A Theory of the Spring Persistence Barrier on ENSO. Part II: Persistence Barriers in SST and Ocean Heat Content

YISHUAI JIN^{a,b} AND ZHENGYU LIU^c

^a *Key Laboratory of Physical Oceanography, Ministry of Education, Ocean University of China, Qingdao, China*

^b *Open Studio for Ocean-Climate-Isotope Modeling, Pilot National Laboratory for Marine Science and Technology (Qingdao), Qingdao, China*

^c *Atmospheric Science Program, Department of Geography, The Ohio State University, Columbus, Ohio*

(Manuscript received 26 October 2020, in final form 9 February 2021)

ABSTRACT: In this paper, we investigate the potential factors that control the relationship between the El Niño–Southern Oscillation (ENSO) persistence barriers (PBs) in sea surface temperature (SST) and ocean heat content (OHC) and apply them to explain observational ENSO PBs. With the addition of seasonal growth rate in SST in the neutral recharge oscillator (NRO) model, approximate analytical solutions of autocorrelation functions for SST and OHC suggest strictly that the timing of PB for OHC leads that of SST by half a year and the strengths of the two PBs are the same. The numerical solutions of the NRO model also show a similar relationship. The role of ENSO growth rate with regard to PBs in SST and OHC is then identified in the damped and unstable ENSO regime. Therefore, it is suggested that for the observational ENSO, the seasonally varying ENSO growth rate in SST controls PBs in SST and OHC simultaneously.

KEYWORDS: ENSO; Climate prediction; Seasonal forecasting; Seasonal cycle; Tropical variability

1. Introduction

A seasonal persistence barrier (PB) has long been observed to occur in the boreal spring–early summer in the variability of tropical Pacific sea surface temperature (SST) (e.g., Niño-3.4; Ren et al. 2016) and the accompanying sea level pressure (Troup 1965; Webster and Yang 1992) and rainfall (Walker and Bliss 1932; Wright 1979). The major feature of this PB in SST is the existence of a band of maximum decline of monthly autocorrelation function (ACF) at a fixed phase (or calendar month), May or June, as seen in the monthly ACF of SST (e.g., Ren et al. 2016; Liu et al. 2019; Figs. 1a,c). Therefore, regardless of the initial month, a damped persistence forecast loses its predictability most rapidly in the following May or June, forming the so-called boreal spring PB of ENSO.

Previous studies also showed a PB in the upper-ocean heat content (OHC) in the tropical Pacific similar to that for SST, except for about half a year phase shift to the boreal winter (Balmaseda et al. 1995; McPhaden 2003; Yu and Kao 2007; Figs. 1b,d). Furthermore, regardless of this phase difference, a stronger spring PB for SST in a period seems to correspond to a stronger winter PB for OHC: the PBs appear stronger in 1958–78 than that in 1979–2000 in both SST and OHC (Figs. 1a,b vs Figs. 1c,d). Although the relationship between SST and OHC in the eastern tropical Pacific has been established both in the models and observations (Jin 1997a; Zhu et al. 2015), what causes this relationship, especially the relationship of PBs in SST and

OHC, is still unknown. Given that ENSO forecast skill often shows improvements across the spring when the observed variations of upper OHC are included as part of the forecast initialization scheme (Smith 1995; Xue et al. 2000; Ren et al. 2018), it is important to understand what determines the relationship of the PBs between SST and OHC in both the phase and strength.

Recent theoretical studies suggested that the ENSO spring PB for SST is caused by the seasonally varying growth rate of SST (Levine and McPhaden 2015; Liu et al. 2019; Jin et al. 2019, 2020). Since the general theory is generic, we can speculate a similar role of the seasonal growth rate in generating the winter PB for OHC. As the seasonal growth rates of both SST and OHC are determined by the common seasonal cycle of the tropical Pacific, we speculate that this common seasonal cycle ultimately causes the PBs in both SST and OHC and, in turn, their relationship.

In Part I of this paper (Jin and Liu 2020, hereafter Part I) we show the role of ENSO period with regard to the timing and strength of SST spring PB. Here in Part II, we investigate the relationship of the PBs between SST and OHC mainly using the neutral recharge oscillator (NRO) model with a seasonally varying growth rate both analytically and numerically. We suggest that the seasonal growth rate in SST leads to both the spring PB for SST and the winter PB for OHC and both PBs are of similar strength. The role of seasonal growth rate is then identified in the damped and self-exciting ENSO regime. This theory allows us to explain the observed relationship of ENSO PBs between SST and OHC.

The paper is arranged as follows. The recharge oscillator model, the definition of PB strength and timing, and the reanalysis data are presented in section 2. In section 3, we explore the effect of seasonal growth rate on PBs of SST and OHC in the NRO model both analytically and numerically. In section 4, we further examine the PBs for ENSO in the damped and self-exciting regime. The role of seasonal growth rate is employed

Denotes content that is immediately available upon publication as open access.

