A New Framework for Estimating and Decomposing the Uncertainty of Climate Projections

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ABSTRACT: Climate projections obtained by running global climate models (GCMs) are subject to multisource uncertainties. The existing framework based on analysis of variance (ANOVA) for decomposing such uncertainties is unable to include the interaction effect between GCM and internal climate variability, which ranks only second to the main effect of GCM in significance. In this study, a three-way ANOVA framework is presented, and all main effects and interaction effects are investigated. The results show that, although the overall uncertainty (O) is mainly contributed by main effects, interaction effects are considerable. Specifically, in the twenty-first century, the global mean (calculated at the grid-cell level and then averaged, and likewise below) relative contributions of all main effects are 54% for precipitation and 82% for temperature; those of all interaction effects are, respectively, 46% and 18%. As the three-way ANOVA cannot investigate the uncertainty components resulting from uncertainty sources, it is improved by deducing the relationship between uncertainty components resulting from uncertainty sources and those resulting from the main effects and interaction effects. By the improved three-way ANOVA, O is decomposed into uncertainty components resulting from the emission scenario (S), GCM (M), and internal climate variability (V). The results reveal that O is mainly contributed by M in the twenty-first century for precipitation, and by M before the 2060s whereas by S thereafter for temperature. The robustness of the V characterization is explored by investigating the variation of V on the number of included ensemble members. The extent of the underestimation of the V contribution is roughly an average of 4% for precipitation and 1% for temperature.

KEYWORDS: Climate change; Climate prediction; Uncertainty; Climate models; Internal variability

1. Introduction

The evidence is clear that the global climate system has changed over the past century and will continue to change in the twenty-first century (Arias et al. 2021; Cox and Stephenson 2007; IPCC 2013). Therefore, a scientific communication challenge is to formulate climate change adaptation and mitigation policies, and this formulation largely relies on future climate change projections. Global climate models (GCMs), which are complex mathematical representations of processes in the ocean–atmosphere–land–biosphere–cryosphere system, are widely applied to future climate projections (Cess et al. 1996, 1990; Chou and Suarez 1994; Entekhabi and Eagleson 1989; Slingo et al. 1996; IPCC 2007, 2013; Wood et al. 1992). These climate projections, however, are subject to large and multisource uncertainties, making it hard to identify the actual climate change signals amidst the background of uncertainties (Chen and Brissette 2019; Cox and Stephenson 2007; Deser et al. 2012, 2014, 2020; Hawkins and Sutton 2009, 2011; Yip et al. 2011).

The overall uncertainty in climate change projections arises from the assumption of future greenhouse gas emission scenarios and the inaccuracy of climate models, as well as the internal variability of the climate system. Due to the unknown natural forcing and human activities of the future, in each Coupled Model Intercomparison Project (CMIP), GCMs in the projection period are forced by a set of assumed emission scenarios, such as the shared socioeconomic pathway (SSP)—representative concentration pathway (RCP) matrix in the CMIP phase 6 (CMIP6) archive (Eyring et al. 2016; O’Neill et al. 2017). However, various emission scenarios indicate different climate projections, which results in the existence of emission scenario uncertainty. Even though GCMs are state-of-the-art tools for simulating the global climate system, the simulated climate system is somewhat different from the actual climate system (H. Chen et al. 2020; Z. Chen et al. 2020; Fan et al. 2020; Tokarska et al. 2020; Vignesh et al. 2020; Zelinka et al. 2020), and this inaccuracy results in the global climate model uncertainty. To investigate this uncertainty component, a great number of GCMs were developed. The numerous GCMs result in a wide range of climate projections, among which the difference or variance can be used to characterize the global climate model uncertainty. The uncertainty component arising from the internal variability of the climate system, which is due to the natural, unforced, and chaotic fluctuations [such as El Niño–Southern Oscillation (Grothe et al. 2020), North Atlantic Oscillation (Börgel et al. 2020), and Pacific decadal oscillation (Wu et al. 2011)] in the actual climate system, is termed internal climate variability uncertainty. In the CMIP6 archive, some multimember GCM ensembles were
developed and carried out to characterize the internal variability of the climate system (Eyring et al. 2016). In one particular GCM ensemble, each member [also referred to as “realization” in previous studies, such as that by Yip et al. (2011)] is subject to the same emission scenario but begins from a slightly different initial atmosphere state (Eyring et al. 2016).

Since clarifying uncertainty sources can provide important scientific support for enhancing the credibility of future projection results and resolve relevant scientific questions for subsequent modeling applications (Miao et al. 2022), a considerable number of studies were carried out for this aim in the past two decades. For instance, in the study by Cox and Stephenson (2007), the overall uncertainty of global mean temperature projections was estimated and decomposed using a conceptual framework. In those by Hawkins and Sutton (2009, 2011), the temperature and precipitation projections were decomposed into references, signals, and residuals using a polynomial fitting approach. Then, the emission scenario uncertainty was characterized by the interscenario variance of signals; the global climate model uncertainty was characterized by the intermodel variance of signals; the internal climate variability uncertainty was characterized by the variance of residuals across all emission scenarios, climate models, and projection lead times; and the overall uncertainty was characterized by the sum of the three uncertainty components. Even though that framework was revisited and demonstrated to work well at the global scale in the study by Lehner et al. (2020), the interaction effects between or among uncertainty sources were neglected (Yip et al. 2011).

To overcome this deficiency, Yip et al. (2011) proposed a model-based framework in which the analysis of variance (ANOVA) (Hogg and Ledolter 1987; Yates 1938) was utilized. In that framework, the internal climate variability uncertainty was characterized by the intermember variance. The interaction effect between emission scenario and GCM was investigated and demonstrated to be an important uncertainty source, especially at long projection lead times (Yip et al. 2011). However, the interaction effects between or among other uncertainty sources were not explored, since the ANOVA used is a two-way ANOVA in which the analysis of variance is incapable of decomposing the uncertainty components resulting from the three sources.

Motivated by the abovementioned reasons, the purposes of this study are to 1) introduce the framework of three-way ANOVA, which is capable of illuminating the uncertainty components resulting from all main effects and interaction effects, and 2) improve this framework to estimate and decompose the uncertainty components resulting from the three uncertainty sources (emission scenario, GCM, and internal climate variability). The main goals are achieved through the following steps: 1) investigating whether the interaction effect between GCM and internal climate variability is considerable; 2) comparing the framework of improved three-way ANOVA with those proved by Yip et al. (2011), Hawkins and Sutton (2009), and Lehner et al. (2020) to confirm its reasonability; 3) using the improved three-way ANOVA framework to estimate and decompose the uncertainty of climate projections; and 4) investigating whether the internal climate variability uncertainty and global climate model uncertainty are robustly characterized or not.

2. Materials and methods

a. Data source

In this study, the climate projections derived from 3 emission scenarios, 17 GCMs, and 2–50 ensemble members (or realizations) in the CMIP6 archive (Eyring et al. 2016) are utilized. They are all downloaded from the Earth System Grid Federation node at https://esgf-node.llnl.gov/search/cmip6/. The three future emission scenarios are SSP1-2.6, SSP2-4.5, and SSP5-8.5, respectively, representing a low-concentration scenario, a medium stabilization concentration scenario, and a high-concentration scenario (O’Neill et al. 2017). The 17 GCMs (Table 1) are forced by the historical forcing during the period 1950–2014 and by the three emission scenarios during the period 2015–2100. For one particular emission scenario, the ensemble members (Table 1) of a GCM are subject to identical radiative forcing but start from slightly different atmosphere states (Eyring et al. 2016). The reason for using fewer GCMs than other studies (e.g., Hawkins and Sutton 2009, 2011; Zhuang et al. 2019; Lehner et al. 2020) is because there are only...
17 multimember GCM ensembles including at least 2 members for each radiative forcing in the CMIP6 archive (last access: 10 July 2023). Although previous studies (Chen et al. 2016; Wang et al. 2018) pointed out that about 10 GCMs are generally required to robustly characterize the global climate model uncertainty, the influence of lesser GCMs on this characterization will be investigated in this study.

