Estimates of regional and global average sea level change remain a focus of climate change research. One complication in obtaining coherent estimates is that geodetic datasets measure different aspects of the sea level field. Satellite altimetry constrains changes in the sea surface height (SSH; or absolute sea level), whereas tide gauge data provide a measure of changes in SSH relative to the crust (i.e., relative sea level). The latter is a direct measure of changes in ocean volume (and the combined impacts of ice sheet melt and steric effects), but the former is not since it does not account for crustal deformation. Nevertheless, the literature commonly conflates the two estimates by directly comparing them. We demonstrate that using satellite altimetry records to estimate global ocean volume changes can lead to biases that can exceed 15%. The level of bias will depend on the relative contributions to sea level changes from the Antarctic and Greenland Ice Sheets. The bias is also more sensitive to the detailed geometry of mass flux from the Antarctic Ice Sheet than the Greenland Ice Sheet due to rotational effects on sea level. Finally, in a regional sense, altimetry estimates should not be compared to relative sea level changes because radial crustal motions driven by polar ice mass flux are nonnegligible globally.
A complication in comparing these distinct observations of sea level change is that they are defined relative to different coordinate systems. Both GRACE and altimetry measurements are tied to the International Terrestrial Reference Frame (ITRF; e.g., Altamimi et al. 2016) through data processing. Namely, for long-term changes considered in this manuscript, this should place the measurements in a center of mass (CM) frame. [On shorter time scales, the altimetry measurements may be more aligned with a center of figure (CF) frame (e.g., Melachroinos et al. 2013; Ray et al. 2013).] Ideally, RSL would be measured globally, in which the resulting estimate of the change in thickness of the ocean would be independent of any reference frame.

In addition, there are significant differences in how rotational feedback is handled in each of the observation types. The International Earth Rotation and Reference Services System (IERS) conventions (Petit and Luzum 2010) specify how geodetic data should be processed, and we specifically make note here of the ocean pole tide correction. The primary purpose of the pole tide correction is to remove the effects of the 14-month Chandler wobble and annual variations of the pole. However, several studies have noted problems with this correction in the 2010 conventions (i.e., Wahr et al. 2015; Desai et al. 2015; King and Watson 2014) and suggested only the effects of a constant, long-term mean pole motion, presumably due to glacial isostatic adjustment (GIA), be left in the data. If the data were processed in such a manner, then any rotational feedback due to recent increased melting from Greenland and Antarctica would also be removed. However, altimetry data have not always been processed according to the IERS conventions (Desai et al. 2015). As such, we consider two end cases for altimetry: one in which the rotational feedback is included in the calculation of SSH change, and one in which this feedback has been removed. We note that tide gauge data are rarely processed in accord with conventions [e.g., no component of the pole tide signal is removed from the Permanent Service for Mean Sea Level (PSMSL) data, a difference that should be kept in mind when combining GPS and tide gauge measurements]. Thus, we include rotational feedback in the examples considered here.

In a static sea level theory (Farrell and Clark 1976), which deals only with the mass component of sea level change (i.e., no thermal expansion), it is assumed that the sea surface remains an equipotential, although the value of that equipotential may change with time. As a consequence, in this theory there is a subtle, but important difference between altimeter measurements of SSH changes and GRACE observations of geoid changes. In particular, GRACE measures perturbations of the reference geoid, whereas altimeter observations track the sea surface and need not remain on the same equipotential through time (Farrell and Clark 1976; Dahlen 1976; Tamisiea 2011).

Following the above discussion, we can write relative sea level change \( \Delta \text{SL} \) as

\[
\Delta \text{SL} = \Delta \text{ASL} - \Delta R, \tag{1}
\]

where \( \Delta \text{ASL} \) is the change in absolute sea level (or SSH), and \( \Delta R \) is the change in the height of the solid surface, commonly referred to as vertical land motion (VLM). Relative sea level changes do not require the specification of a coordinate system, but predictions of \( \Delta \text{ASL} \) require that the two bounding surfaces on the right-hand side of the equation are generated in a consistent coordinate system. Changes in the SSH, or \( \Delta \text{SL} \), can be decomposed into a term associated with perturbations in the geoid \( \Delta G \) plus a geographically uniform shift \( \Delta \Phi/g \), that accounts for the fact (as discussed above) that the equipotential defining SSH can evolve in time (Farrell and Clark 1976; Dahlen 1976; Tamisiea 2011):

\[
\Delta \text{ASL} = \Delta G + \frac{\Delta \Phi}{g}, \tag{2}
\]

where \( \Delta \Phi \) represents the perturbation in the equipotential that defines the sea surface.

