Inferring Optical Depth of Broken Clouds from Landsat Data

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(Manuscript received 2 November 1994, in final form 11 May 1995)

ABSTRACT

Optical depths $\tau_{pp}$ for broken, shallow clouds over ocean were inferred from Landsat cloud reflectances $R_{cl}$ (0.83 $\mu$m) with horizontal resolution of 28.5 m. The values $\tau_{pp}$ were obtained by applying an inverse, homogeneous, plane-parallel radiance model to each pixel value of $R_{cl}$. The primary objective of this study was to estimate optical depth errors incurred by the homogeneous, plane-parallel, independent pixel paradigm. This was achieved by computing reflectances $R_{mc}$ with a 3D Monte Carlo photon transport algorithm that employed $\tau_{pp}$ and cloud geometric thicknesses $h > 0$.

A single cloud was isolated for study in which the solar zenith angle was 30° and average $\tau_{pp}$ was 5.8. This cloud measured about 1.2 km in diameter but $h$ had to be estimated. In the Monte Carlo simulations, $h$ was set to be uniform for the entire cloud. For $h$ between 150 and 300 m, cloud-average reflectance $R_{mc}$ was about 15% less than $R_{cl}$. It was found that use of $\tau_{pp}^{(MC)}$ in the Monte Carlo algorithm yielded $R_{mc} \approx R_{cl}$. For $h = 225$ m, $1/8(h = 225) = 1.11$, and this increased average $\tau_{pp}$ to $\approx 8.0$, which was a 35% increase. At the pixel level, however, random errors associated with fields of $R_{mc} - R_{cl}$ were reduced only slightly when $\tau_{pp}^{(MC)}$ was used rather than $\tau_{pp}$. Finally, $\tau_{pp}^{(MC)}$ was applied to numerous neighboring clouds. When the aspect (height to width) ratio $A$ of neighboring clouds was assumed to be constant, $\tau_{pp}$ for each cloud received a unique scaling, and this yielded Landsat mean reflectances to within 4% for $A < 0.3$. This suggested that grid-averaged $\tau_{pp}$ was likely about 4 rather than 3, as was the plane-parallel, independent pixel estimate.

1. Introduction

All climate models use plane-parallel solutions of the radiative transfer equation to compute solar fluxes for cloudy atmospheres (e.g., Brengleb 1992; McFarlane et al. 1992). Clouds, however, are not this simple and theoretical studies (e.g., Davies 1978; Welch and Wielicki 1985; Stephens 1988; Davis et al. 1990; Barker 1992; Cahalan et al. 1994a) have demonstrated that inhomogeneous and homogeneous clouds transport solar radiation differently. This concerns climate modelers because climate model performances are assessed increasingly with cloud optical properties derived from application of plane-parallel, homogeneous theory to satellite data. The obvious concern is the potential for large systematic biases associated with plane-parallel theory (Welch and Wielicki 1985; Cahalan et al. 1994a).

Recently, Harshvardhan et al. (1994) attempted to infer cloud optical depth $\tau_{pp}$ by applying an inverse plane-parallel, homogeneous, independent pixel radiative transfer model to Landsat near-infrared cloud reflectances $R_{cl}$. Thus, horizontal transport of photons between pixels was neglected. Since the horizontal resolution of the Landsat pixels was $\approx 28.5$ m and cloud geometric thicknesses $h$ are often greater than 100 m, it is likely that in many situations horizontal transport cannot be neglected. The prime objective of the present study was to infer $\tau_{pp}$ for broken, shallow clouds using plane-parallel theory and $R_{cl}$ and establish whether $R_{cl}$ can be reproduced when $\tau_{pp}$ values are used in a 3D Monte Carlo photon transport algorithm with $h > 0$. Landsat imagery was used rather than artificially generated clouds because Landsat reflectances apply to real clouds and the integrity of artificial cumulus clouds is uncertain. Overcast clouds were not considered here but are the subject of future work.