Among the GCMs listed in Table 1, there are four single-model initial-condition large ensembles (SMILEs). The first SMILE is the Australian Community Climate and Earth System Simulator including Earth System Model version 1.5 (ACCESS-ESM1-5) (Ziehn et al. 2020) developed by the Commonwealth Scientific and Industrial Research Organization. There are 40 members ranging from “r1i1p1f1” to “r4i1p1f1” in this SMILE. The second SMILE is the Canadian Earth System Model version 5 (CanESM5) (Swart et al. 2019) developed by the Canadian Centre for Climate Modelling and Analysis. CanESM5 submitted 25 realizations for the historical and tier 1 SSP experiments for each of the “p1” and “p2” model variants, for a total of 50 realizations. The third SMILE is the Model for Interdisciplinary Research on Climate version 6 (MIROC6) (Tatebe et al. 2019) cooperatively developed by the Center for Climate System Research, University of Tokyo, Japan Agency for Marine-Earth Science and Technology, and the National Institute for Environmental Studies. There are 50 members ranging from “r1i1p1f1” to “r50i1p1f1” in this SMILE. The last SMILE is the Max Planck Institute for Meteorological Earth System Model version 1.2 Low Resolution (MPI-ESM1-2-LR) (Mauritsen et al. 2019) developed by the Max Planck Institute for Meteorology. There are 30 members ranging from “r1i1p1f1” to “r30i1p1f1” in this SMILE. All ensemble members in the four SMILEs are forced by the historical forcing during the period 1950–2014 and by the SSP1-2.6, SSP2-4.5, and SSP5-8.5 emission scenarios during the period 2015–2100. More information about the four SMILEs can be found in their references.

### Methods

#### 1) Climate Indices

Since precipitation and temperature are the most highly scrutinized and concerned climate variables in climate change studies (Arias et al. 2021; IPCC 2013), the uncertainties of annual mean precipitation (P; mm day$^{-1}$) and annual mean near-surface air temperature (T; °C) projections are investigated in this study. The two variables are individually calculated for each year at each GCM grid scale and then interpolated into a regular 1° longitude × 1° latitude grid using the bilinear interpolation approach.

The mean relative change (for P; %) or change (for T; °C) is calculated using a 10-yr moving average with the reference period being 1971–2000, following the studies by Hawkins and Sutton (2009, 2011), Yip et al. (2011), and Seneviratne and Hauser (2020). It is noteworthy that the use of anomalies (deviations of climate variables from their normal values) narrows the uncertainty of the original values of the model outputs by removing systematic error (Stroback and Bel 2017) so that the global climate model uncertainty will not be inundated by systematic error. In addition, the use of the 10-yr moving average weakens the interannual variation so that the interdecadal variation (interdecadal variation is more concerned in climate change studies) can be properly characterized.

#### 2) Uncertainty of Climate Projections

As listed in Table 1, there are 14 (10) GCMs including at least 5 (6) ensemble members (or realizations) for each forcing. As the global climate model uncertainty is usually larger than internal climate variability (Hawkins and Sutton 2009, 2011; Lehner et al. 2020; Yip et al. 2011; Zhang and Chen 2021), 3 emission scenarios, 14 GCMs, and 5 ensemble members are utilized when investigating the uncertainty of climate projections. However, to take full advantage of ensemble members and reduce the error related to the chosen sampling.

### Table 1. Information regarding the GCMs and ensemble members used in this study.

<table>
<thead>
<tr>
<th>Name of GCM</th>
<th>No. of members</th>
<th>Names of members</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACCESS-CM2</td>
<td>5</td>
<td>r1i1p1f1, r2i1p1f1, r3i1p1f1, r4i1p1f1, r5i1p1f1</td>
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<tr>
<td>ACCESS-ESM1-5</td>
<td>40</td>
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<tr>
<td>CanESM5</td>
<td>50</td>
<td>r1i1p1f1, r1i2p2f1, r2i1p1f1, r2i1p2f1, ..., r4i1p1f1, r4i1p2f1, r5i1p1f1, r5i1p2f1</td>
</tr>
<tr>
<td>CNRM-CM6-1</td>
<td>6</td>
<td>r1i1p2f, r2i1p2f, r3i1p2f, r4i1p2f, r5i1p2f, r6i1p2f</td>
</tr>
<tr>
<td>CNRM-ESM2-1</td>
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<td>r1i1p2f, r2i1p2f, r3i1p2f, r4i1p2f, r5i1p2f</td>
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<tr>
<td>EC-Earth3</td>
<td>6</td>
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<td>EC-Earth3-Veg</td>
<td>7</td>
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<td>MIROC6</td>
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<tr>
<td>MPI-ESM1-2-LR</td>
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<td>MRI-ESM2-0</td>
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<td>NESM3</td>
<td>2</td>
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<tr>
<td>UKESM1-0-LL</td>
<td>5</td>
<td>r1i1p1f, r2i1p1f, r3i1p1f, r4i1p1f, r5i1p1f</td>
</tr>
</tbody>
</table>

b. Methods

1) Climate Indices

Since precipitation and temperature are the most highly scrutinized and concerned climate variables in climate change studies (Arias et al. 2021; IPCC 2013), the uncertainties of annual mean precipitation (P; mm day$^{-1}$) and annual mean near-surface air temperature (T; °C) projections are investigated in this study. The two variables are individually calculated for each year at each GCM grid scale and then interpolated into a regular 1° longitude × 1° latitude grid using the bilinear interpolation approach.

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process, the selection strategy raised in the study by Milinski et al. (2020) is used in this study. Specifically, the 14 GCMs of which the amount of ensemble members is no less than 5 are first selected. Then, 100 investigations are carried out independently each using the 3 emission scenarios, 14 GCMs, and 5 ensemble members, and their mean result is analyzed and displayed. In each investigation, for one particular GCM including only five members, all members are utilized; for that including more than five members, however, the used five members are selected randomly and as nonrepetitively as possible from all members of this GCM.

The uncertainty of climate projections is estimated and decomposed by the frameworks of two-way ANOVA proposed by Yip et al. (2011) and improved three-way ANOVA presented in this study, which is illuminated as follows.

(i) Two-way ANOVA

Before illuminating the framework proposed by Yip et al. (2011), a brief introduction to the original two-way ANOVA is presented. Two-way ANOVA is derived from the one-way ANOVA proposed by Yates (1938). By the two-way ANOVA, the influence of two explanatory variables (e.g., A and B) on the quantitative outcome (e.g., D) can be investigated. Before applying the two-way ANOVA, an orthogonal experiment should be carried out. In this experiment, A and B are, respectively, set to several levels (e.g., a levels and b levels), and the orthogonal combinations (a × b) of levels of A and B are carried out. Usually, this experiment is repeated several times (e.g., c times) to take the random error into account, thus deriving a × b × c combinations in total. Because there are two different explanatory variables, the effects on the outcome of a change in one variable may not depend on the level of the other variable (additive model) or may depend on the level of the other variable (interaction model). Usually, the interaction model is applied, and an F test can be carried out to determine the significance of the interaction effect (e.g., AB). The opposite of the interaction effect is the main effect, which may be termed as the individual influence of one particular explanatory variable (e.g., A or B) on the quantitative outcome (e.g., D). In the two-way ANOVA, the sum of squared deviations is decomposed as follows:

\[
SS_D = SS_A + SS_B + SS_{AB} + SS_C.
\]  

(1)

where SS_D is the sum of squared deviations of D, with the associated degrees of freedom being \(n \times b \times c - 1\); SS_A is the sum of squared deviations for the main effect of A, with associated degrees of freedom being \(a - 1\); SS_B is the sum of squared deviations for the main effect of B, with associated degrees of freedom being \(b - 1\); SS_A\(B\) is the sum of squared deviations for the interaction effect AB, with associated degrees of freedom being \((a - 1) \times (b - 1)\); and SS_C is the sum of squared deviations for random error, with associated degrees of freedom being \(a \times b \times (c - 1)\).