Any observation of \( \Delta \text{SL} \) will reflect a combination of signals due to sea level changes associated with the last ice age, or GIA, local effects (e.g., tectonics, sediment compaction, ocean dynamics, hydrology, etc.), and any signals due to modern climate change. Accordingly, we can make these contributions explicit by rewriting Eq. (1) in the form

\[
\Delta \text{SL} = \Delta \text{ASL}_{\text{GIA}} - \Delta R_{\text{GIA}} + \Delta \text{ASL}_{\text{clim}} - \Delta R_{\text{clim}} + \Delta \text{SL}_{\text{local}}, \tag{3}
\]

In principle, both the GIA and modern climate signals include the impact on sea level of ice mass changes and steric effects; however, GIA predictions ignore the latter as being negligible in comparison to combined eustatic and deformational effects on sea level. This study focuses solely on present-day sea level signals due to modern polar ice sheet melting, and we do not consider steric and local effects. This modern process impacts mass redistributed both within the ocean and cryosphere, and it involves changes in SSH (\( \Delta \text{ASL}_{\text{clim}} \)) and vertical land motion (\( \Delta \text{R}_{\text{clim}} \)).

There is growing understanding that rigorous comparisons of the various measurements associated with sea level change, including GPS observations of VLM, requires a precise interpretation of the associated dataset (e.g., Chambers et al. 2010; Tamisiea 2011; Wahr et al. 2015; Frederikse et al. 2017). For example, recognizing the distinction between SSH and geoid changes...
[Eq. (2)] has clarified significant differences in published estimates of global mean sea level changes based on GRACE gravity data (Chambers et al. 2010; Tamisiea 2011). However, a number of important issues associated with such comparisons remain unexamined. For example, recent estimates of global mean sea level change from altimetry records are performed by averaging the altimeter record over the area of ocean sampled by the dataset and correcting for a suitably averaged GIA signal (Ablain et al. 2015; Cazenave et al. 2014; Nerem et al. 2010; Prandi et al. 2009; Dieng et al. 2017; Nerem et al. 2018). Following the above equations, we write this estimate as \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \). These published estimates [with the exception of Nerem et al. (2010)] are then commonly assumed to be the same as an estimate of \( \Delta \text{SL}_{\text{clim}} \), which might be derived, for example, from tide gauge data (e.g., Jevrejeva et al. 2008; Hay et al. 2015; Dangendorf et al. 2017). For example, Prandi et al. (2009) compare tide gauge estimates of global mean sea level (GMSL) with altimeter estimates of GMSL, conflating a volumetric change with changes in sea surface height. Similarly, Ablain et al. (2015) attempt to validate satellite-derived estimates of global mean sea level with Argo and GRACE estimates of ocean volume change. This comparison lacks rigor, since changes in gravity reflect a mass redistribution, which will have deformational effects of the ocean floor—and the latter is not accounted for in altimeter data. Finally, Dieng et al. (2017) and Nerem et al. (2018) attempt to close the sea level budget by comparing an ensemble of globally averaged altimetry measurements (i.e., \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \)) to the sum of contributors to global mean sea level (land water storage, steric, ice melt), the latter components being a volumetric change (\( \langle \Delta \text{SL}_{\text{clim}} \rangle \)). We argue that this is not formally correct because \( \langle \Delta \text{SL}_{\text{clim}} \rangle \) represents the actual change in ocean volume, whereas \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \) does not account for the impact of changes in VLM driven by modern melting and effects associated with land water storage (i.e., \( \Delta R_{\text{clim}} \)). The appropriateness of treating \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \) as a measure of volume change has been discussed in Spada (2017). Simply put, is it correct to treat an estimate of \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \) derived from altimetry records as an accurate measure of the net volume change of the ocean associated with ice mass flux and steric effects? Moreover, is the error (or bias) incurred by making this assumption dependent on the geometry of ice mass flux? That is, does the level of bias depend on the relative mass flux from each of the ice sheets and the geometry of this mass flux? Finally, what is the extent of the bias when regional estimates are made and compared? The goal of this study is to quantitatively address these questions.

Recent literature has begun to address the biases discussed above. For example, Frederikse et al. (2017b) use a global reconstruction of ice melt and hydrology to estimate regional- and global-scale biases between absolute sea level and relative sea level, the fields we have defined as \( \langle \Delta \text{ASL}_{\text{clim}} \rangle \) and \( \langle \Delta \text{SL}_{\text{clim}} \rangle \), respectively. They find the differences between these two averages to be on the order of 0.1 mm yr\(^{-1}\), with regional differences being up to an order of magnitude larger. Here, we consider biases in altimetry estimates of GMSL associated with individual ice sheet mass flux, inferred from GRACE. We examine the sensitivity of those estimates to the geometry of ice mass flux by also considering more simplistic melt scenarios. Furthermore, we explore the impact on the bias of rotational signals in sea level change.

