Soil Moisture Estimation Using Thermal Inertia: Potential and Sensitivity to Data Conditions

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ABSTRACT

Thermal inertia retrieval using a thermal infrared remote sensing technique has been examined as a possible method for estimating soil moisture. This method is an application of the theory that thermal inertia highly correlates with soil water content. This study shows a method for retrieving thermal inertia from a heat budget model of the earth’s surface using radiative surface temperatures, insolation, and meteorological data observed in field experiments. In bare to sparsely vegetated areas, this method has the potential to estimate subsurface soil moisture with a precision of $6\% - 4\%$ of the daily volumetric soil moisture content at a significance level of 5%, which is enough to roughly classify thermal inertia estimates into a few levels of soil moisture (e.g., wet, middle, and dry). The analysis also includes an examination of the practical performance of the thermal inertia estimation according to the temporal resolution of the data, assuming the use of satellite and routine meteorological data. It is found that the following combination of data can achieve the precision given above: radiative surface temperature from geostationary/multiple polar orbiting satellites, insolation retrieved from geostationary satellite data, and routine meteorological data.

1. Introduction

Thermal inertia is the square root of the product of the volumetric heat capacity and the thermal conductivity and represents the temporal stability of the temperature of materials. Thermal inertia has been used for estimating the soil moisture of the subsurface layer because the magnitude of the thermal inertia of water and that of other materials are sufficiently different from each other, and it can be theoretically shown that thermal inertia and soil water content have a significant positive correlation. Price (1977) attempted to develop maps of the thermal inertia distribution using daily maximum and minimum radiative temperatures of the earth’s surface measured by polar orbiting satellites and meteorological data. Subsequently, remote sensing researchers have proposed various algorithms for mapping the thermal inertia distribution mainly using the radiative surface temperature ("surface temperature" hereafter) (e.g., Price 1985; Xue and Cracknell 1995; Sobrino and El Kharraz 1999). Recently, many studies have shown the potential utility of the thermal inertia method (Verhoef 2004; Murray and Verhoef 2007; Minacapilli et al. 2009; Lu et al. 2009). In particular, Verstraeten et al. (2006) showed good agreement between the observed soil water content and the 10-day average value of thermal inertia, which was estimated using daytime and nighttime surface temperatures and visible wavelength data acquired by Meteosat and an algorithm using the apparent thermal inertia.

Matsushima (2006, 2007) showed significant positive correlations between thermal inertia and subsurface soil moisture over vegetated surfaces in Mongolia. The thermal inertia was retrieved from a surface heat budget model that uses surface temperatures measured by polar orbiting satellites and meteorological data. This model is categorized as the two-source model (e.g., Anderson et al. 1997; Norman et al. 2000; Caparrini et al. 2004) and originated from Kondo and Watanabe (1992). This

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model comprehensively considered the surface heat budget (e.g., van de Griend et al. 1985), not just the radiation and ground heat flux as in other models. Hence, the thermal inertia and ground heat flux are estimated simultaneously with other parameters such as the bulk transfer coefficients of heat and the evaporation efficiency. The results of Matsushima (2006, 2007) did not show sufficient quantitative resolution to estimate soil water content based on values of the thermal inertia. This may be due to the rough temporal resolution of the surface temperature acquisition, which was limited to once or twice during the daytime. In the present study, we focus on the issue of the temporal resolution of data required for quantitative estimation of the soil water content using thermal inertia. This paper shows the potential of retrieving subsurface thermal inertia and sensitivity to data conditions such as frequency of data acquisition, especially surface temperature, and input data biases based on field data obtained over nearly bare soil in Japan and a sparsely vegetated surface in Mongolia.