It should be noted that when \(c = 1\), SS_C is equal to 0. The main effects deviations (SS_A and SS_B) are calculated as in one-way interaction, just ignoring the other factor. Then, the interaction effect deviations (SS_{AB}) are calculated by using the main effects to construct the best “parallel pattern” means and then looking at the deviations of the actual cell means from the best “parallel pattern means.” If SS_{AB} is equal to 0, the interaction model is converted to an additive model. In other words, there is no interaction effect between the two explanatory variables.

After the decomposition, the F statistics of the main effects of A(F_A) and B(F_B) and the interaction effect of AB(F_{AB}) are calculated as follows:

\[
\begin{align*}
F_A &= \frac{SS_A}{a - 1} / \frac{SS_C}{a \times b \times (c - 1)} \\
F_B &= \frac{SS_B}{b - 1} / \frac{SS_C}{a \times b \times (c - 1)} \\
F_{AB} &= \frac{SS_{AB}}{(a - 1) \times (b - 1)} / \frac{SS_C}{a \times b \times (c - 1)}
\end{align*}
\]  

(2)

Then, the \(p\) value for each F statistic can be derived and the significance of each effect can be confirmed. If the interaction effect is not significant, it may be reasonable to not investigate the influence of AB on D.

In the two-way ANOVA framework proposed by Yip et al. (2011), the ensemble members (or realizations) are treated as the replicates of the orthogonal experiment, and the overall variance (equal to the mean value of squared deviations of the outcome) is termed as overall uncertainty (hereinafter \(O^{2\text{way}}\), where the superscript “\(2\text{way}\)” indicates that \(O^{2\text{way}}\) is estimated and decomposed via the two-way ANOVA framework; the same superscript is also used below) and then decomposed. Specifically, \(O^{2\text{way}}\) is decomposed into the uncertainty components resulting from the main effect of the emission scenario (hereinafter \(S^{2\text{way}}\)), the main effect of the GCM (hereinafter \(M^{2\text{way}}\)), the interaction effect between the emission scenario and GCM (hereinafter \(V^{2\text{way}}\)), and the residual (hereinafter \(R^{2\text{way}}\)). It should be noted that \(V^{2\text{way}}\) is termed as internal climate variability uncertainty in the study by Yip et al. (2011) but not in this study. The uncertainty estimating and decomposing processes in the two-way ANOVA are summarized as Eqs. (3) and (4).

The means of relative change (for \(P\)) or change (for \(T\)) \(x(s, m, r)\) are calculated as

\[
\begin{align*}
x(s, m, \bullet) &= \frac{1}{N_r} \sum_{r=1}^{N_r} x(s, m, r) \\
x(s, \bullet, m) &= \frac{1}{N_m} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} x(s, m, r) \\
x(\bullet, m, \bullet) &= \frac{1}{N_s} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} x(s, m, r) \quad \\
x(\bullet, \bullet, \bullet) &= \frac{1}{N_s \times N_m \times N_r} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} x(s, m, r)
\end{align*}
\]  

(3)

(4)
where \(x(s, m, r)\) is the relative change (for \(P\)) or change (for \(T\)) for scenario \(s\), GCM \(m\), and realization (also ensemble member) \(r\); \(x(s, m, \bullet)\) is the mean over all the members for scenario \(s\) and GCM \(m\); \(x(\bullet, m, \bullet)\) is the mean over all the GCMs and members for scenario \(s\); \(x(\bullet, \bullet, \bullet)\) is the mean over all the scenarios and members for GCM \(m\); \(x(\bullet, \bullet, \bullet)\) is the mean over all the scenarios, GCMs, and members; the \(s\), \(m\), and \(r\) terms are, respectively, the serial numbers of emission scenarios, GCMs, and ensemble members; and \(N_s\), \(N_m\), and \(N_r\) are the amounts of them.

The \(S_{\text{2way}}^\text{2way}\), \(M_{\text{2way}}^\text{2way}\), \(V_{\text{2way}}^\text{2way}\), and \(O_{\text{2way}}^\text{2way}\) are calculated as follows:

\[
\begin{align*}
S_{\text{2way}} &= \frac{1}{N_s} \sum_{s=1}^{N_s} [x(s, \bullet, \bullet) - x(\bullet, \bullet, \bullet)]^2 \\
M_{\text{2way}} &= \frac{1}{N_m} \sum_{m=1}^{N_m} [x(\bullet, m, \bullet) - x(\bullet, \bullet, \bullet)]^2 \\
SM_{\text{2way}} &= \frac{1}{N_s \times N_m} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} [x(s, m, \bullet) - x(s, \bullet, \bullet) - x(\bullet, m, \bullet) + x(\bullet, \bullet, \bullet)]^2 \\
V_{\text{2way}} &= \frac{1}{N_s \times N_m \times N_r} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} [x(s, m, r) - x(s, \bullet, \bullet)]^2 \\
O_{\text{2way}} &= \frac{1}{N_s \times N_m \times N_r} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} [x(s, m, r) - x(\bullet, \bullet, \bullet)]^2
\end{align*}
\]

(iii) Three-way ANOVA

In the three-way ANOVA presented in this study, the ensemble members are treated as the levels of internal climate variability rather than the replicates of the orthogonal experiment. The overall variance is termed as overall uncertainty (hereinafter \(O_{\text{3way}}\) where the superscript “3way” indicates that \(O_{\text{3way}}\) is estimated and decomposed via the three-way ANOVA framework; the same superscript is used below) and then decomposed as well. To be more specific, \(O_{\text{3way}}\) is decomposed into the uncertainty components resulting from the main effect of emission scenario (hereinafter \(S_{\text{3way}}\)), the main effect of GCM (hereinafter \(M_{\text{3way}}\)), the main effect of internal climate variability (hereinafter \(V_{\text{3way}}\)), the interaction effect between emission scenario and GCM (hereinafter \(SM_{\text{3way}}\)), the interaction effect between emission scenario and internal climate variability (hereinafter \(MV_{\text{3way}}\)), and the interaction effect among the emission scenario, GCM, and internal climate variability (hereinafter \(SV_{\text{3way}}\)). The uncertainty estimating and decomposing processes in the three-way ANOVA are summarized in Eqs. (5) and (6).

The means of \(x(s, m, r)\) are calculated as

\[
\begin{align*}
x(s, m, \bullet) &= \frac{1}{N_r} \sum_{r=1}^{N_r} x(s, m, r) \\
x(s, \bullet, r) &= \frac{1}{N_m} \sum_{m=1}^{N_m} x(s, m, r) \\
x(\bullet, m, r) &= \frac{1}{N_s} \sum_{s=1}^{N_s} x(s, m, r) \\
x(s, \bullet, \bullet) &= \frac{1}{N_m \times N_r} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} x(s, m, r) \\
x(\bullet, m, \bullet) &= \frac{1}{N_s \times N_r} \sum_{s=1}^{N_s} \sum_{r=1}^{N_r} x(s, m, r) \\
x(\bullet, \bullet, r) &= \frac{1}{N_s \times N_m} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} x(s, m, r) \\
x(\bullet, \bullet, \bullet) &= \frac{1}{N_s \times N_m \times N_r} \sum_{s=1}^{N_s} \sum_{m=1}^{N_m} \sum_{r=1}^{N_r} x(s, m, r)
\end{align*}
\]

where \(x(s, \bullet, r)\) is the mean over all the GCMs for scenario \(s\) and member \(r\); \(x(\bullet, m, r)\) is the mean over all the scenarios for GCM \(m\) and member \(r\); and \(x(\bullet, \bullet, r)\) is the mean over all the scenarios and GCMs for member \(r\); the meaning of other notations is the same as Eq. (3).

