Correlation Dimensions of Local, Short-Term Climatic Attractors from Observations and a Global Climate Model

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ABSTRACT

Correlation dimensions ν were estimated for 500-mb geopotential heights over eastern North America as predicted by the Atmospheric Environment Service–Canadian Climate Centre general circulation climate model (GCM). A value of 6.0 to 7.0 was obtained, which agrees to within experimental error with a previously estimated value determined from measured data. This agreement is encouraging for it may suggest that on short time scales, the GCM is capable of producing fluctuations on a regional scale that resemble those of the real atmosphere. For western Europe, however, observed and GCM time series of 500-mb geopotential heights yielded values for ν of about 8.0 and 6.0, respectively, but display similar power spectra. In contrast, over the tropical Pacific, ν from GCM data either does not exist or exceeds 11. These results show that topologies of local climate attractors vary over the globe, thus making it difficult to speak of a global attractor.

1. Introduction

Over the past two or three decades, much research has been put into development and validation of general circulation climate models (GCMs). The focus of GCM validation has been on how well their equilibrium solutions account for the seasonal cycle and spatial distributions of fields such as mean temperature, precipitation, and top-of-the-atmosphere radiation budgets (McFarlane et al. 1992; Slingo and Slingo 1991). While equilibrium solutions of GCMs with prescribed sea surface temperatures appear to be confined to a single climate state (clear of catastrophic folds in equilibrium space), they meander in a seemingly aperiodic manner on an attractor. Currently, the existence and dimension of earth's climatic attractor(s) have been in question (Nicolis and Nicolis 1984; Grassberger 1986; Fraedrich 1986). The preferred method of approaching the problem has been to compute the correlation dimension ν of a presumed attractor (Essex et al. 1987; Keppenne and Nicolis 1989; Zeng et al. 1992). Lorenz (1991) reinforced the notion that while such computations of ν say little about global climate in general, they may indicate something about the nature of the dynamics of a localized system and perhaps its relation to the rest of the system. The objective of this study was to calculate ν for decadal time series of 500-mb geopotential heights for selected regions of the Atmospheric Environment Service–Canadian Climate Centre (AES–CCC) GCM and the real atmosphere.

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2. Methodology and data

It has been postulated that some essential topological features of a nonlinear system's attractor can be estimated from a finite time series of a system variable $\{x_i\}$ where $i = 1, \cdots, M$ (Packard et al. 1980). Thus far, attention has been directed at estimating the correlation dimension ν of short-term, local climatic attractors (e.g., Hense 1987; Tsonis and Elsner 1988; aforementioned references). The correlation function of an attractor's projection into D-dimensional space (the embedding dimension) is defined as

$$C(r, D) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Phi(r - |X_i - X_j|_D) \quad i \neq j, \quad (1)$$

where $(X_i \mid i = 1, \cdots, N \leq M)$ are sets of D-dimensional points formed from (x_i), $|X_i - X_j|_D$ is Euclidean distance, r is radius of D-dimensional spheres around X_i, and Φ is the conventional Heavyside function (Grassberger and Procaccia 1983). The scaling region is defined as the range of r satisfying

$$C(r,D) \sim r^{p(D)}, \quad (2)$$

where p is the scaling exponent. For true random noise, $p = D$ for all D; the attractor fills a volume of D-dimensional phase space. If, however, for increasing D, p becomes independent of D and equals ν, the correlation dimension of the attractor is ν (e.g., embedding a line with $\nu = 1$ in two- or three-dimensional space does not alter ν).

Recently, Essex et al. (1987) used 36 years of estimated 500-mb geopotential heights (obtained by objective analysis) recorded daily at 1200 UTC for 9 grid points in eastern North America. In total, they used 107 756 values. Discontinuities in the methods of inferring 500-mb geopotentials are weak for North America (Lambert 1990) and likely do not influence estimates of ν. They used two methods for estimating ν. The first method concatenated the 9 series and the familiar method of time delays (e.g., Nicolis and Nicolis 1984; Fraedrich 1986) was used to create D-dimensional points in reconstructed phase space ($M - N$ increases with D). The second method was conceptually quite different: the jth point in D-dimensional phase space was defined as $(x_{1,j}, x_{2,j}, \ldots, x_{D,j})$ where x_{ij} is the jth measurement in the ith time series. Thus, increasing the embedding dimension is achieved by introducing a new time series as a new coordinate ($N = M$ for all D). Essex et al. (1987) demonstrated that ν is not sensitive to the definition of X_i.