Corresponding author: Yishuai Jin, jinyishuai@126.com; Zhengyu Liu, liu.7022@osu.edu

DOI: 10.1175/JCLI-D-20-0820.1

© 2021 American Meteorological Society. For information regarding reuse of this content and general copyright information, consult the AMS Copyright Policy (www.ametsoc.org/PUBSReuseLicenses).

FIG. 1. Persistence map (lagged autocorrelation) of monthly anomalies in (a) Niño-3.4 SST and (b) OHC for 1958–78. The black circles on the persistence map mark the lag month of maximum autocorrelation decline for different initial months. PB strength is quantified by S_{B_1} [Eq. (2.5)]. (c), (d) As in (a) and (b), respectively, but for 1979–2000.

to explain the relationship of ENSO PBs in the observation in section 5. A summary and discussion are given in section 6.

2. Model, method, and data

a. The parametric recharge oscillator model

Our ENSO model is based on the recharge oscillator model (Jin 1997a,b). It contains cubic and quadratic nonlinearity as well as stochastic forcing. This recharge oscillator captures the dynamic relationship between the western equatorial Pacific OHC anomaly (H) and eastern equatorial Pacific SST anomaly (T). Particularly, it describes the discharge of the equatorial heat content in the warm phase (i.e., El Niño) and recharge at the cold phase (i.e., La Niña) and can be written as follows (Chen and Jin 2020):

$$\frac{dT}{dt} = -\lambda(t)T + \omega_0 H + \sigma \xi - cT^3 + bT^2, \quad (2.1)$$

$$\frac{dH}{dt} = -\omega_0 T, \quad (2.2)$$

$$\frac{d\xi}{dt} = -d\xi + w(t), \quad (2.3)$$

where Eq. (2.1) indicates the combined effects of relaxation of SST anomaly (negative feedback), advective, Ekman upwelling,

thermocline positive feedbacks, and nonlinear terms to SST anomaly. Equation (2.2) suggests a description of the basinwide equatorial oceanic adjustment (low frequency). This recharge oscillator model is able to reveal the main features of the spring PB of SST, which has been identified in Levine and McPhaden (2015). The term ω_0 in this model is the ENSO linear frequency set as $\omega_0 = (2\pi/48) \text{ month}^{-1}$ (Levine and McPhaden 2015); σ is the noise amplitude, with $w(t)$ being a white noise and ξ being a red noise with a decay time scale of $1/r$. Note here that seasonally varying noise magnitude (i.e., σ) takes part in the PB (e.g., Figs. 4 and 5 in Moon and Wettlaufer 2017). However, in the tropical Pacific, seasonal growth rate leads to ENSO spring PB while the seasonal noise enhances this spring PB in some extent (e.g., Fig. 5 in Liu et al. 2019). As such, noise forcing is constant (i.e., σ is constant) in this paper. Moreover, different kinds of noise play a less important role in PB (Jin et al. 2021). Here we use red noise to be consistent with Levine and McPhaden (2015). The quadratic term (b) and cubic term (c) roughly represent the nonlinear dynamic heating from both nonlinear advection and upwelling, and from the upscale effects of ENSO modulation of tropical instability waves (An 2008). The seasonal change is originated from the damping rate λ (or growth rate $-\lambda$) of SST anomaly as $\lambda(t) = R_0 + \lambda_0 \sin(\omega_A t)$, with $\omega_A = (2\pi/12) \text{ month}^{-1}$ as the annual frequency. This seasonal cycle has been shown to lead to the phase locking of ENSO variance (Chen and Jin 2020).

FIG. 2. The numerical solutions of persistence map for $\lambda_e = 1/\pi$ in NRO model for (a) SST and (b) OHC. The black circles on the persistence map mark the lag month of maximum autocorrelation decline for different initial months. PB strength is quantified by S_{B_1} [Eq. (2.5)]. (c),(d) As in (a) and (b), respectively, but for analytical solutions.

and spring PB in SST (Levine and McPhaden 2015). Here we further show that this seasonally varying growth rate also causes the winter PB of OHC.

The approximate analytical solutions of ACFs in SST and OHC are obtained in the NRO, from the neutrally stable, unforced case of the recharge model with $R_0 = 0$, $\sigma = 0$, $c = 0$, $b = 0$. Previous studies have shown that the approximate analytical solution of variance in this neutral model is able to explain the ENSO phase locking of variance (Stein et al. 2014). As such, this neutral status of recharge oscillator may also be able to study ENSO PBs in SST and OHC. Here we further derive the corresponding ACFs of SST and OHC and in turn study the relationship of the PBs in SST and OHC. The robustness of the results of the NRO model will be confirmed numerically in the recharge model in the damped regime (with noise forcing, $R_0 > 0$, $\sigma > 0$, $c = 0$, $b = 0$) and self-exciting unstable regime ($R_0 < 0$, $\sigma = 0$, $c > 0$, $b = 0$). All the numerical results are from the last 500 years of a 1000-yr run. The numerical model Eqs. (2.1)–(2.3) are solved in the time step of 4 h.