The \(S_{\text{3way}}\), \(M_{\text{3way}}\), \(V_{\text{3way}}\), \(SM_{\text{3way}}\), \(SV_{\text{3way}}\), \(MV_{\text{3way}}\), \(SMV_{\text{3way}}\), and \(O_{\text{3way}}\) are calculated as
As described in Eqs. (3)–(6), the formulas to calculate the overall uncertainty and the uncertainty components resulting from the main effect of emission scenario, the main effect of GCM, and the interaction effect between emission and GCM in the three-way ANOVA presented in this study are consistent with those in the two-way ANOVA proposed by Yip et al. (2011). Therefore, it can be expected that the magnitudes of \( S \), \( M \), and \( V \) in the three-way ANOVA presented in this study are consistent with those in the two-way ANOVA proposed by Yip et al. (2011). Therefore, the relationship among \( S \), \( M \), and \( V \) is as follows: \( O^{3\text{way}} = S + M + V \). (7)

When decomposing the uncertainty of \( x(s, m, r) \) via the three-way ANOVA,

\[
O^{3\text{way}} = S^{3\text{way}} + M^{3\text{way}} + V^{3\text{way}} + SM^{3\text{way}} + SV^{3\text{way}} + MV^{3\text{way}} + SMV^{3\text{way}}.
\]

Therefore, the relationship among \( S \), \( V \), \( M \), \( S^{3\text{way}} \), \( M^{3\text{way}} \), \( V^{3\text{way}} \), \( SM^{3\text{way}} \), \( SV^{3\text{way}} \), \( MV^{3\text{way}} \), and \( SMV^{3\text{way}} \) can be assumed as follows:

\[
\begin{align*}
S &= S^{3\text{way}} + a_1^{s,m,r} \times SM^{3\text{way}} + a_2^{s,m,r} \times SV^{3\text{way}} + a_3^{s,m,r} \times SMV^{3\text{way}} \\
M &= M^{3\text{way}} + a_4^{s,m,r} \times SM^{3\text{way}} + a_5^{s,m,r} \times MV^{3\text{way}} + a_6^{s,m,r} \times SMV^{3\text{way}} \\
V &= V^{3\text{way}} + a_7^{s,m,r} \times SMV^{3\text{way}} + a_8^{s,m,r} \times MV^{3\text{way}} + a_9^{s,m,r} \times SMV^{3\text{way}}.
\end{align*}
\]

where \( a_1^{s,m,r}, \ldots, a_9^{s,m,r} \) are undetermined coefficients; the superscript “s,m,r” indicates the uncertainty components are those of \( x(s, m, r) \); the subscripts 1 to 9 are the series number of coefficients, and so on below.
Likewise, for the uncertainty components of \( x(m, s, r) \), \( x(r, m, s) \), \( x(s, r, m) \), \( x(m, r, s) \), and \( x(r, s, m) \), the following relationship can be assumed:

\[
\begin{align*}
S &= S_{3way} + a_{1}^{m,s,r} \times MS_{3way} + a_{2}^{m,s,r} \times SV_{3way} + a_{3}^{m,s,r} \times MVS_{3way} \\
M &= M_{3way} + a_{4}^{m,s,r} \times SM_{3way} + a_{5}^{m,s,r} \times MV_{3way} + a_{6}^{m,s,r} \times MSV_{3way} \\
V &= V_{3way} + a_{7}^{m,s,r} \times SV_{3way} + a_{8}^{m,s,r} \times MV_{3way} + a_{9}^{m,s,r} \times MSV_{3way} \\
&\vdots
\end{align*}
\]

where \( a_{1}^{m,s,r}, \ldots, a_{9}^{m,s,r} \) are undetermined coefficients as well.

According to Eqs. (3)–(6), the following relationship can be calculated:

\[
\begin{align*}
SM_{3way} &= MS_{3way} \\
SV_{3way} &= VS_{3way} \\
MV_{3way} &= VM_{3way} \\
SMV_{3way} &= MSV_{3way} = VMS_{3way} = SVM_{3way} = MVS_{3way} = VSM_{3way}
\end{align*}
\]

Finally, the following relationship can be obtained by solving the simultaneous equations of Eqs. (7)–(11):

\[
\begin{align*}
S &= S_{3way} + (SM_{3way} + SV_{3way})/2 + SMV_{3way}/3 \\
M &= M_{3way} + (SM_{3way} + MV_{3way})/2 + SMV_{3way}/3 \\
V &= V_{3way} + (SV_{3way} + MV_{3way})/2 + SMV_{3way}/3
\end{align*}
\]

Equations (5), (6), and (12) are the improved three-way ANOVA. By this framework, the overall uncertainty \( O_{3way} \) of climate projections can be decomposed into seven components resulting from three main effects \( S_{3way}, M_{3way}, \) and \( V_{3way} \), and four interaction effects \( SM_{3way}, SV_{3way}, MV_{3way}, \) and \( SMV_{3way} \), or into three components resulting from emission scenario \( S \), GCM \( M \), and internal climate variability \( V \). In addition, these uncertainty components are all independent of the dimension orders of \( x(s, m, r) \) or \( x(m, s, r) \), and so on.

The program code (in MATLAB) of the improved three-way ANOVA framework is available at https://www.researchgate.net/publication/374535701_anova3way.

(iv) Relative contributions of uncertainty components

The fraction of each uncertainty component to the overall uncertainty is termed the relative contribution (in %). It should be noted that the sum of relative contributions of \( S_{3way}, M_{3way}, V_{3way}, SM_{3way}, SV_{3way}, MV_{3way}, \) and \( SMV_{3way} \) is equal to 100%. Likewise, the sum of relative contributions of \( S, M, \) and \( V \) is equal to 100% as well.

3) REASONABILITY OF THE IMPROVED THREE-WAY ANOVA

To investigate the reasonability of the improved three-way ANOVA in estimating the uncertainty components resulting from the three sources, a rough comparison between this framework and those proposed by Hawkins and Sutton (2009, hereinafter HS09) and Lehner et al. (2020, hereinafter Le20) is carried out. To reduce the systematic error caused by the difference among the used datasets, datasets that are as similar as possible are utilized in this section. Specifically, in the framework of improved three-way ANOVA, outputs of the first and last ensemble members for each of the 17 GCMs (Table 1) under the 3 emission scenarios are utilized. In the framework of HS09, outputs of the first member for each of the 17 GCMs under the 3 emission scenarios are utilized. In the framework of Le20, the magnitudes of \( S \) and \( M \) are equal to those in the HS09 and the magnitude of \( V \) is calculated using the 50 members of CanESM5 under the 3 emission scenarios. More information about the frameworks of HS09 and Le20 can be found in their references (Hawkins and Sutton 2009; Lehner et al. 2020). However, it should be noted that this comparison is imperfect to a large extent since the required dataset of one particular framework is largely different from that of another one.

4) ROBUSTNESS OF THE V AND M CHARACTERIZATIONS

In this study, only 5 ensemble members for each of the 14 GCMs are included when investigating the uncertainty of climate projections. Therefore, the characterizations of \( V \) and \( M \) may be less robust.

To explore the robustness of the \( V \) characterization, the variation of \( V \) on the number of included members is investigated. More specifically, the uncertainty of climate projections, derived from the orthogonal combinations of three emission scenarios (SSP1-2.6, SSP2-4.5, and SSP5-8.5), four GCMs (ACCESS-ESM1-5, CanESM5, MIROC6, and MPI-ESM1-2-LR), and various numbers (2, 3, ..., and 30) of included members selected
from the four SMILEs, is estimated and decomposed via the improved three-way ANOVA [Eqs. (5), (6), (12)]. When choosing the N included members, however, it may be impossible to traverse all combinations of the included members, since the number of combinations can be extremely large. For instance, there are more than $1.26 \times 10^{14}$ combinations when choosing 25 included members from the 50 members of CanESM5. Therefore, only 100 random combinations are traversed for each number of included members. Subsequently, the mean magnitude and relative contribution of $V$ across all meteorological grids and projection lead times (2020–96) are calculated for each combination in each number of included members. Then, the variations of magnitudes and contributions on the number of included members are illuminated, and whether the characterization of $V$ is robust or not can be determined.