Since 500-mb geopotential heights are highly correlated in space (correlation of about 0.5 out to about 800 km) (Bergman 1979), this will tend to create an overabundance of close points in phase space that may lead to an underestimation of ν. Attempting to overcome this by considering grid points farther apart may not be helpful, for the attractor may begin to blur (loss of locality). Because of these potential, and seemingly unavoidable, sources of error built into the experiment, we have chosen to duplicate Essex et al.'s experiment as closely as possible using data from the AES–CCC GCM 20-year simulation of present-day climate. This is a spectral GCM with seasonally forced sea surface temperature (SST) and a diurnal cycle (Boer et al. 1984). Model resolution is approximately 5.5° × 5.5° which is approximately the resolution of the objective analysis dataset. It is important to compare data of similar resolutions. Viswanathan et al. (1991) computed values of ν for presumably measured surface temperature from one site and surface temperature of the corresponding grid box from a NWP model. They found markedly different values of ν. Depending on the resolution of the model, however, this may not be too surprising since the model values are more a regional value than a point measurement.

Since data are saved every 18 GCM hours, each time series has 9733 points. Therefore, 11 grid points were selected from eastern North America totaling 107 063 values. While each time series was longer than those used by Keppenne and Nicolis (1989), estimates of ν_i for individual series were deemed unreliable due to an insufficient number of data points. According to Nerenburg and Essex's (1990) analysis, errors attributed by Keppenne and Nicolis to their estimates of ν_i are about an order of magnitude too small. The current study employs Essex et al.'s (1987) second method of estimating ν. The nature of errors in this method is as yet unknown.

3. Results

Figure 1 shows the value of p [Eq. (2)] at all r in the scaling region for $D = 9$ [cf. Essex et al.'s (1987) Fig. 2]. The range of r, from about 400 to 900, for which p is 6.3 ± 0.1 (obtained by linear regression with the uncertainty being the standard error) suggests that the critical embedding dimension has not been exceeded and the experiment is free of the depopulation (small r) and saturation (large r) regimes (Nerenburg and Essex 1990). The critical embedding dimension is the value of D at which the scaling region is squeezed out of existence by depopulation and saturation of $C(r, D)$. Figure 2 shows that for the North American GCM data p levels off for increasing D, thus implying that $\nu \approx 6.0$ to 7.0. This is encouraging, for it agrees with the value estimated by Essex et al. (1987). This may imply that on time scales substantially shorter than deep ocean and cryospheric response times, internal fluctuations of the AES–CCC GCM and the real atmosphere bear a resemblance for eastern North Amer-

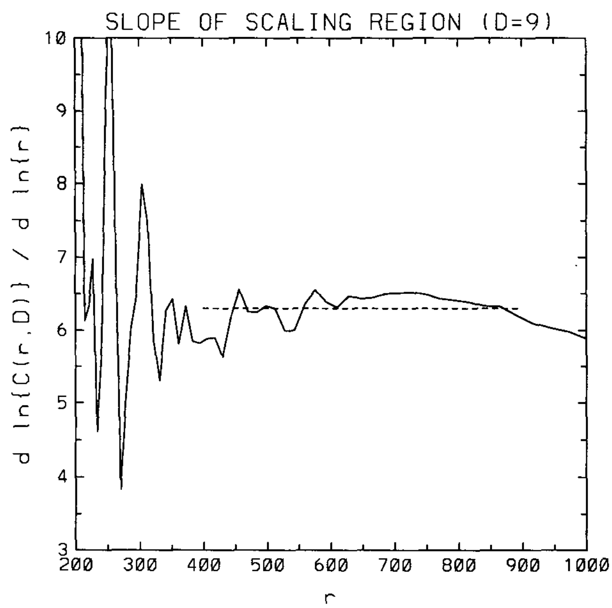

FIG. 1. Plot of $d\ln[C(r, D)]/d\ln r$ (solid line) for embedding dimension D of 9 for GCM 500-mb data over eastern North America. The horizontal broken line represents the estimate of p [Eq. (2)] and spans approximately the scaling region. The effects of depopulation and saturation are seen for $r <$ ~400 and $>$ ~900, respectively.

