Spatial Distribution of Generation of Lorenz's Available Potential Energy in a Global Climate Model

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ABSTRACT

Previous studies have estimated global available potential energy (APE) and global APE generation, but no study has focused on the geographic distribution of contributions to global APE and APE generation. To obtain the information needed for this analysis, simulations were performed using the NCAR CESM1.0.4 climate model. Based on these simulation results, maps of the spatial and seasonal distribution of APE contributions and APE generation in the atmosphere were obtained from the analysis. APE is generated by processes that cool relatively cool areas or warm relatively warm areas. It was found that there are two regions of the mid- to upper troposphere that contribute primarily to APE generation: 1) the tropics, especially the western tropical Pacific, owing largely to latent heat released in the intertropical convergence zone, and 2) the polar regions, especially in the relatively cold polar night, where longwave cooling is not offset by shortwave warming. It was also found that these qualitative results are largely insensitive to the assumptions examined regarding the treatment of topography in the atmosphere. Further, the analysis was extended to calculate how APE and APE generation is changed in a $4 \times \text{CO}_2$ climate relative to a $1 \times \text{CO}_2$ climate. It was found that in the high- CO_2 climate, APE decreased by 7.0% and APE generation decreased by 10.1%. This is consistent with expectations based on decreased equator-to-pole temperature gradients in warmer climates. The methods, results, and analysis presented here should prove useful in helping to build a better understanding of controls on atmospheric kinetic energy.

1. Introduction

Edward Lorenz developed the concept of available potential energy (APE; Lorenz 1955) based on Margules's idea of available kinetic energy (Margules 1910). Since that time, there have been many studies evaluating the global averages of APE and APE generation. For a recent review on Lorenz's concept of APE and subsequent extensions, with pointers to the relevant literature, please see Tailleux (2013). Fundamentally, APE is the difference between the potential energy of the

atmosphere in a given state relative to the potential energy of the atmosphere in a specified rest state. Positive contributions to APE generation are energy fluxes that increase the potential energy of the given state more than the associated rest state (with negative contributions increasing the potential energy of the given state less than the associated rest state). Various approaches have been offered in defining the rest state (e.g., Dutton and Johnson 1967; Dutton 1973; Pearce 1978). As long as the rest state is clearly defined, there is no right or wrong definition of the rest state, but rather definitions may be more or less useful depending on the context. For example, under Lorenz's (1955) definition, parcels in the atmosphere are adiabatically moved in the transition from the given state to the rest state without any transfer of energy among parcels. In contrast, Tailleux (2013) and others have provided definitions of rest state involving nonadiabatic processes that transfer energy among air parcels. Here, we focus on classical (Lorenz 1955) adiabatic definitions of APE and APE generation. Further, it has been shown that local fluxes of available potential energy involve terms that can be

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larger than the terms considered here, however these terms do not add or subtract from the global APE (see [Tailleux 2013](#) and references therein). Since our study is concerned with local contributions to global APE generation our analysis is limited to terms that add to or subtract from global APE.

Another choice in the definition of APE and APE generation is the treatment of moist processes. Lorenz's classical 1955 definition, which we call "dry APE," considers an idealized atmosphere in which condensation is viewed as a source of energy to the atmosphere and evaporation is considered as a sink of energy to the atmosphere. Evaporation and condensation are ignored during the redistribution of atmospheric parcels to the rest state. Later studies (e.g., [Lorenz 1978](#); [Pauluis 2007](#); [Tailleux 2013](#)) consider the latent heat of water vapor as contributing to atmospheric potential energy, and various approaches were developed to address issues related to condensation or evaporation of water during the theoretical redistribution of parcels to their rest state (e.g., [Pauluis 2007](#)). In this analysis we consider only the definition of dry APE as can be found in [Lorenz \(1955\)](#). Note, however, that since we are performing our computations on a realistic atmosphere as represented in a climate model, water vapor cannot be eliminated from the atmosphere. We therefore treat this circumstance by considering water vapor as air mass. Also, according to Lorenz's definition, dry APE does include energy related to condensation and evaporation as heating and cooling sources. Thus, in this treatment dry APE ignores moist processes only in the redistribution of the given atmosphere to its rest state.

The original definition of [Lorenz \(1955\)](#) considered a flat topography; however, the inclusion of nonflat topographies complicates analysis of APE under classical definitions ([Taylor 1979](#); [Koehler 1986](#)). Here, we consider different treatments of the atmosphere with flat and "realistic" (i.e., Earthlike) topographies.

In total, our calculations consider two different treatments: dry APE in a world with flat topography (further on referred to as dry APE over flat topography) and dry APE over Earthlike topography [further on referred to as dry APE over Earthlike (or uneven) topography].