A similar investigation is carried out to determine the robustness of the $M$ characterization. However, the used climate projections are converted to those derived from the orthogonal combinations of three emission scenarios, various numbers (2, 3, …, and 17) of GCMs (all listed in Table 1), and two ensemble members (the first and last one for each GCM). And the random selection of included members is converted to that of included GCMs.

### 3. Results

#### a. Interaction effect between GCM and internal climate variability

To investigate the interaction effect between GCM and internal climate variability, the global mean relative change of $P$ and change of $T$ in a projection period (2021–50) relative to the reference period (1971–2000) are calculated for five ensemble members (from “r1i1p1f1” to “r5i1p1f1”) of six GCMs (ACCESS-CM2, ACCESS-ESM1-5, CanESM5, MIROC6, MPI-ESM1-2-LR, and MRI-ESM2-0) in the three emission scenarios and displayed in Fig. 1, following the second figure in the study by Yip et al. (2011). It should be noted that, in this study, the global mean values are all calculated at the grid-cell level and then averaged, which is a bit different from what Hawkins and Sutton (2009) and Lehner et al. (2020) did (calculating global mean $P$ and $T$ and then average the global mean values). This difference is due to the fact that what is analyzed in this study are the $P$ and $T$ themselves rather than the global mean $P$ and $T$. The results reveal that most lines in each subgraph are not parallel, indicating that the intermember difference is largely GCM-dependent. Taking the mean relative change of $P$ under the SSP5-8.5 scenario (c: $P$/SSP5-8.5) for instance, ACCESS-ESM1-5 is 2.2% lower than MRI-ESM2-0 for “r1i1p1f1,” but 1.7% higher than it for “r5i1p1f1.” Similar instances are also witnessed in other subgraphs. Therefore, there is an apparent interaction effect between GCM and internal climate variability.

In addition, the significance of the main effects or interaction effects at the 0.05 significance level is investigated when estimating and decomposing the uncertainty of climate projections via the three-way ANOVA (Hogg and Ledolter 1987; Yates 1938). The significance of the interaction effect among emission scenario, GCM, and internal climate variability is not investigated, since the significance test in ANOVA cannot be performed when the variance is completely decomposed.
The significance percentage of the main effects or interaction effects across all meteorological grids and projection lead times (2020–96) is presented in Table 2. The results show that the significance percentage of the interaction effect between GCM and internal climate variability is up to 91.4% for $P$ and 97.6% for $T$. In addition, this interaction effect ranks only second to the main effect of GCM in significance. Therefore, the interaction effect between GCM and internal climate variability is considerable. Further, it is more reasonable to use the three-way ANOVA, which is capable of accounting for all main effects and interaction effects, when investigating the uncertainty of climate projections.

Mathematically, interaction is said to occur if the separate effects do not combine additively (de González and Cox 2007; Yip et al. 2011). In essence, it occurs because the response of one particular factor (e.g., GCM) to another factor (e.g., emission scenario) is choice-dependent. In terms of the interaction effect between GCM and internal climate variability, it occurs since the intermember difference simulated by one particular GCM is not identically consistent with that simulated by another GCM (Lehner et al. 2020). For instance, in the projection lead times (2020–96), the global mean intermember variances simulated by the four SMILEs (ACCESS-ESM1-5, CanESM5, MIROC6, and MPI-ESM1-2-LR) are, respectively, 55.0%, 39.7%, 48.4%, and 96.5% for $P$, and 0.15%, 0.16%, 0.11%, and 0.09% for $T$. This difference is because the initial conditions of ensemble members for one particular GCM are not identically consistent with those for another GCM (Deser et al. 2020; Eyring et al. 2016; Lehner et al. 2020) on the one hand. On the other hand, it is also because the responses of different GCMs to identical initial conditions may differ from one another since the climate projections are largely GCM-dependent (H. Chen et al. 2020; Z. Chen et al. 2020; Fan et al. 2020; Tokarska et al. 2020; Xin et al. 2020; Zelinka et al. 2020).

### b. Uncertainty of climate projections

#### 1) Uncertainty resulting from the main effects and interaction effects

(i) Uncertainty magnitudes

When investigating uncertainty magnitudes, the similarity and difference between the two-way ANOVA proposed by Yip et al. (2011) and the three-way ANOVA presented in this study are also illuminated. To demonstrate the conjecture that the magnitudes of $S_{3\text{way}}$, $M_{3\text{way}}$, $SM_{3\text{way}}$, and $O_{3\text{way}}$ decomposed via the three-way ANOVA are, respectively, equal to those of $S_{2\text{way}}$, $M_{2\text{way}}$, $SM_{2\text{way}}$, and $O_{2\text{way}}$ decomposed via the two-way ANOVA, the global mean magnitudes of them are calculated and displayed in Fig. 2. The results reveal that this conjecture is correct for each climate variable in each projection lead time. In addition, the uncertainty magnitudes generally increase along with the projection lead times, indicating that the climate projections generally diverge with time. This is consistent with most previous studies (Cox and Stephenson 2007; Deser et al. 2012; Hawkins and Sutton 2009, 2011; Knutti et al. 2008; Lehner et al. 2020; Xu et al. 2019; Yip et al. 2011; Zhang and Chen 2021; Zhou et al. 2020), indicating that the three-way ANOVA is reasonable in estimating the magnitudes of these uncertainty components. For $P$, $SM_{3\text{way}}$ is lower than $M_{3\text{way}}$ and higher than $S_{3\text{way}}$ throughout the twenty-first century. For $T$, $M_{3\text{way}}$ is higher than $S_{3\text{way}}$ before 2079, whereas an opposite pattern is observed thereafter; $SM_{3\text{way}}$ is higher than $S_{3\text{way}}$ before 2035 while lower

### Table 2. The significance (0.05 level) percentage of the main effects or interaction effects across all meteorological grids and projection lead times (2020–96).

<table>
<thead>
<tr>
<th>Effect</th>
<th>Main effect</th>
<th>Interaction effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Emission scenario (%)</td>
<td>GCM (%)</td>
</tr>
<tr>
<td>$P$</td>
<td>64.4</td>
<td>99.8</td>
</tr>
<tr>
<td>$T$</td>
<td>88.3</td>
<td>100.0</td>
</tr>
</tbody>
</table>

![Fig. 2](image-url)
than $S_{3\text{way}}$ since then, and it is always lower than $M_{3\text{way}}$ in the twenty-first century.

To demonstrate the conjecture that $V_{2\text{way}}$ is equal to the sum of $V_{3\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$, the global mean magnitudes of $V_{3\text{way}}$, $V_{2\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$ are calculated and displayed in Fig. 3. The results reveal that this conjecture is also correct for each climate variable in each projection lead time. For $P$, $M_{3\text{way}}$ and $SMV_{3\text{way}}$ are generally comparable, and they are apparently higher than the $SM_{3\text{way}}$ and $V_{3\text{way}}$, which are comparable as well. For $T$, $MV_{3\text{way}}$ is about 2.6 times that of $SMV_{3\text{way}}$, $V_{3\text{way}}$ and $SV_{3\text{way}}$ are generally comparable, and they are all lower than $SM_{3\text{way}}$. In addition, $V_{3\text{way}}$, $V_{2\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$ are generally time-variant, indicating that it may be more reasonable to estimate these uncertainty components for each time slice in the projection period. However, the temporal variabilities of these uncertainty components are much smaller than those of $S_{3\text{way}}$, $M_{3\text{way}}$, $SM_{3\text{way}}$, and $O_{3\text{way}}$. Similar results are also observed in other studies (Lehner et al. 2020; Yip et al. 2011; Zhou et al. 2020; Zhuan et al. 2019), indicating the robustness of this conclusion.

