Fundamental Behavior of ENSO Phase Locking

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ABSTRACT

El Niño–Southern Oscillation (ENSO) events tend to peak at the end of the calendar year, a phenomenon called ENSO phase locking. This phase locking is a fundamental ENSO property that is determined by its basic dynamics. The conceptual ENSO recharge oscillator (RO) model is adopted to examine the ENSO phase-locking behavior in terms of its peak time, strength of phase locking, and asymmetry between El Niño and La Niña events. The RO model reproduces the main phase-locking characteristics found in observations, and the results show that the phase locking of ENSO is mainly dominated by the seasonal modulation of ENSO growth/decay rate. In addition, the linear/nonlinear mechanism of ENSO phase preference/phase locking is investigated using RO model. The difference between the nonlinear phase-locking mechanism and linear phase-preference mechanism is largely smoothed out in the presence of noise forcing. Further, the impact on ENSO phase locking from annual cycle modulation of the growth/decay rate, stochastic forcing, nonlinearity, and linear frequency are examined in the RO model. The preferred month of ENSO peak time depends critically on the phase and strength of the seasonal modulation of the ENSO growth/decay rate. Furthermore, the strength of phase locking is mainly controlled by the linear growth/decay rate, the amplitude of seasonal modulation of growth/decay rate, the amplitude of noise, the SST-dependent factor of multiplicative noise, and the linear frequency. The asymmetry of the sharpness of ENSO phase locking is induced by the asymmetric effect of state-dependent noise forcing in El Niño and La Niña events.

1. Introduction

El Niño–Southern Oscillation (ENSO) is the dominant climate mode of variability in the tropical Pacific on interannual time scales, having a significant impact on global weather and climate. The onset of ENSO events generally occurs during boreal spring and summer, while most events peak in boreal winter, with the amplitudes decaying in the next spring. Although several negative feedbacks could explain how the coupled ocean–atmosphere system oscillates, for example, via a recharge–discharge oscillator (Jin 1996, 1997a,b) or delayed oscillators (Suarez and Schopf 1988), the underlying physical mechanisms for phase locking of ENSO peak to the boreal winter season still remain elusive. Two possible mechanisms have been proposed to explain the phase locking of ENSO variance: (i) the frequency locking of ENSO to periodic forcing due to nonlinear interaction of the seasonal forcing and the inherent ENSO cycle (Jin et al. 1994; Tziperman et al. 1994) and (ii) the seasonal modulation of ENSO’s coupled instabilities (Philander et al. 1984; Hirst 1986). The purpose of this study is to examine the nature of ENSO phase locking and controlling factors.

Some studies attribute the ENSO phase locking to a competition between the ENSO linear frequency and the tendency to phase lock to a preferred season due to the nonlinear interaction between the annual cycle and ENSO cycle (Tziperman et al. 1994; Jin et al. 1996; Neelin et al. 2000). The linear frequency of ENSO is nonlinearily adjusted so that ENSO’s phase can match the preferred season for each event (Neelin et al. 2000). An alternate hypothesis is that the annual cycle in the climate background can modulate ENSO and to cause El Niño/La Niña events to mature preferentially in the boreal winter. Jin et al. (1996) showed that the annual cycle in the cold tongue alone could give rise to ENSO’s phase locking in the winter. Using a linear Zebiak–Cane (ZC) model (Cane and Zebiak 1985; Zebiak and Cane 1987), Thompson and Battisti (2000) showed that the growth/decay of singular vectors provides a strong tendency to peak in the boreal winter. They suggested that the annual cycle of the ocean in the basic state is
sufficient to generate the ENSO phase locking without considering either nonlinearity or an annual cycle of the stochastic noise. Simple model studies also suggest that ENSO events tend to peak after the maximum growth/decay rate (An and Jin 2011; Stein et al. 2010), and the seasonal modulation of the growth/decay rate is responsible for the phase synchronization of ENSO to the annual cycle (Stein et al. 2014). Several factors could induce a strong seasonal variation of coupled instabilities, which is responsible for the evolution of ENSO, among them the strong annual cycle at the equator, intertropical convergence zone (ITCZ) movement (Philander 1983), sea surface temperature (SST) gradient, thermocline depth, and strength of upwelling (Battisti 1988). Together these factors make the instability strongest during the summer to fall (Philander 1983; Tziperman et al. 1997; Li 1997). Using the ZC model, Tziperman et al. (1997) showed that El Niño’s phase locking is mainly controlled by the basic-state wind divergence that is associated with the seasonal movement of ITCZ. An and Wang (2001) further showed that the seasonal variation of the mean thermocline depth is essential for simulating realistic ENSO phase locking in the ZC model, especially for La Niña events.