b. Definition of PB strength and timing

The ACF r is a function of initial months m and lag months τ (Ren et al. 2016; Jin et al. 2020). Following Liu et al. (2019), the strength and timing of PB are defined from the ACF as follows. First, for a calendar month m , we identify $\tau_B(m)$ as the specific lag of maximum ACF decline, which is calculated as the lag gradient in the time step of 1 month as

$$S_B(m) = \left\{ \frac{r[m, \tau_B(m) - 1] - r[m, \tau_B(m) + 1]}{2} \right\} = \max_{\tau} \left\{ \frac{r(m, \tau - 1) - r(m, \tau + 1)}{2} \right\}, \quad (2.4)$$

where $S_B(m)$ is the maximum gradient for every initial month m . Second, the total intensity of PB (S_{B_1}) is estimated as

$$S_{B_1} = \sum_{m=1}^{12} S_B(m). \quad (2.5)$$

FIG. 3. (a)–(d) As in Figs. 2a–d, respectively, but for $\lambda_e = 2/\pi$.

For each calendar month m , the timing of PB can be defined as

$$\gamma(m) = m + \tau_B(m). \quad (2.6)$$

The average timing of PB is defined as

$$\bar{\gamma} = \frac{1}{12} \sum_{m=1}^{12} \gamma(m). \quad (2.7)$$

c. Data

The observational temperature dataset used here in this study is the monthly Simple Ocean Data Assimilation (SODA; $0.5^\circ \times 0.5^\circ$). We use SODA 2.1.6 (from 1958 to 2008; Carton and Giese 2008) to identify some general features of PBs. The SODA data are publicly available and can be downloaded from http://apdrc.soest.hawaii.edu/datadoc/soda_2.1.6.php. The Niño-3.4 (5°S – 5°N , 170° – 120°W) index is used to represent ENSO activities in SST. The OHC (5°S – 5°N , 120°E – 80°W) (Yu and Kao 2007) is defined as the ocean temperatures averaged in the upper 50–300 m. Other measures of the OHC [e.g., warm water volume in McPhaden

(2003)] give similar persistence maps (not shown). Here the robustness of the results in the recharge oscillator model will also be identified through the observations. All monthly data used here have their climatological seasonal cycle and linear trends removed.

3. The role of seasonal growth rate in ENSO PBs in the NRO model

We now show that the seasonal cycle of ENSO growth rate determines the relationship between the PBs of SST and OHC analytically and numerically in the NRO model.

a. Analytical solution of ACFs in the NRO model

For the linear scenario of the NRO model ($R_0 = 0$, $\sigma = 0$, $c = 0$, $b = 0$) in Eqs. (2.1)–(2.3), an approximate analytical solution of T can be obtained under a weak annual cycle of the growth rate (i.e., small amplitude of this seasonal growth rate) or small λ_e (i.e., nondimensional amplitude of damping rate) [$\lambda(t) = R_0 + \lambda_0 \sin(\omega_A t)$, $\lambda_e = (\lambda_0/\omega_A) \ll 1$]. Using the perturbation method, one special solution of T is (An and Jin 2011; Chen and Jin 2020)

FIG. 4. (a) The modulation of PB timing [Eq. (2.7)] with seasonal amplitude λ_ε . Here for clarity, we use λ_0 in the x axis; $\lambda_0 = 0.1\text{--}0.35 \text{ month}^{-1}$ indicates that λ_ε changes from $0.6/\pi$ to $2.1/\pi$. The blue triangle is the numerical solution for SST while the blue square is for OHC; the green symbols are the same as the blue ones, but for the analytical solutions. (b) As in (a), but for PB strength.

$$T(t) \approx \sin(\omega_0 t) - \lambda_\varepsilon A \sin(\omega_A t) \cos(\omega_0 t) + \lambda_\varepsilon B \cos(\omega_A t) \sin(\omega_0 t), \quad (3.1)$$

where $A = \omega_0 \omega_A / (\omega_A^2 - 4\omega_0^2)$ and $B = (\omega_A^2 - 2\omega_0^2) / (\omega_A^2 - 4\omega_0^2)$ are nondimensional parameters.

The term $T(t)$ is a function of year y and the calendar month m (from 1 to 12 to represent January to December). The seasonally varying variance $\sigma_T^2(m)$ can be derived as (Stein et al. 2014)

$$\sigma_T^2(m) = E[T^2(y, m)] = \frac{1}{2} + \lambda_\varepsilon B \cos(\omega_A m). \quad (3.2)$$

Equation (3.2) shows that the variance always reaches the maximum in December ($m = 12$), which indicates that the seasonal growth rate of ENSO SST anomaly plays an important role in the phase locking of variance (Stein et al. 2014).

