \((q, \theta_A)\) since they are surface fluxes. Note that a perfect match would only work on an aquaplanet since the precipitation over land will result in a river runoff somewhere else. The \( E - P \) pattern is concentrated along the surface branch in \( q - \theta_A \) space. In that space, the \( E - P - R \) pattern is positive at low temperatures, corresponding to mid-to-high latitudes; negative at temperatures between 10° and 25°C, corresponding to the subtropics; and positive at high temperature and specific humidity values typically found in the tropics.

In \( q - \theta_A \) space, the \( E - P - R \) pattern is similarly distributed in temperature but is spread in salinity. The \( E - P - R \) pattern is strongly positive near the highest potential temperatures of the conveyor belt cell and is strongly negative near the lowest. At high latitudes, \( E - P - R \) includes an important contribution from glacial melt.

Because surface air temperatures correspond almost one-to-one to sea surface temperature we argue that these patterns offer a way to link the hydrothermal and thermohaline streamfunctions. The \( E - P - R \) pattern indicates that the freshening in the tropical component of the thermohaline circulation is linked to the high temperature and high specific humidity tip of the hydrothermal streamfunction. At this tip, the hydrothermal streamfunction veers toward decreasing specific humidity along a moist isentrope (black solid line in Fig. 6). Such moist adiabatic motions indicate precipitation; the tropical freshening pattern in \( S - \theta_O \) space is therefore closely associated to precipitation in the tropical atmosphere. We find that the stream lines of the hydrothermal circulation are parallel to the tropical thermohaline circulation along the 27°C line. This parallelism indicates that they are connected and that the two circulations are connected along these branches. The link between the two streamfunctions through the \( E - P - R \) pattern also shows that the increasing salinity in the tropical oceanic cell and increase in specific humidity in the atmospheric cell are connected and occur in the subtropics.

In the hydrothermal circulation, air flowing toward low temperatures is drier than the air flowing toward high temperatures. This implies a net freshwater transport across the isotherms toward high temperatures. This freshwater transport is computed by integrating the streamfunction over the specific humidity with Eq. (8) and is represented by the green line in Fig. 5. This freshwater transport represents the ascension of humid air and has a maximum “upward” transport of 4 Sv of freshwater across the 32°C isotherm. This, in turn, is balanced by precipitation. This precipitation creates a net freshwater transport toward lower temperatures in the atmosphere. It is important to note that this freshwater transport between different atmospheric temperatures (driven by precipitation), is not modeled as a flux and is therefore not present in the Fig. 5.
The equivalent freshwater transport across isotherms in the ocean is computed with Eq. (9) and is represented by the red line in Fig. 5. The conveyor belt cell transports freshwater toward high temperatures and the tropical cell in the opposite direction. The freshwater transport of the ocean thermohaline cells are balanced by the evaporation and precipitation as shown by the blue curve in Fig. 5. It corresponds to the accumulative $E - P - R$ from cold to warm. The break-even point is at 23°C, where the net $E - P - R$ is zero when integrating over all the temperatures above 23°C as well as when integrating below. This temperature is also nearly exactly the same as where the freshwater transport of the ocean thermohaline cells changes direction.

In the conveyor belt cell, the flow toward high temperatures is more saline than the flow toward low temperatures. This cell therefore transports freshwater toward high temperatures. This freshwater transport is necessarily balanced by negative $E - P$ at high and midlatitudes and net precipitation at low latitudes except within the ITCZ. The tropical thermohaline cell has the reverse freshwater transport and is driven by net precipitation within the ITCZ and evaporation in the rest of the tropics. This is due to a freshwater transport within the Hadley and Walker cells that leads to moist air convergence at the ITCZ and over the warm water pool of the Pacific, respectively.

There is also a heat transport associated with both the thermohaline and the hydrothermal circulations across the ocean.
isohalines and isohumes, respectively. The heat transport of the ocean conveyor belt cell is due to the upper branch transporting warm water from the fresher Pacific to the more saline tropical Atlantic and a return flow at cold temperatures. This heat transport in the ocean conveyor belt cell is balanced by the heat gain from the atmosphere when the water flows from the Southern Ocean to the tropics minus the heat loss when the water travels from the tropics to the North Atlantic. The maximum heat transport of this cell is 1.2 PW toward higher salinities across the 34.9 g kg\(^{-1}\) isohaline. The tropical cell transports heat in the opposite direction from high to low salinities, with a maximum heat transport of 0.6 PW. This mainly corresponds to that warm equatorial surface waters flowing from the more saline central Pacific to the fresher western Pacific and a colder return flow.

