Climate Model Biases and Modification of the Climate Change Signal by Intensity-Dependent Bias Correction

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ABSTRACT

Climate change impact research and risk assessment require accurate estimates of the climate change signal (CCS). Raw climate model data include systematic biases that affect the CCS of high-impact variables such as daily precipitation and wind speed. This paper presents a novel, general, and extensible analytical theory of the effect of these biases on the CCS of the distribution mean and quantiles. The theory reveals that misrepresented model intensities and probability of nonzero (positive) events have the potential to distort raw model CCS estimates. We test the analytical description in a challenging application of bias correction and downscaling to daily precipitation over alpine terrain, where the output of 15 regional climate models (RCMs) is reduced to local weather stations. The theoretically predicted CCS modification well approximates the modification by the bias correction method, even for the station–RCM combinations with the largest absolute modifications. These results demonstrate that the CCS modification by bias correction is a direct consequence of removing model biases. Therefore, provided that application of intensity-dependent bias correction is scientifically appropriate, the CCS modification should be a desirable effect. The analytical theory can be used as a tool to 1) detect model biases with high potential to distort the CCS and 2) efficiently generate novel, improved CCS datasets. The latter are highly relevant for the development of appropriate climate change adaptation, mitigation, and resilience strategies. Future research needs to focus on developing process-based bias corrections that depend on simulated intensities rather than preserving the raw model CCS.

1. Introduction

Global climate models (GCMs) are the primary source of information about future climate change (IPCC 2013). Owing to the high computational demand, their spatiotemporal resolution is restricted and, hence,

their output is of limited use for climate change impact research and risk assessment. To resolve this fundamental scale gap, regional climate models (RCMs) nested within GCMs are used to downscale GCM-based projections to regional scales. The higher resolution combined with more realistic parameterizations at fine scales allows RCMs to better reproduce the local mechanisms that shape regional climates (Laprise 2008; Rummukainen 2010). Both GCM and RCM fields can exhibit substantial systematic differences from gridded observational data (Christensen et al. 2008; Sillmann et al. 2013; Kotlarski et al. 2014). Such discrepancies between simulated and observed fields are commonly referred to as biases (Pan et al. 2001). Model output is recommended to be statistically processed (bias corrected) before application (Wood et al. 2004; Fowler

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et al. 2007; Teutschbein and Seibert 2012), although this practice is controversially discussed (Ehret et al. 2012).

A quantitative measure for climate change is the additive or multiplicative difference (ratio) between a climate statistic for a future scenario period and a historical reference period. This difference is known as the climate change signal (CCS). Generally, model biases do not cancel out in the calculation of the CCS (Buser et al. 2009; Gobiet et al. 2015), therefore the CCS is also biased. The purpose of this paper is to provide more evidence to support the hypothesis that the CCS modification due to bias correction is a desirable consequence of removing model biases. To this end, we derive and test a systematic analytical theory of the effect of bias correction on the CCS. The theory allows a quantitative analysis of the CCS modification and can serve as a tool to efficiently and effectively generate novel, improved CCS estimates. The latter will form the base for supporting climate change adaptation, mitigation, and resilience strategies for end users, stakeholders, and policy makers.

The model bias at a fixed location is determined by the climate state; hence, it is time dependent. Recent research (Christensen et al. 2008; Boberg and Christensen 2012; Gobiet et al. 2015) suggests that temperature and precipitation biases of climate models are well approximated as being dependent only on the magnitude of the simulated/observed values—a feature named *intensity dependence*. For example, higher precipitation amounts tend to have larger biases. Consequently, bias correction methods that apply individual corrections to different model intensities are able to successfully debias not only temperature and precipitation (Maraun et al. 2010; Piani et al. 2010; Teutschbein and Seibert 2012; Themeßl et al. 2012; Ivanov and Kotlarski 2017) but also other climate variables, including wind speed (Wilcke et al. 2013). Ivanov and Kotlarski (2017) provided new evidence that such methods also improve the joint temperature–precipitation distribution and preserve the temperature–precipitation temporal association as in the raw climate model data. Based on pseudoreality experiments, Vrac et al. (2007), Maraun (2012), Rätty et al. (2014), and Ivanov and Kotlarski (2017) showed that in most regions intensity-dependent bias correction methods are relatively stable in the long term (i.e., for time scales beyond the maximum period of available observations). Assuming that the intensity-dependent model bias does not explicitly depend on time (i.e., that it is *stationary*), these methods use a function of intensity, called the *correction function* to describe the intensity-dependent corrections. Different simulated intensities correspond to different climate processes, for which climate models have different skills (e.g., Ivanov et al. 2018a,b). This provides a partial

physical justification for the intensity dependence of the correction function, the derivation of which is purely statistical. The view that the stationarity assumption is not overly restrictive is supported by the successful application of such methods in various contexts. Despite that, the validity of the stationarity assumption is subject to scientific debate (e.g., Switanek et al. 2017).

The question about the effect of intensity-dependent bias correction on the CCS has been controversially discussed. In practical applications, the raw model CCS of common climate statistics is generally preserved. However, for specific geographic regions, climate models, and statistics, the alteration of the CCS is comparable to the CCS itself (Hagemann et al. 2011; Boberg and Christensen 2012; Dosio et al. 2012; Themeßl et al. 2012; Räisänen and Rätty 2013; Maurer and Pierce 2014; Cannon et al. 2015; Ivanov and Kotlarski 2017). Whether or not this is a beneficial effect is debatable. For instance, the CCS modification may influence the model climate sensitivity and distort physical scaling relationships between meteorological variables such as temperature and precipitation. Hempel et al. (2013), Cannon et al. (2015), Pierce et al. (2015), and Switanek et al. (2017) developed bias correction methods that aim at preserving the raw model CCS. These methods use corrections that *explicitly* depend on the model *scenario* climatology and, hence, on time. Such bias correction methods lack physical justification for the assumed nonstationary biases. Furthermore, the validity of the raw model CCS is questioned by the fact that model biases can considerably distort the projected CCS (Buser et al. 2009; Gobiet et al. 2015). Boberg and Christensen (2012) and Gobiet et al. (2015) argue that the modification of the CCS of mean temperature by bias correction is meaningful as it alleviates the effect of intensity-dependent model biases on the CCS.

Thus, bias correction methods that keep the raw model CCS unavoidably assume artificial nonstationarities, whereas those that assume intensity dependence have some physical grounds, but modify the CCS. This controversial discussion is so far unresolved because the CCS modification by bias correction has been investigated mostly empirically. Currently, the mechanisms of that modification are analytically understood only for the distribution mean of variables that have no zero values (also known as *interval variables*) such as temperature (Hagemann et al. 2011; Gobiet et al. 2015). However, not only the distribution mean but also other statistics, including characteristics of extreme events, are of high relevance to applications. Current knowledge is not directly transferable to variables that have a natural zero limit (*ratio variables*; Finkelstein and Leaning 1984), such as (sub)daily precipitation and

wind speed, which are of primary importance for assessment of the future severe flooding (Christensen and Christensen 2003) and wind energy potential (Pryor and Barthelmie 2010). This is because zeros are treated differently than positive values. Apart from recent evidence that contains qualitative speculations (Hagemann et al. 2011; Themeßl et al. 2012), the problem is yet unexplored.

This paper is structured as follows. Section 2 presents the analytical theory of intensity-dependent bias correction. In section 3, we test the theory for daily precipitation over alpine terrain. Conclusions are drawn and a short outlook given in section 4.