The covariance function $S_T(m, \tau)$ and then the ACF $r_T(m, \tau)$ of SST between $T(y, m)$ and $T(y, m + \tau)$ can be further derived as (more details can be checked in the appendix of Part I):

$$r_T(m, \tau) \approx \cos \omega_0 \tau + \lambda_\varepsilon A \sin \omega_0 \tau \{ \sin[\omega_A(m + \tau)] - \sin(\omega_A m) \}. \quad (3.3)$$

By using the relationship between SST and OHC [Eq. (2.2)], the time series of OHC can be derived as

$$H(t) = \cos(\omega_0 t) + \frac{1}{2} \lambda_\varepsilon \omega_0 \left[\frac{B - A}{\omega_A + \omega_0} \cos(\omega_A + \omega_0)t - \frac{B + A}{\omega_A - \omega_0} \cos(\omega_A - \omega_0)t \right]. \quad (3.4)$$

The variance of OHC $\sigma_H^2(m)$ can be obtained as follows:

$$\sigma_H^2(m) = \frac{1}{2} + \lambda_\varepsilon \omega_0 \left(\cos \omega_A m \frac{1}{2} \right) \left[\frac{(B - A)}{\omega_A + \omega_0} - \frac{(A + B)}{\omega_A - \omega_0} \right]. \quad (3.5)$$

As $A > 0$, $B > 0$, $\omega_A > \omega_0 > 0$, so $\{ -[(A + B)/(\omega_A - \omega_0)] + [(-A + B)/(\omega_A + \omega_0)] \} < 0$.

According to Eq. (3.5), the maximum variance of OHC occurs in June ($m = 6$). This may explain the seasonal variance of OHC in the tropical Pacific (e.g., Fig. 5b in Yu and Kao 2007). Meanwhile, the analytical solution of T variance [Eq. (3.2)] shows that the minimum variance occurs in June ($m = 6$). According to Eq. (3.5), the minimum variance of H occurs in December ($m = 12$). This half-year relationship of minimum variance may give an explanation why the PB of SST is exactly out of phase with that of OHC (Yu and Kao 2007).

To understand the relationship of PBs in SST and OHC thoroughly, the ACF of OHC $r_H(m, \tau)$ can also be derived as (more details can be checked in the appendix)

$$r_H(m, \tau) \approx \cos \omega_0 \tau + \lambda_\varepsilon A \sin \omega_0 \tau \{ \sin(\omega_A m) - \sin[\omega_A(m + \tau)] \}. \quad (3.6)$$

A comparison of Eqs. (3.3) and (3.6) shows a robust relationship between the ACFs of SST and OHC as

$$r_T(m, \tau) = r_H(m + 6, \tau). \quad (3.7)$$

That is, the ACF of SST is out of phase with, or lags by half a year, the OHC in the annual cycle of initial forecast time. Therefore, according to Eq. (2.4), the PB of SST is also exactly out of phase with that of OHC. With the SST PB happens in late spring to early summer of ENSO, the PB for OHC therefore occurs half a year later in winter, as in the observations (Figs. 1a–d). We note that this 180° out-of-phase relation in the annual cycle of the

FIG. 5. (a),(b) As in Figs. 2a and 2b, but for the damped recharge oscillator model. (c),(d) As in (a) and (b), respectively, but for $\lambda_e = 2/\pi$.

two ACFs, and in turn PBs, is not obvious directly from the equations themselves. Indeed, Eq. (2.2) only states that H always lags T by 90° at all time scales. Nevertheless, our solution shows that this out-of-phase relation in autocorrelation is a general result of the seasonal growth rate, at least in the weak amplitude limit in the context of the NRO model. Moreover, as the PB strength is the accumulation of the gradient of ACF [Eq. (2.5)], Eq. (3.7) shows that PB strength of SST equals that of OHC. As such, the seasonally varying growth rate of SST anomalies controls the PBs of both SST and OHC such that the two PBs have the opposite phases and equal strength. Note here that according to Eq. (3.7), this relationship is robust for different ENSO periods (larger than two years).

b. Numerical solution of PBs in the NRO model

Here, we show that the approximate analytical solution matches the numerical solution well. Furthermore, the features of PBs in light of the analytical solutions derived above are discussed in the numerical solution of NRO model.

First, these analytical solutions of ACFs in Eqs. (3.3) and (3.6) capture the major features of the ACFs in the numerical solution of NRO model, as compared in Fig. 2. For the example of $\lambda_e = 1/\pi$, $\omega_0 = (2\pi/48)$ month⁻¹ [different ENSO periods show a similar relationship (not shown); Chen and Jin 2020],

the numerical solution (Fig. 2a) shows that a PB for SST occurs in the early summer and its strength is 2.00. The analytical solution (Fig. 2c) also indicates a PB of SST in the early summer, although PB strength is slightly weaker (1.83). Similarly, the analytical solution of OHC suggests a close timing and weaker PB strength (boreal winter and PB strength 1.83; Fig. 2d) to the numerical solution (boreal winter and PB strength 1.99; Fig. 2b). The weaker PB strengths for the analytical solutions are due to the neglect of a low-order cross-correlation between the annual cycle of growth rate and ENSO frequency.