The atmospheric cell transports heat toward drier air with a maximum transport of 16.6 PW over the 1.2 g kg\(^{-1}\) isohume. This is significantly more than the meridional heat fluxes reported by Döös and Nilsson (2011). This larger transport is found here because the hydrothermal cell includes heat transports from the meridional overturning in both hemispheres as well as the zonal transports from zonally asymmetric circulations such as the Walker circulation (Kjellsson et al. 2014; Kjellsson 2015). This heat transport across isohumes can be understood as, on one hand, the ascent of air in ITCZ, which becomes drier and, on the other hand, the surface equatorward transport of air getting warmer as it gets more humid. Most of this heat transport is lost to outer space and only a minor part is lost to the ocean. This differential loss explains the big difference between the heat transports across isohumes in the atmosphere and that across isohalines in the ocean.

d. Defining a universal temperature vs salinity or moisture scale

The humidity scale is commonly translated into moist static energy and hence has an energetic meaning equivalent to dry static energy (the two can be presented on the kelvin scale). We thus aim to convert the salinity scale from \(S\) to \(q\) such that a given \(\psi\) and \(\Delta q\) in the ocean and atmosphere requires the same loss and gain of moisture, respectively, and hence the same latent heat energy. The freshwater transport of the atmosphere and ocean given by Eqs. (8) and (9) can be approximated as follows:

\[
F_A \approx \Delta q \psi_A \text{kg} \left(\frac{1000 \text{ g}}{\text{kg}}\right)^{-1} \quad \text{and} \quad F_O \approx -\psi_O \Delta S / S_r,
\]

where we have divided the atmospheric transport by 1000 since specific humidity is in grams per kilogram.

Evaporation is a loss of freshwater from the ocean and a gain of freshwater to the atmosphere. The freshwater transport across an isotherm in atmosphere must be equal of opposite sign to that of the ocean if the total ocean–atmosphere freshwater cycle is to be closed:

\[
F_A = -F_O.
\]

This leads to

\[
\frac{\psi_O \Delta S}{S_r} = \Delta q \psi_A \left(\frac{1000 \text{ g}}{\text{kg}}\right)^{-1} = \Delta q \psi_A^* \gamma.
\]

If we set \(\psi_A = \psi_O\), we then obtain a scaling between \(\Delta q\) and \(\Delta S\) so that the two streamfunctions occupy the same space in \(\theta_A-q\) as in \(\theta_O-S\) coordinates. Hence the scaling becomes

\[
\frac{\Delta q}{\Delta S} = \frac{1}{\frac{S_r}{\gamma}} \approx 7.1.
\]

This provides a scaling between \(dq\) and \(dS\) but not between \(q\) and \(S\). An absolute reference point between \(q\) and \(S\) might be found in a highly idealized single basin ocean. This is, however, beyond the scope of this paper.

To summarize, the thermodynamic coordinates and streamfunctions have been rescaled such that the flow for both the atmosphere and ocean represent equivalent conversions of dry heat and moisture.

e. Comparing the hydrothermal and thermohaline circulations on the same scale

The two streamfunctions computed with Eqs. (3) and (C2) can now be projected together in Fig. 7 by using Eqs. (7) and (15). Equation (15) only determines the ratio of the \(\Delta q\) to \(\Delta S\) but no specific reference point. The two streamfunction can therefore still be shifted along the \(q\) axis relative to each other as shown in Fig. 7.

Since the Pacific is on average fresher than the Atlantic, we argue that 1) it is reasonable to use the left panel in Fig. 7 when merging the atmosphere with the Pacific part of the conveyor-belt cell, and 2) it is more reasonable to use the right panel in Fig. 7 when merging the atmosphere with the Atlantic.

