NOTES AND CORRESPONDENCE

Testing for Change in the Frequency of El Niño Events

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ABSTRACT

This note describes and applies a test for trend in the frequency of El Niño events over the period 1525–1987. Although there appears to have been a significant increase in frequency over this period, this result is consistent with an overall increase in the completeness of the historical record. When the analysis is repeated for the later part of the period and for strong events alone, no significant trends are found.

1. Introduction

The El Niño–Southern Oscillation (ENSO) is the dominant source of internal variability in the global climate system on the annual-to-decadal timescale (e.g., Philander 1990). There has been considerable recent interest in understanding the historical behavior of ENSO (e.g., Diaz and Markgraf 1992). Progress in this area would contribute to an understanding of the physical processes involved in ENSO, as well as its connection to other climatic processes. It would also reduce a significant source of variability in historical climate data and provide information about likely variability in the future. Finally, it would provide a baseline against which the significance of apparent changes in ENSO can be measured.

To address issues of the historical behavior of ENSO, Quinn and his coworkers have used a variety of documentary sources to construct a record of the occurrences of El Niño events extending back almost 500 years (Quinn et al. 1987; Quinn 1992; Quinn and Neal 1992). Details of the construction of this remarkable record and of potential sources of error are provided in the original papers. It has been suggested that this record supports the view that the frequency of El Niño events has not remained constant over the period. For example, Enfield and Cid (1991) identified what they believed to be significant fluctuations in the frequency of El Niño events, although they found no secular trend, while Anderson (1992) believed that the record suggests changes in frequency on the timescale of many decades. The purpose of this note is to describe and apply a formal statistical test for trend in the frequency of El Niño events.

2. Testing for trend

In statistical terminology, the times of occurrence of El Niño events can be modeled as a point process (e.g., Cox and Isham 1980). While a point process is strictly defined over continuous time, the discrete (i.e., annual) timescale of the record of El Niño events is sufficiently fine compared to the length of the record to justify this approximation. A basic property of a point process is its intensity function:

$$\lambda(t) = \lim_{\delta \to 0} [\text{prob}(\text{event } \epsilon (t \pm \delta/2))/\delta], \qquad (1)$$

where t denotes time. This note focuses on testing the null hypothesis H_0: $\lambda(t) = \lambda$ (i.e., that the intensity of the point process of El Niño events is stationary).

Suppose that during a period of observation $(0, T)$, events occur at ordered times $t_1, t_2, \ldots, t_n$. In broad terms, H_0 is rejected if the distribution of these events over the observation period is uneven. However, even if H_0 is true, the inherent irregularity in the timing of events ensures that some parts of the observation period will contain more events than other parts. The statistical problem is to determine whether the distribution of events is too uneven to be consistent with H_0.

Tests for stationarity of a point process are described in Cox and Lewis (1978). In most cases, these tests make the additional assumption that, under H_0, the events follow a Poisson process. This is clearly not the case for El Niño events, which tend to occur at intervals more regular than random. Tests are also available for specific parametric alternatives to H_0, such as

$$\lambda(t) = \exp(\alpha_0 + \alpha_1 t).$$

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If no basis exists for specifying an alternative model a priori, the following approach, which is based on standard methods for testing goodness of fit (e.g., D'Agostino and Stephens 1986), provides a general test for trend in a point process.

For convenience, suppose that the period of observation begins immediately following an event (so that the first event in the record is omitted) and concludes with an event. Let $N(t)$ be the number of events occurring in the period $(0, t)$ and consider a plot of $N(t)/n$ against t/T. This plot will increase from 0 to 1 through a sequence of n steps of height $1/n$ occurring at t_i/T, $i = 1, 2, \cdots, n$. If the underlying point process is stationary, then this plot will tend to fluctuate around the 45° line. However, if the underlying point process is nonstationary, this plot will tend to wander away from the 45° line. One measure of the deviation of the plot from the 45° line is the Kolmogorov–Smirnov statistic:

$$D = \max_{0 \leq t < T} |N(t)/n - t/T| = \max_{1 \leq i < n} [(i/n - t_i/T), \quad (t_i/T - (i-1)/n)], \quad (2)$$

which is the maximum deviation of the plot from the 45° line. An alternative measure is the Cramér–von Mises statistic:

$$\omega^2 = \int_0^T (N(t)/n - t/T)^2 dt = 1/12n^2 + \sum_{i=1}^{n} [t_i/T - (2i-1)/2n]^2/n, \quad (3)$$

which is the integrated squared distance between the plot and the 45° line.