Although the analytical solutions are derived in the limit of $\lambda_e \ll 1$, it is still a good approximation for the full numerical solution for rather larger λ_e . We first increase λ_e to $2/\pi$, the timings of PB in SST and OHC show a little shift (one month earlier) with their phase difference still about 6 months apart both numerically (Fig. 3a vs Fig. 3b) and analytically (Fig. 3c vs Fig. 3d). In addition, the PBs in SST and OHC show a similar strength, although the strength tends to be somewhat weaker in the analytical solution. This has been seen in the example of $\lambda_e = 1/\pi$ in Fig. 2. For $\lambda_e = 2/\pi$, the numerical solution suggests that the PB intensity is 3.17 for SST and 3.13 for OHC (Figs. 3a,b). The analytical solution (Figs. 3c,d) gives a weaker strength than that in numerical solution, but PB intensities

FIG. 6. As in Fig. 5, but for PBs in the self-exciting recharge oscillator model.

remain the same (2.34). A comparison of Figs. 2 and 3 shows that a larger seasonal amplitude (Fig. 2 vs Fig. 3) leads to a stronger PB strength, which is consistent with the role of seasonal amplitude in PB strength in Langevin equation (Liu et al. 2019).

These features discussed above are generally valid for more general growth rates. To study the phase difference between the PBs of SST and OHC systematically, we vary λ_ε from $0.6/\pi$ to $2.1/\pi$ ($\lambda_0 = 0.1\text{--}0.35\text{ month}^{-1}$) to see the modulation of the timing and strength of PBs. Both the numerical solution and the analytical solution suggest that a PB occurs in about July for SST and January for OHC [according to Eq. (2.7)] such that the phase difference is about half a year (Fig. 4a). Therefore, with the seasonally varying growth rate, the phase difference of PB in SST and OHC is robust in NRO model. Meanwhile, the underestimation of the PB strength of the analytical solution relative to the numerical solution increases with the λ_ε (Fig. 4b). It is reasonable because the analytical solution is derived under the assumption of small λ_ε . However, in the numerical solution, the intensities of the two PBs of SST and OHC remain almost the same, as in the analytical solution (blue squares vs blue triangles or green squares vs green triangles in Fig. 4b).

In sum, our study of the NRO model here shows that with the seasonal growth rate of ENSO SST anomalies, the phase difference between the PBs of SST and OHC is about half a year while their strengths are comparable. These features of the PBs

of SST and OHC will be shown to be robust when the ENSO is generalized to the damped regime and self-exciting regime.

4. Role of seasonal growth rate in the damped and self-exciting recharge oscillator model

We now examine the relationship of PBs in SST and OHC in the recharge oscillator model in the damped regime ($R_0 > 0$, $\sigma > 0$, $c = 0$, $b = 0$) and self-exciting regime ($R_0 < 0$, $\sigma = 0$, $c > 0$, $b = 0$).

In the damped regime, with the addition of the seasonal growth rate in SST, PB timing of OHC leads that of SST for about one to two seasons, while two PBs show a close strength. Parameters set for the observational ENSO are (Chen and Jin 2020) $R_0 = -0.1\text{ month}^{-1}$, $\lambda_\varepsilon = 1/\pi$, $\sigma = (1/9)\text{ month}^{-1}$, and $d = 0.66\text{ month}^{-1}$. A clear PB occurs in April (Fig. 5a), indicating that the seasonal growth rate of SST anomalies plays an important role in causing spring PB for SST. It is consistent with the results of Levine and McPhaden (2015) using the damped recharge oscillator model. Meanwhile, this seasonal growth rate also leads to a winter PB for OHC (December), as shown in Fig. 5b. When we further increase the seasonal amplitude of growth rate ($\lambda_\varepsilon = 2/\pi$), PBs also suggest the phase difference for about six months and both of them are strengthened (3.14 for SST and 2.22 for OHC in Figs. 5c,d). The larger different PB strengths between SST and OHC compared with the NRO model are caused by the annual mean damping rate R_0 . In all,

the PBs in the damped regime show similar features with that in the NRO model.

In the self-exciting regime, the seasonally varying growth rate also controls the relationship of PB between SST and OHC. Here we set the parameters for realistic ENSO as $R_0 = 0.1 \text{ month}^{-1}$, $\lambda_e = 1/\pi$, $c = 1/62.3 \text{ month}^{-1}$, and $b = 0$ (Chen and Jin 2020). With the seasonal growth rate existing in the self-exciting recharge oscillator model, a clear PB feature is found in the early summer for SST and in the early winter for OHC (Figs. 6a,b). As the seasonal amplitude is increased ($\lambda_e = 2/\pi$), the phase difference of PBs in SST and OHC is almost unchanged (Figs. 6c,d). On the other hand, both of them are strengthened (from 2.16 to 3.27 for SST and from 1.91 to 2.71 for OHC). Therefore, the features of PBs in the unstable regime are consistent with that in the NRO model.

5. Understanding the observational ENSO PBs in SST and OHC

The recharge oscillator model shows analytically and numerically that the seasonal growth rate of SST anomalies plays an overwhelming role in determining the relationship of ENSO PBs between SST and OHC. This may explain the relationship of the two PBs in the observation.