When both circulations are cast in comparable coordinates, the atmospheric surface branch and the warming–salinifying branches of the conveyor belt cell and the tropical cell follow similar curves. The atmospheric flow is bound below by the Clausius–Clapeyron relation for near-surface air. The correspondence with the oceanic branch indicates that it too is linked to the Clausius–Clapeyron relationship. In the next section we discuss why water-mass transformation from cold fresh high latitudes to warm saline low-latitude water may be set by the Clausius–Clapeyron relationship.
The subtropics have net evaporation where the conveyor belt remains constant in temperature but increases in salinity. This transport might be from the tropical Pacific, flowing through the Indonesian Archipelago and leaking by the Agulhas Current and rings into the tropical Atlantic. Note that the tropical Atlantic has a much higher salinity than the tropical Pacific. It is this salinity difference between ocean basins that leads the thermohaline conveyor belt cell to sink in the Atlantic. The transformation from warm and high-salinity tropical water to cold, fresher water in the North Atlantic follows and opposes the warming and salinifying curve but at higher salinity.

The weak and cold polar cell will need further investigation. We hypothesize, however, that it is the sum of two effects. One effect corresponds to the seasonal cycle in sea ice growth and melt at the ocean surface. Autumn sea ice growth leads are associated with cooling and salinification owing to brine rejection, while spring sea ice melt is associated with warming and freshening, thus forming a cycle in $S$–$uO$ space (Kjellsson et al. 2015). The other effect mirrors the production of AABW in the Southern Ocean close to Antarctica. After being produced, AABW fills the abyssal World Ocean and is slowly mixed with the NADW and returns back to the Southern Ocean as both warmer and more saline waters compared to when it downwells close to Antarctica. The water is then driven by the southward Ekman transport and cools down by heat loss to the atmosphere and freshens by mixing and precipitation as seen in Fig. 6. This weak and cold polar cell has no clear connection to its atmospheric counterpart and only surfaces in the southern part of the Southern Ocean.

f. Geographical correspondence of the warming and moistening branches

The clockwise conveyor belt cell circulation is projected into both geographical and atmospheric hydrothermal coordinates in Fig. 8. The first step when doing this projection is to select the stream layer between 3 and 9 Sv in the conveyor belt as shown in the top panels of Fig. 8. This stream layer is then colored to differentiate the various $T$–$S$ intervals by following the rainbow as in Döös et al. (2012). The colors change gradually and are ordered as a clock, making one 12-h revolution a complete conveyor belt cycle. These temperature–salinity intervals are then reprojected in geographical space in the middle panels of Fig. 8. The left-hand panels of Fig. 8 show the warming phase of the conveyor belt with water flowing from the Southern Ocean toward the tropics, and the right-hand panels show the cooling part with waters flowing from the tropical Atlantic to the northern North Atlantic. The return from the North Atlantic as NADW to the Southern Ocean is not illustrated in Fig. 8 since only the surface water masses are used here. Had we showed this return, it would be filling the entire abyssal ocean with green color. This return would correspond to the NADW and also to some extent to AABW. In the bottom panels of Fig. 8 we have projected the air masses just above the sea surface $\theta$–$q$ space and colored them the same as the water masses beneath. The left-hand bottom panel of Fig. 8 shows how
the atmosphere just above the Clausius–Clapeyron line follows a continuous path toward warmer and more humid air in direct connection with the warming branch of the conveyor belt cell. The reverse cooling path from red to green shows the same continuous pattern corresponding to water flowing from the tropics to the North Atlantic. It is important to note here that it is only in the warming phase that the hydrothermal streamfunction moves in the same direction as the thermohaline conveyor belt cell. In the cooling branch the ocean gets colder and fresher owing to precipitation and hence involves air from higher altitudes.

g. Why does the warming branch of the thermohaline cell follow the moistening branch of the hydrothermal circulation?

Our analysis of the thermohaline and hydrothermal circulations hints at a correspondence between the warming and moistening branch of the atmosphere, which follows and is bounded below by the Clausius–Clapeyron
relationship (Fig. 7), and the warming and increasingly saline branches of the ocean’s tropical and conveyor cells. However, such a correspondence relies on our coordinate transformation from salinity to an equivalent moisture coordinate resulting in a factor of 7. Here we will use a simple physical model to explain why changes in $S$ in the ocean could follow changes in $q$ in the atmosphere under warming and evaporation and, second, why there can be a factor of 7 relating the two.