APE and APE generation, as classically defined, are global properties; thus, allocating contributions of APE and APE generation to specific locations requires the development of an accounting approach that defines local contributions to APE and APE generation in an internally consistent way. For example, let us assume that we have a dry atmosphere that is adiabatically rearranged to its rest state, with a parcel with low potential temperature at high altitude (low pressure) in the given state redistributed to a lower altitude (higher

pressure) in the rest state. Globally integrated, the contributions of each parcel to global APE should sum to the difference between the potential energy of the atmosphere in the given state minus the potential energy in the rest state. One could define local contributions to APE in this case as the difference in potential energy of a parcel in the given state minus the potential energy of that parcel in the rest state. (Under this definition, parcels that were lifted during the redistribution would make negative contributions to APE.) However, one could have alternatively defined the local contribution to APE in terms of the work that must be performed on a parcel to move it from its location in the given state to its location in the rest state. In the example above, work would need to be performed on the parcel to bring it adiabatically from lower to higher pressure conditions (and consequently from lower to higher temperature conditions); thus, in this example this parcel would be said to make a negative contribution to APE. Following [Grotjahn \(1993\)](#), which builds off of [Lorenz's \(1955\)](#) derivation of global APE, we adopt this "work based" approach. The rationale for this choice is that APE represents the part of total atmospheric potential energy that can be used to do work on the atmosphere; any work that is done by the atmosphere in the rearrangement to its rest state is the energy that is available to do work on the atmosphere in the given state.

Using the above formulated definitions for APE and the rest state, our work distinguishes itself in contrast to previous studies further in several ways: Our study is based on a complete and many-year-averaged dataset from a climate model covering the whole atmosphere. The climate model used is the NCAR Community Earth System Model, version 1.0.4. The majority of studies on APE are based on a combination of measured data and idealized assumptions for the atmospheric parameters, such as temperature and pressure profiles (e.g., [Min and Horn 1982](#); [Siegmund 1994](#)). Measured data have the virtue of being closely related to the real world, but they are inherently spatially and temporally incomplete. Further, the results of our work consist of three-dimensional maps of the whole atmosphere showing the contributions to APE and APE generation at every atmosphere grid point consistent with Lorenz's classical definitions of APE. The previous work that is most similar to our study in terms of geographic distribution of APE is [Siegmund \(1994\)](#), which includes zonal mean plots of APE and APE generation for the months of January and July from an early version of the European Centre for Medium-Range Weather Forecasts (ECMWF) model. In contrast to our work, [Siegmund \(1994\)](#) considers Earth as having a flat topography.

Furthermore, our work provides a tool that can be used to investigate how various climate forcing factors affect APE generation. As an example, we apply our calculations to an atmosphere with 4 times the CO₂ content as our base case (having preindustrial CO₂ concentration) and then compare the calculated APE contribution and APE generation fields with our calculations for the base case.

2. Model and methods

a. Climate model

We use the NCAR Community Earth System Model, version 1.0.4 (CESM1.0.4; Gent et al. 2011). In the configurations used here, version 4 of the Community Atmosphere Model (CAM4; Neale et al. 2013) is coupled to version 4 of the Community Land Model (CLM4; Lawrence et al. 2011), version 4 of the Los Alamos Sea Ice Model (CICE4; Hunke and Lipscomb 2008), and a slab ocean model. A $1.9^\circ \times 2.5^\circ$ (latitude $\times$ longitude) grid is used for the atmosphere and land model. The atmosphere model has 26 nearly horizontal levels in the vertical. The ice and ocean models use a grid with a displaced pole and a nominal resolution of 1° (gx1v6). In this configuration, atmospheric calculations employ the finite-volume dynamical core. From the grid resolution, it follows that the atmosphere data have $26 \times 96 \times 144$, or 359 424, grid cells in total.

Climate model data produced in this study contain instantaneous values of pressure, temperature, humidity, and heating rate. For each of our calculations we took 60 yr of simulated climate model data, of which the first 50 yr were omitted and the final 10 yr used for analysis. Since APE is subject to interannual and intra-annual variations, we averaged our results separately for each season as well as for an annual mean. For each season in each year, we took 20 snapshots of the atmosphere in time steps of 109.5 h. Every seasonal average over the 10 yr is thus based on 200 atmospheric snapshots and the annual mean on the total of 800 snapshots.

b. Lorenz's concept of APE

A more detailed exposition of our methods related to contributions to APE and APE generation can be found in the supplemental material. Here we provide a summary overview.

1) APE

The derivation of APE begins with the assessment of potential energy in the atmosphere where the potential energy consists of geopotential energy and internal energy. Since the classical concept of APE as formulated by Lorenz (1955) treats the atmosphere as being dry, latent heat does not contribute to the internal energy.

