Reply to “Comments on ‘Global and Regional Entropy Production by Radiation Estimated from Satellite Observations’”

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ABSTRACT: This reply addresses a comment on the study by Kato and Rose (herein referred to as KR2020). The comment raises four points of criticism. These are 1) on notations used, 2) on a steady-state assumption made, 3) on the result of entropy production change with Earth’s albedo, and 4) disputing the statement that a simple energy balance model cannot produce absorption temperature change with Earth’s albedo. We concur on points 2 and 3 raised by the comment and recognize the significance of entropy storage due to ocean heating in the analysis of how entropy production changes with the shortwave absorptivity of Earth. Once entropy storage is considered, the results of KR2020 indicate that the increase of entropy production rate by irreversible processes, including by radiative processes, is smaller than the increase of entropy storage when absorptivity is increased. This is a manifestation of the primary contribution of positive top-of-atmosphere net irradiances (i.e., energy input to Earth) to heating the ocean and is consistent with an energy budget perspective. Once entropy storage is separated, the entropy production by irreversible processes increases with the shortwave absorptivity.

KEYWORDS: Budgets; Energy budget/balance; Entropy

1. Entropy production rate and changes due to Earth’s absorptivity

The central point of the comment by Gibbins and Haigh (2021, hereinafter GH2021) is to recognize the significance of entropy storage within the Earth system, and that hence Earth is not in a steady state. Here, we summarize the main point of the comment. The rest of the three criticisms are addressed after this main point is discussed. Notations used in this reply follow those used in Kato and Rose (2020, hereinafter KR2020).

The entropy budget $S$ of Earth is [Eq. (14) of KR2020]

$$
\frac{dS}{dt} = \frac{Q_a}{T_e} - \frac{Q_e}{T_a} + \Sigma_{irr},
$$

where $Q_a$ is the heating rate due to absorption of shortwave irradiance and $-Q_e$ is the cooling rate due to longwave emission to space. For a steady-state condition, $dS/dt = 0$. When, however, the global annual mean net top-of-atmosphere (TOA) irradiance is not zero (i.e., $F_{net}^{TOA} \neq 0$), Earth is either warming or cooling depending on the sign of $F_{net}^{TOA}$. Therefore, Earth is not in a steady state and $dS/dt \neq 0$. Current TOA irradiance observations by Clouds and the Earth’s Radiant Energy System (CERES) instruments indicate that the TOA net irradiance is positive, and when a nonsteady state is considered, therefore, Eqs. (20) and (23) of KR2020 are

$$
\dot{S}_{irr} - \frac{dS}{dt} = \frac{Q_a}{T_e} - \frac{Q_e}{T_a} + \Sigma_{irr},
$$

and

$$
\dot{S}_{irr} - \frac{dS}{dt} = - \left( \frac{F_{net}^{atm,SW}}{T_{atm}} + \frac{F_{net}^{atm,LW}}{T_{atm}} + \frac{F_{net}^{sfc,SW}}{T_{sfc}} + \frac{F_{net}^{sfc,LW}}{T_{sfc}} \right).
$$

The annual and global mean net TOA irradiance from July 2005 through June 2015 is positive $0.71 \pm 0.10$ W m$^{-2}$ (Loeb et al. 2018a) so that Earth is warming. The positive net irradiance (i.e., it is defined as downward is positive) contributes to heating ocean, melting ice, warming land, and creating an increasingly warmer and moister atmosphere. Therefore, when the relationship between Earth absorptivity and entropy production by irreversible processes $\Sigma_{irr}$ is estimated (Fig. 8 of KR2020), the entropy storage $dS/dt$ due to heating ocean needs to be taken into account.

We concur with the comment that Earth is not in a steady state so that the rate of entropy storage is not negligible. Because the $dS/dt$ term is positive in Eqs. (2) and (3), the annual global mean entropy productions by irreversible processes of 76 mW m$^{-2}$ K$^{-1}$ and by irreversible nonradiative processes of 49 mW m$^{-2}$ K$^{-1}$ estimated in KR2020 include the $-dS/dt$ term. The size of the storage term is estimated roughly by dividing 0.71 W m$^{-2}$ by the global mean ocean skin temperature of 292 K (Reynolds et al. 2002), which gives the entropy production of 2.4 mW m$^{-2}$ K$^{-1}$. Although this is the climatological value of the entropy storage, subtracting this value from the left side of Eq. (3) introduces a bias, as suggested by the comment (GH2021), because the TOA net irradiance from SYN1deg is different (1.3 W m$^{-2}$) from the observed NET TOA irradiance of 0.71 W m$^{-2}$. Because the variability of the right side of Eq. (2) is primarily due to the variability of irradiances, the scaling approach discussed in the comment is a
TABLE 1. Slope of linear regression line with Earth absorptivity\textsuperscript{a} and 95% confidence interval.