2. Methods and data

a. Model

The linear surface heat budget model of Matsushima (2007) was employed in this study. Details of the model can be found in Matsushima (2007). This model is a two-source model consisting of a vegetation canopy and an underlying soil surface. This model requires diurnal time series of insolation, longwave radiation, air temperature, wind speed, and specific humidity as input variables. In addition, the following parameters are required: the bulk transfer coefficients of heat, evaporation efficiency, thermal inertia, albedo, daily average of the cosine of the solar zenith angle, leaf area index, view angle of surface temperature observations, and canopy height. The model calculates time series of the surface temperatures of the two layers employing the finite difference method, mainly using

$$\frac{d}{dt} \begin{bmatrix} T_c' \\ T_g' \end{bmatrix} = \mathbf{A} \begin{bmatrix} T_c' \\ T_g' \end{bmatrix} + \mathbf{B} \begin{bmatrix} S' \\ L' \\ q_a' \\ U' \end{bmatrix},$$

where $T_c$ is the canopy surface temperature, $T_g$ is the underlying surface temperature, $\omega = 2\pi/86400$ is the insolation, $L$ is the downward longwave radiation, $T_a$ is the air temperature, $q_a$ is the specific humidity, $U$ is the wind speed, and $t$ is time. The prime sign denotes difference from the daily average. Matrices $\mathbf{A}$ and $\mathbf{B}$ include parameters that are listed above. In the derivation of Eq. (1), the force-restore formulation (Stull 1988) is implemented for the ground heat flux $G_g$ as

$$G_g = C_g \left( \frac{dT_g'}{dt} + \omega T_g' \right),$$

in which

$$C_g = \frac{\sqrt{c_g \rho_g \lambda_g}}{2\omega} = \frac{P_g}{\sqrt{2\omega}},$$

where $P_g$ is the subsurface thermal inertia, $\omega$ is the angular velocity of the earth’s rotation assuming a day of exactly 24 h [i.e., 86 400 s ($\omega = 2\pi/86400 = 7.272 \times 10^{-5}$ s$^{-1}$)], and $c_g$, $\rho_g$, and $\lambda_g$ are the specific heat, density, and thermal conductivity of the soil, respectively. The surface heat fluxes were calculated using the estimated surface temperatures. The time step for integrating Eq. (1) was 60 s, and the third-order Runge–Kutta method was used. The surface temperature in the model was calculated using the canopy and underlying surface temperatures, the leaf area index, the surface view angle, and the emissivity. An optimization algorithm was required for searching for the best fit of the estimated surface temperatures to those of the observations. The simplex algorithm was employed in the model (Press et al. 1986). The maximum number of iterations employed for one optimization was 180. Even if an optimization had not converged completely, the iteration of the optimization was terminated to save calculation time. This is justified because convergence was sufficient after 100–150 iterations for almost all optimizations. The bulk transfer coefficients, evaporation efficiency, and thermal inertia were adopted as the optimization parameters. All parameters including the optimized parameters were kept constant in the calculation of one 24-h period. Among the optimized parameters, we focus on the thermal inertia in this paper.

b. Site description

One of the field observation sites was a flat sand dune field in Tottori, Japan, located in the Arid Land Research Center of Tottori University (35.34°N, 134.45°E). The observation period was from 10 July to 18 October 2007 (101 days). This site was a 7 $\times$ 7 m$^2$ square. Grass that grew in the square was manually removed periodically to maintain almost-bare soil surface conditions. The ground outside of the square was covered by permeable short grass. In the center of the square, a four-component radiometer (Eko MR-40), a combination of thermometer and a hygrometer (Vaisala HMP45A),...
and a combination of a wind vane and a cup anemometer (Young YG3002) were installed on a steel pole. Five T-type thermocouples (depth: 2, 5, 10, 20, and 50 cm), eight amplitude-domain reflectometry (ADR)-type soil moisture sensors [Delta ML2x, depth: 0–5 (five sensors), 10, 20, and 50 cm], and a heat flux plate (depth: 5 cm) were buried underground on the western side of the pole. A rain gauge was located on the ground on the eastern side of the square. All data were measured every 10 s, averaged over 1 min, and the 1-min average data were recorded by a datalogger (Campbell CR23X with AT25T and AM16/32).