Let $X_i = t_i - t_{i-1}$, $i = 1, 2, \cdots, n$ be the intervals between successive events, with $t_0 = 0$. Under the additional assumption that, under H_0, these intervals are serially independent, the significance level of the observed value of D or ω^2 can be estimated by the following randomization procedure. Conditional on the observed intervals, a new realization of the point process is generated according to $t_i^* = t_{i-1}^* + X_i^*$, $i = 1, 2, \cdots, n$, where $X_1^*, X_2^*, \cdots, X_n^*$ is a random permutation of the observed intervals and $t_0^* = 0$. The value of the test statistic (D or ω^2) is calculated for this realization, the procedure is repeated a large number of times, and the significance level of the observed value of the test statistic is estimated by the proportion of simulated realizations for which the value of the test statistic exceeds the observed value.

By assuming that, under H_0, all possible permutations of the intervals are equally likely, this procedure attributes any persistent variation in the frequency of El Niño events to trend. Positive serial dependence among the intervals will result in a stronger tendency for the plot of $N(t)/n$ against t/T to wander away from the 45° line than under independence, exaggerating the significance of D and ω^2, and making the randomization test conservative. It is possible, in principle, to devise a test for trend that allows for the possibility of serial dependence. However, such a test would require some kind of explicit model of the joint distribution of intervals (e.g., Wold 1948).

3. Application

In this section, the methods outlined in section 2 are applied to the record of El Niño events given in Table 6.2 of Quinn (1992). In Fig. 1, $N(t)/n$ is plotted against t/T for 117 El Niño events in the period 1525–1987. In broad terms, this plot suggests an overall increase in the frequency of events with time. The value of D for this plot is 0.0845. Of 10 000 simulated realizations, only 32 had values of D that exceeded 0.0845, so the estimated significance level is 0.0032. The value of ω^2 for this plot is 0.00176. Of 10 000 simulated realizations, only 78 had values of ω^2 that exceeded 0.00176, so the estimated significance level is 0.0078. Note that the tests based on D and ω^2 are not independent. Based on conventional notions of statistical significance, the null hypothesis of stationarity is rejected using either test statistic.

It is natural to question the completeness of the early part of the record of El Niño events. In particular, the overall increase in the frequency of recorded events indicated by Fig. 1 is consistent with an overall increase in the completeness of the record. Two further analyses were performed in an effort to avoid this problem. First, the analysis was repeated using only the 56 events during the period 1803–1987 in the hope that the later part of the record is more nearly complete. The beginning of this period was chosen to coincide with the first moderate event identified in an earlier record compiled by Quinn et al. (1987), thereby reflecting some increased degree of confidence by these authors. In Fig. 2, $N(t)/n$ is plotted against t/T for this part of the

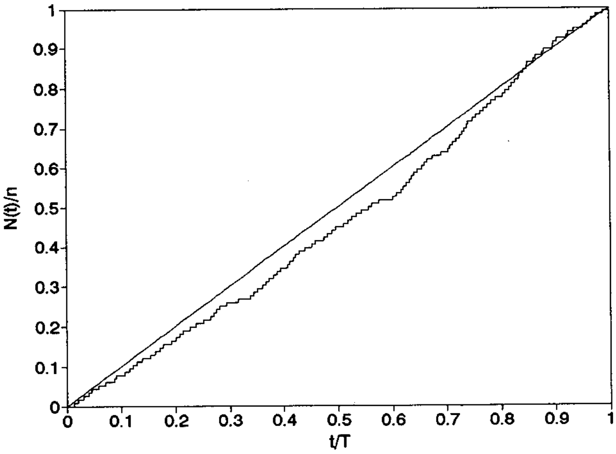

FIG. 1. Plot of $N(t)/t$ against t/T for all events, 1525–1987.

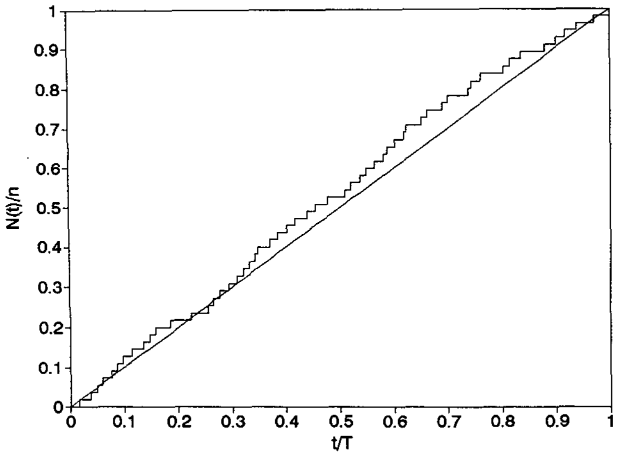

FIG. 2. Plot of $N(t)/n$ against t/T for all events, 1803–1987.