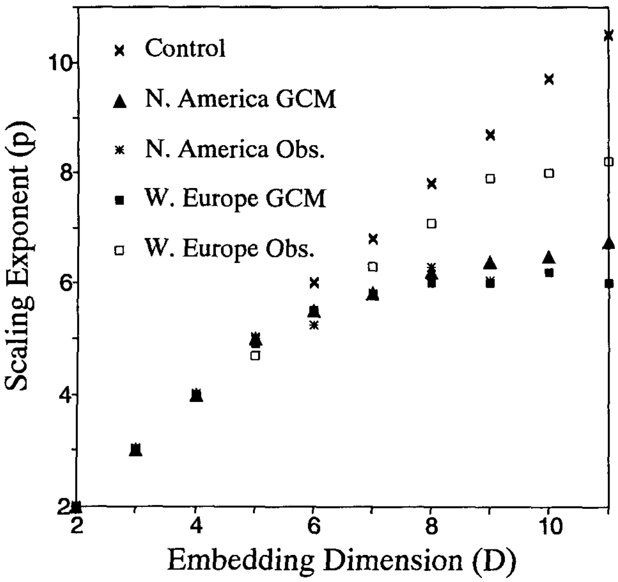

FIG. 2. Scaling exponent p as a function of embedding dimension D for a random control case (9733 points) and for GCM and observed 500-mb geopotential heights for North America and western Europe. The estimated values of ν are 6.0–7.0 for both GCM and observed North American data and 6.0–6.5 for western Europe (GCM), and ~8 for western Europe (observed).

ica at least. Note that in this paper, when $|dp/dD| \leqslant 0.2$ for at least two consecutive values of D, the corresponding mean value of p is taken to be an estimate of ν.

These results are now extended by redoing the procedure using observed and GCM 500-mb geopotential heights for western Europe. The observed time series were extracted from the same dataset used by Essex et al. (1987) (except extended such that each series contains 15 000 points). Again, the GCM data were extracted from the AES–CCC GCM 20-year simulation.

Figure 2 shows p as a function of D for western Europe observed 500-mb geopotential heights. Although the scaling regions were not as well defined as in the North American case, it was inferred that $\nu \sim 8.0$ which is consistent with Keppenne and Nicolis's (1989) values. The estimate of ν for the corresponding GCM time series, however, is only ~6.0 which is quite different than the observed value but similar to both North American estimates. A later version of the AES–CCC GCM (McFarlane et al. 1992) displays a marked absence of storm trajectories for all of southern Europe and the Mediterranean region (R. Laprise 1992, personal communication). If this lack of variability is present in the GCM data used here, this may help explain the low value of ν obtained from model data.

Figure 3 shows power spectra for observed and modeled data averaged for the 11 grids. For both spectra the annual cycles are clearly visible and their overall forms are similar, with the exception of frequencies greater than ~0.3 cycles/day where power associated with GCM fluctuations decreases too fast. This plot demonstrates that time series with similar power spectra need not have similar values of ν (assuming that ~25% is a significant fractional difference for ν). Another notable difference for the two spectra in Fig. 3 is the aliasing of the diurnal cycle into the GCM spectrum. On account of the weakness of the diurnal signal at 500 mb, however, it is expected that the 18-h sampling of GCM data increases the estimate of ν only slightly (probably $\ll 1$). Despite this, however, the GCM's value of ν is still too low.

As a test to see whether the GCM is bound to produce $\nu \sim 6$ everywhere, ν was computed for 500-mb geopotential height for the equatorial Pacific (16°S, 180°W; 16°N, 140°W). Not only does the climate of this region differ greatly from the climate of the previous two cases, few estimates of ν have been made with data relevant to outside the Northern Hemisphere midlatitudes (Hense 1987). In this case, the algorithm failed to converge to an estimate of ν. This suggests that either ν does not exist (a space filling attractor) or it is well in excess of 11 (the maximum tested for). This at least demonstrates that the GCM produces values of ν other than 6. Unfortunately, there are too few observations of this region to get an accurate estimate ν for the real atmosphere. Also, this demonstration is in accord with the results of Zeng et al. (1992). They found that for time series of surface temperature, ν is spatially variable and sometimes very large (>11).