2. Data and method

a. Data

We use observation and model data for daily precipitation that have been studied in a recent publication by Ivanov and Kotlarski (2017). The observations for the 30-yr calibration period 1980–2009 stem from 27 stations of the Swiss National Basic Climatological Network (Swiss NBCN; Begert et al. 2007, updated). (Figure B1 and Table B1 in appendix B show a geographic map and a list of the complete station names and locations, respectively.) The model data were extracted from the database of the ENSEMBLES project (<http://ensembles-eu.metoffice.com>; van der Linden and Mitchell 2009) for the region of Switzerland and the 1970–2099 period. They cover a European domain at a horizontal resolution of approximately 25 km and were generated assuming the IPCC SRES A1B emission scenario (IPCC 2000). Ivanov and Kotlarski (2017) have used data from the ENSEMBLES project to enable comparison of the methodology against the Swiss Climate Change Scenarios Initiative (Appenzeller et al. 2011), which is based on the same data. The model fields were reduced to the station locations by inverse distance weighting interpolation using the four nearest grid points. (Table B2 in appendix B lists the 15 GCM–RCM model chains employed.) The RCM data were postprocessed according to the empirical quantile mapping (QM) method selected in Ivanov and Kotlarski (2017). That method uses a 91-day moving window to calibrate a nonlinear transfer function that varies with the day-of-year for percentiles of order up to 0.99. The transfer function is linearly interpolated between percentiles and constantly extrapolated for quantiles of order above 0.99.

b. Analytical theory

This section introduces the basic ideas of the theoretical considerations and provides a summary of the final results. For detailed analysis that uses the mathematical apparatus of probability calculus (Billingsley

2012), the reader is referred to section 1 of the online supplemental material.

A meteorological variable takes values from intervals of the real line $\mathbb{R}$. Therefore, it is modeled by a continuous random variable X , defined on the sample probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the sample space of elementary outcomes ω , $\mathcal{F}$ is a σ -field of subsets (events) of Ω , and $\mathbb{P}$ is a probability measure on $\mathcal{F}$. Intensity-dependent bias correction methods use a function of intensity f , called the *transfer function*, which operates on raw model output (X) and yields bias-corrected data (Y). Given stationarity, f does not explicitly depend on time and is a univariate increasing function of the model intensity x . The function $f(x)$ is derived from the model (M) and observation (O) data in a historical period with available observations, called the *calibration period*. It is then applied to the model output (S) for a future period, referred to as the *scenario period*, to obtain the bias-corrected data (T) in the scenario period. We use the variable couple (X, Y) to generalize results valid for any of the random variable couples (M, O) or (S, T) . The transfer function can be written in the form $f(x) = x + g(x)$, where $g(x)$ is the bias correction for intensity x . Hereafter, we refer to $g(x)$ as the *correction function*. Obviously, $-g(x)$ is the additive model bias at intensity x . This twofold interpretation of the correction function entails that the distortion of the CCS by intensity-dependent model biases and the modification of the CCS by bias correction are two facets of the same problem. As we will see, if the model intensity spectra for the two climatological periods are different, then intensity-dependent model biases do not cancel. For any climate statistic, the CCS modification is equal to the change of the model bias (of the same statistic) between the two climatological periods.

In this work, for simplicity, we follow a linearized approach, which assumes a linear correction function $g(x) = ax + b$, where $a > -1$ and b are real constants. Thus, the model bias has an intensity-dependent ($-ax$) and a uniform component ($-b$). The transfer function is also linear:

$$Y = f(X) = (a + 1)X + b. \quad (1)$$

The first term on the right-hand side describes the removal of intensity-dependent model biases. Taking the variance of Eq. (1), we get that $-a$ is the relative model bias of the standard deviation. Thus, the removal of intensity-dependent biases adjusts Y 's variance through scaling X by $a + 1$. Therefore, the first term on the right-hand side will be referred to as the *scaled* term of Eq. (1); the second term adjusts the level of Y by subtracting the uniform bias component $-b$ and will hence be referred to as the *level* term of Eq. (1).

QM defines the transfer function as the quantile–quantile (QQ) curve (Ivanov and Kotlarski 2017). To test whether linearity is plausible, we consider QQ plots of the model and observed data at the different stations. For one of these plots, shown in Fig. 1a, linearity appears to be a good approximation. The high values of the coefficient of determination shown in Fig. 1b support this conclusion for all stations.

In the following, the mean value, cumulative distribution function (CDF), and quantile of order α of a random variable X are denoted by $\bar{X}$, F_X , and x_α , respectively. The conditional distribution of X given a positive event is modeled by the random variable $X_{>0}$, which is the restriction of X to the event $\{X > 0\}$. To avoid ambiguity, X will alternatively be referred to as the “unconditional X ” and $X_{>0}$ as the “conditional X .” For the distribution mean, we define the model bias in the calibration period (B):

$$B = \begin{cases} \bar{M} - \bar{O}, & \text{additive case} \\ \frac{\bar{M}}{\bar{O}} - 1, & \text{multiplicative case} \end{cases} \quad (2)$$

and the raw model CCS (R):

$$R = \begin{cases} \bar{S} - \bar{M}, & \text{additive case} \\ \frac{\bar{S}}{\bar{M}} - 1, & \text{multiplicative case.} \end{cases} \quad (3)$$

The CCS obtained from bias-corrected data will be referred to as bias-free CCS (C):

$$C = \begin{cases} \bar{T} - \bar{O}, & \text{additive case} \\ \frac{\bar{T}}{\bar{O}} - 1, & \text{multiplicative case.} \end{cases} \quad (4)$$

For the quantile of order α , the calibration period model bias (B_α) is analogously defined by means of m_α and o_α , the raw model CCS (R_α) by s_α and m_α , and the bias-free CCS (C_α) by t_α and o_α . The definitions are analogous also for the conditional mean and quantiles, only the respective symbols carry the additional subscript “>0”.

1) INTERVAL VARIABLES

The bias-free CCS estimates obtained according to the theory in this subsection will be referred to as *interval estimates*.

(i) CCS of the distribution mean

Expressing b from the averaged calibration period version of Eq. (1) and substituting it in the averaged scenario period version, we obtain

$$\bar{T} = (a + 1)(\bar{S} - \bar{M}) + \bar{O}. \quad (5)$$

The first term on the right-hand side is the difference between the variance-adjusted scenario and calibration period means and can be interpreted as the variance-adjusted CCS. It stems from the scaled term of Eq. (1), describes the removal of intensity-dependent model biases, and will hereafter be referred to as the scaled term of Eq. (5). Thus, the bias-corrected scenario mean is obtained by adding the variance-adjusted CCS to the calibration period observed mean. The latter stems from the level term of Eq. (1) and will hereafter be referred to as the level term of Eq. (5).

Substituting Eqs. (2), (3), and (5) in definition (4), for the bias-free CCS we obtain

$$C = \begin{cases} (a + 1)R, & \text{additive case} \\ (a + 1)R(B + 1), & \text{multiplicative case.} \end{cases} \quad (6)$$

As the observed mean cancels, C only has a scaled component that is derived from the scaled term of Eq. (5) and describes the removal of intensity-dependent model biases. In the additive case, the variance-adjusted calibration period mean $(a + 1)\bar{M}$ cancels, which eliminates the dependence of C on the bias of the mean B ; C is obtained by variance-adjusting R through scaling by $a + 1$ (Hagemann et al. 2011; Boberg and Christensen 2012; Gobiet et al. 2015). In the multiplicative case, the intensity-dependent bias correction includes a correction for the bias of the mean $(a + 1)RB$ in addition to the variance adjustment $(a + 1)R$. The value of R controls how the correction for the bias of the mean affects the CCS. If $R = 0$, this correction has no effect; if $R < 0$, the effect is opposite to B , and vice versa. From Eq. (6) it follows that C can substantially deviate from R , especially for large values of B in the multiplicative case.

(ii) CCS of the quantile of order α

As the transfer function is increasing, x_α is transformed exactly to y_α ; that is,

$$y_\alpha = f(x_\alpha). \quad (7)$$

This allows us to write Eq. (1) and hence also Eq. (5) for the quantile of order α . Thus, the expressions for C_α can be obtained from Eq. (6) by replacing C , R , and B by C_α , R_α , and B_α , respectively. The interpretation of the respective terms and the implications for the CCS are as for the distribution mean.