Consider equatorward motion in the atmospheric boundary layer and surface layers of the ocean (take for example the equatorward branches of the Hadley cell and the ocean’s subtropical gyres). The meridional mass transport is $Q_A$ for the atmosphere and $Q_O$ for the ocean. Equatorward atmospheric flows increase in moisture by $\Delta q$ such that the moisture flux from the ocean to the atmosphere is $Q_A \Delta q$. This loss of freshwater from the ocean causes an approximate salinity change of

$$\Delta S = \frac{S_R}{1000 \text{ g kg}^{-1}} \frac{Q_A}{Q_O} \Delta q. \quad (16)$$

If we make the additional assumption that the temperature of the atmospheric boundary layer and the ocean’s surface layer are the same (as evident in Fig. 1) then Eq. (16) becomes

$$\frac{\Delta S}{\Delta \theta} = \frac{S_R}{1000 \text{ g kg}^{-1}} \frac{Q_A}{Q_O} \frac{\Delta q}{\Delta \theta}. \quad (17)$$

Hence, as long as the ratio of the strength of the atmospheric and ocean circulations remains constant, a change in $q$ for the atmosphere is likely to be proportional to a change in $S$ for the ocean for a given change in potential temperature. This implies that if the atmosphere follows the Clausius–Clapeyron relationship (which defines $\Delta q/\Delta \theta$) then the salinifying branch of the thermohaline circulation will follow an equivalent line set by the scale factor $(35/1000) \times (Q_A/Q_O)$. This relation applies to the thermohaline circulation branches that only experience evaporation since precipitation will reduce $S$. Equation (17) has the same form as Eq. (15) but with the ratio $Q_A/Q_O$ replacing $C_{PA}/C_{PO} \approx 4$. The reason the evaporating branch of the thermohaline circulation in the left panels of Fig. 8 follows Clausius–Clapeyron could be because $Q_A/Q_O \approx 4$.

It is not straightforward to define branches of the hydrothermal circulation that are inside the surface boundary layer. Some of the streamlines associated with the highest absolute $\psi_A$ do not occupy the boundary layer near the ocean surface since they are far from the saturation water vapor concentration. Naïvely, the absolute maximum streamfunction $\psi_A$ at 450 Sv is of order 5 times the difference between the maximum $\psi_O$ (the tropical cell) and minimum $\psi_O$ (the conveyor cell) at 70 Sv. It is therefore likely that not all of these 450 Sv of the hydrothermal cell are inside the surface boundary layer.

Held (2001) proposes that the near-surface atmospheric and oceanic mass transports should be equivalent ($Q_A/Q_O = 1$) close to the equator (the atmosphere being poleward and the ocean equatorward). This may partly explain why, at high temperatures the Conveyor Cell of the thermohaline circulation shows strong salification (e.g., due to moisture gain by equatorward flowing air) but very little temperature gain (since the flow is poleward). However at subtropical latitudes we expect an equatorward near-surface component of the ocean circulation. An independent estimate of $Q_A/Q_O$ comes from Czaja and Marshall (2006). They reported a value of $Q_A/Q_O$ between $<1$ near the equator and as large as 10 at 40°S, although at most latitudes values were between 3 and 5 (see Czaja and Marshall 2006, their Fig. 10).