There are many different hypotheses that can be tested when attempting to recreate satellite imagery with a radiative transfer algorithm. For example, one
may attempt to recreate imagery on a pixel by pixel basis. This is obviously the most stringent test of the 
quality of inferred $\tau_{pp}$ or some transformed field $\bar{\tau}_{pp}$. A less stringent test is whether reflectances averaged 
over individual clouds can be reproduced. This would constitute getting the mean reflectance correct for possibly 
hundreds of contiguous pixels. This paper focused on recreating average reflectances for individual clouds 
and ensembles of clouds in a field.

The structure of this paper is as follows. Section 2 contains a brief outline of the experimental design. In 
section 3, the Landsat scene to be used is presented and cloud reflectances $R_{\text{cloud}}$ for the near-infrared channel are 
extracted. In section 4, the plane-parallel, homogeneous radiance model is defined and $\tau_{pp}$ values are derived 
from the Landsat scene. Section 5 has two main parts. In the first part, values of $\tau_{pp}$ for a selected cloud 
are used in a 3D Monte Carlo photon transport algorithm and the resulting reflectances $R_{\text{mc}}$ are compared 
to $R_{\text{cloud}}$. Then $\tau_{pp}$ values are transformed as a function of $h$, such that when averaged over the cloud, $R_{\text{mc}} \approx R_{\text{cloud}}$. In the second part of section 5, this transformation is shown to be suitable for ensembles of clouds in the scene. Concluding remarks are made in section 6.

2. Experimental design

This section highlights the main steps for assessing the quality of inferred cloud optical depths. Detailed 
discussions are reserved for subsequent sections.

First, the oceanic contribution to Landsat reflectances was roughly accounted for and removed leaving 
approximate cloud reflectances $R_{\text{cloud}}$. Then an inverse, plane-parallel, nadir radiance model was applied to 
each $R_{\text{cloud}}$, which yielded arrays of plane-parallel optical depth $\tau_{pp}$. Cloud geometric thickness $h$ was then estimated. 
Then $\tau_{pp}$ and $h$ were supplied to a 3D Monte Carlo algorithm that accounts for photon transport between 
between pixels (Barker and Davies 1992), and nadir reflectances $R_{\text{mc}}$ were computed and compared to $R_{\text{cloud}}$. If 
when averaged over the cloud, $R_{\text{mc}} \approx R_{\text{cloud}}$, it can be assumed that the plane-parallel and independent pixel 
approximations are adequate. Conversely, if for a reasonable range of $h$, $R_{\text{mc}} \neq R_{\text{cloud}}$, the approximations are 
deemed to be poor. It should be recognized that in the limit as $h \rightarrow 0$, $R_{\text{mc}} \rightarrow R_{\text{cloud}}$ (as do $R_{\text{mc}} \rightarrow R_{\text{cloud}}$), as clouds then become effectively plane-parallel in the Monte 
Carlo experiment. If the plane-parallel and independent pixel approximations are inadequate, transformed fields 
of optical depth $\bar{\tau}_{pp}$ may be tested to determine whether they can reproduce $R_{\text{cloud}}$.

3. Landsat scene: Cloud reflectances

All reflectances reported in this paper are nadir bidirectional reflectances normalized to a Lambertian reflector as

$$R = \frac{\pi I}{\mu_0 S},$$

where $I$ is nadir radiance (calibrated using coefficients given by Markham and Barker 1986), $\mu_0$ is cosine of 
the solar zenith angle $\theta_0$, and $S$ is the time-dependent solar constant (Harshvardhan et al. 1994). Figure 1 
shows Landsat reflectances $R_{\text{LS}}$ (channel 4; narrow band centered at 0.83 $\mu$m) for an image consisting of 
512 x 512 pixels ($\sim 15$ km)$^2$ with $\theta_0 \approx 30^\circ$. Also shown in Fig. 1 is the image's spatial coherence plot 
(Coakley and Bretherton 1982) obtained from corresponding 11.5-$\mu$m imagery. The well-defined arch implies single-layer clouds with near constant cloud-top temperatures.