The other site was a flat field with some senescent grass in Bayan Unjuul, Mongolia (47.04°N, 105.95°E). Data analyzed in this study were obtained between 27 April and 19 June 2008 (54 days) during the 2008 Dust–Vegetation Interaction Experiment (DUVEX) (Shinoda et al. 2010). The soil texture was silty loamy to sand. Grass covered approximately 7.2% of the site, but there were no green leaves during the period of the analysis because of lack of rainfall. This site was fenced and covered an area of 50 × 35 m². Cup anemometers, thermometers, hygrometers, a wind vane, and an ultrasonic anemometer were installed on multiple poles at the site. T-type thermocouples (depth: 1, 2, 5, 10, and 15 cm), time-domain reflectometry (TDR)-type soil moisture sensors Imko TRIME-EZ, depth: 2.5 (five sensors), 5, 10, and 15 cm], and three ground heat flux plates (depth = 1 cm) were buried underground beside the poles. Details of the observations in Bayan Unjuul are given in Shinoda et al. (2010) and Kimura and Shinoda (2010).

In the statistical assessments of the thermal inertia estimates, values were deemed to be realistic if they fell within the range of experimentally determined values and were deemed to be anomalous if they fell outside of this range. According to experimental results using the oven-dried method, values of the thermal inertia of sand in Tottori were approximately 400–1800 J m⁻² s⁻¹/² K⁻¹ for a volumetric soil water content of approximately 0%–5% (Kamichika 1988), and those of silty loam with sand in the Kherlen River watershed, Mongolia (46.5°–48.5°N, 108°–114°E) were approximately 1000–1800 J m⁻² s⁻¹/² K⁻¹ for a volumetric soil water content from approximately 10% to saturation (Sugita et al. 2008). The Kherlen River watershed was classified as mainly typical steppe zone and some forest steppe and dry steppe zones (Sugita et al. 2007). The soil, climate, and vegetation of the typical steppe are similar to those in Bayan Unjuul. Therefore, the realistic ranges of the thermal inertia were defined as 400–3000 J m⁻² s⁻¹/² K⁻¹ in the Tottori case and 600–2000 J m⁻² s⁻¹/² K⁻¹ in the Bayan Unjuul case in order to take into account the ranges of the soil water content that were actually observed in the field experiments and the variability in the final solutions of the optimization. Estimated values falling outside of the above-mentioned ranges were defined as anomalous and excluded from further statistical analysis.

c. Data

Time series of the insolation, downward longwave radiation, air temperature, specific humidity, and wind speed were used as inputs to the model. The upward longwave radiation was converted to surface temperature using the Stefan–Boltzmann law. In this conversion, a simplified correction for atmospheric water vapor developed by Kondo (2000) was employed because the measurements of the upward longwave radiation were made over a wide range of wavelengths between 3 and 100 μm, which includes strong water vapor absorption bands. The atmospheric correction was a set of empirical parameterizations formulated as a function of precipitable water w (in mm), given as

\[ T_s = \sqrt{\frac{L^1 - e_a \sigma T_a^4}{1 - e_a}}, \]

where

\[ e_a = 0.02(\log_{10} w + 4) + 0.01 \quad w < 0.001, \]
\[ e_a = 0.075(\log_{10} w + 3) + 0.03 \quad w < 0.01, \]
\[ e_a = 0.155(\log_{10} w + 2) + 0.105 \quad w < 0.1, \]
\[ e_a = 0.22(\log_{10} w + 1) + 0.24 \quad w \geq 0.1, \]

where \( T_s \) is the converted surface brightness temperature, \( L^1 \) is the upward longwave radiation measured by the radiometer, \( \sigma \) is the Stefan–Boltzmann constant, \( T_a \) is the average air temperature between the surface and the radiometer, and \( e_a \) is the coefficient for the atmospheric correction. The precipitable water was calculated as the multiplication of the specific humidity, the air density, and the height of the radiometer.