The above studies demonstrated that the annual cycle in the cold tongue could determine the ENSO phase locking in the winter; however, other processes associated with seasonal variation in the warm pool might also be critical. Wang et al. (1999) found that the Philippine anticyclone that develops in later fall to winter during El Niño developing phase provides the winter easterly winds anomaly onto the equator to cause ENSO to peak at the wintertime. An and Wang (2001) also indicated that the annual variation of wind anomalies in the western Pacific is important to ENSO phase locking. Vecchi and Harrison (2006) suggested the role of seasonal migration of warm pool in shifting the equatorial winds response to control ENSO peak time in the winter. The southward movement of zonal winds can contribute to the rapid termination of El Niño events by weakening the equatorial westerlies with upwelling oceanic Kelvin waves (Harrison and Vecchi 1999; Lengaigne et al. 2006; Ohba and Ueda 2009) and by changing the discharge process of heat content (McGregor et al. 2012, 2013). The southward shift of zonal winds and in the western Pacific Philippine anticyclone occurs because of the interaction between the ENSO cycle and the western Pacific annual cycle, called “combination mode” by Stuecker et al. (2013).

It is becoming clear that the seasonal modulation of the ENSO growth/decay rate is fundamental for ENSO phase locking (An and Jin 2011; Stein et al. 2014; Li 1997), though whether the cold tongue or warm pool more decisively provide the associated seasonal modulation of ENSO’s growth/decay rate remains elusive. In this paper, we will utilize the simple recharge oscillator (RO) paradigm to revisit the questions about what are the key factors that control the mechanism of phase locking of ENSO, in terms of its peak time, strength of phase locking, and asymmetry of phase locking between El Niño and La Niña events. Our results show that the simple RO model can reproduce the main phase-locking characteristics found in observations, and the phase locking is mainly controlled by the seasonal modulation of the ENSO growth/decay rate. Moreover, it is suggested that the peak time histogram measure of ENSO phase locking, a common measure for capturing the ENSO phase-locking feature, can be used to characterize the phase-locking behavior better when compared with seasonal variance measure.

The paper is arranged as follows: section 2 describes the observation dataset and RO model settings. Section 3 discusses the linear/nonlinear mechanisms of ENSO phase preference/phase locking in the RO model. Section 4 elucidates the overall features of ENSO phase locking in observations and simulations. In section 5, the parameters associated with growth/decay rate, stochastic noise, nonlinearity, and ENSO linear frequency are examined to investigate the sensitivity of ENSO phase locking in terms of peak time and strength. Section 6 gives a summary and some discussion.

2. Data and method

a. The parametric recharge oscillator model

The conceptual model we consider here is a simple recharge oscillator model (Jin 1997a,b; Burgers et al. 2005) with cubic nonlinearity, quadratic nonlinearity, and additive and multiplicative noise. The recharge oscillator captures the dynamic relationship between the western equatorial Pacific thermocline anomalies ($h_w$) and eastern equatorial Pacific sea surface temperature anomalies ($T$) by the following form:

$$\frac{dT}{dt} = RT + \omega_0 h_w + \alpha \xi [1 + BH(T)T] - cT^3 + bT^2. \quad (1)$$

$$\frac{dh_w}{dt} = -r h_w - \omega_0 T. \quad (2)$$

$$\frac{d\xi}{dt} = -m \xi + w(t), \quad \text{and}$$

$$R = R_0 - R_a \sin(\omega_a t - \varphi). \quad (4)$$

In this RO framework, $R$ and $r$ are the growth/decay rate and $\omega_0$ is the ENSO linear frequency. These three
parameters are associated with the Bjerknes–Wyrtki–Jin (BWJ) index that determines the linear growth/decay rate \((R_0 - r)/2\) (Jin et al. 2006) and frequency of ENSO \(\sqrt{\omega_0^2 - (R + r)^2/4}\) (Lu et al. 2018). The annual cycle modulations \((R_a)\) are included in the growth/decay rate with the frequency of \(\omega_a\) and phase of \(\phi\) reflecting the influence of the annual cycle modulation in a basic state on SST growth/decay. We also consider state-dependent stochastic forcing as in Jin et al. (2007). \(\sigma\) is the noise amplitude, \(w(t)\) is white noise term and \(\xi\) is normalized Gaussianly distributed red noise with a decay time scale of \(1/m\). This stochastic forcing term \(\sigma \xi [1 + BH(T)T]\) represents the effect of fast atmospheric wind anomalies on the eastern equatorial SST anomalies (Jin et al. 2007; Levine and Jin 2010; Levine et al. 2016). We use the SST-dependent factor of multiplicative noise (parameter \(B\)) with a Heaviside function \(H(T)\), which is defined as being zero when \(T < 0\) and one when \(T \geq 0\), to represent the state-dependent noise forcing due to the asymmetry of interannual modulation of westerly wind bursts (WWB) or Madden–Julian oscillation (MJO) via SST anomalies. When \(B = 0\), the system is forced only by additive noise, and there is no asymmetry from noise term. Invoking threshold dependence in these symmetry-breaking processes only modifies the near-linear dependence.