Ocean reflectance contributions were removed as follows. It was assumed that Landsat reflectances can be defined as

$$R_{\text{LS}} = R_{\text{cloud}} + \frac{T_{\text{cloud}}}{1 - \alpha_s},$$

where $R_{\text{cloud}}$ is cloud bidirectional reflectance for $\theta_0 \approx 30^\circ$ and viewing zenith angle $\theta$, $T_{\text{cloud}}$ is cloud flux transmittance for direct solar radiation incident at $\theta_0 \approx 30^\circ$, $r_{\text{cloud}}$ and $t_{\text{cloud}}$ are cloud flux reflectance and transmittance for diffuse irradiance (i.e., for radiation involved in multiple reflections between surface and cloud), and $\alpha_s$ is ocean albedo. Since absorption by 
water vapor and cloud droplets is negligible at 0.83 $\mu$m, and assuming that direct and diffuse quantities are approximately equal ($R_{\text{cloud}} = 1 - T_{\text{cloud}} \approx 1 - t_{\text{cloud}} = r_{\text{cloud}}$),

$R_{\text{cloud}}$ can be expressed as

$$R_{\text{cloud}} \approx \frac{R_{\text{LS}} - \alpha_s}{1 - \alpha_s(2 - R_{\text{LS}})}.$$  

The most severe approximation used here is $R_{\text{cloud}} \approx 1 - T_{\text{cloud}}$. For $\theta_0 \approx 30^\circ$, it is more likely that $R_{\text{cloud}} \approx 0.8(1 - T_{\text{cloud}})$ since clouds tend to reflect relatively little toward the zenith (Davies 1984). This approximation, however, results in less than a 1% error in $R_{\text{cloud}}$. Because $\alpha_s$ is small, $R_{\text{cloud}}$ differs significantly from $R_{\text{LS}}$ only near thin edges of clouds. Thus, $\alpha_s$ was set to the clear-sky bidirectional reflectance of 0.003 rather than 0.06, as expected for $\alpha_s$ given isotropic irradiance, since for thin 
cloud edges much direct solar radiation is transmitted. Hence, the error in $R_{\text{cloud}}$ resulting from use of $\alpha_s \approx 0.003$ in the denominator of (1) is again less than $\sim$1%. Note that in this formulation and elsewhere in this paper, Rayleigh scattering and aerosol effects were neglected 
as were all potential adjacency effects (Diner and Martonchik 1984). Hence, inferred fields of $\tau_{pp}$ will be smoothed slightly in addition to smoothing expected due to the independent pixel assumption. Hereinafter, 
arrays of $R_{\text{cloud}}$ represent Landsat reflectances.
4. Inferring plane-parallel optical depths

a. Inverse plane-parallel algorithm

Following Harshvardhan et al. (1994), the cloud drop size distribution for all clouds in this study was assumed to be lognormal with effective radius $10 \ \mu m$ and effective variance $0.1 \ \mu m^2$. Droplets were assumed to be pure water and spherical. The associated scattering phase function was derived from Mie scattering theory.

Since the intention of this work was to use inferred $\tau_{pp}$ in a 3D Monte Carlo algorithm, it was necessary, for consistency, to derive a relationship between $\tau_{pp}$ and nadir bidirectional reflectances for the Monte Carlo algorithm. This meant producing the bidirectional reflectance curve $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$. In order for Monte Carlo reflectances to resemble Landsat imagery, only those photons reflected into an extremely narrow cone about the zenith were of concern. Given the high resolution (very narrow cone) of Landsat imagery, an astronomical number of photons would be needed to achieve a confident rendition of $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$. Clearly, the wider the cone, the easier it is to produce $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$. But, if it is too wide, the correspondence between $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$ and Landsat reflectances will degrade.

Figure 2 shows 3D Monte Carlo reflectances $R_{mc}(\theta')$ at three locations on top of a complex, inhomogeneous cloud as a function of angular half-width of a cone about the zenith $\theta'$ into which reflected photons were collected. These reflectances were normalized to a Lambertian reflector and defined as

$$R_{mc}(\theta') = \frac{\sum_{l=1}^{\theta'} \sum_{\phi_l} N(\theta_l, \phi_l)}{N_p (1 - \cos^2 \theta')},$$

