Initial Results of a New Composite-Weighted Algorithm for Dual-Polarized
X-Band Rainfall Estimation

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ABSTRACT

Data analyses for the mobile Iowa X-band polarimetric (XPOL) radar from a long-duration rain event that occurred during the NASA Iowa Flood Studies (IFloodS) field campaign are presented. A network of six 2D video disdrometers (2DVDs) is used to derive four rain-rate estimators for the XPOL-5 radar. The rain accumulation validations with a collocated network of twin and triple tipping-bucket rain gauges have highlighted the need for combined algorithms because no single estimator was found to be sufficient for all cases considered. A combined version of weighted and composite algorithms is introduced, including a new \( R(A_h, Z_{dr}) \) rainfall estimator for X band, where \( A_h \) is the specific attenuation for horizontal polarization and \( Z_{dr} \) is the differential reflectivity. Based on measurement and algorithm errors, the weights are derived to be as piecewise constant functions over reflectivity values. The weights are later turned into continuous functions using smoothing splines. A methodology to derive the weights in near–real time is proposed for the composite-weighted algorithm. Comparisons of 2-h accumulations and 8-h event totals obtained from the XPOL-5 with 12 rain gauges have shown 10%–40% improvement in normalized bias over individual rainfall estimators. The analyses have enabled the development of rain-rate estimators for the Iowa XPOL.

1. Introduction

The use of multiple X-band dual-polarimetric radars to study the precipitation structure has accelerated in the past decade, largely due to its lower cost, small size, and ability to provide high-resolution observations of the lower troposphere. Other advantages include the continuous improvement in X-band procedures for rainfall estimation, multisensor data fusion for overlapping coverage areas, and effective attenuation correction [for a review, see Bringi et al. (2007)]. In hydrologic research, the improved radar observations of rainfall are especially useful for study of the structure of the rainfall field (Krajewski and Smith 2002; Gebremichael and Testik 2013). These observations can help close the measurement gap (Villarini and Krajewski 2010) of radar-based rainfall with the point estimates of in situ instruments such as rain gauges (Ciach and Krajewski 1999) and disdrometers (Jaffrain and Berne 2012).

Early X-band weather radar network applications were primarily of interest to the atmospheric science community, for example, extreme weather phenomena (McLaughlin et al. 2009). Although precipitation observations from these meteorology-oriented networks helped to quantify the measurement uncertainties in
radar rainfall (Seo et al. 2010), these radars were inadequate to study the significant variability of rainfall at small scales (e.g., Krajewski et al. 2003, 2010; Cunha et al. 2015; Seo et al. 2015). In recent years, deployment of X-band radars that focus exclusively on hydrological applications has gained momentum (e.g., Purdy et al. 2005; Uijlenhoet and Berne 2008; van de Beek et al. 2010; Schneebebeli et al. 2013; Scipion et al. 2013). Most of these studies employ radars mounted high above the ground (e.g., towers and rooftops). The mobile X-band radars in these studies do not operate in a network topology.

Mishra et al. (2016) used the University of Iowa X-band polarimetric (XPOL) radar system to study rainfall variability at high spatiotemporal resolutions by also allowing measurements close to the ground over watersheds of interest. One of the first X-band networks used exclusively for hydrology studies, the Iowa XPOL radar system consists of four nearly identical units mounted on mobile trailers and designated as XPOL-2, XPOL-3, XPOL-4, and XPOL-5. The maiden deployment of Iowa XPOLs occurred during May–June 2013 in the Iowa Flood Studies (IFloodS) field campaign. IFloodS was the first field campaign of the NASA Global Precipitation Measurement (GPM) mission Ground Validation (Hou et al. 2008) program, which was dedicated solely to hydrological research. Mishra et al. (2013, 2014, 2016) provided details of the IFloodS deployment of XPOL along with the radar systems’ engineering design, other scientific objectives, calibration data, and initial data analyses.

In this paper, we focus on validating the XPOL system’s rainfall estimates by comparing them with the estimates obtained from the IFloodS collocated rain gauge network and disdrometers. Hydrologists conventionally rely on dense rain gauge networks to characterize the extreme spatial variability of rainfall (Ciach and Krajewski 2006; Villarini et al. 2008). Researchers have also used disdrometers to estimate this variability (Tapiador and Checa 2010; Tokay and Bashor 2010; Jaffrain and Berne 2012). Several other researchers (Diss et al. 2009; Moreau et al. 2009; van de Beek et al. 2010; Matrosov et al. 2013; Giangrande et al. 2014) have conducted similar comparative analyses with rain gauges for dual-polarimetric tower-mounted X-band radars. In particular, Vieux and Imgarten (2012) studied the scale dependence of hydrological uncertainties by comparing rainfall estimates of tower-mounted Center for Collaborative Adaptive Sensing of the Atmosphere (CASA) X-band radars with a dense rain gauge network. In the case of mobile X-band radars, Schneebebeli and Berne (2012) devised rain-rate algorithms but did not evaluate the results with a rain gauge network. A recent study by Oh et al. (2016) employed a dense rain gauge network to evaluate rain rate for a mobile X-band radar, but they did not develop any new estimators.

We intend to develop the rainfall estimators specifically for mobile Iowa XPOLs and then validate the results with a rain gauge network. We based our initial evaluation on an XPOL-5 radar that had the benefit of deployment in close proximity to various instruments: an S-band radar, dense rain gauge network, and several optical disdrometers. In future research, we will apply these new algorithms to estimate rainfall from other XPOLs to drive hydrological studies.