5. Conclusions and discussion

We have presented global atmospheric and oceanic circulation in thermodynamic coordinates and linked the two by relating seawater salinity to the specific humidity of air. This has permitted the superimposition of the atmospheric hydrothermal circulation on the oceanic thermohaline circulation, which depicts, in temperature–humidity–salinity space, some of the interactions between the atmospheric and oceanic realms. The dominating feature of the atmospheric hydrothermal circulation is a strong counterclockwise cell with a surface branch that can be linked to ocean surface branches of both the conveyor belt cell (Fig. 9) and the tropical cell (Fig. 10).
We have presented evidence suggesting the Clausius–Clapeyron relationship links the atmospheric hydrothermal cell with the oceanic thermohaline cells. When relatively dry air enters the planetary boundary layer over the ocean it picks up sensible heat and moisture. This moistening of dry air drives evaporation at the surface and thus increases sea surface salinity. We suggest that this mechanism connects the warming branch of the conveyor belt cell with the surface branch of the hydrothermal cell. The atmosphere and the ocean have, at the surface, similar temperatures. These temperatures increase as the water in the conveyor belt travels from the southern to the tropical Atlantic. Because of the strong dependency of the Clausius–Clapeyron relationship to temperature, increasing temperatures leads to increased evaporation. This increased evaporation drives increases in specific humidity as well as increases in salinity of the surface waters.

Also the tropical thermohaline cell has a branch that we suggest is linked to the surface branch of the hydrothermal cell in the same way. This corresponds to the water that, after being upwelled in the eastern tropical Pacific, warms up together with the surface air as these travel westward by the South Equatorial Current and the trade winds. The air at the surface then follows the Clausius–Clapeyron relationship and increases its specific humidity by evaporation, a process that in turn increases the salinity of the surface waters as illustrated by the number 3 in Fig. 10.

The present study has focused on present-day climate by using data from the last two decades from a historical simulation by the climate Earth system model EC-Earth. The thermohaline and hydrothermal streamfunctions have in the past been used in order to study both past and future climates (Ballarotta et al. 2014; Kjellsson 2015). The oceanic and atmospheric circulations were, however, analyzed separately in these studies. Kjellsson (2015) showed that a warmer climate extends the hydrothermal atmospheric cell along the Clausius–Clapeyron line. The hydrothermohaline framework would therefore be suited to study the impact of anthropogenic forcing on the oceanic circulation, especially since the hydrothermal circulation has been shown to respond strongly along the Clausius–Clapeyron line. Future work could include investigations to what extent the thermohaline stream lines are parallel to the Clausius–Clapeyron line using more detailed and quantitative diagnostics. This could include calculating the thermohaline streamfunctions for different regions to better isolate different parts of the cells (e.g., the warming branch of the conveyor belt cell, which we have suggested to be strongly related to the Clausius–Clapeyron relationship). This could also be done by investigating how the thermohaline cells expand or contract along a Clausius–Clapeyron line in warmer or colder climates. This response to a different climate could be rapid since most of the ocean mass conversion is in the shallow ocean and especially in the tropics, where the ocean adjustment to the atmosphere is particularly fast owing to the equatorial dynamics.

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APPENDIX A

Mass–Volume Conservation in Tracer Coordinates

We here show that the hydrothermal–thermohaline stream lines are closed if the atmosphere–ocean data conserve heat, latent heat–salt, and mass. The derivation is similar to that by Laliberté et al. (2015). The mass of a point in $q$–$\theta_A$ space is defined as

$$M(q, \theta_A) = \int_\Omega \delta(q - q') \delta(\theta_A - \theta_A') dM.$$  \hspace{1cm} (A1)

The $\theta_A$-derivative of the tendency $\dot{\theta}_A$ is given by the following:

$$\frac{\partial \dot{\theta}_A}{\partial \theta_A} = \frac{\partial}{\partial \theta_A} \left[ \int_\Omega \delta(q - q') \delta(\theta_A - \theta_A') \frac{D\theta_A'}{Dt} dM \right]$$

$$= -\frac{\partial}{\partial \theta_A} \left[ \int_\Omega \delta(q - q') \frac{D\mu(\theta_A - \theta_A')}{Dt} dM \right]$$

$$= -\int_\Omega \delta(q - q') \frac{D\delta(\theta_A - \theta_A')}{Dt} dM,$$  \hspace{1cm} (A2)

where the Dirac function $\delta(\theta_A - \theta_A')$ is the derivative of the Heaviside function $\mu(\theta_A - \theta_A')$ so that $D_\mu(\theta_A - \theta_A') = \delta(\theta_A - \theta_A') D_\mu(\theta_A - \theta_A')$. Equation (A2) states that $\dot{\theta}_A(q, \theta_A)$ is related to the total mass at $q' = q$ and $\theta_A' < \theta_A$; that is, the mass flux at $q$–$\theta_A$ is equal in magnitude but opposite in sign to the mass change at all $\theta_A'$ values below. This is analogous to a barotropic streamfunction, where the northward mass flux through a latitude line $\phi$ is equal in magnitude but opposite in sign to the rate of change in total mass south of $\phi$. Furthermore, Eq. (A3) yields