where $\theta$ and $\phi$ are zenith and azimuth angles (1° intervals), $N(\theta, \phi)$ is the number of photons reflected from a region $\mathcal{R}$ on top of the cloud into an elemental angular bin, and $N_p$ is the total number of photons injected into $\mathcal{R}$. All Monte Carlo experiments reported in this paper, except those in section 5b, were conducted with at least 100 000 photons per pixel. Experiments in section 5b used 100 photons per pixel. The albedo of $\mathcal{R}$ is simply $R_{mc}(\theta' = \pi/2)$. The important point in Fig. 2 is that while reflectance varies with location, for $\theta'$ less than about 10°, $R_{mc}(\theta')$ is almost independent of $\theta'$ and seemingly equal to $R_{mc}(\theta' \to 0^\circ)$. For this reason, and yet wanting to err on the side of caution, $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$ was defined as $R_{mc}(\theta' = 5^\circ)$, in which $R_{mc}(\theta' = 5^\circ)$ was for homogeneous, plane-parallel clouds with photons injected at $\theta_0 = 30^\circ$. Figure 3 shows $\tau_{pp}$ as a function of $R_{pp}(\tau_{pp}|\theta_0 = 30^\circ)$. Thus, it is this functional relationship that when applied to each $R_{cd}$ creates arrays of plane-parallel, independent pixel cloud optical
Fig. 2. Monte Carlo nadir reflectances for three individual pixels atop an inhomogeneous cloud found in Fig. 1 as a function of angular half-width $\theta'$ of cones about the zenith. Stippled regions indicate ± (one standard deviation) about the mean (arising from a finite number of injected photons). As $\theta'$ decreases, decreasing numbers of photons were collected, and thus the widths of the stippled regions increased. Stippled regions (a), (b), and (c) refer to pixels progressively closer to cloud center. The line labeled "average" is actually ± (one standard deviation) about mean cloud reflectance (i.e., the average of all pixels). The cloud was approximately 50 × 50 pixels and 200 000 photons were injected on each pixel.

depths $\tau_{pp}$. Furthermore, all Monte Carlo reflectances reported here are $R_{mc}(\theta' = 5^\circ)$.

b. Plane-parallel optical depths

Figure 4 shows the relative frequency distribution of $\tau_{pp}$ associated with values of $R_{cld}$ extracted from Fig. 1. This frequency distribution corresponds very well with a gamma density function (Barker 1995). The mean and standard deviation of $\tau_{pp}$ for clouds only are about 3.0 and 3.9. This field of $\tau_{pp}$ was used in the 3D Monte Carlo algorithm and results are presented in section 5b.

A single cloud (64 × 64 pixels) was chosen from Fig. 1 for intense examination. Figure 5 shows $\tau_{pp}$ for this particular cloud. The average $\tau_{pp}$ for this cloud is 5.8 and the relative frequency distribution of $\tau_{pp}$ is shown in Fig. 4. The maximum value of $\tau_{pp}$ is $\sim 25$, which, as required, corresponds to a reflectance far less than Landsat's peak reflectance. Had the image contained reflectances in excess of Landsat's peak reflectance, $\tau_{pp}$ for those pixels would have been constant and problematic for two reasons. First, $\tau_{pp}$ for the saturated pixels would have been clearly wrong and so too would their Monte Carlo reflectances. Second, because reflectances of neighboring nonsaturated pixels depend somewhat on radiative transfer within the saturated pixels, incorrect values of $\tau_{pp}$ for saturated pixels would induce errors to Monte Carlo reflectances for adjacent unsaturated pixels. Also shown in Fig. 5 is the cloud's image at 11.5 μm (channel 6). This suggests that this cloud (which measured approximately 1.2 km

Fig. 3. Plane-parallel, homogeneous cloud optical depth $\tau_{pp}$ as a function of corresponding Monte Carlo nadir reflectance using cones of angular half-width about the zenith of 5° and a solar zenith angle of 30°. This function was applied to Landsat cloud reflectances to obtain fields of $\tau_{pp}$ for subsequent use in the Monte Carlo algorithm.
across) consisted of two smaller clouds each about 600 m in diameter.