Overall, the three-way ANOVA presented in this study is consistent with the two-way ANOVA proposed by Yip et al. (2011) in estimating the $S_{3\text{way}}$, $M_{3\text{way}}, SM_{3\text{way}}, SM_{3\text{way}}/SM_{2\text{way}}$, and $O_{3\text{way}}/O_{2\text{way}}$ on the other hand, the $V_{3\text{way}}$ estimated in the two-way ANOVA is further decomposed into the $V_{3\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$ in the three-way ANOVA.

(ii) Uncertainty contributions

The global mean relative contribution of each uncertainty component to $O_{3\text{way}}$ is calculated and displayed in Fig. 4. The results reveal that, for $P$, $O_{3\text{way}}$ is mainly contributed by the interaction effects ($SM_{3\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$) before 2048 and by the main effects ($S_{3\text{way}}$, $M_{3\text{way}}$ and $V_{3\text{way}}$) since then. More specially, the sum of the relative contributions of interaction effects decreases from 68.5% in 2020 to 31.2% in 2096. Oppositely, that of main effects increases from 31.5% in 2020 to 68.8% in 2096. For $T$, $O_{3\text{way}}$ is mainly contributed by the main effects, whereas the sum of the relative contributions of interaction effects is nonnegligible. To be more specific, the sum of relative contributions of main effects increases from 56.6% in 2020 to 87.9% in 2060 rapidly and then to 94.0% in 2096 gradually. On the contrary, that of interaction effects decreases from 43.4% in 2020 to 12.1% in 2060 rapidly and then to 6.0% in 2096 gradually. Overall, the results highlight that it is necessary to account for the interaction effects, although the overall uncertainty is mainly contributed by the main effects.

In terms of the main effects, for $P$, $M_{3\text{way}}$ is the most important uncertainty source throughout the projection period; the relative contribution of $S_{3\text{way}}$ is lower than that of $V_{3\text{way}}$ before 2041 and overtakes it thereafter. Specifically, the relative contribution of $M_{3\text{way}}$ increases from 28.5% in 2020 to 47.6% in 2067 and then decreases to 39.5% in 2096. That of $S_{3\text{way}}$ increases from 0.5% in 2020 to 28.6% in 2096. That of $V_{3\text{way}}$, however, decreases from 2.4% in 2020 to 0.8% in 2096. For

![Fig. 3. The global mean magnitudes of $V_{2\text{way}}$, $V_{3\text{way}}$, $SV_{3\text{way}}$, $MV_{3\text{way}}$, and $SMV_{3\text{way}}$ for (a) $P$ and (b) $T$.](image1)

![Fig. 4. The global mean relative contributions of uncertainty components to the overall uncertainty for (a) $P$ and (b) $T$.](image2)
$T$, $M^{3\text{way}}$ is the most important uncertainty source before 2068 and ranks only second to $S^{3\text{way}}$ since then. Specifically, the relative contribution of $M^{3\text{way}}$ first increases from 54.5% in 2020 to 67.6% in 2038 and then decreases to 25.2% in 2096, which is similar to that in the study by Yip et al. (2011) (increasing from 73% in 2005 to 85% in 2020, then decreasing to 38% in 2065 rapidly and to 29% in 2095 gradually). That of $S^{3\text{way}}$ increases from 0.3% in 2020 to 68.7% in 2096, which is very close to that in the study by Yip et al. (2011) (increasing from 0% in 2005 to 1% in 2022 gradually and then to 66% in 2095 rapidly). That of $V^{3\text{way}}$ decreases from 1.8% in 2020 to 0.1% in 2096. The ranks of uncertainty contributions for the two climate variables are generally consistent with those in previous studies (Cox and Stephenson 2007; Deser et al. 2012; Hawkins and Sutton 2009, 2011; Lehner et al. 2020; Wu et al. 2022; Xu et al. 2019; Yip et al. 2011), indicating that these conclusions are reasonable. However, the relative contribution of $V^{3\text{way}}$ presented in this study is apparently lower than that of $V^{3\text{way}}$ in the study by Yip et al. (2011). This is expected, since $V^{3\text{way}}$ is further decomposed into $V^{3\text{way}}, SV^{3\text{way}}, MV^{3\text{way}},$ and $SMV^{3\text{way}}$ in this study.

In terms of the interaction effects, for $P$, the uncertainty is mainly contributed by $MV^{3\text{way}}$ and $SMV^{3\text{way}}$ before 2089 whereas by $MV^{3\text{way}}$ and $SM^{3\text{way}}$ thereafter. Specifically, the relative contributions of $MV^{3\text{way}}$ (decreasing from 32.2% in 2020 to 9.7% in 2096) and $SMV^{3\text{way}}$ (decreasing from 27.2% in 2020 to 9.3% in 2096) display a rapid decreasing trend along with the projection lead times. However, the opposite is true for the relative contribution of $SM^{3\text{way}}$ (increasing from 7.0% in 2020 to 11.4% in 2096). For $T$, the most important uncertainty source is $MV^{3\text{way}}$ before 2072 but $SM^{3\text{way}}$ since then. To be more specific, the relative contribution of $MV^{3\text{way}}$ (decreasing from 26.2% in 2020 to 1.3% in 2096) is higher than that of $SMV^{3\text{way}}$ (decreasing from 12.8% in 2020 to 0.7% in 2096) throughout the projection period. That of $SM^{3\text{way}}$ ranges between 2.4% and 4.0% in the projection period. That of $V^{3\text{way}}$ is the smallest for both $P$ (0.7%–2.2%) and $T$ (0.1%–1.0%). Overall, it is of great importance to account for $MV^{3\text{way}}$, especially in the early projection lead times.

2) UNCERTAINTY RESULTING FROM THE THREE SOURCES

(i) Temporal variability of uncertainty magnitudes and contributions

To investigate the temporal variability of uncertainty components resulting from emission scenario, GCM, and internal climate variability, the global mean magnitudes and relative contributions of $S$, $M$, and $V$ decomposed via the improved three-way ANOVA are calculated and displayed in Fig. 5.

In terms of the uncertainty magnitudes, both $S$ and $M$ exhibit a rapidly increasing trend along with the projection lead times for $P$ and $T$. Specifically, the magnitudes of $S$ and $M$ in 2096 are, respectively, 48.0 (205.5) and 18.6 (6.9) times those in 2020 for $P(T)$. The magnitude of $V$, however, is largely variable-dependent. For $P$, it first increases from 24.2%² in 2020 to 45.7%² in 2069 gradually and then increases to 118.1%² in 2089 rapidly, whereas it decreases to 101.3%² in 2096 rapidly. This general increasing trend is similar to that obtained by Pendergrass et al. (2017) and can be interpreted as being related to an increase in moisture that is partially mitigated by weakening circulation (Pendergrass et al. 2017). For $T$, it increases from 0.080°C² in 2020 to 0.104°C² in 2043 first and then decreases to 0.067°C² in 2085, whereafter it finally increases to 0.072°C² in 2096. This general stable (or slightly decreasing) trend is consistent with most previous studies (e.g., Hawkins and Sutton 2009; Yip et al. 2011; Lehner et al. 2020). However, it is worth mentioning that a likely decline trend of low-frequency global mean surface air temperature (GMST)
variability in a warmer climate is observed in the study by Brown et al. (2017). In addition, Brown et al. (2017) found that unforced GMST variability is dependent on the background climatological conditions, and thus, climate model control simulations run under perpetual preindustrial conditions may have only limited relevance for understanding the unforced GMST variability of the future. In other words, it may be worth further effort to investigate the trend of internal climate variability for temperature, which is beyond the scope of this study.