2) RATIO VARIABLES

Ratio variables have a natural zero limit. Typical meteorological ratio variables are daily precipitation and

FIG. 1. Testing the linearity assumption. (a) QQ plot of the 99 observed vs model percentiles together with the minimum and maximum values (blue circles) for the OTL-C station-model chain combination and the 1980–2009 period in (left) winter and (right) summer. The solid dark blue and the black lines represent the linear fit and the identity line, respectively; the coefficient of determination (R^2) is indicated. (b) Average coefficient of determination of the linear fits to the QQ plots over the 15 ENSEMBLES model chains for each of the 27 considered stations of the Swiss National Basic Climatological Network (NBCN) in (left) winter and (right) summer. Red curves mark country boundaries. The elevation isolines of 500, 1500, and 3000 m above mean sea level are displayed by gray contours with growing thickness. See also Fig. B1 and Tables B1 and B2 in appendix B.

wind speed. They are zero on dry/calm days and positive otherwise. Denote the positive-event probability of the random variable X by η_X . For daily precipitation, the observed positive-event probability η_O can often be below 0.5. Biases of ratio variable statistics are due to a misrepresentation not only of intensities but also of the positive-event probability. The transfer function [Eq. (1)] is only defined for positive values, so it cannot correct η_M and η_S . The idea is, if $\eta_M > \eta_O$, to set the number of positive events that are in excess to zero, and to artificially convert the necessary proportion of the zeroes to positive values in the opposite case ($\eta_M < \eta_O$). In this work, the first situation will be referred to as the “threshold case” and the second as the “frequency adaptation” (FA) case (Themeßl et al. 2012; Ivanov and Kotlarski 2017). For daily precipitation and state-of-the-art RCMs, the threshold case is more common due to the “drizzling effect” (e.g., Frei et al. 2006). The FA case seldom occurs and is usually related to the “summer drying” of state-of-the-art RCMs in some parts of southeast Europe (Themeßl et al. 2012). The process of adjusting the positive-event probability is referred to as the “drizzle correction” because, as we will show later, it manipulates the lower tail of the intensity distribution. Thus, the bias correction of a ratio variable is generally a two-step procedure, involving 1) drizzle correction and 2) intensity-dependent bias correction with the transfer function. If the latter is expressed by Eq. (1), $-a$ is the relative model bias of the standard deviation *on positive events*.

In the threshold case, all model values smaller than the threshold value of $\mu = F_M^{-1}[F_O(0)]$ are set to zero and the rest of them are transformed by Eq. (1). Hence, the adjusted positive-event probability in the scenario period is

$$\eta_T = 1 - F_S(\mu). \quad (8)$$

We provide the theory for the threshold case in this subsection and for the FA case in appendix A. The bias-free CCS estimates obtained according to the ratio theory will be referred to as *ratio estimates*.

(i) *CCS of the distribution mean*

The basic equation for the bias-corrected unconditional mean is

$$\bar{Y} = (a + 1)(\bar{X} - \bar{W}_X) + b\eta_Y, \quad (9)$$

where $\bar{W}_X = \int_{\{X < \mu\}} X d\mathbb{P}$ is the average drizzle correction. Expressing b from the calibration period version of Eq. (9) and substituting it in the scenario period version of the same equation, we obtain

$$\bar{T} = (a + 1)[\bar{S} - (\xi + 1)\bar{M}] + (\xi + 1)\bar{O} + (a + 1)\varepsilon, \quad (10)$$

where

$$\xi = \frac{\eta_T}{\eta_O} - 1 \quad (11)$$

is the bias-free CCS of the positive-event probability and

$$\varepsilon = (\xi + 1)\bar{W}_M - \bar{W}_S \quad (12)$$

quantifies the effect of drizzle correction by the weighted difference between the average drizzle correction terms for the calibration and scenario periods. The definition and interpretation of the scaled and level terms of Eq. (10) are analogous to those for the respective terms of Eq. (5), only that they additionally depend on the CCS of the positive-event probability ξ . The term proportional to ε describes the process of drizzle correction and will be referred to as the ε term of Eq. (10). Substituting Eqs. (2), (3), and (10) in definition (4), for the bias-free CCS we obtain

$$C = \begin{cases} (a + 1)(R - \xi B) - a\xi\bar{O} + (a + 1)\varepsilon, & \text{additive case} \\ (a + 1)[R + (R - \xi)B] - a\xi + (a + 1)\frac{\varepsilon}{\bar{O}}, & \text{multiplicative case.} \end{cases} \quad (13)$$

The terms proportional to R and/or B stem from the scaled term of Eq. (10) and constitute the CCS scaled component. The latter describes the correction for intensity-dependent model biases by the sum of the simulated CCS R and the correction for the bias of the distribution mean B , which is scaled by $-\xi$ in the additive and $R - \xi$ in the multiplicative case; by analogy to R in the interval theory [Eq. (6)], the scaling coefficient

controls how the correction for the bias of the mean affects the CCS. Note that the effect of this correction on the multiplicative CCS depends on the balance between the simulated CCS R and the simulated CCS of the positive-event probability ξ . The term proportional to ε stems from the ε term of Eq. (10) and is referred to as the ε component of C . The remaining term originates from the level term Eq. (10) and constitutes the level

component of C : it is an adjustment dependent on the bias-free CCS of the positive-event probability ξ (and on the observed mean $\overline{O}$ in the additive case). The corrections described by the three CCS components are affected by the variance adjustment through the factor a . A comparison to Eq. (6) reveals that the absolute CCS modification $|C - R|$ for a ratio variable can

be substantially larger than for an interval variable with the same a and R ; this is due to the additional dependence on B and ξ in the additive and on ξ in the multiplicative case.

The bias-free CCS of the conditional mean is derived analogously, using Eq. (10) and relations between statistics of the unconditional and conditional distribution:

$$C_{>0} = \begin{cases} (a+1)[(\psi_s + 1)R_{>0} + (\psi_s - \psi_c)B_{>0}] + (a+1)(\psi_s - \psi_c)\overline{O}_{>0} + (a+1)\varepsilon_{>0}, & \text{additive case} \\ (a+1)\{(\psi_s + 1)R_{>0} + [(\psi_s + 1)R_{>0} + \psi_s - \psi_c]B_{>0}\} + (a+1)(\psi_s - \psi_c) + (a+1)\frac{\varepsilon_{>0}}{\overline{O}_{>0}}, & \text{multiplicative case.} \end{cases} \quad (14)$$

Here, the effect of the drizzle correction is quantified by

$$\varepsilon_{>0} = (\psi_c + 1)\overline{W}_{M_{>0}} - (\psi_s + 1)\overline{W}_{S_{>0}}, \quad (15)$$

where $\overline{W}_{X_{>0}} = \overline{W}_X \eta_X^{-1}$, and

$$\psi_c = \frac{\eta_M}{\eta_O} - 1 \quad \text{and} \quad \psi_s = \frac{\eta_S}{\eta_T} - 1 \quad (16)$$

are the multiplicative biases of the positive-event probability in the calibration and scenario periods, respectively. The definitions and interpretations of the scaled, level, and ε components of $C_{>0}$ as well as the potential for CCS modification are analogous to those for the CCS of the unconditional mean [Eq. (13)]. The effect of the raw model CCS R is weighted by the bias of the positive-event probability in the scenario period $\psi_s + 1$. The role of $-\xi$ as a quantifier of the effect of the positive-event probability change is overtaken by the CCS of the positive-event probability bias $\psi_s - \psi_c$. More specifically, the correction for the bias of the conditional mean is proportional to $\psi_s - \psi_c$ in the additive and $(\psi_s + 1)R_{>0} + \psi_s - \psi_c$ in the multiplicative case, with the corresponding implications for the dependence of $C_{>0}$ on $B_{>0}$.