Integrating both geopotential energy and internal energy over a whole atmospheric column and building the ratio between the two shows that this ratio is constant with a value of approximately 2/5. Therefore, it is convenient to consider the sum of both energies. This sum can be treated as a single form of energy and was named total potential energy (TPE) by Margules (1910).

Using the ideal gas law, the hydrostatic equation, and the relationship between the heat capacities and gas constant, it can be shown that for an atmospheric column the total enthalpy is equal to the total potential energy [see the supplemental material and, e.g., Holton (2004)]. This equality enables us to choose the form of energy under consideration (i.e., enthalpy or total potential energy) in our study of APE.

In terms of the rest state we apply the definition as given by Lorenz (1955). The underlying physical assumption for this rest state is that surfaces of equal pressure and equal potential temperature are parallel to the geopotential field and therefore stratified and parallel to each other. Thus, in the reference state, pressure will be constant on an isentrope and potential temperature will be constant on an isobar.

With this definition of a rest state, the so-called exact formula of Lorenz (1955) for APE can be defined (see supplemental material). It is an expression for the global “dry” available potential energy per square meter for an atmosphere overlying flat topography. The equation shows that dry APE can be understood as the result of a divergence: the deviation of pressure on an isentrope from the average pressure on that same isentrope is what contributes to the available portion of the total potential energy.

The exact formula of Lorenz (1955) reveals that explicit calculations of APE can be performed by having knowledge only about the two thermodynamic state variables, temperature and pressure, in the given-state atmosphere over the entire volume of interest, which in our case is global in scale.

2) APE GENERATION

Lorenz's energy cycle contains three energy transformation rates, all of which naturally have to equal each other in the long-term average. These three rates are, explicitly, the generation of APE, the conversion of APE to kinetic energy, and the dissipation of kinetic energy.

In this energy cycle, APE is generated by spatial and temporal variability in the intensity of the solar radiation that is heating the atmosphere. This uneven heating warms already warm places (i.e., the tropics) and lets already cool places (i.e., the poles) cool down further, thus leading to an increase of the meridional temperature gradient. In terms of Lorenz's concept of APE, this leads to an increasing

deviation of the state of the atmosphere from its rest state, which is equivalent to an increase of APE.

The role of the atmosphere in this context can be seen as a heat engine which drives the energy cycle (e.g., Wallace and Hobbs 2006). The rate of input energy to the system at the top of the atmosphere is the solar constant of around 340 W m^{-2} (net incoming solar radiation is around 240 W m^{-2} ; Wallace and Hobbs 2006). In the context of the heat engine, the rate of output energy is the generation rate of APE, which can be estimated by identifying the efficiency factor of the atmospheric heat engine.

Using the exact formula for APE, Lorenz (1967) derives an efficiency factor starting from the rate of change of APE with time, where two terms representing rates of APE generation G and APE conversion C can be identified:

$$\frac{\partial \text{APE}}{\partial t} = \overline{G} - \overline{C}, \quad (1)$$

where the bar over G and C stands for averages of both quantities over time. At each location, APE is generated when warm places are heated or cool places cooled and destroyed when warm places are cooled or cool places are heated. As described in the supplemental material, the rate of generation of APE can be calculated if the diabatic heating for every parcel, the rest-state pressure or temperature, and the given-state pressure or temperature are known.

c. Implementation of mathematical concepts of APE and its generation

Lorenz's exact formula allows us to obtain only a global average estimate of dry APE for an atmosphere over flat topography. With a limited volume method, as found in Grotjahn (1993), APE calculations can be applied to an atmosphere consisting of discrete air parcels, therefore allowing one to locate the contribution from each discrete air parcel. Using Grotjahn's (1993) method we implemented our APE calculations in an algorithm that computes the contribution to the total amount of APE and its generation coming from each air parcel in the atmosphere (here assumed to be a grid cell of the three-dimensional climate model). Thus, the computations result in 3D fields with the computed distributions of APE and APE generation for latitude, longitude, and altitude.

We implemented computations of dry APE for atmospheres over both flat and Earthlike topographies. These computations are explained below and in greater detail in the supplementary material. Since the computations for the treatment of Earthlike topography already include the computations for flat topography, the latter will not be explained separately.

The Python computer program used to perform the analysis described here is provided in the supplemental material.

DRY APE IN AN ATMOSPHERE OVER EARTHLIKE TOPOGRAPHY

We are considering dry APE in this case. To have a "dry treatment" of the atmosphere, we treat the mass of water that is in a model grid cell as if it were air and neglect the addition or subtraction of moist processes to the internal energy during adiabatic redistribution. Note that a dry APE treatment still takes latent heat fluxes as external heating terms into consideration since they play a major role for the generation of APE.