In terms of the relative contributions, for \( P \) and \( T \), \( S \) exhibits a stable and then increasing tendency; \( M \) exhibits a first increasing and then decreasing tendency; and \( V \) goes down with time in the projection period. To be more specific, for \( P \), the relative contribution of \( S \) ranges between 12.9% and 14.3% during the period 2020–50 and then increases to 37.8% in 2096; that of \( M \) first increases from 57.2% in 2020 to 66.0% in 2052 and then decreases to 53.1% in 2096; that of \( V \) decreases from 28.6% in 2020 to 9.1% in 2096. For \( T \), the relative contribution of \( S \) ranges between 6.4% and 7.6% during the period 2020–35 and then increases to 70.9% in 2096 rapidly; that of \( M \) increases from 73.6% in 2020 to 79.6% in 2035 and then decreases to 28.1% in 2096; that of \( V \) decreases from 19.7% in 2020 to 1.0% in 2096. In terms of the whole projection period, the mean relative contribution of \( S \) for \( T \) (34.6%) is slightly higher than it is for \( P \) (20.6%), which is probably because the temperature is more sensitive to different emission scenarios (Papalexiou et al. 2020). However, an opposite pattern is observed for the relative contribution of \( V \) (an average of 17.7% for \( P \) and 7.1% for \( T \)). Similar results are found in other studies (Wu et al. 2022; Zhang and Chen 2021) as well. In short, the overall uncertainty is mainly contributed by global climate model uncertainty throughout the projection period for \( P \). For \( T \), it is mainly contributed by global climate model uncertainty before 2069, whereas by emission scenario uncertainty since then.

(ii) Spatial patterns of uncertainty contributions

To investigate the spatial patterns of uncertainty contributions in different projection lead times, the projection periods 2031–50, 2051–70, and 2071–90 are chosen to represent the near, middle, and far futures. Then, the multidecadal mean relative contribution of each uncertainty component to the overall uncertainty in each projection period is calculated and displayed in Fig. 6 for \( P \) and Fig. 7 for \( T \). The results reveal that the uncertainty contributions generally exhibit similar spatial variability patterns in different projection periods. To put it in another way, the temporal variability of one particular uncertainty contribution does not alter its overall spatial variability pattern.

For \( P \), the relative contribution of \( S \) is generally higher in the Arctic Ocean in the north of North America, Hudson Bay and Strait, the middle Atlantic Ocean in the northeast of South America, the southern Pacific Ocean in the west of Chile, the Indian Ocean in the southwest of Oceania, most of Southern Ocean, and part of Antarctica, and is lower in the Arctic Ocean in the north of Kamchatka Peninsula, the North Atlantic Ocean.
south of Greenland, eastern Europe north of the Black Sea and north of the Caspian Sea, the middle Atlantic Ocean in the west of central Africa, and the Indian Ocean between Indonesia and Australia. An opposite spatial pattern is observed for the relative contribution of $M$. The disorganized spatial variability patterns of the relative contributions of $S$ and $M$ are mainly due to the complex and inconsistent response of precipitation in different regions to the booming emission of greenhouse gases (Arias et al. 2021; IPCC 2007, 2013). The relative contribution of $V$ is generally higher in the midlatitudes and lower in the low and high latitudes. In comparison with those of $S$ and $M$, the relative contribution of $V$ exhibits a more regular spatial variability pattern. This is because $V$ is inherent to the climate system and is less affected by external forcing.

For $T$, the relative contribution of $S$ is generally higher in the low and middle latitudes except for central Africa, and lower in the Arctic Ocean, North Atlantic Ocean, and Southern Ocean. The spatial variability pattern of the $M$ contribution is almost contrary to that of the $S$ contribution. In comparison with $P$, the spatial variability patterns of the relative contributions of $S$ and $M$ are more regular for $T$. This is mainly because the response of temperature in various regions to the increasing radiative forcing is more harmonious than that of precipitation (Arias et al. 2021; IPCC 2007, 2013). The relative contribution of $V$ is lower in the low latitudes and part of the North Pole, and higher in the middle latitudes, the North Atlantic Ocean or Arctic Ocean near the Kamchatka Peninsula, and part of the Southern Ocean and Antarctic polar region.

c. Reasonability of the improved three-way ANOVA

To investigate the reasonability of the improved three-way ANOVA, a comparison of this framework with the HS09 and Le20 frameworks is carried out, and the global mean magnitudes and relative contributions of uncertainty components estimated via the three frameworks are calculated and displayed in Fig. 8 for $P$ and Fig. 9 for $T$. The results reveal that, for $T$, the magnitudes and relative contributions of uncertainty components in the improved three-way ANOVA are very close to those in the HS09 or Le20 in general. For instance, the magnitudes of $S$ and $O$ in the improved three-way ANOVA are almost equal to those in the HS09 and Le20 before 2057 and slightly higher than them since then. The magnitude of $M$ in the improved three-way ANOVA is very close to those in the HS09 and Le20 before 2057 and slightly higher than them since then. The relative contribution of $S$ in the improved three-way ANOVA is slightly higher than those in HS09 or Le20 before 2048 and almost equal to them afterward. The relative contributions of $M$ and $V$ in the improved three-way ANOVA are very close to those in HS09, especially in the long projection lead times. The relative contribution of $S$ in the improved three-way ANOVA is very close to those in HS09 or Le20 before 2057 and slightly higher than them since then. The magnitude of $V$ in the improved three-way ANOVA is between those in HS09 and Le20 for all projection lead times. The relative contribution of $S$ in the three-way ANOVA is slightly higher than that in HS09 and Le20 before 2048 and almost equal to them afterward. The relative contributions of $M$ and $V$ in the improved three-way ANOVA are very close to those in HS09, especially in the long projection lead times. For $P$, the uncertainty magnitudes or contributions in the improved three-way ANOVA are generally similar to those in HS09 or Le20. For instance, the magnitudes of $M$ and $O$ in the improved three-way ANOVA are very close to those in HS09 and Le20 before the 2060s. The relative contribution of $M$ in the three-way ANOVA is slightly higher than that in

![Figure 7](image-url)
HS09 before the 2040s and very close to it since then. However, differences in the uncertainty magnitudes and contributions between the improved three-way ANOVA and HS09 or Le20 are observed. For instance, the magnitude and relative contribution of $S$ in the improved three-way ANOVA are higher than those in HS09 or Le20 throughout the projection period. The magnitudes of $M$ and $O$ in the improved three-way ANOVA are slightly higher than those in HS09 and Le20 after the 2060s. The magnitude and relative contribution of $V$ in the improved three-way ANOVA are generally lower than those in HS09 and Le20. Even so, these differences can be termed small and reasonable. For instance, the mean difference in the $S$ contribution between the improved three-way ANOVA and HS09 is an average of 9.7%, which is slightly smaller than the difference in the $M$ contribution between HS09 and Le20 (an average of 9.9%). The mean difference in the $V$ contribution between the improved three-way ANOVA and HS09 is an average of 11.1%, which is slightly smaller than that between HS09 and Le20 (an average of 11.6%) as well. The smaller role of $V$ in the improved three-way ANOVA than those in HS09 and Le20 is mainly due to the fact that only two ensemble members are utilized in the improved three-way ANOVA. Using more ensemble members will enlarge the role of $V$ but will also enlarge the systematic error related to the difference among the used datasets at the same time. Overall, the improved three-way ANOVA is reasonable in estimating the uncertainty components resulting from the three uncertainty sources.

d. Robustness of the $V$ and $M$ characterization

To investigate the robustness of the $V$ characterization, the variation of $V$ on the number of included ensemble members is explored. Specifically, the mean magnitudes and relative contributions of $V$ across all meteorological grids and the projection lead times (2020–96) are calculated for each combination of included members in each number of members. Afterward, the probability distribution of the mean magnitudes or relative contributions of $V$ of all combinations is displayed via a boxplot for each number of included members and each climate variable in Fig. 10. The results reveal that when the number of included members is lower than five, the mean magnitudes and relative contributions of $V$ increase rapidly when more members are included. In other words, the magnitudes and relative contributions of $V$ may be largely underestimated when the number of included members is lower than five. When the number of included members increases from 5 to 10, the mean magnitudes and relative contributions of $V$ remain increasing, but the increasing ratios are much gentler. When 10–15 members are included, the increases in magnitudes and relative contributions of $V$ are almost negligible. Therefore, it can be concluded that the magnitudes and relative contributions of $V$ illuminated in section 3b(2) are underestimated to some extent since only five members for each GCM are included. Nevertheless, the extent of this underestimation can
be quantified roughly. Specifically, the underestimating extent of the magnitudes (relative contributions) of V is an average of 5.3%±(3.5%) for P and 0.01°C² (1.0%) for T.