5. Results

a. Single cloud

All results in this subsection pertain to the single cloud shown in Fig. 5. As mentioned earlier, if this field of \( \tau_{pp} \) was used in a Monte Carlo algorithm and \( h \) was set arbitrarily small, the Landsat image would be reproduced. This, however, would be terribly unrealistic as the implied extinction coefficient \( \beta \) would be arbitrarily large. Note that when \( \tau_{pp} \) values were used as input for the Monte Carlo code with \( h > 0 \), each cloud pixel was assumed to have the same \( h \) and each pixel column was assumed to be homogeneous. Hence, a tacit assumption was that changes to \( \beta \) were manifest by changes to cloud drop number concentration (since the phase function was invariant also). Following Minnis et al.'s (1992) work on stratocumulus during FIRE, \( h \approx -45.6 + 84.3 \tau^{1/2} \) (m), and so, given the range of \( \tau_{pp} \) for this cloud, it is highly probable that \( h \) was between 100 and 350 m. Given that both small clouds constituting the cloud in Fig. 5 were \( \sim 600 \) m across, this means an aspect ratio (cloud height divided by width) for the smaller clouds is between about 0.15 to 0.6 and an overall cloud aspect ratio \( A \) is between about 0.08 and 0.3.

Figure 6a shows absolute differences between Monte Carlo reflectances \( R_{mc} \) for \( h = 285 \) m (\( A \approx 0.22 \)) and \( R_{old} \). Not only were individual pixel reflectances estimated poorly, but cloud-averaged reflectance \( R_{mc} \) was 0.053 less than \( R_{old} \), which is about a 20% underestimation (see Fig. 6b). Furthermore, the standard deviation of pixel reflectance differences was 0.055. It is interesting to note that the few regions of Fig. 6 where \( R_{mc} > R_{old} \) correspond to clouds near cloud sides that were irradiated directly (photons were injected at 173° clockwise from north). This is because in the Monte Carlo experiments, edges of this cloud were certainly too thick geometrically, and so the numbers of photons injected into the sides of the outermost columns of this cloud were too large. Also, note that the greatest underestimations of \( R_{old} \) occurred near, but not at, cloud edges. These regions represent zones where in reality there was a large net horizontal flux of photons toward thin edge regions. Hence, \( \tau_{pp} \) are particularly low in these regions but also, the Monte Carlo simulation further enhanced this because again these regions experienced a net horizontal flux of photons toward thin edges. Near the center of the cloud, however, underestimates were not as large because locally the cloud was closer to plane parallel than it was near its edges.

The quality of the plane-parallel, independent pixel assumptions can be assessed by applying the plane-parallel, homogeneous relation in Fig. 3 to \( R_{mc} \) as obtained using \( \tau_{pp} \) and \( h > 0 \). This gives effective plane-
parallel optical depths, \( \tau_{mc} \) \( = R_{pp}^{-1} (R_{mc}) \), corresponding to the Monte Carlo reflectances. If the assumptions are correct, \( \tau_{mc} \) will equal \( \tau_{pp} \). Figure 7, however, shows scatterplots of \( \ln(\tau_{mc}) \) against corresponding values of \( \ln(\tau_{pp}) \) for various values of \( h \). In general, values of \( \tau_{mc} \) were smaller than \( \tau_{pp} \) (the more so as \( h \) increases), with greatest discrepancies for large \( \tau_{pp} \).

Since the points in Fig. 7 are distributed quite evenly with approximately constant and symmetric distributions of conditional residuals for \( \ln(\tau_{mc}) \), conditions are suitable for application of least-squares linear regression. After regressing the data and transforming back to linear coordinates, \( \tau_{mc} \) is related approximately to \( \tau_{pp} \) as

\[
\tau_{mc} = \gamma \tau_{pp}^{\delta},
\]

where \( \ln(\gamma) \) is the intercept and \( \delta \) the slope of the regression line. Since \( \gamma \) was statistically indistinguishable from 1, the regressions were forced through the origin thus making \( \gamma = 1 \) (in all cases, the goodness of fit parameter exceeded 0.93). This left the simple relation \( \tau_{mc} = \tau_{pp}^{\delta} \) where \( \delta \) is at least a function of \( h \). The fact that \( \tau_{pp} < \tau_{mc} < 1 \) signifies that thin regions experienced net horizontal influxes of photons from neighboring thick regions (the more so as \( h \) increases).