The wind speed data used as model input were derived from the original measured values, which were converted to virtual values at a height of 10 m using the logarithmic law and assuming a roughness length of 0.01 m. In the Bayan Unjuul case, wind speed data were mainly values measured by the cup anemometer; however, values from the ultrasonic anemometer were used when cup anemometer values were unavailable.

The daily averages of the observed values of the ground heat flux were used in the calculations for the Tottori
d. Sensitivity to temporal resolution

The simulations, which were a total of 17 runs, consisted of one run in which the input was given to the model with the full temporal resolution of the original data (run 1), and runs in which the temporal resolution was degraded (runs 2–17) (see Table 1). The intervals and timings of the input and surface temperature data other than run 1 were assigned as acquisition times of actual satellite observations. The interval of the insolation (60 min) was assigned as that of a geostationary satellite, assuming use of an algorithm of estimating insolation from the satellite data (Kawamura et al. 1998). The interval of the surface meteorology was assigned as that of the routine meteorological observations (180 min). The surface temperature interval in run 2 was assigned as that of a geostationary satellite (60 min), and the intervals in runs 3–17 were assigned as those of polar orbiting satellite sensors: Terra and Aqua Moderate Resolution Imaging Spectroradiometer (MODIS). The observation times of Terra were assigned as 1000 and 2200 h, and those of Aqua were as 0100 and 1300 h, which were around the equatorial crossing times of the satellites. The use of surface temperatures more than three times a day and some of twice a day includes assuming combination use of Terra and Aqua. Wang et al. (2006) conducted a similar analysis in which one daytime observation of each of Terra or Aqua and combinations of daytime and nighttime observations were examined. Times in Table 1 are in local standard time (LST). In the Bayan Unjuul case, however, the times in Table 1 should be subtracted by 1 h because LST in Bayan Unjuul is approximately 1 h faster than local solar time there. The model required input data every minute, which was the integration time step of the model. In run 1, the original 1-min data were used for input variables. In other runs, 10-min averages were used according to the data interval of the given run. The time series of 10-min averages were interpolated into 1-min intervals using the Akima method (Akima 1970), and it is these interpolated time series that were used as input variables. Surface temperatures, which were the variables to be optimized, were input as 10-min averages from 0300 to 2100 h LST every 30 and 60 min in runs 1 and 2, respectively. In other runs, nine 1-min values of surface temperature around each input time were given to the model. Using nine data values at one time was necessary in order to satisfy the degrees of freedom condition of the optimization and because surface temperatures in nature are subject to fluctuations.

e. Sensitivity to input variables and parameters

Sensitivity of the thermal inertia to input variables and parameters was tested for runs 2, 5, 12, 16, and 17. The variables and parameters that were modified to simulate various conditions in the tests are listed in Table 2. Tests are named tests 1–13, and test 1 is the control. As explained above, examinations of the sensitivity of the results to the data acquisition frequency, that is the temporal resolution of data, are denoted by run, while examinations of the sensitivity of the results to data anomalies are denoted by test. Conditions of the sensitivity tests were based on unavailability of the data in practical cases (tests 2–4), parameter setting associated with the ground heat flux formulation (tests 5 and 6), and typical measurement and estimation errors (tests 7–12). Sensitivity to a combination of possible errors of multiple input variables was also examined (test 13). In tests 7–10 and 13, factors multiplied by control values of input variables were set so that they were changed randomly in a certain range at every time step. This corresponded with errors of observed values being different in every observation time. The ranges of random change in individual input variables are also listed in Table 2. The same sets of runs and tests were investigated in the Tottori and Bayan Unjuul cases; those for the
Tottori case are identified by the symbol “A” (e.g., run 1A) and those for the Bayan Unjuul case are identified by the symbol “B” (e.g., run 2B).