The quadratic \((b)\) and cubic \((c)\) nonlinear terms roughly represent nonlinear dynamic heating (NDH) from both deterministic nonlinear advection and upwelling, and from the upscale effect of ENSO modulation of tropical instability waves (TIWs) (An 2008). In this RO model framework, we summarize all the processes that generate the ENSO asymmetry into two main symmetric-breaking processes, the symmetric-breaking nonlinearity in either NDH or ENSO state that depends on deterministic coupled feedbacks. Without loss of generality, these two symmetric-breaking processes are related to two parameters \(b\) and \(B\), respectively.

To illustrate the fundamental behavior of ENSO dynamics of the RO model, we consider an idealized case. Here, we set \(R_0 - r = -(1/23)\) month\(^{-1}\), \(R_a = (1/6.75)\) month\(^{-1}\), \(r = (1/11.5)\) month\(^{-1}\), \(\omega_0 = (2\pi/43)\) month\(^{-1}\), \(\sigma = (1/9)\) month\(^{-1}\), \(m = (1/1.5)\) month\(^{-1}\), \(c = (1/62.3)\) month\(^{-1}\) C\(^{-2}\), \(b = (1/56.5)\) month\(^{-1}\) C\(^{-1}\), \(\omega_a = (2\pi/12)\) month\(^{-1}\), and \(B = 0.9\). The choice of parameters is determined from the BWJ index in observations (Jin et al. 2006; S. Zhao and F.-F. Jin 2019, unpublished manuscript). All the simulated results are from the last 20,000 years of the 21,000-yr model run with 1-day time step. Monthly and pentad (5 day) averages are used to analyze in this study. The El Niño (La Niña) events in the model are defined as occurring when the simulated temperature is larger than one standard deviation (smaller than one negative standard deviation).

b. Observations

The observed SST dataset used in this study is the monthly Hadley Centre Global Sea Ice and Sea Surface Temperature (HadISST; Rayner et al. 2003) dataset from 1870 to 2018. Multiple centered 30-yr base periods are used to define the “anomaly” as suggested by Climate Prediction Center (Lindsey 2013). The El Niño (La Niña) events are defined as occurring when the 3-month running averaged SST anomalies (SSTA) in the Niño-3.4 region (5°N–5°S, 120°–170°W) is larger than 0.5°C (smaller than −0.5°C). In this study, only moderate and strong El Niño and La Niña events with 3-month mean Niño-3.4 SST anomalies at or above 1.0°C are discussed. Interpolated pentad observed temperature is also calculated in this study to overcome the lack of ENSO sampling and compare with the model.

3. Linear/nonlinear mechanism of ENSO phase preference/phase locking

As previously mentioned, there two possible mechanisms can lead to the preferred boreal winter occurrence of ENSO peak. One is a linear mechanism, the seasonal modulation of ENSO’s coupled instabilities. Another one is a nonlinear mechanism, the frequency locking of ENSO due to nonlinear interaction of the seasonal cycle and the inherent ENSO cycle. In this section, we will examine these two mechanisms using the RO model.

For the linear scenario of RO model, nonlinear terms in Eqs. (1) and (2) are set to zero \((c = 0, b = 0)\). An approximate analytical solution was obtained for the unforced case, neutrally stable regime of the RO model, where \(\sigma = 0\), \(R_0 = 0\), and \(r = 0\). For a small value of \(R_a\), and using the perturbation method, the solution is (An and Jin 2011)

\[ T = C \left\{ \sin(\omega_0 t + \phi_0) - \frac{R_a}{\omega_a^2 - 4\omega_0^2} \left[ \omega_0 \sin(\omega_a t) \cos(\omega_a t + \phi_a) - \frac{\omega_a^2 - 2\omega_0^2}{\omega_a} \cos(\omega_a t) \sin(\omega_0 t + \phi_0) \right] \right\} , \]
where \( C \) and \( \phi_0 \) are constants dependent on the initial conditions. Setting \( R_a = (1/6) \text{ month}^{-1}, \phi_0 = (2\pi/48) \text{ month}^{-1} \). Numerical integration of the linear RO model is performed and compared with the analytical solution in Figs. 1a and 1b. Both of the integrations start with the same initial conditions. The numerical solution is matched to the analytical solution well for first a few ENSO cycles. Considerable differences between numerical and analytical solutions appear afterward owing to a high-order correction to the ENSO frequency due to the presence of the annual cycle modulation in the growth/decay rate. The effect of the annual modulation from term associated with \( R_a \) is beyond second-order solution from the perturbation method (An and Jin 2011). To demonstrate that there is a phase preference in these linear solutions, the histogram of ENSO peak time and seasonal variance are shown in Figs. 1c and 1d. For the analytical solution, the histogram is calculated by considering the phase dependence on different initial conditions (\( \phi_0 \) from 0 to \( 2\pi \)). The seasonal variance is normalized by its maximum variance.