At this stage, one is faced with deciding whether to parameterize \( \delta \) as a function of \( h \) or \( A \). It is conceivable that when \( A \) is small, \( \delta \) depends most on \( h \) (i.e., intercolumnar transport of photons: see Cahalan 1989) due to the lack of importance of cloud sides. If, however, \( A \) is large, cloud sides are then important and \( \delta \) probably depends most on \( A \). Since the clouds addressed in this study had aspect ratios probably less than 0.2, it is likely appropriate to parameterize \( \delta \) as a function of \( h \).

Figure 8 shows a log-log plot of \( \delta \) as a function of \( h \). For \( h \) less than about 100 m, \( \delta \) is >0.95 meaning that the plane-parallel, independent pixel approximations are almost valid (as mentioned before, as \( h \) goes to 0, the pixels are effectively plane-parallel despite the fact that they are just 28.5 m wide). As \( h \) increases, \( \delta \) decreases, meaning that the more the cloud resembles a narrow column, the more radiation leaks out its sides, thus making it appear, from nadir, to become thinner. Since \( h \) was expected to be in the range of 100 to 350 m, this translates into \( \delta \) between about 0.82 and 0.95.

Also shown in Fig. 8 are two values of \( \delta \) for experiments in which \( \beta \) was set constant at 30 km\(^{-1}\) and \( h \) was allowed to vary locally but with a flat cloud base. Clearly, overall reflectances were closer to plane-parallel values than were corresponding reflectances for constant \( h \). This was due mostly to edge regions being closer to plane-parallel.

Having established an approximate relationship between \( \tau_{mc} \) and \( \tau_{pp} \) and having demonstrated that the plane-parallel, independent pixel model may be
seriously inadequate if \( h \) exceeds about 300 m, the following question was addressed: if \( \tau_{pp}^{1/6(h)} \) values are used in the Monte Carlo algorithm, can the original Landsat reflectances be reproduced? This simple transformation, however, cannot be expected to be a panacea since it does not discriminate between pixels near cloud edge and center, nor does it distinguish between pixels on illuminated and shaded sides of a cloud. Furthermore, its express reliance on \( \tau_{pp} \) is disconcerting having just demonstrated that \( \tau_{pp} \) can be inadequate. It should be kept in mind, however, that the main objective here was not to recreate the exact imagery but rather to recreate cloud-averaged reflectances. Clearly, the hope is that cloud-averaged \( \tau_{pp}^{1/6(h)} \) is significantly closer to the actual value than is the corresponding average of \( \tau_{pp} \). What is interesting about \( \tau_{pp}^{1/6(h)} \) is that its inverse was proposed by Davis et al. (1990) for relating the reduced optical depth needed for a plane-parallel algorithm to yield the same albedo as 3D monofractal clouds: they referred to \( \tau_{pp}^{(6-1)/6} \) as the packing factor.

Before using \( \tau_{pp}^{1/6(h)} \) in the Monte Carlo algorithm, consider the dependence of cloud-averaged extinction coefficient \( \beta = \tau_{pp}^{1/6(h)} h^{-1} \) on \( h \) given that

\[
\overline{\tau_{pp}^{1/6(h)}} = \int_0^\infty p(\tau_{pp}) \tau_{pp}^{1/6(h)} d\tau_{pp},
\]

(5)
b. Ensembles of clouds

In this subsection, adjusted plane-parallel optical depths $\tau_{pp}^{1/\delta(h)}$ were applied to numerous clouds (those shown in Fig. 1) surrounding the cloud used to define $\delta(h)$ (Fig. 8). Then, overall nadir reflectances for clouds only were computed and compared to the corresponding Landsat reflectance. The cloud fraction for this field was 0.30.

This experiment could have proceeded several ways. In reality, each cloud has its own unique $\delta(h)$ function. To determine these, however, would be completely impractical and intractable. Simpler approaches include assuming that $\delta(h)$, as shown in Fig. 8, applies to all clouds as follows: either (i) assume that all clouds have the same $h$ and hence all optical depths are adjusted as $\tau_{pp}^{1/\delta(h)}$, or (ii) assume that the overall aspect ratio $A$ of each cloud is constant ($h$ depends on cloud area) and apply to each cloud a unique value of $\delta(h)$. Results are presented for both assumptions.

