NOTES AND CORRESPONDENCE

On Obtaining Daily Climatological Values from Monthly Means

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ABSTRACT

A method is proposed for deriving daily climatological values that are consistent with a given set of monthly means or monthly totals. It involves making an adjustment to a harmonic analysis of the monthly values. The method appears to provide reasonable results even under difficult circumstances.

1. Introduction

Climatologies are often given in terms of mean values for each month. Even when individual daily values are available, there will generally be large day-to-day fluctuations that reflect sampling variations rather than real climatological differences. Monthly means or monthly totals smooth out most of the unwanted noise. (There is nothing magic about months as the averaging period, but it is the one most frequently used. For many purposes shorter periods, say 10 days, might be preferred. The discussion that follows can be adapted to these shorter periods as well.) Passing a smooth curve through these monthly values can then give daily values of a climatology of monthly running means. For many purposes, however, one is interested in daily values rather than running means. Using the raw monthly means in their place will create very substantial discontinuities between the last day of one month and the beginning of the next. One could also interpolate linearly between the monthly means. This eliminates the discontinuities, but introduces systematic biases. In particular the annual maxima and minima will be understated in the daily values. The true daily values must exceed (and be less than) the monthly mean for at least some part of the month. However, linearly interpolated daily values will always lie within the envelope of monthly means; the implied daily values can never exceed the maximum monthly value (or be less than the minimum monthly value). Daily values thus will generally be less (greater) than the given monthly mean in the month representing the maximum (minimum) of the annual cycle.

This has long been recognized, but we know of no standard method for accomplishing it. H. C. S. Thom (unpublished and undated manuscript[1]) described a rather complex procedure based on Lagrangian periodic and aperiodic interpolation formulas. The current practice of the National Climatic Data Service (NCDC) is to use "natural spline function(s)". They fit cubic splines (Grenville 1960) to the monthly means (or totals) and make manual adjustments so that the averages agree exactly[2] with the given monthly means rather than the midmonthly values (R. Quayle, N. Guttman and J. Owensby, private communication) as in the original spline fit. The results, insofar as we have been able to ascertain from various examples included in the *Climatography of the United States, Daily Normals of Temperature, Heating and Cooling Degree Days and Precipitation 1951–1980* appear to be quite reasonable. Nevertheless a somewhat less complex method, and one that does not require subjective adjustments, seems desirable.

We have tried several alternate procedures for converting a climatology of monthly means into one of daily values. Some, like the spline method, were based on fitting polynomial functions in time. We have settled, however, on a simple method based on a harmonic fit as most adequate.

2. Derivation

Let us begin by representing the (unknown) daily climatology by the sum of harmonic components:

$$y(t) = a_0 + \sum_j [a_j \cos(2\pi jt/12) + b_j \sin(2\pi jt/12)]. \tag{1}$$

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[1] "Daily Normals from Monthly Normals," probably written in the mid-1960s; Thom's affiliation is given as Environmental Data Service, ESSA.

[2] Almost too exactly. No roundoff errors are permitted.

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For later convenience we have chosen to express time, t, in units of months. (We will treat months as equal in length. In effect, days in February will be longer than days in January. This has no substantial effect on the results.) Clearly y is periodic in 12 months.

Now we integrate y between the limits $T \pm 1/2$ to obtain the average value of y in month number T,

$$\begin{aligned}
\bar{y}(T) &= a_0 + \sum_j (12/2\pi j)\{a_j[\sin(2\pi j(T+1/2)/12) \\
&\quad - \sin(2\pi j(T-1/2)/12)] \\
&\quad - b_j[\cos(2\pi j(T+1/2)/12) \\
&\quad - \cos(2\pi j(T-1/2)/12)]\} \\
&= a_0 + \sum_j (12/\pi j)\sin(\pi j/12) \\
&\quad \times [a_j\cos(2\pi jT/12) + b_j\sin(2\pi jT/12)]. \quad (2)
\end{aligned}$$

This can be rewritten

$$\bar{y}(T) = A_0 + \sum_j [(A_j\cos(2\pi jT/12) + B_j\sin(2\pi jT/12)] \quad (3)$$

where

$$A_j = (\pi j/12)^{-1}\sin(\pi j/12)\,a_j \quad (4)$$

and

$$B_j = (\pi j/12)^{-1}\sin(\pi j/12)\,b_j. \quad (5)$$

This means that if we choose coefficients A_j and B_j such that $\bar{y}(T)$ takes on known corresponding climatological monthly mean values (Y_T), the curve defined by (1) will have the property of having the desired monthly averages. If the Y_T are monthly totals rather than averages, one simply divides by the number of days in the respective months and proceeds as before.