**a. Common X-band rainfall estimators**

Hydrologists have conducted substantial research to devise algorithms to estimate rain rates using radar observables [e.g., Bringi and Chandrasekar (2001, and references therein) for developments prior to 2001 and Hong and Gourley (2014) for recent updates]. Here we include a brief review of these algorithms with an emphasis on X-band radars. The most common variables used to estimate rain rate \( R \) (mm h\(^{-1}\)) are the horizontal radar reflectivity \( Z_h \) (mm\(^6\) m\(^{-3}\)), differential reflectivity \( Z_{dr} \) (dimensionless, in linear units), and specific differential phase \( K_{dp} \) (deg km\(^{-1}\)). The first variable, \( Z_h \), is sensitive to calibration errors but easy to observe. Therefore, the rainfall estimator \( R(Z_h) \) is the simplest choice for S-band radars (Seo et al. 2010). At X band, signal attenuation leads to biases in \( Z_h \) (Park et al. 2005), so X-band rainfall estimation algorithms usually rely on the second variable, \( K_{dp} \), because of its insusceptibility to attenuation and the higher sensitivity of propagation phase at X band (Matrosov et al. 2002). Park et al. (2005) rely on an X-band rain-rate algorithm that uses only \( Z_h \) where an estimator of the form \( R(Z_h) = aZ_h^b \) (here, \( Z_h \) is expressed in mm\(^6\) m\(^{-3}\)) admits different values for the set (indicated by curly brackets) of constants \([a, b]\) in case of stratiform and convective types of precipitation. Along with Maki et al. (2005) and Matrosov et al. (2005, 2006), Park et al. (2004) also provide X-band \( K_{dp} \)-only estimators of rain rate, wherein the estimator \( R(K_{dp}) = aK_{dp}^b \) takes the values of the constants \([a, b]\) in the range 12–20 and 0.76–0.85, respectively. The third variable, \( Z_{dr} \), is closely related to drop size diameter; however, the estimator \( R(Z_{dr}) \) is infeasible in practice because \( Z_{dr} \) is a relative measurement. The \( Z_{dr} \)-based algorithms, therefore, usually employ another absolute measurement variable such as \( Z_h \) or \( K_{dp} \). While this increase in the number of variables equips an algorithm with more information on the drop size distribution (DSD), it also acquires more sources of measurement errors.

The twin-measurement estimator \( R(Z_h, Z_{dr}) = aZ_h^{0.1}Z_{dr} \), where \( Z_{dr} \) is a dimensionless ratio (in linear
units here), has been widely evaluated; Bringi and Chandrasekar (2001, p. 538) detail the values of the constants $a$, $b$, and $c$ for X-band radars. At X band, this estimator is less efficient because of significant bias in $Z_d$ caused by differential attenuation. The $R(Z_h, Z_{dr})$ estimator is also sensitive to the presence of hail, graupel, and bright band (Ryzhkov and Zrnič 1995). For an X-band radar, Oh et al. (2016) found the improvement in rainfall estimation using $R(Z_h, Z_{dr})$ to be incremental compared to $R(Z_h)$. The other twin-measurement estimator is modeled as $R(K_{dp}, Z_{dr}) = a K_{dp}^{b} Z_{dr}^{c}$; it has the advantage of reduced dependence on absolute radar calibration. The $R(K_{dp}, Z_{dr})$ estimator is not widely used for X band because of accentuated $Z_{dr}$ error at higher frequencies. A few X-band rain retrieval assessments (Gosset et al. 2010; Matrosov 2010) conclude that $R(K_{dp}, Z_{dr})$ is unable to outperform $R(K_{dp})$, yet improvement in X-band results with $R(K_{dp}, Z_{dr})$ continues to be reported (Schneebeli and Berne 2012). The estimator $R(Z_h, K_{dp})$ (Bertonasco et al. 2013) has not been extensively evaluated for X band.

Rain-rate algorithms that employ all three variables ($Z_h$, $Z_{dr}$, and $K_{dp}$) have also been proposed for X band. These have shown better performance than $R(Z_h)$ and $R(K_{dp})$ algorithms (Anagnostou et al. 2004; Matrosov et al. 2002). This estimator is usually expressed as $R(Z_h, Z_{dr}, K_{dp}) = a Z_h^{b} Z_{dr}^{c} K_{dp}^{d}$, where $a$ is a function of the altitude and $b$, $c$, and $d$ are constants. However, the increased number of variables causes concerns on accrual of further measurement uncertainties and usage of this estimator is less common.

b. X-band estimators based on $A_h$

A trade-off between measurement uncertainties and various calibration biases suggests $R(K_{dp})$ as the only plausible choice for X-band rain-rate estimation. However, the computation of $K_{dp}$ in the presence of noise does not result in a robust estimate, especially in light rain (Goddard et al. 1994). Therefore, Ryzhkov et al. (2014) recently investigated the use of specific attenuation (for horizontal polarization) $A_h$ to estimate rainfall because $A_h$ is less sensitive to DSD variability. Their estimator yielded robust S-band estimates, even though the attenuation at S band is insignificant. Their X-band estimates using $R(A_h)$ agreed with the rain gauges. More recently, Mishra et al. (2016) compared the performance of $R(A_h)$ for mobile X-band radars with $R(Z_h)$ and $R(K_{dp})$. They tuned the attenuation correction procedure as detailed by Bringi et al. (2011) for a C-band radar to the mobile X-band system.