(ii) CCS of the quantile of order α

As the transfer function is increasing and only applied to positive values, Eq. (7) is valid, but only if both x_α and y_α are positive, that is $\alpha \geq 1 - \eta_Y$. Quantiles of smaller orders are zero in at least one of the variables X and Y .

Their bias-free CCS is trivial, from climatological perspective uninteresting, and, hence, not considered. We need the analog of Eq. (10) for a positive quantile of order α . However, there is no certainty that the unconditional quantile of order α is positive in all variables involved. Fortunately, by construction the quantiles of order α for the conditional distributions $F_{Y_{>0}}(y)$ and $F_{X_{\geq \mu}}(x)$ are image and preimage in transformation (1):

$$y_{>0,\alpha} = f(x_{\geq \mu,\alpha}) = (a+1)x_{\geq \mu,\alpha} + b. \quad (17)$$

This allows us to work with α in the real $(0, 1)$ interval. The definition and interpretation of the scaled and level terms of Eq. (17) are analogous to those for the respective terms of Eq. (1). We express b from the calibration period version of Eq. (17) and substitute it in the scenario period version of the same equation. Then, using links between quantile orders in the different distributions, we express the quantile of order α :

$$t_\alpha = (a+1)(s_\alpha - m_\beta) + o_\beta, \quad (18)$$

where

$$\beta = 1 - (1 - \alpha)(\xi + 1)^{-1}. \quad (19)$$

As the condition $\beta > \alpha$ is equivalent to $\xi > 0$, β describes the effect of changing positive-event probability.

Substituting Eqs. (2), (3), and (18) in definition (4), for the bias-free CCS we finally obtain

$$C_\alpha = \begin{cases} (a+1)R_\alpha + o_{\beta\alpha} - (a+1)m_{\beta\alpha}, & \text{additive case} \\ (a+1)[R_\alpha + (R_\alpha - m_{\beta\alpha})B_\alpha] + o_{\beta\alpha} - (a+1)m_{\beta\alpha}, & \text{multiplicative case,} \end{cases} \quad (20)$$

where

$$x_{\beta\alpha} = \begin{cases} x_\beta - x_\alpha, & \text{additive case} \\ \frac{x_\beta}{x_\alpha} - 1, & \text{multiplicative case.} \end{cases} \quad (21)$$

Equation (20) shows that the bias-free CCS C_α is a sum of a scaled and a level component. The scaled component stems from the scaled term of Eq. (17) and contains the terms proportional to R_α and/or B_α ; the level component consists of the remaining terms. In the additive case, the variance-adjusted quantile $(a+1)m_\alpha$ is canceled, which eliminates the dependence on B_α ; so the scaled

component only describes the variance adjustment of the simulated CCS R_α . The interpretations of the two components, the implications for the dependence of C_α on B_α , and the potential for CCS modification are analogous to those for Eq. (13); note only that the role of ξ is overtaken by $m_{\beta\alpha}$ and the level component is the weighted difference $o_{\beta\alpha} - (a+1)m_{\beta\alpha}$. Equation (20) is valid for a quantile order α , such that all other quantile orders in the equation correspond to positive quantiles.

The bias-free CCS of the conditional quantile of order α is analogically derived based on Eq. (17) and links quantile orders in the different distributions and definitions (2), (3), and (4):

$$C_{>0,\alpha} = \begin{cases} (a+1)R_{>0,\alpha} + (a+1)(s_{>0,\beta_s\alpha} - m_{>0,\beta_c\alpha}), & \text{additive case} \\ (a+1)\{(s_{>0,\beta_s\alpha} + 1)R_{>0,\alpha} + [(s_{>0,\beta_s\alpha} + 1)R_{>0,\alpha} + s_{>0,\beta_s\alpha} - m_{>0,\beta_c\alpha}]B_{>0,\alpha}\} & \text{multiplicative case,} \\ + (a+1)(s_{>0,\beta_s\alpha} - m_{>0,\beta_c\alpha}), & \end{cases} \quad (22)$$

where

$$\begin{aligned} \beta_c &= 1 - (1 - \alpha)(\psi_c + 1)^{-1}, \\ \beta_s &= 1 - (1 - \alpha)(\psi_s + 1)^{-1}. \end{aligned} \quad (23)$$

As $\beta_c > \alpha$ is equivalent to $\psi_c > 0$, β_c describes the effect of the positive-event probability bias in the calibration period. Analogously, β_s describes the effect of the positive-event probability bias in the scenario period. The definition and interpretation of the scaled and level components of Eq. (22), the implications for the dependence of $C_{>0,\alpha}$ on $B_{>0,\alpha}$, and the potential for CCS modification are similar to those for Eq. (14); note only that the roles of ψ_s and ψ_c in Eq. (14) are overtaken by $s_{>0,\beta_s\alpha}$ and $m_{>0,\beta_c\alpha}$, respectively. As with Eq. (20), Eq. (22) is valid for a quantile order α , such that all other quantile orders in the equation correspond to positive quantiles.

The theory developed in this section is a tool for quantitative analysis of the CCS modification due to bias correction. It allows end users to recognize model biases with high potential to distort the simulated CCS. It provides a new opportunity to efficiently estimate the bias-free CCS without performing the bias correction, provided that the bias correction assumptions hold and the model biases and raw model CCS of certain climate statistics are known. As we demonstrate in the next section, the CCS estimates are adequate despite the seemingly restrictive theoretical assumptions.

3. Results and discussion

Here, we analyze the ability of the linearized analytical theory to predict the CCS and its modification for the bias correction and downscaling example described in section 2a. The CCS is for the scenario period 2070–99 relative to the reference period 1980–2009. The analysis focuses on the multiplicative CCS as it is more commonly used for ratio variables. Section 2 in the online supplemental material includes results for the additive CCS. We only test the threshold case as η_M always exceeds η_O . Discrepancies between the QM bias-corrected and the ratio CCS are attributable to the nonlinearity, seasonal variation, and constant extrapolation of the QM transfer function. In the following, the station-model chain combinations will be referred to as “cases”; unless otherwise stated, the qualifier “large” will be with respect to the sample of the cases.

a. Performance of the theory with respect to the CCS

The distributions of the raw, QM bias-corrected, ratio, and interval estimates of the multiplicative CCS are displayed in Figs. 2 and 3 for the distribution mean and quantile of order 0.9, respectively. The box-and-whisker plots summarize the 405 values for the 27 stations and 15 model chains. Figures S1 and S2 in the supplemental material display analogous results for the additive CCS. Compared to the distribution of the interval CCS estimates, the distribution of the ratio estimates more

FIG. 2. Multiplicative CCS (dimensionless) between the 2070–99 and 1980–2009 climatological periods for the 27×15 combinations of precipitation stations and ENSEMBLES model chains in (left) winter and (right) summer. (a) CCS estimates of the unconditional mean, based on raw model output (R), quantile-mapped data (C^{QM}), the interval theory (C^{interval}), and the ratio theory (C^{ratio}) as well as the scaled (sc), level (lc), and ε (ec) components of C^{ratio} [Eq. (13)]. Red, green, and blue circles show the three station–model chain combinations with the largest $|C^{\text{QM}} - R|$ in decreasing order. Each box is defined by the lower and upper quartiles, the intermediate line segment is the median, and the whiskers extend to the data extremes. A dotted line marks the zero level. See also Tables B1 and B2 in appendix B. (b) As in (a), but for the conditional mean; the components of $C^{\text{ratio}}_{>0}$ [Eq. (14)] in summer are normalized by a factor of 5 and displayed in gray.

closely follows that of the QM bias-corrected estimates. In summer, the ratio theory tends to slightly overestimate the QM CCS, whereas the interval theory strongly underestimates the QM CCS of conditional statistics (right panels of Figs. 2, 3, S1, and S2).