Similarly, the variation of M on the number of included GCMs is explored to investigate the robustness of the M characterization. The probability distribution of the mean M magnitudes or contributions of all combinations for each number of included GCMs is displayed in Fig. 11. The result shows that, when the number of GCMs is higher than 13, the increases in magnitudes and relative contributions of M are very small for both P and T. Therefore, it can be concluded that the characterization of M in section 3b(2) is robust since 14 GCMs are included.

4. Discussion

In this study, a new framework based on the improved three-way ANOVA is presented to estimate and decompose the uncertainty of climate projections. In comparison with the frameworks proposed by Hawkins and Sutton (2009) and Lehner et al. (2020), this framework is capable of accounting for the interaction effects between or among the three uncertainty sources. In comparison with the two-way ANOVA proposed by Yip et al. (2011), this framework is not only capable of accounting for each main effect and interaction effect but also capable of estimating uncertainty components resulting from the three sources. Even though the magnitudes and relative contributions of V are demonstrated to be underestimated, the extent of this underestimation can be quantified roughly. Therefore, the improved three-way ANOVA is a promising framework for estimating and decomposing the uncertainty of climate projections.

The past few years have witnessed an explosion in the development and usage of SMILEs, in which the amounts of ensemble members generally range between 10 and 100 (Chen and Brissette 2019; J. Chen et al. 2020; Chen et al. 2019, 2021; Deser et al. 2020; Gu et al. 2019; Lehner et al. 2020; Maher et al. 2018; Zhou et al. 2020; Zhu et al. 2018, 2019). And it has been widely accepted that the usage of multiple SMILEs facilitates a more robust characterization of the internal climate variability (Deser et al. 2020; Lehner et al. 2020; Milinski et al. 2020). Even though the amount of SMILEs is small so far (Deser et al. 2020; Lehner et al. 2020), more SMILEs will be developed and carried out in the foreseeable future. In addition, the improved three-way ANOVA is capable of utilizing multiple SMILEs. Therefore, a more robust estimation of the internal climate variability uncertainty will come true with the advance of SMILEs and the improved three-way ANOVA presented in this study.

The improved three-way ANOVA can also be generalized into an improved multiway ANOVA that can be utilized to estimate and decompose the uncertainty propagating along with the impact modeling chains. In previous studies (Chegwidden et al. 2019; Troin et al. 2018; Wang et al. 2020; Zhang et al. 2022), the used frameworks are based on traditional multiway ANOVA, and the overall uncertainty is decomposed into uncertainty components resulting from the main effects of all uncertainty sources and the interaction effects between or among

FIG. 9. As in Fig. 8, but for T.
them. However, the number of uncertainty components can be very large, which may make it hard to analyze the result. Taking that based on traditional six-way ANOVA, for instance, there are up to 63 (6 main effects and 57 interaction effects) uncertainty components. If the sum of relative contributions of interaction effects is comparable with or larger than that of main effects, it can be almost impossible to determine the relative importance of the uncertainty sources. If using frameworks based on the improved multiway ANOVA, however, the uncertainty component resulting from each uncertainty source can be illustrated and the rank of the uncertainty sources can be determined easily. Therefore, the improved Multiway ANOVA may be widely used to investigate the uncertainty propagating along with the impact modeling chains.

Apart from the improved three-way ANOVA, the interaction effects are noteworthy as well. Even though the interaction effect between emission scenario and GCM was emphasized to be an important contribution to the overall uncertainty in the study by Yip et al. (2011), interaction effects are usually omitted in most studies (e.g., Lehner et al. 2020; Wu et al. 2022). This is probably because the relative contribution of the interaction effect between emission scenario and GCM is too small to affect the major conclusions (Reintges et al. 2017; Woldemeskel et al. 2016; Yip et al. 2011). It should be acknowledged that this conclusion is also demonstrated to be reasonable in this study (the relative contribution of this interaction effect is an average of 7.7% for precipitation and 3.1% for temperature). However, the interaction effect between GCM and internal climate variability is found to be only next to the main effect of GCM in significance. And the relative contribution of this interaction effect is an average of 19.6% for precipitation and 9.2% for temperature. In addition, there are four interaction effects in total, and their sum contribution (an average of 46.4% for precipitation and 17.6% for temperature) can be even comparable to that of all main effects (an average of 53.6% for precipitation and 82.4% for temperature). Overall, it is more reasonable to take the interaction effects into consideration when investigating the uncertainty of climate projections, especially for precipitation.

5. Conclusions

This study is inspired by the discovery that the interaction effect between GCM and internal climate variability is considerable. To account for this interaction effect, the three-way ANOVA, which is capable of accounting for all main effects and interaction effects, is presented. More importantly, it is further improved to estimate the uncertainty components resulting from emission scenario, GCM, and internal climate variability. The following conclusions can be drawn:
1) It is of great importance to take the interaction effect between GCM and internal climate variability into consideration since it ranks only second to the main effect of GCM in significance.

2) Even though the overall uncertainty is mainly contributed by the main effects, it is necessary to account for the interaction effects, especially for $P$. Specifically, the sum of relative contributions of all main effects (interaction effects) is an average of 54% (46%) for $P$ and 82% (18%) for $T$, respectively.

3) In terms of the uncertainty magnitudes, both $S$ and $M$ exhibit a rapidly increasing trend along with the projection lead times for $P$ and $T$, whereas $V$ exhibits an increasing and then decreasing (increasing in general) trend for $P$ and an increasing–decreasing–increasing (stable in general) trend for $T$. In terms of the uncertainty contributions, the overall uncertainty is mainly contributed by $M$ in the twenty-first century for $P$, and by $M$ before the 2060s whereas by $S$ thereafter for $T$. Specifically, for $P$, the mean relative contributions of $S$, $M$, and $V$ are 21%, 61%, and 18%, respectively. For $T$, they are 35%, 58%, and 7%, respectively.

4) The underestimating extent of the relative contribution of $V$ is roughly an average of 4% for $P$ and 1% for $T$ in this study. The characterization of $M$ is robust.

Overall, this study highlights that it is necessary to use the improved three-way ANOVA when investigating the uncertainty of climate projections since this framework is not only able to account for the uncertainty components resulting from all main effects and interaction effects but also able to estimate uncertainty components resulting from the three uncertainty sources. In addition, more GCMs including more ensemble members are still required to robustly characterize the internal climate variability uncertainty.

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Data availability statement. The climate projections derived from the orthogonal combination of the three emission scenarios, 10 GCMs, and five ensemble members in the CMIP6 archive are available from the Earth System Grid Federation node at https://esgf-node.llnl.gov/search/cmip6/. In addition, those derived from the four SMILEs (ACCESS-ESM1-5, CanESM5, MIROC6, and MPI-ESM1-2-LR) are available from this node as well.

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