<table>
<thead>
<tr>
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<th>18 years (2000–18)</th>
<th>20 years (2000–20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NET TOA irradiance (EBAF) (W m(^{-2}))</td>
<td>0.20 ± 0.12</td>
<td>0.20 ± 0.10</td>
</tr>
<tr>
<td>NET\textsuperscript{b} TOA irradiance (SYN initial) (W m(^{-2}))</td>
<td>0.25 ± 0.07</td>
<td>0.28 ± 0.07</td>
</tr>
<tr>
<td>Sea surface temperature (K)</td>
<td>0.05 ± 0.03</td>
<td>0.05 ± 0.03</td>
</tr>
<tr>
<td>((Q_e/T_a) - (Q_e/T_e)) (W m(^{-2}) K(^{-1}) per unit absorptivity)\textsuperscript{c}</td>
<td>(-0.73 ± 0.28)</td>
<td>(-0.85 ± 0.24)</td>
</tr>
<tr>
<td>(dS/dt) (W m(^{-2}) K(^{-1}) per unit absorptivity)\textsuperscript{c}</td>
<td>0.85 ± 0.26</td>
<td>0.97 ± 0.22</td>
</tr>
<tr>
<td>(\Sigma_{irr}) (W m(^{-2}) K(^{-1}) per unit absorptivity)</td>
<td>0.12 ± 0.06</td>
<td>0.12 ± 0.05</td>
</tr>
</tbody>
</table>

\textsuperscript{a}The slope of entropy production rate vs absorptivity is computed with absorptivity anomalies and entropy production rate in milliwatts per meter squared per kelvin multiplied by 1000; i.e., the unit of the slope is watts per meter squared per kelvin.

\textsuperscript{b}NET is defined positive inward; i.e., it is positive when Earth gains energy.

\textsuperscript{c}Note that \((Q_e/T_a) - (Q_e/T_e)\) is computed with adjusted irradiances.

The desired way of estimating climatological values. Therefore, the climatological value of \(\Sigma_{irr}\) is given by

\[
\Sigma_{irr} = \frac{-Q_{e,SYN} F_{net,SW,EBAF}}{T_a} + \frac{Q_{e,SYN} F_{net,SW,SYN}}{T_a} + \frac{Q_{e,SYN} F_{net,LW,EBAF}}{T_e} + \frac{F_{net,SW+LW,EBAF}}{T_{skin}}, \tag{4}
\]

where the SYN and EBAF subscripts indicate the data product used for deriving irradiances. Equation (4) gives \(\Sigma_{irr} = 83\) mW m\(^{-2}\) K\(^{-1}\) with 18 years of SYN1deg (Rutan et al. 2015; Kato et al. 2018) and EBAF (Loeb et al. 2018a) data products. Equation (4) preserves the value of the storage term inferred from the most accurately known global mean energy fluxes derived from ocean temperature measurements and scales computed TOA untuned irradiances, which in turn adjusts \(\Sigma_{irr}\) to within its estimated uncertainty of 10%.

In the analysis of how entropy production changes with absorptivity of Earth, KR2020 used anomalies. Although the slope \((dS/dt)/(dS/dt)\) does not depend on absolute values of entropy storage in the analysis by KR2020, Johnson et al. (2016) demonstrate that anomalies of TOA net irradiance agree well with the variability of ocean heating rates. Although both TOA net shortwave and net longwave irradiances can influence ocean heating rates, a recent study indicates that net shortwave irradiance anomalies that are predominantly caused by low-level cloud fraction anomalies that largely affect net shortwave irradiance at TOA are largely responsible for increasing energy input to oceans (Loeb et al. 2018b). Therefore, the negative slope of entropy production with increasing absorptivity derived in KR2020 of \(-0.73 ± 0.28\) W m\(^{-2}\) K\(^{-1}\) per unit absorptivity is \(d[\Sigma_{irr}]/d[\Sigma_{irr}]\) at a global annual scale, where \(a\) is the absorptivity of Earth and \(\Sigma_{irr}\) is irreversible processes including radiation exchange within the Earth system. This states that when the absorptivity of Earth increases, the entropy storage in the system increases more than the entropy production by irreversible processes. This is consistent with observations showing an excellent agreement between the variability of TOA net irradiance and ocean heating rates (Johnson et al. 2016).