The theoretical versus the QM bias-corrected multiplicative CCS estimates for the distribution mean and

the quantile of order 0.9 are displayed in Figs. 4 and 5, respectively. Analogous results for the additive CCS are shown in Figs. S3 and S4 in the supplemental material. The ratio and interval estimates for the station–model chain combinations are displayed as red and blue circles, respectively, together with the corresponding simple linear regression fits in a darker hue.

FIG. 3. As in Fig. 2, but for the quantile of order $\alpha = 0.9$. The scaled (sc) and level (lc) components of $C^{\text{ratio}}_{0.9}$ and $C^{\text{ratio}}_{>0.9}$ are according to Eqs. (20) and (22), respectively. In the left panel of (a), the green and blue circles overlap in lc. In the right panel of (a), the red and blue circles overlap in sc.

Ideally, the points should lie along the identity line. The coefficient of determination (R^2) and root-mean-square error (RMSE) are provided as goodness-of-fit statistics for both the ratio and interval theories. Visual inspection suggests that the ratio CCS estimate has a strong (close to identity) linear relation to the QM CCS. The high values of R^2 (67%–90%) and the small RMSE (0.05–0.10 for the multiplicative CCS and 0.2–2.57 mm day $^{-1}$ for the additive CCS) support this observation. The ratio theory estimates have higher R^2 and lower RMSE relative to the interval estimates. Again, the advantage of the ratio theory is most obvious in summer and for conditional statistics;

this is attributable to the higher proportion of zero events (dry days) in summer that the interval theory does not take into account. Visual inspection as well as the diagnostic statistics suggest that both theories perform better in winter. In summer, both theories tend to overestimate the negative QM CCS, which can be generalized as a tendency to dampen the CCS (right panels of Figs. 4, 5, S3, and S4).

We conclude that the linearized ratio theory generally provides a good and substantially improved description of the QM bias-corrected CCS compared to the interval theory.

FIG. 4. Test of the analytical theory for the multiplicative CCS (dimensionless) of daily precipitation between the 2070–99 and 1980–2009 climatological periods in (left) winter and (right) summer. (a) Unconditional mean: theoretical CCS estimate [Theoretical C , Eq. (13)] vs the CCS estimate from QM bias-corrected data (C^{QM}). The red and the blue circles indicate the ratio and interval CCS estimates, respectively; each circle marks one of the 27×15 combinations of precipitation stations and ENSEMBLES model chains. The corresponding simple linear fits (dark red and dark blue) as well as the identity line (black) are represented by straight lines. The goodness-of-fit statistics R^2 and RMSE indicate the coefficient of determination and the root mean-square error, respectively. (b) As in (a), but for the theory of the conditional mean [Eq. (14)].

b. Performance of the theory with respect to the CCS modification

1) GENERAL ANALYSIS

For both the ratio and interval theories, columns 3–7 of Table 1 provide the correlations of the theoretical CCS modifications with the modification by the QM method as well as the average absolute theoretical and QM CCS modifications; absolute values prevent mutual

cancellation of opposite errors. The average absolute raw model CCS shown in the last column puts the CCS modification values into context. As can be seen, the QM CCS modification is generally smaller in absolute value than the simulated CCS, so there is no certainty that the theory will be as adequate for the CCS modification as it is for the CCS. Generally, the CCS modification by the ratio theory highly correlates with the QM CCS modification (0.79–0.92); relatively low correlations are observed only for the multiplicative CCS of the

FIG. 5. As in Fig. 4, but for the quantile of order $\alpha = 0.9$. Theoretical $C_{0.9}$ and Theoretical $C_{>0.9}$ are according to Eqs. (20) and (22), respectively.

unconditional mean (0.49) and unconditional quantile of order 0.9 (0.56) in summer. The interval theory estimates have substantially lower correlations (0.11–0.85) with the QM estimates; for the multiplicative CCS of the unconditional mean in summer the correlation is even negative (−0.22). The average absolute modification of the multiplicative CCS is generally small (0.07–0.1), slightly larger (0.12–0.13) for the conditional mean and the quantile of order 0.9 in summer. For the additive

CCS, the modification is generally below or close to 1 mm day^{-1} but approaches 3 mm day^{-1} for the conditional quantile of order 0.9 in summer. The ratio estimates are close to the QM estimates and closer than the interval ones, except for the multiplicative CCS of the unconditional mean in summer. In the latter case, the ratio estimate (0.02) is substantially smaller than the QM estimate (0.08), whereas the interval estimate is precise. A glance at Fig. 4a reveals that the ratio theory

TABLE 1. Statistics of the CCS modification between the 2070–99 and 1980–2009 climatological periods over the sample of 27 stations of the Swiss National Basic Climatological Network (NBCN) and 15 ENSEMBLES model chains for winter (DJF) and summer (JJA). The estimates refer to the multiplicative CCS of the unconditional (UMm) and conditional (CMm) mean, the unconditional (UQm) and conditional (CQm) quantile of order 0.9, the additive CCS of the unconditional (UMa) and conditional (CMa) mean, and the unconditional (UQa) and conditional (CQa) quantile of order 0.9. The term r^{ratio} is the correlation coefficient between the CCS modification $C^{\text{ratio}} - R$ predicted by the ratio theory and the actual CCS modification $C^{\text{QM}} - R$ by the empirical QM method; r^{interval} is the respective correlation coefficient for the interval theory. The next three columns are the mean absolute CCS modifications predicted the ratio ($|C^{\text{ratio}} - R|$) and interval ($|C^{\text{interval}} - R|$) theories and for the QM method ($|C^{\text{QM}} - R|$). The term $|R|$ is the mean absolute simulated CCS. In the additive cases, the values of the CCS modifications and the simulated CCS are in mm day^{-1} ; all other values are dimensionless.

Statistic	Season	r^{ratio}	r^{interval}	$ C^{\text{ratio}} - R $	$ C^{\text{interval}} - R $	$ C^{\text{QM}} - R $	$ R $
UMm	DJF	0.79	0.58	0.07	0.13	0.10	0.12
	JJA	0.49	−0.22	0.02	0.08	0.08	0.18
CMm	DJF	0.88	0.44	0.08	0.04	0.07	0.11
	JJA	0.79	0.11	0.16	0.07	0.12	0.14
UQm	DJF	0.81	0.76	0.08	0.09	0.09	0.11
	JJA	0.56	0.28	0.05	0.05	0.09	0.18
CQm	DJF	0.86	0.54	0.08	0.03	0.08	0.11
	JJA	0.81	0.24	0.16	0.06	0.13	0.15
UMa	DJF	0.93	0.73	0.35	0.25	0.36	0.70
	JJA	0.92	0.64	0.35	0.30	0.52	0.85
CMa	DJF	0.86	0.27	0.66	0.26	0.60	0.72
	JJA	0.82	0.47	1.01	0.25	1.21	0.72
UQa	DJF	0.88	0.85	0.72	0.62	0.88	1.77
	JJA	0.85	0.71	1.01	0.85	1.73	2.26
CQa	DJF	0.81	0.37	1.63	0.63	1.70	1.79
	JJA	0.81	0.50	2.71	0.71	2.96	1.88

systematically overestimates the negative CCS values, which leads to an underestimation of the absolute CCS modification. The modifications predicted by the interval theory often have a different sign from the QM modifications; this inflates the average absolute CCS modification estimated by the interval theory and artificially makes it closer to the QM estimate. In contrast to the multiplicative CCS, for the additive CCS the absolute modification has a clear seasonality, being larger in summer than in winter. This can be explained by the larger precipitation amounts in summer, which also entail larger biases.