One could use $A_h$ in conjunction with other polarimetric variables for rainfall estimation, although such estimators have not been examined for X-band radars. For example, Keenan et al. (2001a,b) applied $R(A_h)$ to estimate tropical rain with a C-band radar and obtained exceedingly large errors because of the difficulty in estimating $A_h$ at C band. However, the performance across various raindrop axial ratios improved when they combined $A_h$ and $K_{dp}$ to estimate $R$. The same study suggested using $Z_{dr}$ in conjunction with $A_h$ and found that the estimator $R(A_h, Z_{dr})$ was least affected by the biases due to assumed raindrop axial ratio. This estimator took the form $R(A_h, Z_{dr}) = a A_h b 10^{c Z_{dr} d}$, where the values of the constants $a$, $b$, and $c$ were in the ranges from 836 to 1009, 0.98 to 1.01, and $-3.0$ to $-2.1$, respectively. Similarly, algorithms using several polarimetric variables such as $R(Z_h, K_{dp}, A_h)$ and $R(Z_h, Z_{dr}, K_{dp}, A_h)$ have also been suggested for S-band radars (You and Lee 2015), but not comprehensively evaluated for X band.

c. Composite and weighted rainfall estimators

Because no general consensus exists on using any one of the aforementioned methods with assured accuracy, choosing an exact rainfall estimation algorithm for a particular radar deployment remains a subjective craft. Therefore, previous research has used different estimators together and applied them individually in precipitation regions where they have a proven performance record. A number of composite estimators have been previously suggested for S- and C-band radars where the estimator is chosen based on thresholds derived from $Z_h$ alone (Ryzhkov et al. 2005), $Z_h$ with additional polarimetric variables (Gorgucci et al. 2001; Keenan et al. 2001b; Bringi et al. 2004; Silvestro et al. 2009), hydrometeor classification (Giangrande and Ryzhkov 2008), and precipitation dynamics (Le Bouar et al. 2001). Because a composite method introduces discontinuities at threshold values, applying a weighted linear combination of multiple estimators to all precipitation regions might be helpful. The weights in such a combination account for the error characteristic of the estimator at different rainfall rates. Such weighted estimators are currently being tested for S-band radars (Pepler et al. 2011; Pepler and May 2012; Wen et al. 2015).

The plethora of rain-rate estimators, their possible contortions, and relative advantages make it difficult to conclude which algorithm will always be the best. The only foregone conclusion is that the weather radar community will continue to base state-of-the-art algorithms for rainfall estimation on fusing and weighting existing approaches in ways that complement the strengths of each and improve upon the errors for a specific precipitation event. In this paper, we propose an XPOL rainfall estimator that is both composite and weighted in order to represent realistic error variations.
of rain rates within the algorithm. We combine only those estimators that are most suited for X band [namely, \( R(Z_h) \), \( R(K_{dp}) \), \( R(A_h) \), and our proposed \( R(Z_{dr}, A_h) \)] and use data from the recent XPOL deployment in the IFloodS campaign. In the next section, we provide details of the instruments and dataset from the IFloodS. We then analyze the rain-rate estimators using data from the 2D video disdrometers (2DVDs) and XPOL radar in sections 3 and 4, respectively. We compare the XPOL rain rate with that of collocated rain gauges in section 5, and then propose our composite-weighted rain-rate algorithm in section 6.

The novel contributions of this work are as follows: 1) we propose a new composite-weighted rainfall estimator for an X-band system; 2) we obtained the data for this estimator from a mobile radar network that makes measurements over watersheds at high spatiotemporal resolutions (e.g., 75-m range resolution and 5°s\(^{-1}\) scan rate for Iowa XPOLs when compared with 100 m and 12°s\(^{-1}\) for CASA X-band systems, respectively); and 3) for the first time, we suggest, develop, and test an \( R(Z_{dr}, A_h) \) estimator for X-band observations.

## 2. IFloodS instrumentation and dataset

All the data used in this work were collected during the IFloodS campaign (from 1 May to 15 June 2013). The IFloodS experiment involved deployment of an extensive network of ground instruments to observe precipitation in central and northeastern Iowa. NASA and the Iowa Flood Center (IFC) laid down a heavily instrumented network of rain gauges, disdrometers (2DVDs and Autonomous Parsivel Units), and micro rain radars throughout the IFloodS region. The Iowa XPOL radars were deployed in pairs in the Clear Creek and Turkey River Watersheds. The coverage of all XPOLs overlapped with that of NASA’s S-band polarimetric (NPOL) radar. Mishra et al. (2016) presented comprehensive polarimetric data analyses and comparisons of XPOL-2 and XPOL-4 radars for common precipitation events. In this study, we use data from the XPOL-5 radar located closest to the NPOL radar.

### a. Iowa XPOL-5 radar

The Iowa XPOL-5 radar is one of the four radars that constitute the University of Iowa X-band mobile weather radar system [see Mishra et al. (2016) for details regarding radar parameters]. During the IFloodS field campaign, we deployed the XPOL-5 at the Eastern Iowa Airport in Cedar Rapids, Iowa (Fig. 1). Its coverage overlapped with the XPOL-3 radar in the Clear Creek Watershed. The XPOL-5 operated at range resolution of 75 m and scanned over the watershed (90°–310° sector) at the rate of 5°s\(^{-1}\) for both plan position indicator (PPI) and range–height indicator (RHI) scans. When targets of opportunity occurred within the XPOL-5 coverage, its scan sequence was as follows: a low-elevation full PPI surveillance scan at 5°s\(^{-1}\), sector PPI scans at 5°s\(^{-1}\) over the watershed sector in elevations from 2° to 8° in increments of 1°, a birdbath (vertically pointing) scan at 12°s\(^{-1}\), and one RHI each in the direction of the NPOL and XPOL-3 at 5°s\(^{-1}\). Occasionally, the XPOL-5 also operated in increased range oversampling mode, although the range resolution (75 m) did not change.