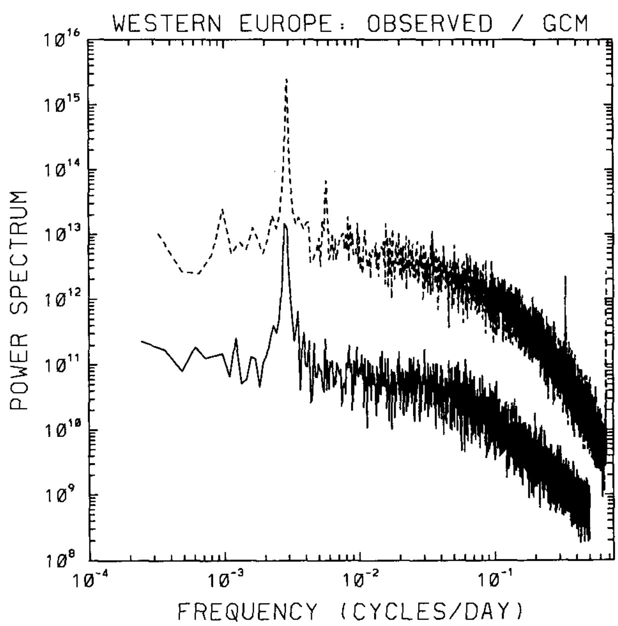

FIG. 3. Average power spectra for 11 observed (solid line) and GCM time series of 500-mb geopotential heights over western Europe. The GCM spectrum was shifted up 2 decades for presentation purposes.

4. Discussion and conclusions

It now appears that evidence is mounting that suggests that for at least midlatitudes local attractors for 500-mb geopotential heights exist and that their dimension is probably between about 6 and 8. The significance of this value, however, is still a mystery. For example, does it signify that as few as 8 or so independent variables are required to capture much of the short-term variability of global climate (e.g., Nicolis and Nicolis 1984)? This is likely false given Lorenz's (1991) demonstration and the fact that ν appears to vary greatly over the globe. Furthermore, given that both midlatitude GCM values of ν computed here are much limited in range relative to those inferred from observations, this prompts the question of whether all existing atmospheric GCMs with specified SSTs will produce $\nu \sim 6$ for most extratropical locations. It could be argued that GCMs in general may tend to produce systematically small values of ν on account of not resolving processes on scales less than several hundred kilometers.

It was somewhat surprising to find that the 500-mb geopotential attractor for the tropical location in the GCM is either space filling or of relatively high dimension (>11). This, however, is not necessarily inconsistent with earlier findings (F. Zwiers 1992, personal communication) that showed that GCM-simulated rainfall in the tropics (which is directly related to convection) had no preferred time of occurrence (an even distribution over the day). Had this distribution been narrower (as in measured data), perhaps the dimension of the attractor would be less.

Over the last 40 years, earth's climate was forced by varying solar constant, aperiodic fluctuations of SSTs, volcanic events, and trace gas changes all superimposed on the annual and diurnal solar cycles. The GCM was forced with only the solar cycles and an annual repetition of monthly averaged SSTs from observations. Despite the GCM's lack of external forcings, which certainly influence local and global climate, the agreement between observed and modeled values of ν for North America is encouraging. Before this agreement can be truely significant, however, more needs to be known about the significance, sensitivity, and meaning of ν and climate simulations should be conducted with at least long-term observed SSTs and known external forcings. The disparity of results for western Europe, however, should be cause for concern. As a final comment, it is not yet clear whether the fair to satisfactory showing of the GCM in this study, in conjunction with other validation studies (e.g., McFarlane et al. 1992; Slingo and Slingo 1991), implies that the GCM can correctly predict climatic changes due to external forcings such as changes in trace gas concentrations.

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