The histograms of ENSO peak time for the linear damped regime are further examined and then compared with that of nonlinear scenario in an unstable regime (Fig. 2). For the linear scenario, stable regime, and unforced case of RO model, we set \( \sigma = 0, R_0 = -(1/10) \text{ month}^{-1}, R_a = (1/6) \text{ month}^{-1}, \phi_0 = (2\pi/48) \text{ month}^{-1}, c = 0, b = 0, \) and \( r = 0 \) in Eqs. (1) and (2). Since it is an exponential damped system, the time series is rescaled by \( e^{-\lambda t} \), where \( \lambda \) is decay constant such that rescaled ENSO evolution has nondecaying evolution. The rescaled solution is very similar to that in the linear neutral case. There is a very similar ENSO “phase preference” in the stable regime as well (Fig. 2a). The ENSO peak occurs in boreal winter with the maximum probability of ENSO peak appears in December. For the nonlinear scenario, unstable regime, and unforced case of RO model, we set \( \sigma = 0, R_0 = (1/10) \text{ month}^{-1}, R_a = (1/6) \text{ month}^{-1}, \phi_0 = (2\pi/48) \text{ month}^{-1}, c = (1/62.3) \text{ month}^{-1}, b = 0, \) and \( r = 0 \) in Eqs. (1) and (2). In this case, the ENSO period is locked to 4 years due to frequency locking, and the month of ENSO peak is perfectly locked to December (Fig. 2b). This is a typical case of so-called phase locking associated with nonlinear frequency-locking mechanism. However, the difference between the nonlinear “phase locking” mechanism and linear “phase preference” mechanism will be largely smoothed out in the presence of noise forcing. As the stochastic forcing is considered [setting \( \sigma = (1/3) \text{ month}^{-1}, B = 0 \)], the histograms show that the maximum probability of ENSO peak time appears in boreal...
winter both in the linear and nonlinear solutions in a very similar manner (Figs. 2c,d). With the stochastic forcing, phase locking and phase preference become equivalent. The term phase preference is more general. Nevertheless, we will use the commonly used term phase locking to refer to the ENSO’s preferred peak at boreal winter in this study.

4. Features of ENSO phase locking

a. Observations

The Niño-3.4 index based on Hadley SST from 1870 to 2018 is displayed in Figs. 3a and 3b for El Niño and La Niña events. The composite of ENSO events is represented by the thick black curve with standard deviation (vertical black bar) for a maximum of 4-yr duration, starting from the previous year [year(−1)] to developing year [year(0)], decaying year [year(1)], and afterward [year(2)]. For averaged El Niño and La Niña evolution (thick black curves), the events have a maximum peak in boreal winter months (November–January). The individual El Niño and La Niña events after 1950 are also shown in Figs. 3a and 3b with thin curves. Although the composite shows a peak in the boreal wintertime, some events have a peak in fall or spring months. The spread of Niño-3.4 index of El Niño events at the peak time is larger than that of La Niña events at the peak time due to the asymmetry of noise effect (Lopez et al. 2013; Lopez and Kirtman 2014; Chen et al. 2015).

A commonly used measure for the ENSO phase-locking behavior is the SSTA variance as shown in Fig. 3c. Another direct measure for capturing the ENSO peak is to examine the histogram of SSTA peak time accounting to the calendar month (Fig. 3d). These two methods both capture the basic characteristics of ENSO phase locking. The peak times of both El Niño and La Niña tend to occur toward the end of the calendar year from November to January. It is worth noting that, in the histogram, the maximum probability of ENSO peak time appears in December for El Niño but in November.
for La Niña (Fig. 3d); however, both the peak month of positive and negative variances occurs preferentially in December (Fig. 3c). Moreover, the spread of peak month is narrower for La Niña than El Niño as shown in the histogram. The asymmetry in the phase locking between El Niño and La Niña and further comparison between histogram and variance measures will be discussed in the later subsection using observations and RO model.