A robust relationship between the PBs of SST and OHC can be found in the observation, as shown in Fig. 1. During the years 1958–78, a strong PB (4.24) for SST occurs in May with a strong PB (4.03) for OHC in January (Figs. 1a,b). For the years 1979–2000, a clear PB feature is found in the early summer for SST and in the boreal winter for OHC (Figs. 1c,d). Both of them are weakened (3.32 for SST and 3.07 for OHC). To study the relationship between PBs in SST and OHC systematically, we calculate the modulation of the phase and strength of the two PBs with time. Figures 7a and 7b show the decadal variations of the phase and strength of PBs in SST and OHC, which is calculated within a 20-yr window moving for the data from 1958 to 2008. It is found that the PB for SST occurs in April–June (black line in Fig. 7a), which lags the PB for OHC (red line in Fig. 7a) by 3–6 months. Regardless of the phase difference, both of them suggest a similar strength change (black line vs red line in Fig. 7b). As such, the relationship between the PBs for SST and OHC is consistent with that in the seasonally varying recharge oscillator model, suggesting that a seasonal cycle of ENSO growth rate in SST anomalies also controls this relationship in the observation.

Particularly, the analytical solutions of NRO model may be applied to explain the decadal modulations of ENSO PBs in the observations. The strength of the PB in SST decreases from 1968 to 1985 and increases after that period (black line in Fig. 7b). Meanwhile, a similar decadal modulation can be found in the PB of OHC (red line in Fig. 7b). This can be identified in the analytical solutions of the NRO model. According to Eq. (3.7), the change of growth rate can lead to the modulation of PBs in SST and OHC simultaneously. For example, when amplitude of growth rate is larger (i.e., λ_e), the SST PB is strengthened and as well as the OHC PB (Fig. 4b). Accordingly, the change of PBs in SST and OHC in the observation may be caused by the modulation of growth rate. This growth rate can be regarded as the Bjerknes (BJ) index (Jin et al. 2006), which is controlled by the background in the tropical Pacific (Jin et al. 2019). This BJ index has been identified to exhibit a

FIG. 7. (a) PB timing [quantified by Eq. (2.7)] of SST (black line) and OHC (red line) calculated with a 20-yr window that moves year by year from 1958 to 2008. The shading is the spread of timing for SST (blue) and OHC (gray), respectively. (b) As in (a), but for PB strength.

decadal modulation (Fang et al. 2019). As such, the decadal change of PBs may be caused by the background in the tropical Pacific, which is consistent with Yu and Kao (2007).

6. Summary and discussion

This paper attempts to understand the potential factor that controls the relationship between ENSO PBs in SST and in OHC. Based on the approximate analytical solution of SST anomaly variance in the NRO model (Stein et al. 2014), we derive the ACFs for SST and OHC, respectively, and explain the relationship between

these two PBs. More specifically, the approximate analytical solution of ACFs explain the phase difference and strength modulation for PBs in SST and OHC. It shows strictly that with addition of the seasonal cycle of ENSO growth rate in SST anomalies, the phase of PB for OHC leads that of SST for half a year. Meanwhile, both of the PBs will have the same strength. The numerical solutions of the NRO model also suggest the similar relationship between two barriers. This relationship is further found to be robust in the recharge model for ENSO in the damped regime and self-exciting regime. As such, it may explain that the seasonal growth rate controls PB relationship (phase and strength) for observational ENSO. Given that this seasonally varying growth rate is controlled by the thermocline feedback and thermodynamic damping (Jin et al. 2019), both the PBs in SST and upper OHC are affected by the seasonality of the background in the tropical Pacific.

It should be noted here that other factors may also control the relationship between the ENSO SST and OHC anomalies. Levine and McPhaden (2015) suggests that the state dependent noise (e.g., westerly wind bursts) will enhance ENSO spring PB for SST, which indicates that it may also change the PB for OHC.

How to separate these effects on ENSO PBs still needs further investigation.

Acknowledgments. We thank the editor and three anonymous reviewers for their helpful comments on this research. This work is supported by Chinese MOST 2017YFA0603801, NSFC41630527, and U.S. NSF AGS-1656907.

APPENDIX

Derivation of Autocorrelation Function of OHC Based on the NRO Model

Let $C_1 = (B - A)/(\omega_A + \omega_0)$, $C_2 = (A + B)/(\omega_A - \omega_0)$, and $C = C_1 - C_2$; accordingly, $\sigma_H^2(m)$ [Eq. (3.5)] can be simplified as

$$\sigma_H^2(m) = \frac{1}{2} + \lambda_\varepsilon C \omega_0 \left(\cos \omega_A m \frac{1}{2} \right). \quad (\text{A1})$$

The covariance $S_H(m, \tau)$ of OHC between $H(y, m)$ and $H(y, m + \tau)$ can be derived as

$$S_H(m, \tau) = \frac{1}{2} \cos \omega_0 \tau \left[1 + \frac{C}{2} \lambda_\varepsilon \omega_0 \{ \cos[\omega_A t(y, m)] + \cos[\omega_A t(y, m + \tau)] \} \right] + \frac{1}{2} \lambda_\varepsilon \omega_0 \frac{C_1 + C_2}{2} \sin \omega_0 \tau \{ \sin[\omega_A t(y, m)] - \sin[\omega_A t(y, m + \tau)] \}. \quad (\text{A2})$$