The computations for the treatment of dry APE over Earthlike topography consist of three steps, of which the first is to compute the rest state needed to compute APE contributions. The steps are explained in more detail below:

- 1) determination of the rest-state pressure assuming Earth's topography is flat;
- 2) adjustment of rest-state pressure and temperature to an uneven (Earthlike) topography after Koehler (1986); and
- 3) computation of dry APE and, from the heating terms in the atmosphere, the generation of dry APE.

For step 1, under Lorenz's classical 1955 definition of APE for a dry atmosphere over flat topography, knowledge of the potential temperature of each parcel is sufficient to determine the rest state. More explicitly, the rest state is found by sorting all parcels by their value of potential temperature, with the lowest potential temperature being at the bottom layer and the highest value being at the top. In the rest state, each parcel is assumed to be spread out evenly over the area of the Earth surface (see Fig. S1 in the supplemental material).

In a rest state of identical air columns, we can compute the pressure at every point in the atmosphere from the knowledge of the total atmospheric mass and the hydrostatic equilibrium that prevails.

The result of step 1 is thus the pressure profile of the atmosphere in the rest state assuming that topography is flat. In the next step, this pressure profile is adjusted to an Earthlike topography.

In step 2, the fundamental difference introduced by an uneven topography compared to a flat one lies in the relationship between pressure and altitude, which makes it more complicated to determine the rest state when there is uneven topography. For example, consider the case in which a parcel of air with the lowest potential temperature sits on a high plain while somewhere else on Earth's surface there is a deep well that

could accommodate that parcel of cold air. The calculation must be able to determine that that parcel of cold air could be contained by the well, and the available potential energy associated with that parcel must reflect that fact. Further, in the flat case, at different locations, air columns are identical. In an atmosphere over uneven topography, however, the air columns are not identical to each other, and the relationship between altitude above the ground and pressure differs. The rest-state pressure depends on the mass above each point of a layer as before, but the amount of air mass that will lie above a particular location will depend on the topography at other locations. In this case, an iterative approach can yield the rest-state distribution from the given-state pressure profile.

Both Taylor (1979) and Koehler (1986) proposed methods to determine the rest state of the atmosphere when topography is uneven, with Taylor's method being less exact than Koehler's. Koehler's (1986) approach is based on the conservation of mass, which must prevail between two isentropes in an adiabatic redistribution. However, since mass has to be conserved and we know how much mass is above each isentrope from the flat case, the pressure in the rest state can be determined iteratively by adjusting an initial pressure profile (e.g., the rest-state pressure profile calculated assuming flat topography) toward mass conservation. Before this numerical solution can be found, however, we must have a definition of average pressure on isentropes in an atmosphere over uneven topography. The problem is that the isentropes are horizontal in the rest state, as before; however, now, as surface topography is uneven, some isentropes intersect Earth's surface. A way has to be defined regarding how to treat isentropes that have intersected the ground, and thus the isentrope does not exist in the atmosphere at all locations. Koehler (1986) follows the procedure introduced by Lorenz (1955) where the average pressure on an isentrope surface intersecting the ground is determined using the pressure profile values for the part of the isentropes that is above ground and using the surface pressure values for the part of the isentrope that is underground. This definition takes care of the case of the cold parcel and the deep well mentioned above.

The adjustment process from flat to Earthlike topography is mathematically described in the supplemental material and follows the approach described by Koehler (1986).

In step 3, the contribution to APE and APE generation coming from each atmospheric parcel is computed. Computing APE is done by reformulating Lorenz's exact equation for APE, which integrates over isentropes to an equation that builds an integral over limited volumes (Grotjahn 1993). Summing Grotjahn's (1993) definition of APE in a parcel over all parcels in the

atmosphere yields the total APE in the atmosphere. By solving the integral over the mass of one parcel only, the parcel's contribution to the total APE is obtained.

As mentioned above, the enthalpy of an atmospheric column in hydrostatic equilibrium equals the total potential energy as the sum of the geopotential and the internal energy in that column. However, this equality does not hold for each individual parcel (if it does not span over a whole atmospheric column). Thus, performing APE calculations where the global average available enthalpy is calculated will result in the same value when the global average of the available potential energy is retrieved, but the geographic distributions of contributions to those totals will differ. Thus, it is critical to define which metric of contributions to global available potential energy is being applied.

We calculate APE generation for each parcel as the product of three terms: 1) the mass of the parcel, 2) the energy flux into the parcel per unit mass, and 3) a term reflecting the efficiency with which energy fluxes contribute to APE generation (see the supplemental material). This efficiency factor depends on the pressure of the parcel in the rest state and the pressure of the parcel in the given state. This term is positive if the parcel would be at lower pressure in the rest state. Thus, energy fluxes into a parcel contribute to APE generation if there is a net energy flux into a parcel that resides at a higher pressure than its pressure in the rest state or if there is a net energy flux out of a parcel that resides at a lower pressure than its pressure in the rest state.