b. Parametric recharge oscillator model

In this subsection, the behavior of ENSO phase locking in an idealized case of parametric recharge oscillator model is examined and compared with observations. Similar to Fig. 3, the monthly averaged simulated temperatures for El Niño and La Niña events are displayed in Figs. 4a and 4b. For averaged El Niño and La Niña evolution, the curves have a maximum in boreal winter months (November–January), which is consistent with observations. The same as in observations, the spread of Niño-3.4 index of El Niño events at the peak time is larger than that of La Niña events at the peak time. The seasonal variation and histogram of peak time are illustrated in Figs. 4c and 4d. The histogram of ENSO peak time shows that the ENSO peak occurs in each month of the calendar year, with the maximum probability of the ENSO peak appearing in boreal winter (November). The maximum growth/decay rate (dashed curve in Fig. 4c) leads ENSO peak about 2–3 months. When the symmetry-breaking processes are considered in the model, the shape of peak time histogram of La Niña is sharper, indicating the strength of phase locking of La Niña (blue bars in Fig. 4d) is larger than that of El Niño (red bars in Fig. 4d). From the analysis above, the simple RO model framework reproduces
the main characteristics of ENSO phase locking found in observations.

c. The histogram of ENSO peak time and seasonal variation of ENSO variance

Although the histogram of ENSO peak time and seasonal variance of ENSO are two commonly used measures for the ENSO phase-locking behavior, the asymmetry of ENSO phase locking captured by histogram is missed in the variance measure. This is evident in the pentad-averaged histogram of ENSO peak time and seasonal variation of ENSO for observations and an idealized RO model simulation shown in Fig. 5. The asymmetry of ENSO peak time between El Niño and La Niña events is evident in the histograms of ENSO peak time both in observations and simulations (Figs. 5c and 5d). Moreover, the spread of peak time is narrower for La Niña than El Niño in the histogram of ENSO peak time, indicating that the strength of ENSO phase locking is also asymmetric, while the shapes of normalized seasonal variance are similar in El Niño and La Niña. Hence, we suggest that the histogram of ENSO peak time is a more informational method to characterize the ENSO phase locking in terms of its peak time, strength, and asymmetry.

5. Parameter dependence of phase locking

To investigate the sensitivity of ENSO phase locking in terms of peak time and strength, the parameters associated with growth/decay rate, noise, nonlinearity, and linear frequency are examined in this section. The pentad-averaged temperature is used in this section to evaluate the features of phase locking accurately, and the histogram of ENSO peak time is smoothed by a
1-month running average. As the schematic diagram of the histogram shown in Fig. 6a, the ENSO peak time is estimated from the maximum peak of the histogram within a 5-month time window centered about the peak. The strength of phase locking is quantified by the sharpness of the histogram of ENSO peak time, defined as the range of the 90% quantile of the peak time distribution centered on its maxima. Pentad-averaged seasonal variance of ENSO in the (c) observations and (d) idealized model case. The seasonal variance is computed separately for El Niño (red bars) and La Niña (blue bars) years and is defined from June to May around peak time and normalized by its maximum variance.

a. Modulation of growth/decay rate

The modulation of growth/decay rate is controlled by three factors: the mean of growth/decay rate ($R_0$ and $r$), the amplitude of annual cycle modulations ($R_a$), and the phase of annual cycle ($\phi$). Figure 7 shows the month of ENSO peak and the sharpness of ENSO phase-locking dependence on the phase of the annual cycle $\phi$, linear growth/decay rate ($R_0 - r)/2$, and amplitude of the annual cycle $R_a$. The main factor of growth/decay rate that determines the ENSO peak time is the phase of the
annual cycle \( \phi \). The maximum growth/decay rate leads the ENSO peak by about 2.5 months, regardless of the phase \( \phi \) (Fig. 7a). Another parameter that can influence the ENSO peak time is the linear growth/decay rate \( (R_0 - r)/2 \) (Fig. 7b). In this experiment, we vary the \( R_0 \) to obtain the linear growth/decay rate, and the \( R_a \) is not zero with the maximum on September–October. When the linear growth/decay rate is close to zero (neutral regime), the maximum probability of ENSO peak time appears in December, while the ENSO peak shifts to November if the system is dissipated or unstable. The maximum probability of ENSO peak time does not depend on the strength of the annual cycle in growth/decay rate \( (R_a) \) if the strength of the annual cycle is large enough (larger than 0.1 month\(^{-1}\)) as shown in Fig. 7c. When the strength of the annual cycle is small, there is no favorable season for ENSO events as seen by the increased standard deviation. It is worth noting that the standard deviation of ENSO peak time is slightly larger for El Niño due to the stronger noise effect on El Niño events.

The strength of phase locking is mainly controlled by the linear growth/decay rate \( (R_0 - r)/2 \) and amplitude of the annual cycle \( R_a \). The phase of the annual cycle \( \phi \) cannot change the strength of phase locking (Fig. 7d), because the sharpness of phase locking remains about 6.2 months for El Niño and 3.5 months for La Niña. The symmetry-breaking processes of ENSO (parameters \( b \) and \( B \)) can generate asymmetry in the sharpness of ENSO phase locking, where La Niña tends to be more sharply phase-locked than El Niño. The sharpness of ENSO phase-locking dependence on linear growth/decay rate (Fig. 7e) shows that the increase of the strength of ENSO phase locking and asymmetry of sharpness of phase locking between El Niño and La Niña are a consequence of the increased instability. Moreover, the spread of ENSO peak and sharpness decrease as the instability increase. When the amplitude of the annual cycle modulation \( R_a \) increases, the strength of ENSO phase locking increases as well with the ENSO SSTA histogram changing from a wide toward a sharp distribution (Fig. 7f).