Several points of nomenclature are important to specify at the onset. First, we will only consider modern climate signals. We will use the terms “vertical land motion” and “crustal displacement” (\( \Delta R \)) interchangeably. Similarly, as we note above, we will consider absolute sea level \( \Delta \text{ASL} \) and sea surface height changes to be interchangeable. Since we do not consider steric effects in this study, the relative sea level change \( \Delta \text{SL} \) we discuss represents the change in the distance between the sea surface and solid surface associated with mass changes alone. In the recent literature, the global mean of this component of relative sea level change has been termed barystatic sea level (Gregory et al. 2013). In the text below, we will use the term “sea level” or “relative sea level” only when referring to relative sea level.

### 2. Methods

The equation governing gravitationally self-consistent sea level changes driven by ice mass flux on a spherically symmetric, linear (Maxwell) viscoelastic Earth model was first derived by Farrell and Clark (1976) for the case of fixed shorelines and a nonrotating system. Since the mid-1990s the theory has been extended to include rotational effects and time-varying shorelines (e.g., Johnston 1993; Milne and Mitrovica 1996; Mitrovica and Milne 2003). Both approaches make use of viscoelastic Love number theory (Peltier 1974) to compute the deformational response of the Earth model to the changing surface mass (ice plus ocean) load. We adopt the pseudospectral algorithm derived by Kendall et al. (2005) to solve the governing sea level equation, and we adopt a truncation at spherical harmonic degree and order 512. In these calculations, the radial profiles of elastic constants and density are prescribed from the seismic model Preliminary Reference Earth Model (PREM; Dziewonski and Anderson 1981). The results shown below involve present-day sea
level changes due to modern melting events, and in this case, we model the Earth’s response as purely elastic. We consider various melting geometries, including uniform melting from the Greenland Ice Sheet (GrIS) and the West Antarctic Ice Sheet (WAIS), as well as nonuniform melt geometries constructed from one of a wide number of GRACE-based estimates of mass flux from the GrIS and Antarctic Ice Sheet (Harig and Simons 2012, 2015). The case of global glacier melting is shown in the online supplemental material.

3. Results and discussion

As discussed above, satellite altimeter missions [e.g., TOPEX/Poseidon, Jason-1, Jason-2, Jason-3, ERS-1, Envisat, Geosat Follow-On (GFO), SARAL] measure changes in SSH relative to a reference ellipsoid and are generally confined to the lower to middle latitudes (i.e., between 66°N and 66°S). We focus here on the bias introduced by assuming that absolute sea level change due to modern climate change ΔASL$_{\text{clim}}$ is the same as ΔSL$_{\text{clim}}$; the latter, as we discussed above, is a measure of the ocean mass and (in the absence of steric effects) volume change whereas the former is not, since it does not include deflections of the second bounding surface of relative sea level, the crust, in response to modern melting.

Figure 1 provides maps of the total ice mass flux over both Greenland and the Antarctic over a period of approximately a decade beginning in 2003 in units of centimeters water equivalent, as inferred from GRACE satellite gravity data (Harig and Simons 2015). Many such maps exist in the literature, and our adoption of these particular estimates simply serves as an illustration of calculations based on melt geometries that are more realistic than, for example, the common assumption in sea level modeling of geographically uniform thinning of the polar ice sheets (Clark and Lingle 1977; Clark and Primus 1987; Conrad and Hager 1997; Mitrovica et al. 2001; Plag 2006; Brunnabend et al. 2015; Spada and Galassi 2016).