3. Results and discussion

Our main analysis is centered on dry available potential energy calculations for an atmosphere over Earthlike topography. Additionally, we applied the dry APE calculations to a dataset for an atmosphere over flat topography to estimate the influence of topography on the distribution and quantity of APE and its generation.

As mentioned in the methods section, APE can be computed either in terms of enthalpy or as the sum of geopotential and internal energy. Here, all maps show results in terms of enthalpy. We use the term given state to refer to the actual state of the atmosphere and rest state to refer to the state of the atmosphere after it has been adiabatically arranged to minimize enthalpy.

Since our calculations result in 3D datasets covering the whole atmosphere, we projected our results into two dimensions in two different ways. The first is a column-integral representation, in which the values for each column of atmosphere are integrated, and the second one is the zonal mean form, where for each point in altitude and latitude the mean over all longitudes is computed.

TABLE 1. Globally averaged values for dry APE, total potential energy (TPE), and dry APE generation G for the treatment of an atmosphere over Earthlike topography.

Season	APE (10^6 J m^{-2})	TPE (10^9 J m^{-2})	G (W m^{-2})
DJF	5.51	2.515	3.72
MAM	5.33	2.518	3.44
JJA	5.45	2.528	3.62
SON	5.39	2.517	3.45
Annual	5.42	2.520	3.56

a. Distribution of dry APE and APE generation over Earthlike topography

1) ANNUAL MEAN

The global average APE, total potential energy, and APE generation for each season and the annual mean can be found in Table 1. Our estimates for global APE in this climate model (3.56 W m^{-2}) are consistent with a previous estimate for an earlier version of this same climate model (Marvel et al. 2013) but are larger than previous observationally based estimates, which tend to be in the range of $2\text{--}3 \text{ W m}^{-2}$ (e.g., Peixoto and Oort 1992). It is beyond the scope of this study to identify reasons why the atmospheric component of CESM1.0.4 generates more available potential energy than inferred from observations.

In the rest state the isentropes are horizontal and stratified. The top-left panel of Fig. 1 shows a zonal mean of the annual potential temperature distribution and indicates the shape of the isentropes in the given-state $1 \times \text{CO}_2$ atmosphere. To arrive at the rest state and thus straighten the isentropes, air parcels at high latitudes are adiabatically lowered and air parcels at low latitudes are adiabatically lifted. Adiabatic ascent (descent) leads to a decrease in temperature and pressure (increase in temperature and pressure). Thus, air parcels in low latitudes will have a lower temperature in their rest state compared to their temperature in the given state, and air parcels in high latitudes experience the opposite temperature change (their rest-state temperature is higher than their temperature in the given state).

Under this definition of local contributions to APE, cold potential temperatures in the midtroposphere near the poles are considered to have negative APE because these parcels would need to be compressed to lower them to the mean level of their isentropes, and this compression would require work to be performed. In contrast, warm potential temperatures in the midtroposphere near the equator are considered to have positive APE because as they are lifted to the mean level of their isentropes, they expand and are able to do work. These patterns can be observed in Fig. 2, which shows the zonal annual mean distribution of APE as well as the vertically

integrated annual mean APE. These figures show that large positive contributions to APE come from low latitudes and low altitudes, whereas in high latitudes in low to mid-altitudes contribute large negative values to the total APE.

Figure 3 shows results for the annual mean distribution of dry APE generation in the atmosphere. In the low latitudes, the column-integral representation reveals areas of strong APE generation (positive, red) being adjacent to areas of strong APE destruction (negative, blue). APE is also generated near both poles. In the low latitudes, APE is generated largely as a result of the latent heat released upon condensation of water vapor in the ITCZ. In the high latitudes, APE is generated largely as a result of longwave radiative cooling. Comparison of the left panel of Fig. 3 with the top panels of Fig. 1 illustrates that APE is generated both by warming in the tropical regions where isentropes lie below their mean pressure level and by cooling in the polar regions where isentropes lie above their mean pressure level.

2) SEASONAL VARIATIONS

APE generation exhibits substantial seasonality (Figs. 4 and 5). In the zonal mean representation, it can be seen that APE generation in the tropics largely follows the meridional movement of the ITCZ. Seasonally, the ITCZ shifts to the cooler hemisphere. The two generation peaks close to each other on both sides of the equator in low altitudes seen in the annual mean (Fig. 3) are the consequence of this meridional movement of a single area with high APE generation rates. Similarly, polar APE generation is largely a seasonal phenomenon, with APE generation reaching its maximum in the polar night, when radiative cooling is not offset by shortwave warming. APE generation reaches particularly large values in the eastern tropical Pacific, which is a region with vigorous convective activity. Additionally, atmospheric heating over the summertime Tibetan Plateau (see Yao and Zhang 2013) is a particularly strong source of APE generation. Further, condensation of rain over tropical forests contributes substantially to APE generation.