The parameters \( R \) in the simple RO model [Eq. (1)] include the six main contributing positive and negative feedbacks that determine the growth/decay of eastern Pacific SSTA (Jin et al. 2006, 2020). The above results indicate that the growth/decay of ENSO not only can control the preferred month of ENSO peak via the phase of seasonal modulation but also can determine the strength of phase locking via the linear growth/decay rate and the amplitude of seasonal modulation.

### b. Stochastic forcing of ENSO

In the previous studies, the ENSO peaks are locked to the same seasons either with or without stochastic forcing (Jin et al. 1996; Blanke et al. 1997; Neelin et al. 2000). Although these results indicate that the growth/decay of ENSO not only can control the preferred month of ENSO peak via the phase of seasonal modulation but also can determine the strength of phase locking via the linear growth/decay rate and the amplitude of seasonal modulation.
both the deterministic and stochastic symmetry-breaking processes can generate ENSO asymmetry, to examine the independent effect of symmetry-breaking processes on phase-locking asymmetry, we set the value of the quadratic nonlinearity $b$ as zero in Eq. (1) when we test the parameter $B$. Both the amplitude of noise $\sigma$ (Fig. 8a) and SST-dependent factor $B$ (Fig. 8b) have little influence on the ENSO’s preferential month when the annual modulation of growth/decay rate is considered, which is consistent with previous studies (Jin et al. 1996; Blanke et al. 1997; Neelin et al. 2000).

In contrast, the sharpness of phase locking is highly dependent on the amplitude $\sigma$ and asymmetry of noise $B$. When the amplitude of stochastic forcing increases (Fig. 8c), for both El Niño and La Niña, the strength of phase locking decreases and the ENSO SSTA histogram changes from a sharp toward a wide distribution. Because the stochastic forcing is a normalized Gaussian distributed red noise and the peak of noise can occur in any month, this leads to a broader distribution of the histogram of SSTA peak time. The effect of noise on ENSO phase locking is also indicated by Neelin et al. (2000), the spread of ENSO peak month is broadened in their hybrid couple model when noise is added. The difference of the sharpness between El Niño and La Niña increases as the noise is stronger due to the asymmetry from the multiplicative noise term. In Fig. 8d, the increased asymmetry of stochastic forcing (parameter $B$) can lead to weaker El Niño phase locking and stronger La Niña phase locking. When the parameter $B$ increases, the increased multiplicative noise will weaken the El Niño phase locking with a broader distribution of ENSO SSTA histogram. Although the multiplicative noise would not directly affect La Niña events due to the Heaviside function $H(T)$, the increased instability associated with higher SST-dependent factor $B$ will strengthen the La Niña phase locking. For the above reasons, the asymmetry of the sharpness between El Niño and La Niña significantly increases as the SST-dependent factor $B$ is larger.

c. Nonlinearity

The irregularity of ENSO may be attributed to nonlinear dynamics (Tziperman et al. 1994; Jin et al. 1994; Chang et al. 1994). The nonlinear processes not only can act to control the growth/decay rate of the system

![Graphs showing the month of ENSO peak and the sharpness of phase-locking dependence on growth/decay rate, linear growth/decay rate, and amplitude of annual cycle for El Niño and La Niña events.](image-url)
effectively, but also have an effect on the asymmetry of ENSO in terms of amplitude, duration, and spatial SST patterns (An and Jin 2004). While the RO model framework is not able to address the pattern asymmetry, the other asymmetries can be addressed qualitatively with the RO model (Levine and Jin 2010). We here summarize all the nonlinear processes into cubic and quadratic nonlinearity with parameter $c$ and $b$, respectively, to investigate the behavior of phase locking associated with nonlinearity.