Figure 11 shows overall Landsat reflectance $R_{mc}$ for the cloud field shown in Fig. 1 as well as $R_{uc}$ for the case in which all pixels had $h = 285$ m [i.e., $1/\delta(h) = 1.15$]. The $R_{uc}$ value of 0.113 is less than the $R_{mc}$ value of 0.135 by $\sim 16\%$. This is because there was many small clouds (cf. Wielicki and Welch 1986; Cahalan and Joseph 1989) with excessive cloud aspect ratios that had too many photons injected onto, and lost too many photons out of, their sides. In this case, the average cloud aspect ratio was about 0.2. For reference, when $h = 285$ m with no scaling of $\tau_{pp}$ (again, average $A \approx 0.2$), $R_{mc}$ was just 0.097 (30% too low).

In the second set of experiments, individual cloud aspect ratio $A$ governed $\delta(h)$ as follows. First, individu-

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Fig. 8. Slopes $\delta$ of the log–log regression lines relating $\tau_{uc}$ to $\tau_{pp}$ (open circles, of which four were shown in Fig. 7) as a function of cloud geometric thickness $h$ for the cloud shown in Fig. 5. The curves fit through the nine points are listed on the plot. Filled circles represent values of $\delta$ applicable to clouds with $\beta$ fixed at 30 km$^{-1}$ but variable $h$ and flat cloud base ($h$ coordinate is mean geometric thickness).

where $p(\tau_{pp})$ is the density function of $\tau_{pp}$. Figure 9 shows a plot of $\beta$ and $\tau_{pp}^{1/\delta(h)}$ as functions of $h$ and suggests that for $h$ between 100 m and 350 m, $\beta$ is between $\sim 25$ and $\sim 62$ km$^{-1}$. For this type of cloud, however, $\beta$ is likely to be between 30 and 50 km$^{-1}$ (e.g., Stephens and Platt 1987; Minnis et al. 1992). This shifts the likely value of $h$ to between 150 and 300 m, implying that the mean optical depth is between 7.5 and 9 rather than 5.8, as was the case for $\tau_{pp}$ [Fig. 4 shows the relative frequency distribution of $\tau_{pp}^{1/\delta(h)}$ for $1/\delta(h) = 1.11$]. Hence, it is conceivable that the plane-parallel, independent pixel approach has, for this cloud, underestimated average optical depth by up to $\sim 35\%$.

Figure 10 shows $R_{uc} - R_{ud}$, where $R_{uc}$ were obtained using $\tau_{pp}^{1/15}$ ($h = 285$ m). The value of $R_{uc}$ differs from $R_{ud}$ by less than 0.003 (or 1.3%). Therefore, this confirms that for this cloud, the simple power-law transformation of $\tau_{pp}$ as a function of $h$ reproduced successfully cloud-averaged Landsat reflectances. There still were, however, problems at the pixel level. First, the standard deviation of the pixel reflectance differences was 0.050 compared to 0.055 for the $\tau_{pp}$ case (Fig. 6). Thus, for the most part, the power transformation simply removed the mean bias error but did little for the random error. Second, as in Fig. 6 and for reasons explained earlier, pixels near illuminated edges were still too reflective. Also, values of $R_{uc}$ near cloud center overestimated $R_{ud}$ often by 5% to 20% and by over 40% near the relatively dark central region discernible in Fig. 5.
Fig. 10. As in Fig. 6 except that Monte Carlo reflectances were derived using $\tau^{1/\delta}$ rather than $\tau_p$. [(a) is absolute difference and (b) is relative difference].