The spatial pattern of APE that we derive for this climate model is consistent with observationally based estimates obtained in previous literature (e.g., Oort 1964; Peixoto and Oort 1992). APE as well as APE generation is larger in Northern Hemisphere winter than during the rest of the year because of a maximum in meridional temperature gradients in this season and because in the night of the polar winter the cold air loses energy to space through longwave cooling without compensating shortwave heating.

b. Effects of considering topography

The results for global average APE and APE generation between different treatments of topography can be found

FIG. 1. (left) Zonal and annual mean potential temperature (K; contours and color shadings) and (right) net heating climate and (bottom) a $4 \times \text{CO}_2$ climate. Note that the contours of constant potential temperature (i.e., isentropes) in the troposphere are closer to the ground in the $4 \times \text{CO}_2$ simulation and midtropospheric tropical heating and polar cooling occurs at a somewhat higher altitude.

in Table 2. The column marked dry-topo minus the column marked dry-flat indicates the influence of topography on dry APE and dry APE generation. At global scale these differences are relatively small. More insight can be gained

by looking at the zonal mean differences between the different atmospheric treatments presented in Fig. 6.

The zonal mean line for the annual contributions to dry APE generation over flat topography shows two peaks on

FIG. 2. Distribution of annual mean contributions to global dry APE for the treatment of an atmosphere over Earthlike topography. (left) Zonal mean APE contributions ($\text{J m}^{-2} \text{Pa}^{-1}$); (right) vertically integrated APE contributions (10^6J m^{-2}). Contributions to APE are greatest near the middle of the tropical troposphere because in an adiabatic redistribution that flattens isentropes, air parcels of this region would be adiabatically lifted, which leads them to expand as they decompress. By expanding, these parcels can do work on the surrounding environment. Under this accounting, contributions to APE are equal to the amount of work that can be done by a parcel of air (or would need to be done to a parcel of air) when moving from its given state to its rest state.

FIG. 3. As in Fig. 2, but for distribution of annual mean dry APE generation in units of (left) $10^{-6} \text{ W m}^{-2} \text{ Pa}^{-1}$ and (right) W m^{-2} . APE generation is greatest near the tropics in the ITCZ where air with relatively warm potential temperatures is heated largely by condensing water vapor and near the poles where air with relatively low potential temperatures is radiatively cooling, resulting in longwave radiation to space.

both sides of the equator in low latitudes. These are the two peaks also observed in the zonal mean map in Fig. 3. Toward both poles, the zonal mean APE generation is strongly increasing until reaching approximately the same

amount as the peaks around the equator. Figure 6 shows the differences in zonal mean vertically integrated contributions to dry APE and APE generation over flat versus Earthlike topography. Consideration of topography tends

FIG. 4. Seasonal changes in the distribution of dry APE generation in an atmosphere over Earthlike topography ($10^{-6} \text{ W m}^{-2} \text{ Pa}^{-1}$). APE generation reaches high values where water vapor is condensing in the ITCZ. The ITCZ moves with the seasons into the colder hemisphere. APE generation also reaches high value in the polar night, where loss of longwave radiation to space is not compensated for by shortwave heating.

FIG. 5. As in Fig. 4, but in units of watts per square meter. In addition to the features seen in Fig. 4, it can be seen that summertime heating by the Tibetan Plateau and condensation during the rainy season over rain forests also contribute to APE generation.

to increase estimates of APE generation in land areas with high topography, although there are also substantial changes in APE generation over the oceans (see supplemental material). These differences in APE generation influence the distribution in contributions to APE.

The qualitative similarities among both treatments that can be seen in Figs. 6a,c indicate that a good qualitative understanding of APE contributions and APE generation can be obtained from any of these cases. (Of course, if the focus of a study is to understand the role of topography in APE generation, then it would be important to implement an appropriate treatment of that factor.)

In the rest of this work, we focus on dry APE in an atmosphere over Earthlike topography.

c. Application to a $4 \times \text{CO}_2$ climate

To investigate the changes in APE and APE generation distribution from an increase in CO_2 concentration in the atmosphere, we applied our APE calculations to climate model data produced for an atmosphere with atmospheric CO_2 concentrations increased to 4 times the preindustrial level. Our analysis is performed for dry APE in an

atmosphere with Earthlike topography, as described in section 2. The primary result is that a fourfold increase in atmospheric CO_2 content in this model results in a 7.0% reduction in global APE and a 10.1% decrease in APE production, suggesting a 3.3% increase in the residence time of APE in the atmosphere (Table 3). A decrease in APE production is consistent with expectations based on decreased equator-to-pole temperature differences in warmer climates. Clearly, there are substantial changes in the western tropical Pacific and tropical Indian Oceans

TABLE 2. Comparison of globally averaged values for seasonal means of APE and APE generation G for the two treatments: dry APE for an atmosphere over Earthlike topography (dry-topo) and a dry APE for an atmosphere over flat topography (dry-flat).