The cubic nonlinear damping term $c$ can significantly control the amplitude of the ENSO system, but there is only little effect on ENSO phase locking in terms of peak time and strength (Figs. 9a and 9c). The month of ENSO peak may slightly move from mid- to late December to early December, and the sharpness of phase locking may only increase less than 1.5 months as parameter $c$ increases from near 0 to 0.035 month$^{-1}$C$^{-2}$. Compared with the cubic nonlinearity term $c$, there is only weak dependence of ENSO amplitude related to the quadratic nonlinearity term $b$; however, quadratic nonlinearity represents the primary source for ENSO symmetric braking, because it tends to favor ENSO warm and cold phase differentially and thus to generate ENSO asymmetry. Here, the SST-dependent factor of multiplicative noise term ($B$) is set as zero in Eq. (1) to examine the independent effect of quadratic nonlinearity term on phase-locking asymmetry. The ENSO peak time is locked to December, and it just moves between early and late December as parameter $b$ is changed (Fig. 9b). Different from SST-dependent factor of noise $B$, another necessary symmetric braking process, quadratic nonlinearity, tends to have a weak impact on the strength of ENSO phase locking (Fig. 9d).

d. ENSO linear frequency and parametric resonance

According to the BWJ index, the frequency of ENSO modes is captured by the Wyrtki frequency index $\sqrt{\omega_0^2 - [(R + r)^2/4]}$ formulated in the RO framework.

![Fig. 8](image.png)

*Fig. 8. As in Fig. 7, but for the dependence on the (a),(c) amplitude of noise and (b),(d) SST-dependent factor.*
We here vary the critical parameter that controls the frequency of RO ENSO to examine how the linear frequency \( (\nu_0) \) may affect the ENSO’s phase locking. Before discussing the ENSO phase-locking dependence on linear frequency, the ENSO amplitude’s dependency on the linear period \( (2\pi/\nu_0) \) is shown in Fig. 10. The magnitude of ENSO is increased as the linear period is longer, but there is a sudden increase of amplitude in the about 2-yr period. This significant value is induced by parametric resonance, which is an important phenomenon of the harmonic oscillator. The parametric resonance is found everywhere in physics and has extensive and various applications. Stein et al. (2014) suggest that there is parametric resonance in the linear RO model system when the ratio of annual cycle frequency to ENSO linear frequency is 2. In our result, the parametric resonance appears in the system as the linear period is near 2 years and the parametric resonance phenomenon is more significant with a larger amplitude of annual cycle in growth/decay rate as shown in Fig. 10. Actually, the parametric resonance also occurs around the 4-yr period in the model without the stochastic forcing (not shown), but the impact of resonance on amplitude in 4-yr linear period is smaller than that in the 2-yr linear period. As the stochastic forcing is added in the system, the parametric resonance in 4-yr linear period is weathered by noise.

In addition to the impact of parametric resonance on ENSO amplitude, the ENSO phase locking in terms of peak time and strength is also influenced by the parametric resonance. The month of the ENSO peak would move to September and October when the parametric resonance occurs with 2-yr ENSO linear period as shown in Fig. 11a. For all periods larger than 2.5 years, the ENSO peak time appears in December with a slight shift. In Fig. 11b, the strength of ENSO phase locking is increased as the ENSO linear period is longer, while there is a sudden increase of phase-locking strength in

Fig. 9. As in Fig. 7, but for the dependence on the (a),(c) cubic nonlinearity and (b),(d) quadratic nonlinearity.
the around 2-yr period both in El Niño and La Niña events. It is worth noting that there is no difference of phase-locking strength between El Niño and La Niña when the ENSO period is close to 1 year, and the asymmetry of phase-locking strength significantly increases when the ENSO period is larger than 2 years.

6. Summary and discussion

In this study, the characteristic of ENSO phase locking in terms of the peak time, the strength, and the asymmetry between El Niño and La Niña events have been investigated using a simple recharge oscillator model framework. ENSO displays a close relationship with the annual cycle, where events usually start from boreal spring, grow during boreal summer and fall, then reach their peak time in boreal winter, with the intensity decaying in the following spring. The results show that the asymmetry of ENSO phase locking captured by histogram is missed in the variance measure. We suggest that the histogram of ENSO peak time is an informational method to characterize the ENSO phase-locking problem compared with variance measure. The histogram of ENSO peak time illustrates that the peaks of both El Niño and La Niña tend to occur toward the end of the calendar year from November to January, and the phase locking of La Niña is stronger than that of El Niño. The simple RO model can reproduce most features of ENSO phase locking found in observations.

The linear/nonlinear mechanism of ENSO phase preference/phase locking is examined using the simple RO model. The difference between the nonlinear phase-locking mechanism and linear phase-preference mechanism will be largely smoothed out in the presence of noise forcing. Further, we use the RO framework to discern the key factors, that is, (i) the annual cycle modulation of the growth/decay rate, (ii) the stochastic forcing, (iii) the nonlinearity, and (iv) the ENSO linear frequency. With sufficient strength of annual cycle $Ra$, the ENSO peak time is mainly controlled by the growth/decay rate particularly in terms of the phase of the annual cycle $\phi$, and the maximum growth/decay rate always leads ENSO peak about 2–3 months. The ENSO peak month does not depend on the intensity of the annual cycle $Ra$ in the growth/decay rate or the linear
growth/decay rate \( (R_0 - r)/2 \). Besides, the stochastic forcing and nonlinear process also cannot change the month of ENSO peak away from boreal wintertime.