To facilitate the analysis of the CCS modification based on the ratio theory, Figs. 2 and 3 as well as Figs. S1 and S2 present additional box-and-whisker plots for the CCS components. QM most substantially and systematically affects the CCS of conditional statistics in summer. The simulated CCS and hence also the scaled component are strongly negative [Eqs. (14) and (22)]. However, the increased tendency of the model to overpredict positive events (wet days) in the future, reflected by $\psi_s - \psi_c > 0$ for the distribution mean and $s_{>0,\beta_s,\alpha} - m_{>0,\beta_s,\alpha} > 0$ for the quantile of order 0.9, inflates the level component; this largely compensates the scaled component and dampens the raw model CCS. Note that the ε component for the distribution mean tends to be negligible compared to the other CCS components.

Thus, the linearized ratio theory well captures the average CCS modification by QM even for the statistics

and seasons with the largest modifications. This indicates that the theoretical assumptions are not too restrictive. Again, the ratio theory is superior to the interval theory and performs better in winter.

2) ANALYSIS OF INDIVIDUAL CASES WITH LARGE CCS MODIFICATIONS

As shown in Table 1, the CCS modification by QM tends to be smaller than the simulated CCS. Therefore, it is instructive to study the mechanisms by which QM modifies the CCS as it does for the few cases with large modifications. We focus on the three station–model chain combinations with the largest absolute modifications of the QM CCS. They are visualized in Figs. 2 and 3 as well as Figs. S1 and S2 with red, green, and blue circles in decreasing order. (The station abbreviations and the short references for the model chains are listed in Tables B1 and B2 in appendix B, respectively.) In the light of the ratio theory, we interpret the CCS components that are large in absolute value with respect to the rest of the sample and/or to the other CCS components; if not all of the large CCS components have the same sign, only those that have the same sign as the CCS modification are considered. To enhance the physical understanding, we further analyze each of these components in more detail.

As seen in the left panel of Fig. 2a, the three largest absolute modifications of the multiplicative CCS of the

unconditional mean in winter are amplifications of the simulated positive CCS. They occur for SAM-A (0.85), SAM-O (0.79), and SIA-A (0.61) and are accurately predicted by Eq. (13). The values of the ε component [Eq. (12)] for the first two cases rank as the first and second largest and even exceed the other CCS components in absolute value. This is due to 1) the large increase of the positive-event probability in the future ($\xi > 0$), 2) the substantial decrease of the average drizzle correction in the scenario relative to the calibration period ($\overline{W_S} < \overline{W_M}$) and 3) the normalization by the observed mean precipitation value of 0.88 mm day^{-1} ; SAM is the only station with a mean observed precipitation below 1 mm day^{-1} in winter. In the third case, the simulated positive CCS is larger than the future increase of the positive-event probability ($R - \xi > 0$); therefore, the correction for the positive bias of the mean ($B > 0$) is positive and amplifies the scaled component; the ε component is large due to points 1 and 2 above. For the three summer cases, OTL-C (−0.37), LUG-C (−0.37), and OTL-E (−0.26), QM amplifies the simulated negative CCS (right panel of Fig. 2a). The theory captures the sign of the CCS modification but largely underestimates its magnitude. The first two cases have large negative scaled components due to the large positive $R - \xi$ values and large negative B values; thus, the CCS modification is due to removal of intensity-dependent model biases. The third case is not interpretable due to poor performance of the theory.

As seen in the left panel of Fig. 2b, the three largest absolute modifications of the multiplicative CCS of the conditional mean in winter are amplifications of the simulated positive CCS. They occur for SAM-A (0.43), SBE-J (0.35), and SAM-J (0.33) and are accurately predicted by Eq. (14). For SAM-A, the scaled component is large positive because the large positive-event probability bias in the scenario period ($\psi_s > 0$) amplifies the effect of the large positive $R_{>0}$; thus, the effect of the simulated CCS by far exceeds the negative correction for the bias of the mean ($B_{>0} > 0$). The lower future drizzle correction ($\overline{W_{S>0}} < \overline{W_{M>0}}$) and positive-event probability bias ($\psi_s < \psi_c$) make the ε component [Eq. (15)] of SAM-A the largest in the sample. Contrary to the majority of the cases in winter (left panel of Fig. 2b), for SBE-J the positive-event probability bias increases in the future ($\psi_s > \psi_c$) and the variability is underestimated ($a > 0$); this inflates the level component and through it also the CCS. For SAM-J, the culprit for the large amplification of the CCS is the ε component that is large due to reasons similar to SAM-A. For each of the summer cases, SIO-L (0.49), BAS-L (0.43), and BER-L (0.42) (right panel of Fig. 2b), the large increase of the positive-event probability bias in the future ($\psi_s > \psi_c$)

inflates the level component; the resulting positive CCS modification is larger in absolute value than the effect of the negative simulated CCS and reverses it.

As seen in the left panel of Fig. 3a, the three largest absolute modifications of the multiplicative CCS of the unconditional quantile of order 0.9 in winter are amplifications of the simulated positive CCS. They occur for SAM-A (0.81), SAM-O (0.74), and OTL-J (0.72) and are accurately predicted by Eq. (20). For each of them, the large positive simulated CCS $R_{0.9}$ overwhelms the effect of higher future positive-event probability ($m_{\beta_{0.9}} > 0$), so that the correction for the positive bias of the quantile of order 0.9 ($B_{0.9} > 0$) is also positive and amplifies the scaled component. The higher future positive-event probability is described by the positive $o_{\beta_{0.9}} - m_{\beta_{0.9}}$ difference, which, together with the negative a , leads to a large positive level component. For each of the summer cases: SIO-J (−0.36), GVE-B (−0.33), and SIO-N (−0.32) (right panel of Fig. 3a), the lower future positive-event probability is reflected by the large negative $o_{\beta_{0.9}} - m_{\beta_{0.9}}$ difference, which makes the level component large negative. The effect is an amplification of the simulated negative CCS, albeit a little underestimated by the theory. For GVE-B, the scaled component is also large and negative due to the simulated large negative CCS that almost balances the effect of lower future positive-event probability ($R_{0.9} - m_{\beta_{0.9}} \approx 0$).

As seen in the left panel of Fig. 3b, the three largest absolute modifications of the multiplicative CCS of the conditional quantile of order 0.9 in winter occur for OTL-K (−0.58), SAM-A (0.55), and OTL-A (−0.54) and are accurately predicted by Eq. (22). For OTL-K and OTL-A, the lower future positive-event probability bias results in the first and second largest negative $s_{>0,\beta_s} - m_{>0,\beta_c}$ differences. This makes level component large negative and strongly dampens the positive simulated CCS $R_{>0,0.9}$. For SAM-A, the lower future positive-event probability bias is reflected by the positive $s_{>0,\beta_s} - m_{>0,\beta_c}$ difference. The latter results in a large positive level component and makes the positive bias of the quantile of order 0.9 ($B_{>0,0.9} > 0$) amplify the positive simulated CCS, thus strongly inflating the scaled component. In each of the summer cases, BAS-L (0.53), CHM-L (0.52), and NEU-L (0.50) (right panel of Fig. 3b), the higher future positive-event probability is reflected by the large positive $s_{>0,\beta_s} - m_{>0,\beta_c}$ difference, which inflates the level component; the resulting positive CCS modification is larger in absolute value than the effect of the negative simulated CCS and reverses it.

The theory successfully predicts the largest absolute modifications of the additive CCS as well (see Figs. S1 and S2). Although the cases are different from those for

the multiplicative CCS, the analysis of the CCS modification is analogous. In contrast to the multiplicative CCS (left panels of Fig. 2), the ε component has no significant role for the cases with the largest absolute modifications of the CCS of the distribution mean in winter (left panels of Fig. S1).