### b. 2DVDs

The 2DVD system (Kruger and Krajewski 2002) uses two orthogonally placed cameras to concurrently observe falling raindrops. It is the most commonly used automated raindrop measurement method and can provide considerable drop counts for shape and size measurements (Gatlin et al. 2015). The 2DVD measurements have been used to investigate the relationships between drop size, shape, and terminal fall velocity (Thurai and Bringi 2005; Thurai et al. 2009), as well as to evaluate weather radar-rainfall estimates (Thurai et al. 2008; Zhang et al. 2008).

During the IFloodS campaign, six 2DVDs—SN37, SN38, SN70, SN25, SN35, and SN36—were strategically deployed locations of the XPOL-5, 2DVDs, triple-bucket gauges, and IFC rain gauges. We deployed XPOL-5 at the Eastern Iowa Airport site with full coverage of the Clear Creek Watershed (shown in white). The gray circle shows 40-km range coverage of XPOL-5 radar. Clockwise from top left, the photographs show XPOL-5 radar, 2DVD, WY triple-bucket gauge, and IFC rain gauge as deployed during IFloodS.

![Fig. 1. Deployment locations of the XPOL-5, 2DVDs, triple-bucket gauges, and IFC rain gauges. We deployed XPOL-5 at the Eastern Iowa Airport site with full coverage of the Clear Creek Watershed (shown in white). The gray circle shows 40-km range coverage of XPOL-5 radar. Clockwise from top left, the photographs show XPOL-5 radar, 2DVD, WY triple-bucket gauge, and IFC rain gauge as deployed during IFloodS.](image)
deployed along the 130° radial relative to the NPOL radar. The XPOL-5 radar had only SN37, SN38, and SN70 within its coverage. Table 1 shows the locations of these 2DVDs relative to the XPOL-5 radar. Each of the two cameras of the 2DVDs contains 632 pixels. The raindrops that exceeded 50% of their terminal fall speed were eliminated to avoid spurious measurements caused by angel clutter or splash drops [see Tokay et al. (2001) for this threshold]. To eliminate noise, the recorded data do not account for minutes that clocked fewer than 10 drops and rain rate below 0.01 mm h\(^{-1}\).

d. University of Wyoming rain gauges

We can further enhance the reliability of a twin tipping-bucket rain gauge by adding a third bucket. Such a triple-bucket gauge is similar to its twin-bucket counterpart in operating principle and data collection method. The University of Wyoming (WY) installed a closely spaced network of four triple-bucket rain gauges (15444, 22390, 15442, and 15443) near the town of Shueyville, Iowa, about 8 km southeast of XPOL-5 (see Fig. 1, Table 1). These gauges were custom designed by adding a third off-the-shelf bucket to an existing twin-bucket gauge. All stations reported 5-min accumulations of precipitation.

e. Rain events

Iowa weather is characterized by wet springs and hot, humid summers. The months of May and June are particularly prone to floods. Most of eastern Iowa is topographically flat. Therefore, our choice of events for this analysis depended only on the availability and quality of data from XPOL-5, gauges, and disdrometers. We studied the 26 May 2013 precipitation case when all the instruments were available. This event was a mixture of stratiform and convective echoes associated with a mesoscale convective system (MCS). The convection began to build in the early morning (0700 UTC) on 26 May along the low-level jet to the south and west of XPOL-5. The XPOL-5 region witnessed the mixture of stratiform and convective systems until 1900 UTC, with the most intense echoes covering most of XPOL-5 region between 1100 and 1300 UTC and 1500 and 1800 UTC. After the MCS passed through, a more intense and widely scattered convection developed and continued until the next day. The XPOL-5 collected uninterrupted data for this event at 75-m range resolution during the period of 25–27 May 2013. Here, we analyze only data collected in PPI mode; the vertical structure of precipitation observed in RHI scans is not within the scope of this study.

3. Rain-rate estimators from 2DVD data

We used data from the six aforementioned 2DVDs installed during the IFloodS campaign to derive various

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Range (km)</th>
<th>Azimuth</th>
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<tbody>
<tr>
<td>IFC gauges</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ATK01</td>
<td>16.13</td>
<td>319.9°</td>
</tr>
<tr>
<td>AGC02</td>
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<td>237.9°</td>
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<tr>
<td>WY gauges</td>
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</tr>
<tr>
<td>22390</td>
<td>8.25</td>
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<tr>
<td>15443</td>
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<tr>
<td>2DVDs</td>
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</tr>
<tr>
<td>SN38</td>
<td>11.96</td>
<td>255.7°</td>
</tr>
<tr>
<td>SN37</td>
<td>30.26</td>
<td>292.7°</td>
</tr>
<tr>
<td>SN70</td>
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<td>149.7°</td>
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</table>
rain-rate algorithms for the XPOL radars. Commensurate with our remarks in sections 1a and 1b, we considered the following four algorithms for the X-band rain-rate estimators:

1. reflectivity-based estimate \[ R(Z_h) \],
2. \( K_{dp} \)-based estimate \[ R(K_{dp}) \],
3. \( R \) from the specific attenuation \[ R(A_h) \], and
4. \( R \) estimated from \( A_h \) and \( Z_{dr} \) \[ R(A_h, Z_{dr}) \].