record. In this case, the plot suggests an elevated frequency during the middle of the observation period. The value of D for this plot is 0.0841. However, of 10 000 simulated realizations, 1519 had values of D larger than 0.0841, so the estimated significance level is only 0.1519. The value of ω^2 for this plot is 0.00154. Of 10 000 simulated realizations, 1459 had values of ω^2 larger than 0.00154, so the estimated significance level is only 0.1459. For this period, the null hypothesis of stationarity cannot be rejected.

As a second attempt to reduce the problem of record incompleteness, the analysis was repeated using the 42 events over the period 1525–1987 that were classified by Quinn (1992) as strong or very strong, with events classified as "moderate plus" or "moderate/strong" not included. In this case, the hope is that the record of strong events is reasonably complete, even over the entire observation period. In Fig. 3, $N(t)/n$ is plotted against t/T for the record of strong events. The plot appears to exhibit no systematic departures from the 45° line. The value of D for this plot is 0.0543. Of 10 000 simulated realizations, 9390 had values of D larger than 0.0543, so the estimated significance level is only 0.9390. The value of ω^2 for this plot is 0.00064. Of 10 000 simulated realizations, 8032 had values of ω^2 larger than 0.00064, so the estimated significance level is only 0.8032. There is, therefore, no evidence of trend in the frequency of strong El Niño events.

The record given in Quinn (1992) represents an update of the record given in Quinn et al. (1987) (see, also, Quinn and Neal 1992). As a check of the robustness of these results, the analysis was repeated using the earlier record. As noted, Quinn et al. (1987) listed no moderate events prior to 1803, so only the overall record since 1803 and the record of strong events during the period 1525–1987 could be analyzed. The plots for these records are extremely similar to the corresponding plots for the earlier records and are not shown. The estimated significance levels of D and ω^2 are 0.2353 and 0.2705, respectively, for the earlier post-1803 record and 0.5743 and 0.6661, respectively, for the earlier record of strong events. These results are in qualitative agreement with the results for the later record.

Finally, to check for the possibility that low-order serial correlation in the intervals could invalidate the randomization procedure, lag-one serial correlations were estimated for the various records. In no case was the estimate significant at the 0.05 level. For the overall record of events, the estimated lag-1 serial correlation was around 0.14, with a slow decay for higher lags.

4. Discussion

The results of the previous section indicate that the trend in the overall record consists of an increase in the frequency of moderate events over the period 1525–1803. The identification and classification of events over this period was based primarily on a subjective assessment of historical documents, using a variety of meteorological, hydrological, and biological indicators. Examples of such indicators include elevated sea levels, flooding in normally arid coastal lowlands, and mass mortality of guano birds (which prey on anchoveta). It seems reasonable to suppose that the probability that an event of a given magnitude goes unrecorded is higher early in the record when the number, quality, and survival of documents are presumably lower. It also seems reasonable to suppose that the probability that an event at a particular time goes unrecorded is higher for a moderate event with a relatively small signature than for a strong event with a relatively large signature. Taken together, these factors are sufficient to account for the results presented here. While it is certainly not possible to rule out a real trend in moderate events

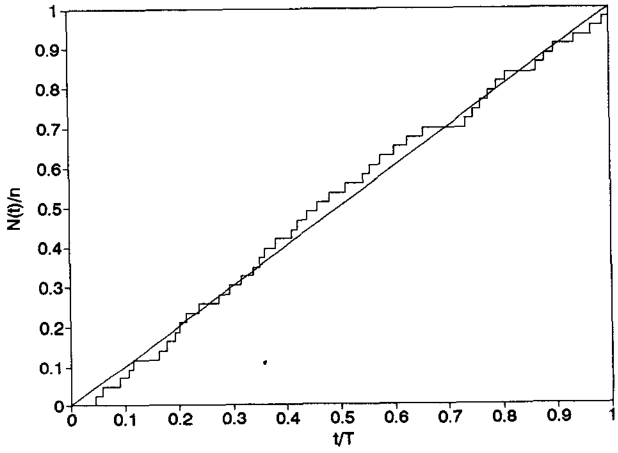

FIG. 3. Plot of $N(t)/n$ against t/T for strong events, 1525–1987.

alone over the period 1525–1803, the tentative conclusion is that there has been no secular trend in the frequency of El Niño events over the period 1525–1987.

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