Although the theory generally performs well, for some cases with substantial CCS modifications it is not that successful. The majority of these cases occur in summer, which is consistent with the observation that the theory performs better in winter (Figs. 2–5; see also Figs. S1–S4). The reason is that in summer a larger proportion of precipitation stems from small-scale convective processes, the parameterization of which in RCMs is highly unlikely to result in linear biases. This is illustrated in the right column of Fig. 1a, which presents the transfer function for the OTL-C case. OTL-C systematically shows some of the highest discrepancies between the theoretical and QM CCS estimates. Crucial for the representation of the precipitation distribution are the deviations from the linear fit in the lower part of the distribution, which describes the majority of precipitation events. The linear fit leads to a substantial overestimation at this range, which results in a general overestimation of precipitation statistics. The constant extrapolation for new extremes, used in the QM method, but not accounted for by the theory, also contributes to the overestimation (right column of Fig. 1a). The transfer function of the QM method varies slowly with the day-of-year that the 91-day moving window is centered on, so the net effect on the CCS should be negligible.

In summary, we showed that the linearized ratio theory successfully predicts the QM bias-corrected CCS and the corresponding modification of the simulated CCS on average and for the individual cases with the largest absolute modifications. The analysis also demonstrated how the theory can be used to reveal and quantify the important mechanisms that modify the CCS. These mechanisms involve bias removal and are not an artifact of the bias correction method. The systematic evaluation in Ivanov and Kotlarski (2017), including pseudoreality experiments to test the stationarity assumption, indicated that the application of bias correction is scientifically appropriate. This strongly suggests that the CCS modification for this particular bias-correction example is a desirable effect.

4. Conclusions and outlook

We develop a linearized analytical description of the mechanisms by which stationary model biases affect the climate change signal (CCS). We show that the same

mechanisms are responsible for the modification of the CCS by intensity-dependent bias correction methods. The issue has so far been solved for interval variables (such as temperature) and the additive CCS of the distribution mean (Hagemann et al. 2011; Gobiet et al. 2015). Our theory is applicable not only to the distribution mean of interval variables but also to ratio variables (such as daily precipitation and wind speed) and distribution quantiles. It also considers multiplicative CCS and statistics of the positive (conditional) distribution. For ratio variables, the model bias of the positive-event probability plays a crucial role as its sign defines two different treatments of the zeroes. Formally, the theory provides simple linear equations that predict the bias-free CCS based on known model biases and raw model CCS of certain climate statistics. The bias-free CCS has a scaled component that describes the removal of intensity-dependent biases and a level component that adjusts the CCS level in accordance with the future change of the positive-event probability or its bias. Adjusting the positive-event probability affects the CCS of the distribution mean and is quantified by an additional ε component. The theoretical approach can be extended in a straightforward manner for known non-stationary/nonlinear biases (e.g., linear with a break point and polynomial) and other climate statistics (variance, temporal autocorrelation coefficients, impact indices, etc.).

In an illustrative application, we test the theory for an empirical quantile mapping (QM) method. QM is employed to bias-correct and downscale 15 ENSEMBLES model chains of 25-km resolution to 27 precipitation stations in the topographically structured terrain of Switzerland. We investigate the effect of bias correction on the CCS between the 2070–99 and 1980–2009 climatological periods and its modification. Taking the values of zero into account, the linearized ratio theory generally provides a good and substantially improved description of the QM bias-corrected CCS compared to the interval theory. The ratio theory well captures the CCS modification by QM even for the station–model chain combinations with the largest modifications and quantifies the underlying mechanisms. Therefore, it can be used to analyze the results of actual bias correction in a linear approximation. In the particular application, we show that the severe amplification of the multiplicative CCS of the distribution mean for few cases in winter is due to an increase of the positive-event probability coupled with a decrease of the drizzle correction in the future. The strong dampening of the negative CCS of the conditional mean in summer is due to the correction for the increased tendency of the model to overpredict positive events (wet days) in the

future. The few occurrences for which the theory performs less than optimally can be explained by the nonlinearity of the QM transfer function and the extrapolation for extremes that did not occur in the calibration period (“new” extremes). This is a challenging test because of the complex alpine topography and the presence of an additional model bias component induced by the scale difference (~ 25 km for the model data versus local scale for the weather stations), which is highly unlikely to be linear with intensity. Therefore, the theoretical description is expected to perform even better for other geographical regions and/or when no implicit downscaling step is involved.

Our theory and results demonstrate that intensity-dependent bias correction modifies the CCS by removing model biases rather than due to mathematical artifacts. This strongly suggests that if scientifically appropriate, this type of bias correction can be trusted not only for correcting climate statistics but also for modifying their CCS. As discussed in the introduction, there is a growing body of evidence in support of the assumptions of stationarity (Vrac et al. 2007; Maraun 2012; Rty et al. 2014; Ivanov and Kotlarski 2017) and intensity dependence (Christensen et al. 2008; Boberg and Christensen 2012; Gobiet et al. 2015) of the model bias, which must hold for bias correction to be reasonable. Further, the climate model must adequately capture changes of relevant physical/chemical/biological processes and large-scale atmospheric circulation patterns for bias correction to be legitimate (Ehret et al. 2012; Maraun et al. 2017).

For end users (impact modelers, planners, engineers, policy makers), the analytical theory is a tool 1) to detect model biases with high potential to distort the simulated CCS and 2) to analyze model output in case bias correction severely changes the raw model CCS. Moreover, this tool provides a new opportunity to efficiently and effectively debias CCS estimates: efficiently, because the estimates can be obtained without actually performing the bias correction, and effectively, because the approximation is adequate. The novel, improved CCS datasets will support adaptation, mitigation, and resilience policies for stakeholders, policy makers, and various sectors, including agriculture, water management, tourism, energy, transport, disaster risk reduction, and infrastructure.

Future work will include testing the theory for model simulations that underestimate the probability of positive events (frequency adaptation case), for variables, other than temperature and precipitation, and for other geographical regions. The theory will be extended to account for the effect of the constant extrapolation used by empirical methods, for nonlinear transfer functions (e.g., Piani et al. 2010) that allow analytical treatment,

for statistics other than mean and quantiles, and for statistics of multimodel ensembles (Gobiet et al. 2015). Future research needs to focus on the development of process-based corrections (e.g., Bellprat et al. 2013; Gmez-Navarro et al. 2018) rather than preserving the raw model CCS.

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APPENDIX A

Linearized Theory for the FA Case

If the model generates less positive events than observed, the simulated zeroes are replaced by random sampling from the empirical distribution of the observed data smaller than the value of $\theta = F_O^{-1}[F_M(0)]$. The process is modeled by the random variable Φ , the CDF of which is equal to the conditional distribution of O given $O < \theta$. Lemma (2.3) in Berkes and Philipp (1979) guarantees the existence of Φ under very general conditions. Thus, the adjusted positive-event probability in the scenario period is given by

$$\eta_T = \eta_\Phi (1 - \eta_S) + \eta_S, \quad (\text{A1})$$

where $\eta_\Phi = (\eta_O - \eta_M)(1 - \eta_M)^{-1}$.

a. CCS of the distribution mean

The basic equation for the bias-corrected unconditional mean is

$$\bar{Y} = (a + 1)\bar{X} + b\eta_X + (1 - \eta_X)\bar{\Phi}, \quad (\text{A2})$$

where the term proportional to $\bar{\Phi}$ describes the process of drizzle correction. Expressing b from the calibration

period version of Eq. (A2) and substituting it in the scenario period version of the same equation, we obtain

$$\bar{T} = (a+1)[\bar{S} - (\xi+1)\bar{M}] + (\xi+1)\bar{O} + \varepsilon, \quad (\text{A3})$$

where

$$\xi = \frac{\eta_S}{\eta_M} - 1 \quad (\text{A4})$$

is the raw model CCS of the positive-event probability [cf. the threshold case, Eq. (11), where ξ is the bias-free CCS] and

$$\varepsilon = -\xi\bar{\Phi}. \quad (\text{A5})$$

Substituting Eqs. (2), (3), and (A3) in definition (4), for the bias-free CCS we finally obtain

$$C = \begin{cases} (a+1)(R - \xi B) - a\xi\bar{O} + \varepsilon, & \text{additive case} \\ (a+1)[R + (R - \xi)B] - a\xi + \frac{\varepsilon}{\bar{O}}, & \text{multiplicative case.} \end{cases} \quad (\text{A6})$$

The definitions and interpretations of the CCS components, the implications for the dependence of C on B , and the potential for CCS modification are analogous to those for the threshold case [Eq. (13)]. Note only that the ε component does not depend on the variability bias a .