Individual cloud areas $N$ (number of pixels) were determined (Wielicki and Welch 1986) using $\tau_p = 0.5$ as the lower threshold, and effective pixel diameters were defined as $2\sqrt{\pi N/\theta}$. Then $h$ were defined as $(2 \times 28.5)\sqrt{\pi N} \approx 32\sqrt{N}$: a unique $\delta(h)$ for each cloud. The curve in Fig. 11 shows Monte Carlo cloud reflectances for $A$ ranging from 0 (plane parallel) to 0.7 (which is certainly much too larger). The main point here is that cloud field reflectances depend weakly on $A$ and are within 4% of $R_{out}$ for $A < 0.3$ (at $A = 0.2$ the results are much better than having each value of $\tau_p$ raised to the power of $1/\delta(h) = 1.15$). Note that several competing factors were at play here. As $A$ increased, clouds became taller and less reflective for $\theta_0 = 30^\circ$ (Davies 1978), but effective cloud fraction and cloud–cloud interactions increased (Welch and Wielicki 1985), as did mean optical depth [because $1/\delta(h)$ increased]. The downward trend in Fig. 11 indicates, therefore, that the impact of increasing $A$ was dominant and also that $\delta(h)$ is not universal even for this limited neighborhood.

In conjunction with Fig. 11, Fig. 12 shows that average cloud optical depth as a function of $A$ ranged from 3 at $A = 0$ to 11 at $A = 0.7$. Since the value of $A$ was probably near 0.1 to 0.2, this means that the true average optical depth was probably between 3.5 and 4.5 rather than 3.

6. Summary and conclusion

The primary objective of this exploratory study was to make a quantitative assessment of errors incurred when retrieving optical depths for shallow, broken clouds from Landsat imagery using plane-parallel, independent pixel models about solar radiative transfer (e.g., Harshvardhan et al. 1994). These assumptions imply no net horizontal transfer of photons and no cloud sides through which photons may enter and exit. The question addressed, therefore, was when optical depths $\tau_p$ are retrieved from plane-parallel, independent pixel models, then used in a 3D Monte Carlo algorithm with finite cloud thickness, can the original Landsat reflectances be reproduced? Real data were used rather than artificially generated clouds because the integrity of artificial cumulus cloud-generation schemes is unknown, whereas Landsat reflectances came from authentic clouds.

An individual cloud was selected for intense Monte Carlo experimentation in which cloud geometric thicknesses $h > 0$ were used. It was established that use of $\tau_p$ can lead to substantial underestimations of cloud-averaged reflectance. Therefore, the original question was modified to test transformed values of $\tau_p$. A simple scaling of optical depth defined as $\tau_p^{1/\delta(h)}$, where $\delta(h) < 1$, led to very good estimates of cloud-averaged reflectance, but results were still quite poor at the pixel
level. This was not surprising given that $\tau_{pp}$ themselves had been deemed inappropriate (but easy to achieve) and that the simple power transformation respects neither the proximity of pixels to cloud sides nor solar irradiance geometry. The hope, however, is that cloud-averaged $\tau_{pp}^{\text{hoch}}$ values are significantly closer to true values than are cloud-averaged $\tau_{pp}$. For this cloud, it appears that mean $\tau_{pp}$ was 20% to 40% too low.

Adjusted values of $\tau_{pp}$ were then applied to numerous clouds surrounding the one used to derive $\delta(h)$. By assuming that cloud aspect ratios were constant, each cloud had its pixel values of $\tau_{pp}$ scaled with a unique $\delta(h)$. In these cases, Monte Carlo reflectances were within 4% of the Landsat value. This was encouraging for it is not necessarily true that the stringency of the tests performed decreases as one attempts to reproduce mean reflectance for an increasing number of clouds. For the cloud field considered here, it is possible that the plane-parallel, independent pixel model underestimated mean optical depth by as much as 30%. However, had the clouds in the Monte Carlo experiments been characterized by variable $h$ rather than constant $h$, the underestimates of $\tau_{pp}$ quoted here may have been reduced to as little as 10% on account of $1/\delta(h)$ being closer to 1 (B. A. Wielicki, personal communication, 1995). This is an important point that deserves close examination.

In conclusion, therefore, this study showed that it may be possible to adjust plane-parallel inferred optical depths by a simple scaling procedure to yield more accurate estimates of cloud- and cloud-field-averaged optical depths. Regarding inference of optical depth on a pixel by pixel basis, however, much more elaborate transformations are needed: simple scaling of plane-parallel values is inadequate.

Acknowledgments. The authors wish to thank Lindsay Parker of Lockheed Engineering and Science Company and Bruce Wielicki of NASA-Langley for supplying the Landsat data. Also, thanks is extended to an anonymous reviewer for helpful and encouraging suggestions.

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