2. Notations used in KR2020

We admit that notations used in KR2020 might be confusing to those who are familiar with notations used for entropy studies, but notations used in KR2020 for entropy balance are consistent with notations used for energy balance. We briefly clarify our notations used in KR2020 here.

Equation (5) of KR2020 expresses entropy balance at TOA.

\[0 < \frac{dS_{atm}}{da} < \frac{dS_{atm}}{dt} \tag{5}\]

The net entropy flux is defined as positive inward. We denote \(J_{net,TOA}\) for entropy export to space, where the symbol \(J\) is used to express entropy carried by radiation. Radiation carries energy and entropy, both of which are absorbed and emitted, but only energy exchange contributes heating and cooling. The entropy fluxes on the right side of Eq. (5) of KR2020 are scalar and positive. The plus sign in front of them indicates increasing entropy (i.e., positive production) and the negative sign indicates decreasing entropy (i.e., negative production). There should be a minus sign in front of \(J_{net}\) because the entropy of outgoing scattered shortwave radiation from Earth is larger than the entropy of incoming shortwave radiation (Wu and Liu 2010). These notations are consistent with notations of the energy balance equations in Eqs. (1)–(4) of KR2020.

Equations (6) and (7) of KR2020 are used to define \(\Sigma_{str}\) and \(\Sigma_{str}\). The right sides of these equations clarify that entropy is produced by heating by radiative and nonradiative processes. Similarly, Eqs. (9)–(11) of KR2020 define \(\Sigma_{abs}, \Sigma_{emt}\), and \(\Sigma_{irr}\) and separate \(\Sigma_{str}\) and \(\Sigma_{str}\) by processes. Because Eq. (9) expresses entropy production in the material by heating due to absorption of shortwave irradiance, we do not include
\[-(4/3)(1 - \alpha)F_{\text{sun}}/T_{\text{sun}}\] on the right side as suggested by the comment. This term is treated separately by Eq. (5), and it appears as a separate term in Eq. (12). The right side of Eq. (12) of KR2020 includes entropy carried by radiation and entropy produced by heating and cooling. In setting up Eq. (14) of KR2020, we imagine a hypothetical boundary of Earth where energy comes in and goes out by radiation and do not ask how Earth exchanges energy through this boundary. When we focus on entropy production by heating and cooling, the terms expressing entropy carried by radiation denoted by \(J\) drop. This leads to the entropy budget Eq. (14) of KR2020, which corresponds to the MS2 case discussed in Bannon (2015) and used in a study by Bannon and Lee (2017), as well as the entropy budget equation of the transfer system of Gibbins and Haigh (2020). Equation (14) is also equivalent to the entropy budget equation for a closed system discussed in, for example, de Groot and Mazur (1984, chapter 3 therein).

The upward longwave irradiance at the \(i\)th layer \(F_{\text{sfc},i}^{\text{LW}}\) in Eq. (26) of KR2020 should be \(F_{\text{emt},i}^{\text{LW}}\) and the irradiance at the surface \(F_{\text{sfc},i}^{\text{LW}}\) should be \(F_{\text{emt},\text{LW}}\), where \(F_{\text{emt},i}^{\text{LW}}\) and \(F_{\text{emt},\text{LW}}\) are respectively the upward emitted longwave irradiance by the \(i\)th layer and surface. The outgoing longwave irradiance in the model is different from the reason suggested by SYN1deg-Month datasets are available online (https://ceres.larc.nasa.gov/order_data.php).

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**Data availability statement.** The Ed4.1 CERES EBAF–TOA (Loeb et al. 2018a) and SYN1deg-Month (Rutan et al. 2015; Kato et al. 2018) datasets are available online (https://ceres.larc.nasa.gov/order_data.php).

**REFERENCES**