Season	APE (10^6 J m^{-2})		G (W m^{-2})	
	Dry-flat	Dry-topo	Dry-flat	Dry-topo
DJF	5.64	5.51	3.72	3.72
MAM	5.38	5.33	3.51	3.44
JJA	5.40	5.45	3.60	3.62
SON	5.39	5.39	3.57	3.45
Annual	5.45	5.42	3.60	3.56

FIG. 6. Influence of an Earthlike topography and moist processes on annual zonal mean APE and APE generation. In all panels dry APE with flat topography is shown in blue and dry APE with Earthlike topography is shown in red. (a) Zonal annual mean vertically integrated contributions to APE (10^6 J m^{-2}). (b) As in (a) but minus results for dry APE with flat topography (10^6 J m^{-2}). (c) As in (a) but for APE generation (W m^{-2}). (d) Difference in zonal annual mean vertically integrated APE generation minus results for dry APE with flat topography (W m^{-2}). Near the South Pole, APE generation is on average larger in the case of the Earthlike topography, and near the equator there is on average more APE generation in the flat case.

that are likely associated with changes in convective systems in these regions.

It is beyond the scope of this study to do a detailed evaluation of changes in contributions to APE and APE production in high- CO_2 climates. Here, we merely indicate how this type of analysis could be applied to help understand changes in APE and APE generation between high- and low- CO_2 climates. The bottom panels of Fig. 1 show zonal mean potential temperature and net heating rates for a $4 \times \text{CO}_2$ climate as simulated by CESM1.0.4. In the $4 \times \text{CO}_2$ climate, the isentropes are lower and midtropospheric heating somewhat higher. Figures 7a,c show zonal means of the vertical integral of contributions to APE and APE generation, respectively, for $4 \times \text{CO}_2$ and $1 \times \text{CO}_2$ climates. The differences between climate states are small relative to latitudinal differences within one climate state. Figures 7b,d show the differences between climate states for APE contributions and APE generation. Intriguingly, zonal mean changes from a quadrupling of CO_2 (Fig. 7d) are similar to changes from the inclusion of topography relative to the flat case (Fig. 6d). Unfortunately, we are not able to unambiguously identify the reasons for this similarity.

It might be thought that surface albedo changes would be a primary contributor to changes in APE generation in a high- CO_2 climate. Loss of sea ice and snow would be expected to lead to additional heating of relatively cold polar surface air masses, which would be hypothesized

to decrease APE production. However, analysis of changes in these surface properties is complicated by the fact that the positions of isentropes are altered in the high- CO_2 climate state. Our results indicate that the largest changes in APE generation occur in the upper troposphere. Figures 8 and 9 show the results for the maps of changes in contributions to APE and APE generation for a $4 \times \text{CO}_2$ climate relative to $1 \times \text{CO}_2$ climate. Changes in the distribution of APE production seen in Fig. 9 (left) are in part the product of a higher radiating altitude in high-latitude regions and higher altitude of condensation in the tropics.

4. Conclusions

We have calculated the geographic and seasonal variation in APE contributions and APE generation in the atmospheric component of the NCAR CESM1.0.4

TABLE 3. Globally averaged values for dry APE and dry APE generation G in a $4 \times \text{CO}_2$ climate.

Season	APE (10^6 J m^{-2})		G (W m^{-2})	
	$1 \times \text{CO}_2$	$4 \times \text{CO}_2$	$1 \times \text{CO}_2$	$4 \times \text{CO}_2$
DJF	5.51	5.17	3.72	3.40
MAM	5.33	4.88	3.44	3.04
JJA	5.45	5.08	3.62	3.26
SON	5.39	5.02	3.45	3.09
Annual	5.42	5.04	3.56	3.20

FIG. 7. Difference in zonal annual mean of dry APE and dry APE generation between a 4-times increased atmospheric CO_2 concentration and a preindustrial CO_2 concentration (W m^{-2}), over Earthlike topography. (a) Zonal annual mean vertically integrated contributions to APE (10^6 J m^{-2}). (b) As in (a), but for zonal annual mean vertically integrated contributions to APE for a $4 \times \text{CO}_2$ climate relative to a $1 \times \text{CO}_2$ climate (10^6 J m^{-2}). (c) As in (a), but for APE generation (W m^{-2}). (d) As in (b), but for difference in APE generation of a $4 \times \text{CO}_2$ climate relative to a $1 \times \text{CO}_2$ climate (W m^{-2}). Colors for lines in all panels: $1 \times \text{CO}_2$ (red) and $4 \times \text{CO}_2$ (purple).

climate model. Maps showing the global average distribution of APE and APE generation are found in Figs. 2 and 3. Further, we have provided figures showing the seasonal variability in the spatial distribution of these quantities (Figs. 4 and 5). We have also shown, for the first time, how the spatial distribution of APE contributions and APE generation may change in a high- CO_2 climate (Figs. 7, 8, and 9).