$$-\int_\Omega \delta(q - q') \frac{D\delta(\theta_A - \theta_A')}{Dt} dM$$

$$= -\int_\Omega \frac{D}{Dt}[\delta(q - q') \delta(\theta_A - \theta_A')] dM$$

$$+ \int_\Omega \delta(\theta_A - \theta_A') \frac{D\delta(q - q')}{Dt} dM,$$  \hspace{1cm} (A4)

by partial integration. Note that the last term in Eq. (A4) is equal to the $q$-derivative of the tendency $\dot{\theta}_q$. It thus follows that

$$\frac{DM(q, \theta_A)}{Dt} + \frac{\partial \dot{\theta}_A}{\partial \theta_A} + \frac{\partial \dot{\theta}_q}{\partial q} = 0.$$  \hspace{1cm} (A5)

Hence, the rate of change of mass at the point $q$–$\theta_A$ is set by the divergence of the tendencies in specific humidity and potential temperature. Furthermore, the material derivative comprises both a local change $\partial / \partial t$ and an advective term $\mathbf{v} \cdot \nabla$. However, the global integral of a gradient is identically zero, so the mass continuity in Eq. (A5) is given by

$$\frac{\partial M}{\partial t} + \frac{\partial \dot{\theta}_A}{\partial \theta_A} + \frac{\partial \dot{\theta}_q}{\partial q} = 0;$$

that is, the rate of change in mass at $q$–$\theta_A$ plus the divergence of the tendencies is zero. It can also be shown that $\partial M/\partial t$ only depends on the global mass integrals of $\partial q/\partial t$ and $\partial \dot{\theta}_q/\partial t$. If $\partial M/\partial t$ is time averaged over a sufficiently long time and there are no trends in specific humidity or potential temperature, then

$$\frac{\partial M}{\partial t} \approx 0 \Rightarrow \frac{\partial \dot{\theta}_A}{\partial \theta_A} + \frac{\partial \dot{\theta}_q}{\partial q} \approx 0.$$  

The derivations above apply for any tracers in a GCM, so they also hold for salinity and potential temperature in the ocean. It thus follows that the thermohaline stream lines are closed if there are no trends in absolute salinity or potential temperature. However, reanalysis data or model outputs seldom conserve any of the tracers, so mass fluxes in hydrothermal or thermohaline space have divergences. These may arise if the model is still drifting (i.e., the spinup period was too short) or if there are inaccuracies in the numerical schemes. Apart from the errors, both global warming and subsampling of multidecadal variations give trends in atmospheric and oceanic potential temperature as well as specific humidity and salinity (Durack et al. 2012; Zika et al. 2015).

APPENDIX B

Previous Formulations of the Thermohaline Streamfunction

The ocean thermohaline streamfunction shown in Fig. 2b is an improvement on the thermohaline streamfunction introduced by Zika et al. (2012) and Döös et al. (2012). In these previous studies, the streamfunction was defined as
\[ \psi(S, T) = -\frac{1}{t_f - t_0} \int_{t_0}^{t_f} \int_{A_{th}(S, T)} \mathbf{V} \cdot d\mathbf{A} \, dt, \quad (B1) \]

where \( A_{th}(S, T) \) is the part of the isothermal surface \( T \) where the salinity is less than \( S \), \( \mathbf{V} \) is the three-dimensional velocity, and \( d\mathbf{A} \) is the surface element on \( A_{th}(S, T) \) pointing toward increasing temperature. The definition presented here improves this definition on two fronts. On one front, by including the time tendency term in Eq. (2), the improved thermohaline streamfunction now takes into account that isotherms and isohalines can, for instance, move in the same direction as the current and hence do not correspond to fluxes across the isotherms or the isohalines. This improvement was first introduced by Groeskamp et al. (2014).