Thus, we determine the A_j and B_j by inverting the Fourier transform in Eq. (3) and solve for the a_j and b_j in (4) and (5), limiting j to $0 \leqslant j \leqslant 6$. In other words we calculate

$$\begin{aligned}
a_0 &= \sum_T Y_T/12 \\
a_j &= [(\pi j/12)/\sin(\pi j/12)] \\
&\quad \times \sum_T [Y_T\cos(2\pi jT/12)/6] \quad j = 1, 5 \\
b_j &= [(\pi j/12)/\sin(\pi j/12)] \\
&\quad \times \sum_T [Y_T\sin(2\pi jT/12)/6] \quad j = 1, 5 \\
a_6 &= [(\pi/2)/\sin(\pi/2)]\sum_T [Y_T\cos(\pi T)/12] \\
b_6 &= 0. \quad (6)
\end{aligned}$$

Because we have treated months as being of equal length, we adjust the lengths of the days in each month. The value of t, in Eq. (1), for the mth day of month number T, is given by $t = (T - 0.5) + (m - 0.5)/D$, where D is the number of days in month T.

Note that the factor that converts the daily to the monthly climatology is just the amplitude response function of a rectangular filter of length one month. Liebmann et al. (1989) followed this procedure but included only the first two harmonics. This results in a smoother annual cycle of daily values which will fit the individual monthly means more loosely.

3. An example

To illustrate the application of this method we use as an example the mean monthly 850 mb temperature at a gridpoint in the northeastern United States taken from the 10-year climate diagnostics data base (CDDB) of the Climate Analysis Center. In Fig. 1 are plotted for comparison the monthly means and the calculated daily climatology. One may question the very minor secondary maximum in mid-January implied by the fit, but this is a minor aberration of the order 0.2°C. The uncorrected fit to the monthly means, obtained by omitting the factors of the form $\theta/\sin\theta$ in (6), is illustrated in Fig. 2. This differs only subtly from Fig. 1. One can nevertheless judge by eye, for example, that the mean monthly temperatures implied by the smooth curve in Fig. 2 are slightly too high in December and February, too low in January.

The representation of the annual cycle implied by the method of Liebmann et al., truncation to two harmonics, is shown in Fig. 3. Because of the smaller number of adjustable parameters, this curve does a relatively poor job of representing the very flat winter minimum implied by the monthly mean values. Differences in the implied daily climatologies between the truncated and untruncated representations exceed 1.25°C in late January.

A more severe test is provided by heating or cooling degree days, or by precipitation amounts in a seasonally very arid climate, when the implied smooth curve can, and does, take on negative values. As a test, we applied the method to heating degree days in Birmingham, Alabama, and compared our calculations, rounded to the nearest whole degree day, with the daily values given by NCDC. There are few differences, none greater than one degree day (the precision of the NCDC published "normals"), and no negative values when the results are given only to the nearest whole degree. (The greatest difference was that monthly sums of our daily values sometimes disagreed with the given monthly totals due to roundoff errors, but the NCDC sums did not.) Similarly when the input was the monthly total precipitation in Sacramento, California, the resulting daily normals, rounded to the nearest 0.01 inch (the preci-

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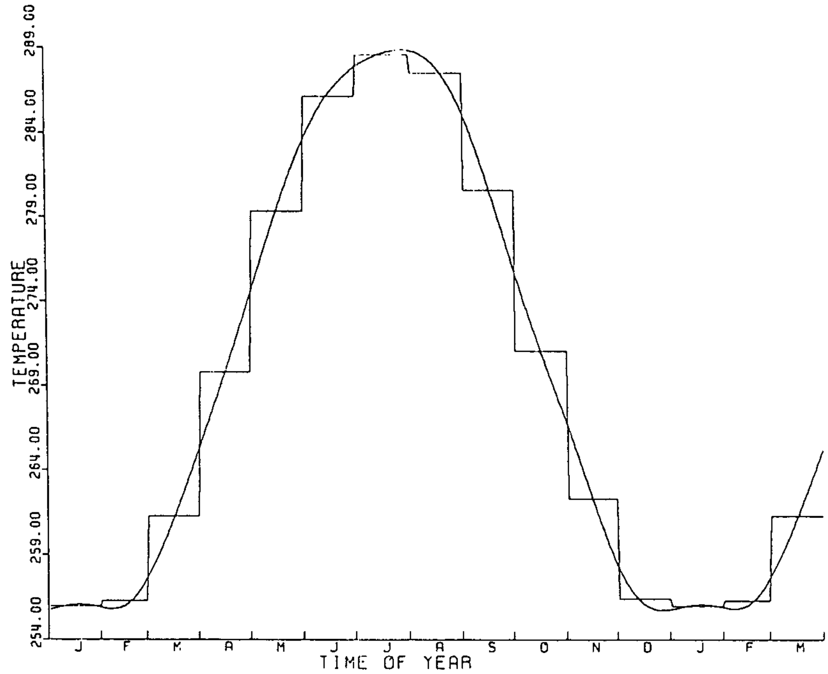