As mentioned earlier, the first two are commonly employed for X band, while the third is more recent. Ryzhkov et al. (2014) described the advantages and disadvantages of \( R(A_h) \) over the first two estimators. Note that, like \( R(K_{dp}) \) and unlike \( R(Z_h) \), the \( R(A_h) \) estimate is not affected by radar miscalibration.

The fourth estimator included in our evaluation has not previously been applied to X-band observations. It is based on \( A_h \) and \( Z_{dr} \), and therefore, it can potentially reduce the errors due to DSD variability. However, this estimator will also have error contributions from two parameters instead of just one, as is the case of the other three estimators. We will explore this further in section 6.

We performed the T-matrix scattering simulations to quantify the sensitivity of the four estimators to natural variations in DSD. Here, we used 1-min DSD data from the 2DVDs as input. We smoothed these 1-min DSD data over 3 min by a uniformly weighted moving-average window (i.e., over three 1-min samples). For the scattering simulations, we used data from a total of four different events (occurring on 20, 25, 26, and 28 May 2013, respectively) from the IFloodS campaign. In addition, we used the reference drop shapes in Thurai et al. (2007) to represent the most probable shapes for drop equi-volume diameters up to 7 mm. We further assume a Gaussian distribution for drop canting angles as that has a standard deviation of 6° (Huang et al. 2008), as well as the ambient temperature set to 20°C for the simulations. Figures 2a–c show the scatterplots of \( Z_h \), \( K_{dp} \), and \( A_h \) derived from the scattering simulations against the measured \( R \) from the 2DVD data. In all three cases, we show the best-fit equations as solid orange lines. Figure 2d shows the \( R \) estimated from a two-parameter fit (the two parameters being \( A_h \), \( Z_{dr} \)) versus the measured \( R \). We included the best-fit equation for \( R(A_h, Z_{dr}) \) case in the legend of Fig. 2d.

We obtained the following function fits corresponding to the four estimators (here, “linear” in the subscript indicates that the variable is expressed in linear units):

1. estimator 1: \( Z_{h,\text{linear}} = 350.3 R^{1.56} \),
2. estimator 2: \( R(K_{dp}) = 15.5 K_{dp}^{0.89} \),
3. estimator 3: \( R(A_h) = 47.6 A_h^{0.856} \); and
4. estimator 4: \( R(Z_{dr}, A_h) = 155.6 A_h Z_{dr,\text{linear}}^{-1.97} \).

The first three estimators show widely distributed scatter highlighting the effect of DSD variability. On the other hand, the scatter for the fourth estimator shows a strong linear relationship with the measured rain rate. The dependence of \( R \) on mass-weighted mean diameter \( D_m \) (see Fig. 2c) largely subsides when \( Z_{dr} \) is incorporated in the estimator (Fig. 2d). This effect is similar to the reduction of scatter seen in \( R(K_{dp}, Z_{dr}) \) estimator described by Ryzhkov and Zrnić (1995).

4. XPOL data over 2DVDs

Before applying the four estimators to XPOL data to derive rainfall rates and rain accumulations, we ensured high data quality parameters for estimation of \( R \). In particular, as we mentioned in section 3, our algorithms use the following parameters: 1) the attenuation-corrected \( Z_h \), 2) \( K_{dp} \), 3) \( A_h \), and 4) the attenuation-corrected \( Z_{dr} \). Prior to and during IFloodS, we calibrated the XPOL radars using solar scans (Mishra et al. 2013), metallic sphere, birdbath scans, and engineering tests of different subsystems (Mishra et al. 2016). Our previous work (Mishra et al. 2016) also describes in detail the attenuation correction algorithm for the measured values of XPOL radar reflectivity and differential reflectivity. To illustrate various data processing steps, Fig. 3 shows sample range profiles of the aforementioned XPOL-5 parameters along the azimuth in the direction of one of the 2DVDs (SN38). We took the range profiles from a 3°-elevation PPI scan at 1116 UTC 26 May 2013. As evident from the range profiles of measured \( Z_h \) and corrected \( Z_h \) shown in Fig. 3a, the radar observables suffer from significant attenuation. Similarly, considerable differential attenuation is evident in the corresponding \( Z_{dr} \) range profiles (Fig. 3b). We calibrated the \( Z_{dr} \) data using the nearest birdbath scans that showed a bias of 0.3 dB. During the birdbath scan, the radar makes a full 360° scan while pointing vertically. Since the \( Z_{dr} \) measured in this position should ideally be zero during light stratiform rain, any deviation from this value will give the measurement of \( Z_{dr} \) bias.

Figure 3c shows the specific attenuation (i.e., \( A_h \)) itself; it is one of the outputs of our attenuation correction algorithm (Mishra et al. 2016). Finally, Fig. 3d shows the range profile of \( K_{dp} \) obtained through our differential phase filtering procedure (Hubbert and Bringi 1995). The processing steps also include the elimination of nonprecipitation echoes from the radar data.