Figure 2a is a prediction of the sea level change ΔSL$_{\text{clim}}$ arising from the GrIS melt geometry in Fig. 1a, commonly described as a sea level fingerprint. The map is normalized to a global relative sea level change of 1 mm and can be scaled linearly for different levels of melt. Figure 2b is the analogous prediction for the SSH change ΔASL$_{\text{clim}}$, and Fig. 2c is their difference (ΔSL$_{\text{clim}}$ − ΔASL$_{\text{clim}}$), which is identical to the negative of the vertical land motion associated with the melting (−ΔR$_{\text{clim}}$). Figure 2c is the field that is being neglected, that is, it is the source of bias, when one considers altimeter measurements of ΔASL$_{\text{clim}}$ as being equivalent to ΔSL$_{\text{clim}}$. Figures 2d and 2e are analogous to Figs. 2b and 2c, respectively, with the exception that the calculation of SSH change does not include rotational feedback. Figure 2f provides a calculation of the mean value of the fields ΔSL$_{\text{clim}}$ (blue line), ΔASL$_{\text{clim}}$ (red line), and ΔASL$_{\text{clim}}$ without rotational feedback (orange line) computed between ±X, where X ranges from 60° to 90° latitude (the red and orange lines are barely distinguishable on the plot). Note that a global (pole to pole) average of the ΔSL$_{\text{clim}}$ field yields a value of 1 mm, which reflects the level of (normalized) melting introduced into the system.

Regardless of whether rotational feedback is included in the calculation of ΔASL$_{\text{clim}}$, the mean ΔASL$_{\text{clim}}$ value ranges from 0.95 for an averaging between 60°N and 60°S latitudes to 0.92 for the pole-to-pole average. The average over the region sampled by most altimetry datasets (±66° latitude) is 0.94, indicating that mean sea level change derived from altimetry data (ΔASL$_{\text{clim}}$)
is a 6% underestimate of the true global mean sea level change ($\Delta SL_{clim}$) associated with the melt event. It is interesting to note that this bias increases as the range of the altimetry measurements is increased, so that it would reach 8% if altimetry measurements were to sample the ocean from pole to pole.

Figure 2 replots the results in Figs. 2a–c within the western Pacific and Indian Ocean, and the North Atlantic. (Figure S1 in the supplemental material provides an analogous set of close-ups for Figs. 2a,d,e.) The former region is in the far field of the GrIS and the negative of the crustal displacement $-\Delta R_{clim}$ shown in Fig. 3c—that is, the error incurred by assuming that $\Delta ASL_{clim}$ is measuring $\Delta SL_{clim}$—is dominated by rotational and ocean loading effects. If we consider the units on the plots to be millimeters per year, then at Perth, in Western Australia, the difference between the relative sea level change (1.00 mm yr$^{-1}$) and absolute sea level change (0.57 mm yr$^{-1}$), that is, the negative of VLM, is 0.43 mm yr$^{-1}$. In contrast, the North Atlantic is in the near field of the GrIS and in this case $\Delta R_{clim}$ (VLM), shown in Fig. 3f, is largely a consequence of gravitational and deformational effects associated with the mass loss over Greenland. In this case, predicted relative sea level decreases by $-0.16$ mm yr$^{-1}$ despite the fact that absolute sea level is predicted to rise by 0.55 mm yr$^{-1}$,
a difference that reflects an uplift of the crust by 0.71 mm yr\(^{-1}\). Clearly, the error incurred in treating \(\Delta ASL_{\text{clim}}\) and \(\Delta SL_{\text{clim}}\) as equivalent can be extreme when one is considering regional sea level.

Figure 4 is analogous to Fig. 2 for predictions associated with mass flux within the Antarctic Ice Sheet (AIS) (Fig. 1b). In this case, the discrepancy between absolute and relative sea level (Fig. 4c), where both include rotational feedback, that is, radial displacement of the crust, is characterized by a large rotational signal (evident in the blue and red zones in the Northern Hemisphere) and gravitational and deformational effects associated with mass changes in the AIS (evident in the predicted signal south of 40°S). In the Northern Hemisphere, the peak discrepancies reach ~0.5 mm yr\(^{-1}\); over North America, altimetry overestimates the relative sea level rise (peak value of 0.56 mm yr\(^{-1}\)), and in Asia it underestimates relative sea level rise (peak value of 0.46 mm yr\(^{-1}\)). In the near field, in particular on Antarctic coasts, the discrepancy can be two orders of magnitude greater.

Figures 4d and 4e consider the case where the field \(\Delta ASL_{\text{clim}}\) does not include rotational feedback, but \(\Delta SL_{\text{clim}}\) does. Over North America, altimetry would now underestimate the relative sea level rise by 0.86 mm yr\(^{-1}\), while in Asia the discrepancy between relative sea level and SSH changes is less than 0.1 mm yr\(^{-1}\). In the near field of the AIS, the results are once again dominated by gravitational and deformational effects, and the signal in Fig. 4e is comparable to that of Fig. 4c. Close-up images of the plots in Fig. 4 over the North Atlantic and western Pacific (analogous to Fig. 3 and Fig. S1) are shown in the supplemental material.