3. Results

a. Influence of the temporal resolution of the data on thermal inertia estimation

The temporal resolution of the data used in run 1 was that of the original data and the highest of all the runs, and the results showed good positive correlations between the thermal inertia and volumetric soil water content in both the Tottori and the Bayan Unjuul cases (Figs. 1 and 2). In the Tottori case, the correlation coefficient of the data samples in the realistic range (93 days) was 0.86, and was 0.80 (53 days) in the Bayan Unjuul case. The trends and scatter in the results of run 2 were similar to those in the results of run 1; however, the results of run 2 were characterized by more scatter than those of run 1 (Fig. 3). On the other hand, the results of run 5, in which only once-a-day daytime surface temperature data were used, showed poor correlation between soil moisture and thermal inertia (Fig. 4). The results of run 16, in which data collection twice during the daytime and once during the night was simulated, were similar to those of run 2 (Fig. 5). The results of run 17 (twice during daytime and twice during the night) were similar to run 2 as well, but not clearly improved on those of run 16. The magnitude of the fluctuations of the thermal inertia calculated in the runs can be quantified by evaluating the correlation coefficient and the root-mean-square error (RMSE) of the thermal inertia. Here, the error of an estimation was defined as the difference between the estimate and the value of the linear regression equation based on the results of run 1 at the same volumetric soil water content. The regression equations in the Tottori and the Bayan Unjuul cases were:

for the Tottori case

$$P = 178\theta + 612 \quad (5a)$$

### Table 2. Conditions of the sensitivity tests.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Control</td>
</tr>
<tr>
<td>2</td>
<td>No daily change of downward longwave radiation ($L_{Y} = 0$)</td>
</tr>
<tr>
<td>3</td>
<td>Daily average of ground heat flux was 0 ($G_{M} = 0$)</td>
</tr>
<tr>
<td>4</td>
<td>Constant value of albedo was 0.25 ($\alpha = 0.25$)</td>
</tr>
<tr>
<td>5</td>
<td>Characteristic period of force-restore formulation was 12 h ($\omega = 1.454 \times 10^{-4}$ s$^{-1}$)</td>
</tr>
<tr>
<td>6</td>
<td>Characteristic period of force-restore formulation was 6 h ($\omega = 2.909 \times 10^{-4}$ s$^{-1}$)</td>
</tr>
<tr>
<td>7</td>
<td>0 to +2 K anomaly in daily average of air temperature ($T_{a} + 0$ to $T_{a} + 2$)</td>
</tr>
<tr>
<td>8</td>
<td>0 to +2 g kg$^{-1}$ anomaly in daily average of specific humidity ($q_{a} + 0$ to $q_{a} + 2$)</td>
</tr>
<tr>
<td>9</td>
<td>0 to 3 times the control values of wind speed (0 to 3U)</td>
</tr>
<tr>
<td>10</td>
<td>1.0 to 1.2 times the control values of downward insolation (1.0$S_{Y}$ to 1.2$S_{Y}$)</td>
</tr>
<tr>
<td>11</td>
<td>+1 K anomaly in surface temperature ($T_{s} + 1$)</td>
</tr>
<tr>
<td>12</td>
<td>+3 K anomaly in surface temperature ($T_{s} + 3$)</td>
</tr>
<tr>
<td>13</td>
<td>Combination of tests 2–4 and 7–11</td>
</tr>
</tbody>
</table>