This classification uses the standard deviation of the differential propagation phase \( \Phi_{dp} \) (over a 10-gate moving window) to generate a binary data mask where a value of 1 indicates “good” data, that is, the presence of meteorological echoes (Fig. 3d). The classification is
based on our use of a threshold of 5° and SNR > 0 dB. To derive $K_{dp}$, we first use the iterative range filter methodology applied to each range profile of $\Phi_{dp}$. The finite impulse response (FIR) range filter is, in essence, a weighted moving-average filter in which the weights are determined by the desired magnitude response of the filter transfer function (or spectrum). Here, the FIR filter coefficients are based on 75-m gate spacing [an example of the filter transfer function for 150-m gate spacing can be found in Hubbert and Bringi (1995)]. The iterative nature of the algorithm described in Hubbert and Bringi (1995) is designed to remove local perturbations in the $\Phi_{dp}$ data (e.g., due to the backscatter differential phase) while maintaining the monotonic increase in the propagation phase with the range along the beam. We use a “telescoping” method to compute the $K_{dp}$ from the iteratively filtered $\Phi_{dp}$ profile; that is, we use a variable number of gates, depending on the $Z_{h}$ value, to determine the slope of a linear least squares fit [see Mishra et al. (2016) for details].

The SN38 2DVD was located at a range of 12 km from XPOL-5 (see Fig. 1). Table 2 lists the values of the XPOL-5 observables at this location (marked by a green line) for the range profiles in Fig. 3. At the SN38 site, the path attenuation is about 3 dB, and the path differential attenuation is about 0.4 dB. The values of the copolar correlation coefficient $r_{hv}$ and the data mask at this

Fig. 2. Scatterplots of the measured rain rate from the 2DVD data against polarimetric variables (a) $Z_{h}$, (b) $K_{dp}$, and (c) $A_{h}$ derived from the scattering simulations at X band. In each of these plots, the orange line represents the best-fit equation. (d) Scatterplot of $R$ obtained from a two-parameter ($A_{h}$ and $Z_{dr}$) fit against the measured $R$ from 2DVD.
location clearly indicate that the returned echoes correspond to precipitation. The significantly high values of $K_{dp}$ and $A_h$ further corroborate this. The outputs from our data processing procedures show excellent agreement with the 2DVD-based estimates. Figure 4 shows the time series comparisons of the XPOL-5 $Z_h$, $Z_{dr}$ (both are corrected for attenuation), $K_{dp}$, and $A_h$ over three 2DVD sites, with the corresponding variables obtained from 1-min DSDs of three disdrometers. Here, we applied spatial smoothing (over 618 azimuth and 62 range gates) to the radar data and temporal averaging of 5 min to the 2DVD-based data. The time series depicts the MCS event of 26 May 2013, which lasted well over 8 h. Throughout the event, we see close resemblance between XPOL-5 measurements and each of the three 2DVDs within its coverage over all four parameters. The $K_{dp}$ and $A_h$ comparisons are of particular interest; their temporal fluctuations are remarkably similar between XPOL-5 and 2DVDs, lending confidence and credence to our data processing techniques.

### 5. Rain-rate comparisons

After validating our data processing and quality control procedures, we can now assess the four rain-rate estimators. To compare our results, we use three 2DVDs and 12 rain gauges (see Table 1 for location details of these instruments) as in situ references.

#### a. Comparisons with 2DVDs

In Fig. 5, we show the time series comparisons of radar rain rates with SN38 2DVD as an example. Here, each panel compares rain-rate estimates from SN38 with that obtained from XPOL-5 via one of the four estimators described in section 3. To eliminate speckle noise from the time series, we applied Lee filtering (Thurai et al. 2012) to the XPOL-5-based estimates. We averaged the 2DVD-based calculations using a moving window of 5 min to eliminate noisy fluctuations. In general, the SN38 time series shows reasonably good agreement with all four time series of XPOL-5. However, no single XPOL-5 estimator clearly outperforms the rest by showing less deviation from the SN38 estimate at all times. For example, at 1100 UTC, $R(K_{dp})$ follows the high peaks of SN38, whereas the traditional $R(Z_h)$ as well as our proposed $R(A_h, Z_{dr})$ algorithm better captures the rapid fall in rain rates between 1200 and 1300 UTC. It is also apparent that none of the estimators alone is sufficient to determine $R$ for all cases. We therefore propose a hybrid or combination of all methods to improve the accuracy.

We consider two possible ways of developing such a hybrid estimator. The first is to formulate a composite algorithm that outputs one of the four estimates based on some predetermined thresholds (or hard limits) for each of the measured radar variables. The second option is to use a weighted average of four rain-rate estimators where the weights remain constant for the entire event and are derived from the error analysis. Figure 6a shows the output of a simplified version of a composite algorithm $R_{\text{composite}}$ based on $Z_h$, $K_{dp}$, $A_h$, and $Z_{dr}$ thresholds.
FIG. 4. The time series of XPOL-5 attenuation-corrected polarimetric variables (a)–(c) $Z_h$, (d)–(f) $Z_{dr}$, (g)–(i) $K_{dp}$, and (j)–(l) $A_h$, compared with that of 2DVDs SN37, SN38, and SN70, respectively.
Figure 6b shows the resulting output when all four estimators are equally weighted to produce the estimator $R_{\text{equal}}$. We note that the composite algorithm improves upon the strengths of $R(Z_h)$ and $R(K_{dp})$, largely following the trend indicated by SN38, albeit with some overestimation of rainfall. On the other hand, the weighted estimator generally agrees with 2DVD estimates with underestimation at high peaks. The total rain accumulation for SN38, as in Figs. 5 and 6, was 28.4 mm when compared with 29 and 36.2 mm for the simple composite estimator $R_{\text{composite}}$ and equally weighted algorithm $R_{\text{equal}}$, respectively.