Most notably, these signals introduce a significant bias if one treats the mean value of \(\Delta ASL_{\text{clim}}\) as interchangeable with \(\Delta SL_{\text{clim}}\). As an example, in the case where rotational feedback is included, the mean value of \(\Delta ASL_{\text{clim}}\) measured by altimetry (i.e., an average within ±66° latitude) would overestimate global mean sea level change \(\langle \Delta SL_{\text{clim}} \rangle\) by 17%. This is reduced to 12% when the SSH change does not include rotational feedback.

FIG. 3. (a)–(c) As in Figs. 2a–c, except that the plots focus on a region covering the western Pacific and Indian Ocean. (d)–(f) As in (a)–(c), but for the North Atlantic Ocean. In (d)–(f), the signal close to Greenland exceeds the color scale on the plot.
This bias is 2–3 times larger than the bias we noted for GrIS melt (Fig. 2f). However, in contrast to Fig. 2f, this bias decreases if altimetry measurements were to sample the ocean from pole to pole.

Next, we explore the extent to which the geometry of mass flux impacts the biases identified in Figs. 2 and 4f. Figures 5a and 5b repeat these earlier calculations under the assumption that mass flux from the GrIS and WAIS is geographically uniform. As noted above, an assumption of uniform melt is common in published sea levelfingerprints. The results in Fig. 5 indicate that the bias introduced in treating $\langle \Delta \text{ASL}_{\text{clim}} \rangle$ as a measure of global mean sea level change is insensitive to the geometry of GrIS melt (cf. Figs. 5a and 2f) but is very sensitive to the geometry of AIS melt. Indeed, the bias in an altimetry measurement of mean sea level change within $\pm 66^\circ$ latitude is $\sim 2\%$ for a uniform WAIS melt scenario, regardless of whether the altimetry measurement is corrected for rotational feedback, an order of magnitude less than the bias predicted on the basis of the GRACE-based AIS ice mass flux shown in Fig. 1b. The greater sensitivity of the level of bias to the details of mass flux across the AIS relative to the GrIS is in part due to the signal in sea level from Earth rotation. Since the AIS is centered at the South Pole, small changes in the geometry of mass flux can lead to large variations in the direction of polar motion, and thus large changes in the geometry of sea level change driven by this polar motion. In contrast, the GrIS is off the rotation axis, and net mass loss from the ice sheet will always displace the pole toward Greenland, leading to a more constrained signal in sea level associated with rotational feedback.

As a final calculation, we consider the bias in estimates of $\langle \Delta \text{ASL}_{\text{clim}} \rangle$ from mean values of $\Delta \text{ASL}_{\text{clim}}$ in the

![Fig. 4](image-url)
case of melting from global glacier systems. Results analogous to Figs. 2 and 4 for the case of mass flux from glaciers tabulated in the Fifth Assessment Report of the Intergovernmental Panel on Climate Change (Vaughan et al. 2013) are shown in Fig. S4. The level of bias is comparable to the GrIS results in Fig. 2. In particular, \( \Delta \text{ASL}_{\text{clim}} \) is 0.94 for an average between 66\(^\circ\)N and 66\(^\circ\)S latitudes, indicating that mean sea level change derived from altimetry data is a 6% underestimate of the true global mean sea level change \( \Delta \text{SL}_{\text{clim}} \) in the glacier tabulation we have adopted.

4. Conclusions

In the introduction, we posed a series of questions. First, is it correct to treat the global-scale mean signal \( \Delta \text{ASL}_{\text{clim}} \) obtained from altimetry records as a measure of the net volume change of the ocean associated with ice mass flux and steric effects? Second, is the bias incurred by making this assumption dependent on the geometry of ice mass flux and the relative size of the mass flux from each of the ice sheets? In regard to the first question, our results demonstrate that using altimetry records to estimate global scale ocean volume changes can lead to biases that exceed 15% (Fig. 4f). Our predictions have also shown that the magnitude of the bias will depend on the relative contributions from Greenland and Antarctic ice mass flux (these biases have opposite sign; see Figs. 2f and 4f) and also on the detailed geometry of Antarctic mass flux (cf. Figs. 4f and 5b). Improving estimates of ocean volume change from altimetry requires, at the very least, independent information regarding net mass flux from the polar ice sheets.

A final question posed in the introduction involves the extent of the bias when regional, rather than global, estimates of relative and absolute sea level change are made and compared. The results in Figs. 2–4 and Figs. S1–S3 indicate that altimetry measurements should never be equated to relative sea level anywhere on the globe because radial crustal displacements associated with polar ice mass flux are nonnegligible globally (Figs. 2c and 4c).

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