### Table 3. RMSE and correlation coefficient of each run in both the Tottori and Bayan Unjuul (BU) cases. Data samples with anomalous values of thermal inertia were not included in the statistics. Symbols ** and * denote that the difference between the correlation coefficients of a given run and that of run 1 cannot be statistically rejected at a significance level of 1% and 5%, respectively.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>RMSE (J m$^{-2}$ s$^{-1/2}$ K$^{-1}$)</th>
<th>Correlation coefficient</th>
<th>No. of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tottori</td>
<td>BU</td>
<td>Tottori</td>
</tr>
<tr>
<td>1</td>
<td>284</td>
<td>127</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>362</td>
<td>176</td>
<td>0.76**</td>
</tr>
<tr>
<td>3</td>
<td>727</td>
<td>405</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>658</td>
<td>257</td>
<td>0.41</td>
</tr>
<tr>
<td>5</td>
<td>613</td>
<td>337</td>
<td>0.52</td>
</tr>
<tr>
<td>6</td>
<td>823</td>
<td>455</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>509</td>
<td>318</td>
<td>0.61</td>
</tr>
<tr>
<td>8</td>
<td>517</td>
<td>269</td>
<td>0.57</td>
</tr>
<tr>
<td>9</td>
<td>875</td>
<td>360</td>
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<tr>
<td>10</td>
<td>625</td>
<td>318</td>
<td>0.55</td>
</tr>
<tr>
<td>11</td>
<td>580</td>
<td>194</td>
<td>0.43</td>
</tr>
<tr>
<td>12</td>
<td>415</td>
<td>302</td>
<td>0.66</td>
</tr>
<tr>
<td>13</td>
<td>430</td>
<td>260</td>
<td>0.72*</td>
</tr>
<tr>
<td>14</td>
<td>586</td>
<td>199</td>
<td>0.54</td>
</tr>
<tr>
<td>15</td>
<td>536</td>
<td>257</td>
<td>0.55</td>
</tr>
<tr>
<td>16</td>
<td>377</td>
<td>206</td>
<td>0.74*</td>
</tr>
<tr>
<td>17</td>
<td>414</td>
<td>245</td>
<td>0.70</td>
</tr>
</tbody>
</table>
for the Bayan Unjuul case $P = 61 \theta + 779,$ \hfill (5b)

where $P$ is the thermal inertia (J m$^{-2}$ s$^{-1/2}$ K$^{-1}$) and $\theta$ is the volumetric soil water content (%). The values calculated from Eqs. (5a) and (5b) were regarded as true values for the purpose of this study. Figure 6 and Table 3 show the correlation coefficients and RMSEs of all runs. Differences in the correlation coefficient and RMSE due to the differences in the temporal resolution of the data, especially in surface temperature, are clearly illustrated. Run 1 showed the best performance, followed by run 2, in which the data collection frequency was equivalent to that of a geostationary satellite. The third best performance, especially in the Tottori case, was that of runs 13 and 16, in which the data collection frequency was equivalent to that of multiple polar orbiting satellites. In particular, the performance of the run using both daytime and nighttime surface temperatures (runs 13, 16, and 17) was better than that of the run using only daytime surface temperatures (run 10) even though the number of surface temperature observations during daytime used was the same. Of runs that simulated three surface temperature observations in diurnal course, runs of twice during daytime (runs 13 and 16) produced better results than those of once during daytime (runs 14 and 15). Runs that simulated one surface temperature observation in diurnal course did not produce good results. The results of runs 4, 8, 11, and 14 were accurate for one field site, but not accurate for the other field site. Therefore, the following conclusions can be drawn in terms of the ability of the proposed method to predict thermal inertia: the performance of run 1 was the best; runs with a data collection frequency of surface temperature that simulated a geostationary satellite and multiple polar orbiting satellites, especially the combination of twice during daytime and once during the night, performed almost as well as the run that used the data with the full temporal resolution (run 1); and results of runs in which the frequency of surface temperature data collection was once or twice
a day were not satisfactory, but results of the combination of once during daytime and once during the night were better.

b. Influence of the input anomaly on thermal inertia estimation

The values of the error statistics for each sensitivity test are shown in Fig. 7. As in the previous subsection, RMSE was calculated with respect to the values from the regression functions evaluated for run 1 [Eqs. (5a) and (5b)]. Values of RMSE and the correlation coefficient were calculated only from data samples that had values of thermal inertia in the realistic range. In terms of both the correlation coefficients and the RMSEs, run 2 for all tests was roughly better than the other runs. On the other hand, all tests of run 5 revealed worse results. The tendency of the error statistics for a given test (2–13) with respect to the runs was roughly the same as that of the control (test 1). Differences in statistical values among the tests were generally not significantly large; however, the model performance for the large surface temperature anomaly test (test 12) was generally the worst for a given run when evaluated in terms of the statistical values. Random bias of the insolation gave a bad influence on test 10 of all runs of the Bayan Unjuul case. The test of combined biases (test 13) did not show maximum sensitivity among other tests except for run 5 of the Tottori case and run 16 of the Bayan Unjuul case. Statistics of some of the tests were slightly better than those of the control in each of the runs; however, no individual test was better than the control in all of the runs.