### b. Comparisons with rain gauges

The 12 gauges were located in the range of 8–24 km from XPOL-5 (see Fig. 1, Table 1). Figure 7a compares the time series of the rainfall rates of one of the gauges, OXF01, located within the Clear Creek Watershed, with the radar-based estimates. Here, for clarity, we provide only two radar-based estimates, that is, $Z$–$R$ and $R(K_{dp})$; we will consider all four estimators later in this subsection. The OXF01 gauge was located at a range of 18.6 km from the radar. We performed the XPOL-5 PPI scans at 3° elevation. This results in the radar pulse volume at 2.6 km height above the gauge site.

As shown in Fig. 7a, the radar-based estimates and the gauge measurements show good agreement during 1100–1300 UTC. But a time lag appears between the two time series during other periods, with the gauge data lagging behind the radar estimates. We attribute this lag partly to the 2.6-km difference in height; if one assumes typical fall speeds of 4–8 m s$^{-1}$ for rain drops, then this height difference results in the gauge lagging behind by 5–10 min. The time lag will be even longer for smaller drops. Since these time lags are significant, rain accumulation would be a more appropriate metric to evaluate the rain-rate estimators.

Figure 7b shows the time series comparison of rain accumulation corresponding to Fig. 7a. Here, we begin accumulation for XPOL-5 at 0830 UTC to avoid the sharp peak in the radar-based $R$ in preceding time instant. Throughout the 8-h time period, we observe that the $Z$–$R$ accumulations are usually within 1 mm of the gauge accumulation. The $R(K_{dp})$ shows a slight overestimation that sometimes reaches a maximum of about 3 mm over the gauge accumulation. The total accumulation from the rain gauge was 21 mm, whereas the $Z$–$R$ and $R(K_{dp})$ methods yielded 22 and 24.5 mm, respectively.

We now examine accumulations from all four estimators and compare them with all 12 rain gauges. Here, we choose 2-h time periods (0800–1000, 1000–1200, 1200–1400, and 1400–1600 UTC) for rain
accumulation. Figure 8a shows that all four estimators are uniformly distributed around the 1:1 line. For this particular event, we note that the $R(A_h)$ values usually lie below the 1:1 line, whereas the $R(K_{dp})$ estimates tend to lie above the same line. The $R(Z_h)$ and $R(A_h, Z_{dr})$ results are more evenly distributed around the line of equality, while the $Z-R$ method exhibits less scatter. Once again, we observe that all estimators have varying distribution and characteristics, highlighting the need for a hybrid algorithm that can leverage the strengths of each. In Fig. 8b, we averaged all four estimates and noticed that the spread is reduced even with this simple equi-weighted estimation. In the next section, we fine-tune the weights by deriving them based on the errors associated with each of the four estimators.

FIG. 7. The time series of (a) rain rate and (b) rain accumulations from the gauge OXF01 (see Fig. 1) compared with that from the corresponding range (~18.6 km) of the XPOL-5 radar. We estimated the XPOL-5 fields using $R(Z_h)$ and $R(K_{dp})$ algorithms. The data correspond to the precipitation event of 26 May 2013.

FIG. 8. Two-hour rain accumulation comparisons of XPOL-5 with the 12 gauges using (a) four estimators individually and (b) an equi-weighted average of the four estimates. The gray curve presents the 1:1 line.
6. Toward a composite-weighted algorithm

As remarked earlier, prior research on S-band rainfall estimation has considered two hybrid methods: composite and weighted estimators. Very recent work by Thurai et al. (2016) using University of Chicago–Illinois State Water Survey (CHILL) X-band data over a heavily instrumented site shows that a simple equi-weighted rainfall estimate was closer to the Pluvio rain gauge measurements than any of the four estimators individually. The same study shows that a simple form of a composite algorithm also agreed well with the Pluvio measurements. In the case of XPOL-5 data, we have already shown the performance of these two hybrid estimators in Fig. 6.

For further improvement of our hybrid estimators, we considered using the weights with the composite algorithm. The rationale for this combination is based on the following statement: while the weights depend on the errors associated with each of the estimators, they can also vary based on the rain intensity itself. To this end, we split the weights into six categories based on the rain intensity partitions: 1) 9–30, 2) 30–35, 3) 35–40, 4) 40–44, 5) 44–49, and 6) ≥49 dBZ. In qualitative terms, they broadly represent light rain, stratiform rain, mixed rain type, convective rain, strong convection, and severe convection, respectively. Here, we assumed that the upper limit value is excluded from the interval, that is, 9–30 dBZ implies the interval ≥9 and <30 dBZ. For the values below 9 dBZ, we used the conventional Z–R algorithm and ignored other estimators.

We then analyzed the errors associated with the four estimators to derive the weights. We considered two components: the measurement error that quantifies variability due to polarimetric observables and the algorithm error that incorporates errors due to the use of a specific estimator. In the sequel, we derived expressions for the measurement errors. On the other hand, we directly computed the algorithm errors from simulations. We explain in section 6c that, in order to account for the combined effect of these two errors, they can be assumed to be uncorrelated.

a. Measurement errors

The measurement errors are related to the radar parameters used in a specific algorithm. In the following, we consider each estimator separately. Here, $\text{var}(\cdot)$ denotes the variance of the quantity inside the brackets. The variable with a bar above denotes the mean value.

We provide the full derivation of measurement errors of estimator 1 in the appendix. The measurement errors for the other three estimators can be derived mutatis mutandis. The key assumptions for each estimator are, however, included below.