APE as defined is a global property, and there are several different possible ways of accounting for how

different locations in the atmosphere contribute to overall APE. We have adopted a definition that emphasizes the amount of work that the expansion of a parcel can do as it is adiabatically moved from its initial (given) state to its rest state (or the negative of the amount of work that must be done on a parcel to adiabatically compress the parcel as it is so moved). We have provided a local definition of APE generation, although this definition depends on the existence of a rest state that is defined in terms of global properties. We refer to

FIG. 8. Differences in annual mean dry APE contributions in a $4 \times \text{CO}_2$ climate relative to a $1 \times \text{CO}_2$ climate for the treatment of an atmosphere over Earthlike topography. (left) Zonal mean APE contributions ($\text{J m}^{-2} \text{ Pa}^{-1}$). (right) Vertically integrated APE contributions (10^6 J m^{-2}). Note the scales are a factor of 10 smaller than the range of the parent plots of APE contributions.

FIG. 9. As in Fig. 8, but for APE generation with units of (left) of $10^{-6} \text{ W m}^{-2} \text{ Pa}^{-1}$ and in (right) of W m^{-2} . Note the scales are a factor of 10 smaller than the range of the parent plots of APE generation.

APE computed here as “dry” APE since our analysis does not consider phase changes that water may undergo in atmospheric redistribution to the rest state. In terms of moist processes, only latent heat fluxes associated with water phase changes that are represented in the climate model are taken into account. In the treatment of the atmospheric data coming from the climate model the mass of water is treated as air.

Our primary analysis considers variation in surface topography, whereas classical treatment from Lorenz (1955) considers only flat topography. The extension of classical theory to uneven topography was made by Koehler (1986), and his extension is applied here. Consideration of uneven, Earthlike topography precludes analytic solution for the rest state, so an iterative solution was applied. Differences between the spatial distributions of APE contributions as calculated for with and without topography are typically small relative to the spatial variation within one treatment of topography (Fig. 6).

We have found that APE generation primarily occurs in two altitude regions of the mid-to-upper troposphere (Fig. 3). There are geographically two regions that primarily contribute to APE generation: 1) the tropics, owing largely to latent heat released in the ITCZ, and 2) the polar regions, owing largely to longwave radiative cooling. Polar APE generation is greater in the polar night, where shortwave warming does not offset longwave cooling (Figs. 4 and 5). Tropical APE generation shifts seasonally with the ITCZ.

Furthermore, we have calculated how APE and APE generation is changed in a $4 \times \text{CO}_2$ climate relative to a $1 \times \text{CO}_2$ (Table 3), where the climate change was calculated using a slab-ocean configuration of the model with specified ocean heat transport. We found that in the

high- CO_2 climate, APE generation decreased by 10.1% and APE decreased by 7.0% relative to the $1 \times \text{CO}_2$ climate. In steady state, kinetic energy dissipation equals APE production, so this implies a 10.1% decrease in kinetic energy dissipation for a high- CO_2 climate in this model. This is consistent with expectations based on decreased equator-to-pole temperature gradients in warmer climates (Serreze and Francis 2006).

Future work can build off of the foundation provided here in at least several different directions. Further studies could investigate a “moist” treatment of the atmosphere resulting in moist APE, where different thermal expansion properties of water vapor relative to air and also phase changes that may occur upon redistribution to the rest state are taken into account. Consideration of moist APE requires an extension to the classical (Lorenz 1955) definition of the rest state, which was provided by Lorenz (1978). In contrast to dry APE, there is, however, no analytical solution for the computation of the rest state for moist APE, and therefore numerical solutions would have to be applied. Further on, the current analysis could be made more detailed by considering the roles of longwave, shortwave, latent, and sensible heating terms. The analysis of climate change scenarios caused by high greenhouse gas concentrations (or perhaps extreme wind power scenarios; e.g., Marvel et al. 2013) could separately analyze the effects of these different heating terms and their interactions with changed altitudes of isentropes in a changed climate and thus determine causes for changes in APE generation.

This work represents a first step toward a more general understanding of APE generation, conversion to kinetic energy, and eventual dissipation. Fundamental questions remain regarding our collective understanding of the atmospheric heat engine (Laliberté et al. 2015). We have

helped to develop this understanding by developing tools and methods to calculate the distributions of APE contributions and APE generations in a climate model of the atmosphere and producing the first maps showing the spatial distributions of these quantities.

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