c. Precision of soil moisture estimation

Regression functions were also formulated for runs 2 and 16 as well as run 1, and inversely expressed as compared to Eq. 5—namely, the independent variable was the thermal inertia and the dependent variable was the volumetric soil water content. The determined regression functions with their standard errors (SE) are as follows:
for run 1 of the Tottori case  
\[ \theta = 0.0056P - 3.44 \quad \text{and} \quad SE = 1.34, \]  
(6a)

for run 1 of the Bayan Unjuul case  
\[ \theta = 0.0164P - 12.8 \quad \text{and} \quad SE = 1.53, \]  
(6b)

for run 2 of the Tottori case  
\[ \theta = 0.0046P - 0.79 \quad \text{and} \quad SE = 1.38, \]  
(6c)

for run 2 of the Bayan Unjuul case  
\[ \theta = 0.0164P - 13.7 \quad \text{and} \quad SE = 1.74, \]  
(6d)

for run 16 of the Tottori case  
\[ \theta = 0.0059P - 3.47 \quad \text{and} \quad SE = 1.60, \]  
(6e)

and

for run 16 of the Bayan Unjuul case  
\[ \theta = 0.0149P - 11.7 \quad \text{and} \quad SE = 1.82, \]  
(6f)

where \( P \) has units of \( \text{J m}^{-2} \text{s}^{-1/2} \text{K}^{-1} \), and \( \theta \) and \( SE \) have units of percent. Formulation of \( SE \) is given as

\[ SE = \sigma_\theta \sqrt{1 - R^2}, \]  
(7)

where \( \sigma_\theta \) is the standard deviation of \( \theta \), and \( R^2 \) is the coefficient of determination. Equations (6a) and (6b) are inverse functions of Eqs. (5a) and (5b), respectively. Judging by the values of the standard errors, the regression functions were able to estimate the volumetric soil water content from thermal inertia within \( \pm 3\% \) in the Tottori case and within \( \pm (3-4)\% \) in the Bayan Unjuul case at a significance level of 5%. This implies that the method investigated in this study has the potential to be a practical technique to classify soil moisture into a few levels (e.g., wet, middle, and dry).

4. Discussion

This paper mainly aims to confirm temporal resolution of the input variables of the model, especially surface temperature. Among the simulations of three surface
temperature observations over the diurnal course, simulations of twice during daytime resulted better than those of once during daytime. An example of the simulation of two surface observations during daytime and once during the night shows better fit to the observed values of surface temperature than the simulation of one surface temperature during daytime and twice during the night (Fig. 8). This difference occurred not just because the observation around the time of maximum surface temperature was removed. In this case, the sea breeze started between 1000 and 1300 h, so that the surface temperature started declining just before noon. On the other hand, the model was not able to adjust itself to the quick change of atmospheric condition because the characteristic period of the force-restore formulation implemented in the model was 24 h. If the shape of surface temperature change during daytime was not sufficiently fitted to the observations, the optimized value of thermal inertia would be overestimated when the amplitude of surface temperature change was underestimated, or underestimated when the amplitude was overestimated.

Results of the present study showed that using surface temperature observations acquired in both daytime and nighttime was important so that the amplitude of surface temperature can be accurately estimated. However, there is an associated problem regarding the atmospheric stability. The formulation of the bulk transfer coefficient of sensible heat flux in the present model does not consider changes in atmospheric stability. In reality, the bulk transfer coefficient is larger during daytime because of unstable conditions than during the night when conditions are more stable. Because in the model the value of the bulk transfer coefficient was set to be constant over the diurnal course and tended to be largely affected by unstable conditions during daytime, an overestimation of the sensible heat flux occurs during the night. This overestimation of nighttime sensible heat flux when the surface temperature is close to the air temperature resulted in a high thermal inertia. This effect may explain why results of the simulation with four times of surface temperature observations (run 17) were comparable to the results with three observations (e.g., run 16). A formulation of the bulk transfer coefficient considering the atmospheric stability is needed.