The sharpness of ENSO phase locking, which is defined as the range of the 90% quantile of the histogram of ENSO peak time distribution centered on its maxima is used to quantify the strength of phase locking, another important characteristic of ENSO phase locking. The sharpness of ENSO phase locking depends on the strength of the seasonal cycle of the growth/decay rate \( (R_a) \) as well as on the linear growth/decay rate regime \([(R_0 - r)/2]\). When the amplitude of the annual cycle modulation increases or the system become more unstable, the strength of ENSO phase locking increases as well with the ENSO SSTA histogram changing from a wide toward a sharp distribution. However, we find there is no dependence on the sharpness of phase locking on the phase of the annual modulation of growth/decay rate \( \phi \). The strength of phase locking also highly depends on the stochastic forcing of ENSO. The amplitude of stochastic forcing \( \sigma \), providing a wider distribution of the histogram of SSTA peak time, leads to the weaker phase locking both in El Niño and La Niña. When the SST-dependent factor \( B \) increases, the raised multiplicative noise decreases the strength of El Niño phase locking with a broader distribution of El Niño phase histogram. Although the multiplicative noise would not directly affect La Niña events due to the threshold function \( H(T) \), the increased instability associated with increased noise strengthen the La Niña phase locking. The stochastic symmetry-breaking processes of ENSO can generate the asymmetry in the strength of ENSO phase locking, where La Niña tends to be more sharply phase-locked than El Niño. Otherwise, the nonlinearity (parameter \( c \) and \( b \)) only has little effect on the sharpness of phase locking.

Finally, it is found that the parametric resonance, appearing in the RO model system when the ration of annual cycle frequency \( \omega_a \) to ENSO linear frequency \( \omega_0 \) is 2, not only can impact on the ENSO amplitude but also can influence on the ENSO phase locking in terms of peak time and strength. Without the parametric resonance, the ENSO peak time does not change with ENSO linear frequency, and the strength of ENSO phase locking is increased as the ENSO linear period is longer. Around the 2-yr ENSO linear period, during which parametric resonance occurs, the month of ENSO peak would shift to September and October, and the strength of phase locking would increase suddenly both in El Niño and La Niña events. The characteristic of parametric resonance needs to be further studied.

In summary, the preferred month of ENSO peak depends critically on the phase \( \phi \) and strength of the seasonal modulation \( (R_a) \) of the ENSO growth/decay rate. Furthermore, the strength of phase locking is mainly controlled by the linear growth/decay rate \([(R_0 - r)/2]\), the amplitude of seasonal modulation of growth/decay rate \( (R_a) \), the amplitude of noise \( \sigma \), the SST-dependent factor of multiplicative noise \( B \), and the ENSO linear frequency \( \omega_0 \). The asymmetry of the strength of ENSO phase locking is induced by the state-dependent stochastic forcing.

It has been noted that the modulation of the ENSO growth/decay rate is fundamental for ENSO phase locking (Philander et al. 1984; Hirst 1986; Li 1997; An and Jin 2011; Stein et al. 2014). Stein et al. (2014) showed that the variance of ENSO is determined by the seasonal
modulation of the growth/decay rate. To delineate the relationship between ENSO seasonal variation and phase locking, we define a normalized SST anomaly $\tilde{T}$ as follows:

$$\tilde{T} = \frac{T}{\sigma_T(m)},$$

(6)

where $T$ is the original temperature time series and $\sigma_T(m)$ is the seasonal standard deviation. By definition, the variance of $\tilde{T}$ is always one. It has no obvious phase locking anymore (Figs. 12a,b), clearly supporting the notion that the phase locking of ENSO is controlled by the seasonal variance. It should be noted that in our RO model, the seasonal variance of ENSO is generated by the seasonal modulation of the growth/decay rate. However, there are other possible factors that could contribute to the seasonal variance of ENSO. For example, the annual cycle of the intensity of noise forcing may also contribute to the seasonal variance and thus the phase locking of ENSO (Lu and Liu 2019).

From previous studies, the ENSO seasonality could arise from the annual cycle of the warm pool and the cold tongue, seasonal meridional migration of the ITCZ and South Pacific convergence zone (SPCZ), the seasonal variation of ENSO-related wind anomalies in the western Pacific, the annual variation in the mean thermocline depth, seasonal change in upwelling, increased activity of WWBs during spring, and enhanced tropical instability waves in the fall. To evaluate their contribution in the RO model framework is to be further studied. Moreover, some global climate models fail to properly simulate the ENSO phase locking with weak seasonal modulation or phase locking to a wrong season (Wittenberg et al. 2006). The mechanism of ENSO’s phase locking and the ability to reproduce the ENSO phase locking in current coupled general circulation models (CGCMs) need to be further examined.

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