The ACF of OHC $r_H(m, \tau)$ can be obtained as

$$r_H(m, \tau) = \frac{\langle H(m), H(m + \tau) \rangle}{\sqrt{\langle H(m), H(m) \rangle \langle H(m + \tau), H(m + \tau) \rangle}} = \frac{\frac{1}{2} \cos \omega_0 \tau \left\{ 1 + \frac{C}{2} \lambda_\varepsilon \omega_0 [\cos(\omega_A m) + \cos(\omega_A (m + \tau))] \right\} + \frac{1}{2} \lambda_\varepsilon \omega_0 \frac{C_1 + C_2}{2} \sin \omega_0 \tau \{ \sin(\omega_A m) - \sin(\omega_A (m + \tau)) \}}{\sqrt{\left[\left(\frac{1}{2} + \frac{1}{2} \lambda_\varepsilon \omega_0 C \cos(\omega_A m) \right) \left\{ \frac{1}{2} + \frac{1}{2} \lambda_\varepsilon \omega_0 C \cos[\omega_A (m + \tau)] \right\} \right]}}. \quad (\text{A3})$$

As $\lambda_\varepsilon \ll 1$, at the leading order, $r_H(m, \tau)$ can be written as

$$r_H(m, \tau) \approx \cos \omega_0 \tau + \lambda_\varepsilon \omega_0 \frac{C_1 + C_2}{2} \sin \omega_0 \tau \{ \sin(\omega_A m) - \sin[\omega_A (m + \tau)] \}, \quad (\text{A4})$$

$$C_1 + C_2 = \frac{(B - A)}{\omega_A + \omega_0} + \frac{(A + B)}{\omega_A - \omega_0} = \frac{(B - A)(\omega_A - \omega_0) + (A + B)(\omega_A + \omega_0)}{\omega_A^2 - \omega_0^2} = \frac{2B\omega_A + 2A\omega_0}{\omega_A^2 - \omega_0^2}. \quad (\text{A5})$$

Take $A = \omega_0 \omega_A / (\omega_A^2 - 4\omega_0^2)$, $B = (\omega_A^2 - 2\omega_0^2) / (\omega_A^2 - 4\omega_0^2)$, to Eq. (A5)

$$C_1 + C_2 = \frac{2\omega_A}{(\omega_A^2 - 4\omega_0^2)}. \quad (\text{A6})$$

So $r_H(m, \tau)$ can be simplified into

$$r_H(m, \tau) \approx \cos \omega_0 \tau + \lambda_\varepsilon A \sin \omega_0 \tau \{ \sin(\omega_A m) - \sin[\omega_A (m + \tau)] \}. \quad (\text{A7})$$

REFERENCES

- An, S.-I., 2008: Interannual variations of the tropical ocean instability wave and ENSO. *J. Climate*, **21**, 3680–3686, <https://doi.org/10.1175/2008JCLI1701.1>.
- , and F.-F. Jin, 2011: Linear solutions for the frequency and amplitude modulation of ENSO by the annual cycle. *Tellus*, **63**, 238–243, <https://doi.org/10.1111/j.1600-0870.2010.00482.x>.
- Balmaseda, M. A., M. K. Davey, and D. L. Anderson, 1995: Decadal and seasonal dependence of ENSO prediction skill. *J. Climate*, **8**, 2705–2715, [https://doi.org/10.1175/1520-0442\(1995\)008<2705:DASDOE>2.0.CO;2](https://doi.org/10.1175/1520-0442(1995)008<2705:DASDOE>2.0.CO;2).
- Carton, J. A., and B. S. Giese, 2008: A reanalysis of ocean climate using Simple Ocean Data Assimilation (SODA). *Mon. Wea. Rev.*, **136**, 2999–3017, <https://doi.org/10.1175/2007MWR1978.1>.
- Chen, H.-C., and F.-F. Jin, 2020: Fundamental behavior of ENSO phase locking. *J. Climate*, **33**, 1953–1968, <https://doi.org/10.1175/JCLI-D-19-0264.1>.