On the second front, the improved thermohaline streamfunction can now handle the divergence stemming from the unsteady nature of the ocean circulation on the time scales studied and the addition–removal of mass in the form of freshwater arising from a varying \( E - P - R \) geographical distribution. The original definition as given by Eq. (B1) would nevertheless produce a streamfunction very similar to the one found in Fig. 2b and can therefore still be useful because it provides a simpler mathematical expression for the thermohaline streamfunction.

The tendencies comprise both a stationary part and an advective part:

\[ \dot{q}(q, \theta_A) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \int_{\Omega} \delta[q - q'(x, y, z, t)] \delta[\theta_A - \theta_A'(x, y, z, t)] \frac{dq'}{dt} g^{-1} \, dx \, dy \, dp \, dt \]

\[ \dot{\theta}_A(q, \theta_A) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \int_{\Omega} \delta[q - q'(x, y, z, t)] \delta[\theta_A - \theta_A'(x, y, z, t)] \frac{d\theta_A}{dt} g^{-1} \, dx \, dy \, dp \, dt, \quad (C1) \]

and the tendencies have the units for \( \dot{q} \) (K \( \cdot \) kg s\(^{-1}\)) and for \( \dot{\theta}_A \) (kg s\(^{-1}\)). Here, \( \Omega \) is the global atmosphere comprising elements \( g^{-1} \, dx \, dy \, dp \), where \( g \) is the gravity acceleration.

We split the field of tendencies \( \dot{q} \) and \( \dot{\theta}_A \) into a rotational and a divergent part:

\[ \nabla_{q, \theta_A}^2 \psi_A = (\Delta q)^2 \frac{\partial^2}{\partial q^2} + (\Delta \theta_A)^2 \frac{\partial^2}{\partial \theta_A^2} \psi_A = (\Delta q)^2 \frac{\partial^2 \dot{\theta}_A}{\partial q \partial \theta_A} - (\Delta \theta_A)^2 \frac{\partial^2 \dot{q}}{\partial q \partial \theta_A}, \]

\[ \nabla_{q, \theta_A}^2 \chi_A = (\Delta q)^2 \frac{\partial^2}{\partial q^2} + (\Delta \theta_A)^2 \frac{\partial^2}{\partial \theta_A^2} \chi_A = \Delta q \Delta \theta_A \left( \frac{\partial \dot{q}}{\partial q} + \frac{\partial \dot{\theta}_A}{\partial \theta_A} \right). \quad (C2) \]

We apply the same SOR algorithm and the boundary conditions \( \dot{\theta}_A = 0 \) and \( \nabla_{q, \theta_A} \chi_A = 0 \). We find that the tendency potential \( \chi_A \) is two orders of magnitude smaller than the streamfunction \( \psi_A \). We conjecture that \( \chi_A \) is more important for the ocean than \( \chi_A \) is for the atmosphere because evaporation and precipitation exert more influence on ocean mass than for the mass of the atmosphere.

Thus the thermohaline streamfunction \( \psi_A \) can be divided into a stationary and an advective component, given by \( \psi_A = \psi_A^{\text{stat}} + \psi_A^{\text{adv}} \). The same applies for the hydrothermal streamfunction, given by \( \psi_A = \psi_A^{\text{stat}} + \psi_A^{\text{adv}} \). This decomposition was presented by Groeskamp et al. (2014) for the thermohaline streamfunction, where the term “diathermohaline streamfunction” was used to emphasize that this gives the transport that truly crosses the isotherms and isohalines. Kjellsson et al. (2014) used this decomposition for the hydrothermal streamfunction and found that the stationary parts are an order of magnitude smaller than the advective parts for the atmosphere. The numerical discretization of the thermohaline and hydrothermal streamfunctions was presented in Döös et al. (2012). Note that our discretization of tracer advection differs somewhat from that in IFS and NEMO. However, we do not expect that our choice of advection scheme influences the results of the paper significantly.

APPENDIX C

Computing the Hydrothermal Streamfunction

In the same way, we bin the tendencies \( \dot{q} \) and \( \dot{\theta}_A \) into \( q - \theta_A \) space in the atmosphere data as