The bias-free CCS of the conditional mean is derived analogously, using Eqs. (A3) and (A4) and relations between statistics of the unconditional and conditional distribution:

$$C_{>0} = \begin{cases} (a+1)(\psi_s + 1)R_{>0} + \frac{\psi_s - \psi_c}{\psi_c + 1}\bar{O}_{>0} + \varepsilon_{>0}, & \text{additive case} \\ (a+1)(\psi_s + 1)R_{>0}(B_{>0} + 1) + \frac{\psi_s - \psi_c}{\psi_c + 1} + \frac{\varepsilon_{>0}}{\bar{O}_{>0}}, & \text{multiplicative case,} \end{cases} \quad (\text{A7})$$

where

$$\varepsilon_{>0} = -\xi \frac{\eta_\Phi}{\eta_T} \bar{\Phi}_{>0}. \quad (\text{A8})$$

In the additive case, the scaled term of the uniform bias is canceled, and hence $C_{>0}$ is independent of $B_{>0}$. In the multiplicative case, the correction for the bias of the conditional mean $B_{>0}$ is proportional to $R_{>0}$, with the respective implications for the dependence of the bias-free CCS $C_{>0}$ on $B_{>0}$. The definitions and interpretations of the CCS components and the potential for CCS modification are analogous to those for the threshold case [Eq. (14)]; note only that both the level and ε components do not depend on the variability bias a .

b. CCS of the quantile of order α

Equation (7) is valid if both x_α and y_α are positive; that is, $\alpha \geq 1 - \eta_X$. The quantiles of order α for the conditional distributions $F_{Y \approx_\theta}(y)$ and $F_{X>0}(x)$ are image and preimage in transformation (1):

$$y_{\approx_\theta, \alpha} = f(x_{>0, \alpha}) = (a+1)x_{>0, \alpha} + b. \quad (\text{A9})$$

The quantile of order α is given by Eq. (18) and the bias-free CCS by Eq. (20). Note that ξ in the expression for β [Eq. (19)] is defined by Eq. (A4).

The bias-free CCS of the conditional quantile of order α is analogically derived based on Eq. (17), links between quantile orders in the different distributions, and definitions (2), (3), (4), and (23):

$$C_{>0, \alpha} = \begin{cases} (a+1)R_{>0, \alpha} + (a+1)(s_{>0, \beta_s, \alpha} - m_{>0, \beta_s, \alpha}) + o_{>0, \beta_{sc}, \alpha}, & \text{additive case} \\ (a+1)\{(s_{>0, \beta_s, \alpha} + 1)R_{>0, \alpha} + [(s_{>0, \beta_s, \alpha} + 1)R_{>0, \alpha} + s_{>0, \beta_s, \alpha} - m_{>0, \beta_s, \alpha}]B_{>0, \alpha}\} \\ + (a+1)(s_{>0, \beta_s, \alpha} - m_{>0, \beta_s, \alpha}) + o_{>0, \beta_{sc}, \alpha} & \text{multiplicative case,} \end{cases} \quad (\text{A10})$$

FIG. B1. Map of the locations of the daily precipitation observation stations (black dots) within the Swiss National Basic Climatological Network (NBCN). Red curves mark country boundaries. The elevation isolines of 500, 1500, and 3000 m above mean sea level are displayed by gray contours with growing thickness. A complete list of the station names and coordinates is given in Table B1 in appendix B.

where

$$\beta_{sc} = 1 - (1 - \alpha) \left(\frac{\psi_s + 1}{\psi_c + 1} \right)^{-1} \quad (\text{A11})$$

As $\beta_{sc} > \alpha$ if and only if $\psi_s > \psi_c$, β_{sc} describes the effect of changing positive effect probability bias in the future. The definitions and interpretations of the CCS components and the implications for the dependence of

TABLE B1. List of the 27 stations of Swiss National Basic Climatological Network (NBCN) considered in the present work.

Station	Abbreviation	Longitude	Latitude	Elevation (m)
Altdorf	ALT	8°37'	46°53'	438
Andermatt	ANT	8°35'	46°38'	1438
Basel / Binningen	BAS	7°35'	47°32'	316
Bern / Zollikofen	BER	7°28'	46°59'	552
Chaumont	CHM	6°59'	47°03'	1136
Château-d'Oex	CHD	7°08'	46°29'	1029
Col du Grand St-Bernard	GSB	7°10'	45°52'	2472
Davos	DAV	9°51'	46°49'	1594
Elm	ELM	9°11'	46°55'	958
Engelberg	ENG	8°25'	46°49'	1035
Geneva-Cointrin	GVE	6°08'	46°15'	420
Grimsel Hospiz	GRH	8°20'	46°34'	1980
Grächen	GRC	7°50'	46°12'	1605
La Chaux-de-Fonds	CDF	6°48'	47°05'	1018
Locarno / Monti	OTL	8°47'	46°10'	366
Lugano	LUG	8°58'	46°00'	273
Lucerne	LUZ	8°18'	47°02'	454
Meiringen	MER	8°10'	46°44'	588
Neuchâtel	NEU	6°57'	47°00'	485
S. Bernardino	SBE	9°11'	46°28'	1638
Samedan	SAM	9°53'	46°32'	1708
Sargans	SAR	9°26'	47°03'	487
Segl-Maria	SIA	9°46'	46°26'	1804
Sion	SIO	7°20'	46°13'	482
St. Gallen	STG	9°24'	47°26'	775
Säntis	SAE	9°21'	47°15'	2502
Zurich / Fluntern	SMA	8°34'	47°23'	555

TABLE B2. List of the 15 employed GCM–RCM model chains from the ENSEMBLES project. For reasons of brevity, in this work model chains are referred to as shown in the column “Short reference.” Expansions of acronyms are available online at <https://www.ametsoc.org/PubsAcronymList>. C4I is the Community Climate Change Consortium for Ireland; DMI is the Danish Meteorological Institute; ICTP is the International Centre for Theoretical Physics; METO-HC is the Met Office Hadley Centre; SMHI is the Swedish Meteorological and Hydrological Institute.

Institution	GCM	RCM	Short reference
C4I	HadCM3Q16	RCA3	A
CNRM	ARPEGE	RM5.1	B
DMI	ARPEGE	HIRHAM5	C
DMI	BCM	HIRHAM5	D
DMI	ECHAM5	HIRHAM5	E
ETHZ	HadCM3Q0	CLM	F
ICTP	ECHAM5	REGCM3	G
KNMI	ECHAM5	RACMO2	H
METO-HC	HadCM3Q0	HadRM3Q0	I
METO-HC	HadCM3Q16	HadRM3Q16	J
METO-HC	HadCM3Q3	HadRM3Q3	K
MPI-M	ECHAM5	REMO	L
SMHI	BCM	RCA	M
SMHI	ECHAM5	RCA	N
SMHI	HadCM3Q3	RCA	O

$C_{>0,\alpha}$ on $B_{>0,\alpha}$ are analogous to those for the threshold case [Eq. (22)]; note only that the role of $m_{>0,\beta,\alpha}$ is overtaken by $m_{>0,\beta,\alpha}$. The additional dependence on $O_{>0,\beta,\alpha}$ further enhances the potential for CCS modification.

APPENDIX B

Data Details

The 27 Swiss National Basic Climatological Network (NBCN) observation stations considered for this study are mapped in Fig. B1 and listed in Table B1. The 15 employed GCM–RCM model chains from the ENSEMBLES project are listed in Table B2.

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