The sensitivity due to a combination of errors of input variables (test 13) revealed errors comparable with the other sensitivity tests. Since the model is essentially a linear model, any effect of biases of input variables is superposed. In test 13, the insolation was enlarged, which caused the thermal inertia to increase. Increments in the air temperature, the surface temperature, and the specific humidity also affected the increase in the thermal inertia. However, increase in the wind speed had an
opposite effect that made the thermal inertia small; this to some extent might have cancelled the increase effect on the thermal inertia.

The sensitivity due to adding anomaly of input variables (tests 7–10 and 13) did not reveal remarkable results compared to those of the other tests, except for test 10 of the Bayan Unjuul case, in which the statistics of all runs were worse than the other tests. This might be because the increment of the insolation was more sensitive to the surface temperature change because of dry conditions of subsurface soil and the surface being relatively flat (short of vegetation). Random anomalies can enhance the estimation errors because insolation usually has only small fluctuations during a clear day with few clouds present (such conditions often occur in Mongolia).

Regression formula of the volumetric soil water content as a function of thermal inertia at Tottori and Bayan Unjuul are significantly different seemingly because of difference of soil texture. Soil at Tottori was that of a sand dune that mainly consisted of rough particles and little silt and clay. On the other hand, at Bayan Unjuul, soil consisted of silt and clay as much as sand. Thermal characteristics of soil are mainly dependent on its porosity if the water content is the same because the thermal characteristics of materials composing soil are not largely scattered (Jury and Horton 2004). Practically, typical relations between soil moisture and thermal inertia for principal soil textures should be determined, and the relations should be applied to actual surface using some surface soil cover map, such as the FAO–UNESCO Soil Map of the World (1:5 000 000) (FAO 2007), as a reference.

5. Conclusions

A method for estimating daily subsurface soil moisture by using the thermal inertia was developed. This method employed a simplified heat budget model of the earth’s surface including a vegetation canopy that incorporated radiative surface temperatures, insolation, and meteorological data. The potential of this method was investigated using field data acquired over a sand dune in Japan and a steppe with short senescent grass leaves in Mongolia. Over bare to sparsely vegetated surfaces, the estimation accuracy of the method was found to be $\pm (3–4\%)$ for the volumetric soil moisture content at a significance level of 5% when the input data and the surface temperature data of the original temporal resolution were used as inputs to the model. In practical terms, the use of surface temperature data collected three times a day, in which twice during daytime and once during the night, insolation data collected once an hour, and meteorological data collected every three hours was enough to estimate subsurface soil moisture with almost the same degree of precision as the estimation using the full temporal resolution data that were available to this study. These data intervals correspond to the use of data from multiple polar orbiting satellites, a geostationary satellite, and routine meteorological observations, respectively. Availability of satellite data and relevant products has been better recently. There are some accurate insolation products with high spatial and temporal resolutions (e.g., Takenaka et al. 2011). One can use multiple and different types of satellite data to obtain a diurnal course of physical quantity. This study can give an indication of necessary data acquisition frequency of, especially, surface temperature by actual satellite observations for estimating subsurface soil moisture with an error as much as that of the potential estimation. The authors think that the present method will be at least useful in studies associated with small scale and high spatial resolution. Additionally, formulation of the bulk transfer coefficient needs to be improved, one needs to examine how soil texture affects the parameters of the empirical function, and how dense vegetation covering soil surfaces affects the precision of thermal inertia estimation. This is all geared toward regional- and global-scale applications and to actual satellite data assimilated into the model.

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REFERENCES