FIG. 1. Monthly means of an arbitrary 850 mb gridpoint, and the area-preserving harmonic fit. The first part of the second year is shown for clarity.

sion of the input data) were all zero or positive and never differed from the published NCDC values by more than 0.01. Interestingly, both sets of daily normals, trying to fit smoothly the broad summer minimum in Sacramento precipitation, suggest a slight midsummer secondary maximum, amplitude only

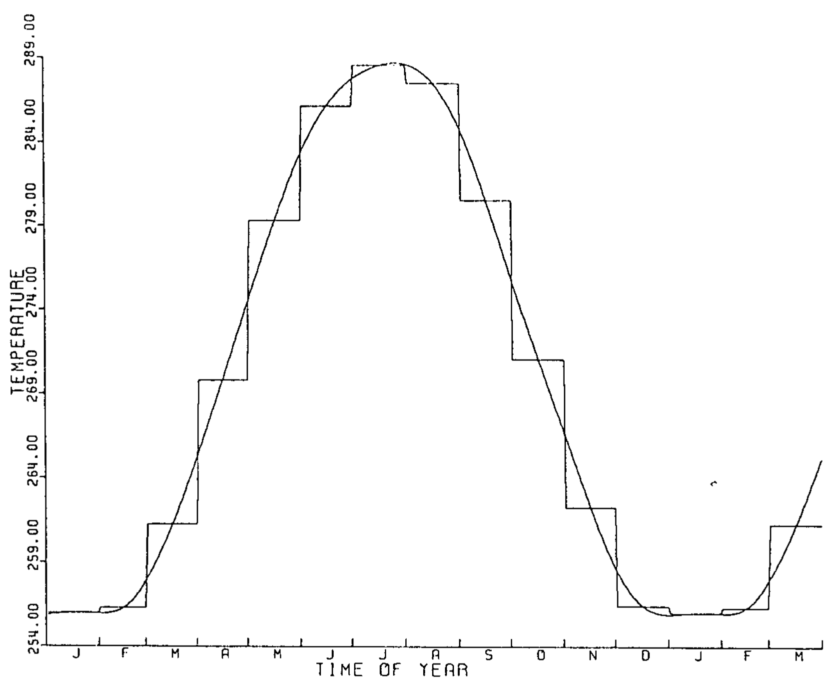

FIG. 2. Exact harmonic fit to the monthly means. The curve passes through the monthly mean value at the midpoint of each month.

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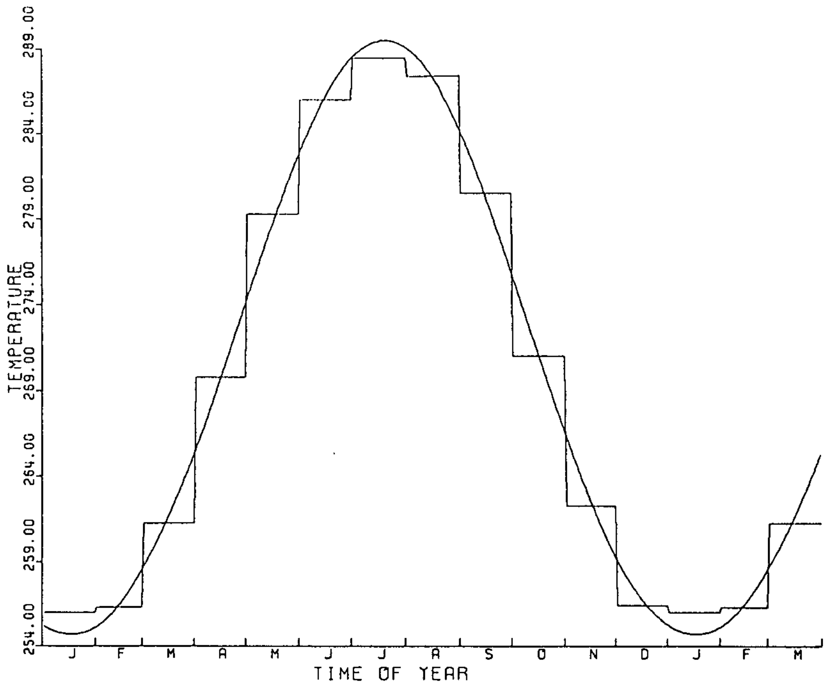

FIG. 3. The adjusted harmonic fit truncated at wavenumber 2.

0.01, similar to the midwinter secondary maximum in temperature shown in Fig. 1.

4. Conclusion

On the basis of these and other similar examples the harmonic method of deriving daily normals from monthly normals, very simple to calculate, seems entirely satisfactory. The annual cycle is defined by 12 adjustable parameters determined on the basis of 12 monthly means. The information available is used fully and correctly.

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