- Fang, X. H., F. Zheng, Z. Y. Liu, and J. Zhu, 2019: Decadal modulation of ENSO spring persistence barrier by thermal damping processes in the observation. *Geophys. Res. Lett.*, **46**, 6892–6899, <https://doi.org/10.1029/2019GL082921>.
- Jin, F.-F., 1997a: An equatorial ocean recharge paradigm for ENSO. Part I: Conceptual model. *J. Atmos. Sci.*, **54**, 811–829, [https://doi.org/10.1175/1520-0469\(1997\)054<0811:AEORPF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<0811:AEORPF>2.0.CO;2).
- , 1997b: An equatorial ocean recharge paradigm for ENSO. Part II: A stripped-down coupled model. *J. Atmos. Sci.*, **54**, 830–847, [https://doi.org/10.1175/1520-0469\(1997\)054<0830:AEORPF>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<0830:AEORPF>2.0.CO;2).
- , S. T. Kim, and L. Bejarano, 2006: A coupled-stability index for ENSO. *Geophys. Res. Lett.*, **33**, L23708, <https://doi.org/10.1029/2006GL027221>.
- Jin, Y., and Z. Liu, 2020: A theory of the spring persistence barrier on ENSO. Part I: The role of ENSO period. *J. Climate*, **34**, 2145–2155, <https://doi.org/10.1175/JCLI-D-20-0540.1>.
- , —, Z. Lu, and C. He, 2019: Seasonal cycle of background in the tropical Pacific as a cause of ENSO spring persistence barrier. *Geophys. Res. Lett.*, **46**, 13 371–13 378, <https://doi.org/10.1029/2019GL085205>.
- , Z. Lu, and Z. Liu, 2020: Controls of spring persistence barrier strength in different ENSO regimes and implications for 21st century changes. *Geophys. Res. Lett.*, **47**, e2020GL088010, <https://doi.org/10.1029/2020GL088010>.
- , Z. Liu, C. He, and Y. Zhao, 2021: On the formation mechanism of the seasonal persistence barrier. *J. Climate*, **34**, 479–494, <https://doi.org/10.1175/JCLI-D-19-0502.1>.
- Levine, A. F. Z., and M. J. McPhaden, 2015: The annual cycle in ENSO growth rate as a cause of the spring predictability barrier. *Geophys. Res. Lett.*, **42**, 5034–5041, <https://doi.org/10.1002/2015GL064309>.
- Liu, Z., Y. Jin, and X. Rong, 2019: A theory for the seasonal predictability barrier: Threshold, timing, and intensity. *J. Climate*, **32**, 423–443, <https://doi.org/10.1175/JCLI-D-18-0383.1>.
- McPhaden, M. J., 2003: Tropical Pacific Ocean heat content variations and ENSO persistence barriers. *Geophys. Res. Lett.*, **30**, 1480, <https://doi.org/10.1029/2003GL016872>.
- Moon, W., and J. S. Wettlaufer, 2017: A unified nonlinear stochastic time series analysis for climate science. *Sci. Rep.*, **7**, 44228, <https://doi.org/10.1038/srep44228>.
- Ren, H.-L., F.-F. Jin, B. Tian, and A. A. Scaife, 2016: Distinct persistence barriers in two types of ENSO. *Geophys. Res. Lett.*, **43**, 10 973–10 979, <https://doi.org/10.1002/2016GL071015>.
- , J. Zuo, and Y. Deng, 2018: Statistical predictability of Niño indices for two types of ENSO. *Climate Dyn.*, **52**, 5361–5382, <https://doi.org/10.1007/s00382-018-4453-3>.
- Smith, N. R., 1995: An improved system for tropical ocean subsurface temperature analyses. *J. Atmos. Oceanic Technol.*, **12**, 850–870, [https://doi.org/10.1175/1520-0426\(1995\)012<0850:AISFTO>2.0.CO;2](https://doi.org/10.1175/1520-0426(1995)012<0850:AISFTO>2.0.CO;2).
- Stein, K., A. Timmermann, N. Schneider, F.-F. Jin, and M. F. Stuecker, 2014: ENSO seasonal synchronization theory. *J. Climate*, **27**, 5285–5310, <https://doi.org/10.1175/JCLI-D-13-00525.1>.
- Troup, A., 1965: The ‘southern oscillation’. *Quart. J. Roy. Meteor. Soc.*, **91**, 490–506, <https://doi.org/10.1002/qj.49709139009>.
- Walker, G. T., and E. W. Bliss, 1932: World weather V. *Mem. Roy. Meteor. Soc.*, **4**, 53–84.
- Webster, P. J., and S. Yang, 1992: Monsoon and ENSO: Selectively interactive systems. *Quart. J. Roy. Meteor. Soc.*, **118**, 877–926, <https://doi.org/10.1002/qj.49711850705>.
- Wright, P. B., 1979: Persistence of rainfall anomalies in the central Pacific. *Nature*, **277**, 371–374, <https://doi.org/10.1038/277371a0>.
- Xue, Y., A. Leetmaa, and M. Ji, 2000: ENSO prediction with Markov models: The impact of sea level. *J. Climate*, **13**, 849–871, [https://doi.org/10.1175/1520-0442\(2000\)013<0849:EPWMMT>2.0.CO;2](https://doi.org/10.1175/1520-0442(2000)013<0849:EPWMMT>2.0.CO;2).
- Yu, J.-Y., and H.-Y. Kao, 2007: Decadal changes of ENSO persistence barrier in SST and ocean heat content indices: 1958–2001. *J. Geophys. Res.*, **112**, D13106, <https://doi.org/10.1029/2006JD007654>.
- Zhu, J., A. Kumar, and B. Huang, 2015: The relationship between thermocline depth and SST anomalies in the eastern equatorial Pacific: Seasonality and decadal variations. *Geophys. Res. Lett.*, **42**, 4507–4515, <https://doi.org/10.1002/2015GL064220>.