1) Estimator 1, $Z–R$ or $R(Z_m)$:

$$\frac{\text{var}(R)}{(R)^2} = \beta^2 \frac{\text{var}(Z_m)}{(Z_m)^2} + (0.46\beta r)^2 \text{var}(A_h), \quad (1a)$$

where $Z_m$ and $Z$ are measured and corrected reflectivities (in linear units), $\beta$ is a constant, and $r$ is the range. We show in the appendix that

$$\frac{\text{var}(R)}{(R)^2} = (0.016) + (A_h r)^2 \left(0.0023 + \frac{0.0078}{K_{dp}^2}\right). \quad (1b)$$

2) Estimator 2, $R(K_{dp})$:

$$\frac{\text{var}(R)}{(R)^2} = (0.89)^2 \frac{\text{var}(K_{dp})}{(K_{dp})^2}. \quad (2a)$$

Assuming $R(K_{dp}) = 15.5K_{dp}^{0.89}$ (see Fig. 2b) and, based on the typical $\Phi_{dp}$ data in rain, the standard deviation of $K_{dp}$ to be $0.3^\circ \text{km}^{-1}$, it can be shown that

$$\frac{\text{var}(R)}{(R)^2} = \left[0.072 \left(\frac{1}{K_{dp}^2}\right)\right]. \quad (2b)$$

3) Estimator 3, $R(A_h)$:

$$\frac{\text{var}(R)}{(R)^2} = (1.06)^2 \text{var}(A_h) \left(A_h^3\right). \quad (3a)$$

We now assume $R(A_h) = 47.6Z_{A_h}^{1.06}$ (see Fig. 2c) and that $A_h$ varies as $Z_{A_h}^{0.8}$ [derivable from Bringi and Chandrasekar (2001, Eq. 7.73c) with $n = 4.5$ and $m = 6$]. Then, it can be shown that

$$\frac{\text{var}(R)}{(R)^2} = (0.029) + \left(0.1 \frac{1}{K_{dp}^2}\right). \quad (3b)$$

4) Estimator 4, $R(A_h, Z_{dr})$:

$$\frac{\text{var}(R)}{(R)^2} = \frac{\text{var}(A_h)}{(A_h^3)} + (0.46\gamma r)^2 \text{var}(A_{dp}). \quad (4a)$$

We now assume $Z_{dr} = Z_{dr,\text{measured}} \exp(0.46A_{dp}r)$ and that $\gamma$ (the same as $A_h$) varies as $Z_{A_h}^{0.8}$. Here, $Z_m$ and $Z_{dr}$ are measured and corrected differential reflectivities (in linear units). At a given range $r$, the path integrated differential attenuation $\text{PIA}_{dp}(r) = 2\int_0^r A_{dp}(u) \, du$, where $A_{dp}$ is the specific differential attenuation and $du$ is the differential of the variable $u$. Then, it can be shown that

$$\frac{\text{var}(R)}{(R)^2} = \left(0.026 + 0.09 \frac{1}{K_{dp}^2}\right) \left[1 + 0.85(A_{dp} r)^2\right]. \quad (4b)$$
We can evaluate the above set of equations either for a particular event or for all events observed by the radar. For example, if we consider the case described in Table 2, then the values of \( \text{var}(R)/(\bar{R})^2 \) are 0.097, 0.36, 0.51, and 0.48 for estimators 1, 2, 3, and 4, respectively. We observed that the corresponding \( R \) over the 2DVD SN38 disdrometer was 2.22 mm h\(^{-1}\). The total rain accumulation for SN38, as in Figs. 5 and 6, was 28.4 mm when compared with 48.1, 39.2, 21.5, and 10.3 for estimators 1, 2, 3, and 4, respectively. On the other hand, one can also evaluate the equations by processing randomly chosen PPI scans to extract the parameters required as input, such as \( K_{dp}, A_h, A_{dp}, \) etc. from each PPI pixel for which the data mask has been marked as "good" (i.e., clutter free, meteorological echoes). We carried out this set of procedures using a sufficient number of PPI scans to ensure converged histograms of \( \text{var}(R)/(\bar{R})^2 \) for each of the estimators. We further derived the histograms for six \( Z_h \)-based partitions for a given pixel. Table 3 shows the mode values of these partitioned histograms for the four estimators. The values show a clear dependence on the \( Z_h \) partition. We also observe a substantial decrease in measurement errors for the higher-intensity precipitation in the cases of \( R(K_{dp}), R(A_h), \) and \( R(A_h, Z_{dr}) \). This is to be expected because all three parameters \( (K_{dp}, A_h, Z_{dr}) \) become more significant and sensitive at higher rain rates.

### Algorithm errors

The scatterplots given in Fig. 2 enable us to quantify the errors arising from the application of the various algorithms. Such an "algorithm error" is a measure of the variability of the individual DSD-based calculations from the mean fits shown for each of the estimators. Here, we follow procedures similar to those presented in Bringi et al. (2011; see their Figs. 6a–c and the accompanying test). In particular, for every reflectivity interval mentioned in the beginning of section 6, we compute the fractional standard error \( \text{FSE} = \sqrt{\text{var}(R_{\text{est}} - R_{\text{disdro}})/R_{\text{disdro}}} \), where \( R_{\text{est}} \) is the estimate obtained by one of the four algorithms from section 6a and \( R_{\text{disdro}} \) is the true rain rate measured by the disdrometer. We can use the resulting outputs of algorithm and measurement errors to derive \( \text{var}(R) \) based on the FSE for the four estimators. We list the FSE for each estimator in Table 4.

#### Derivation of weights

We can assume measurement errors and algorithm errors to be uncorrelated (Bringi and Chandrasekar 2001, Eq. 8.24b). Hence, within a given \( Z_h \) partition, their variances can be added together for each of the estimators to obtain the total error variance. We then use this total variance to derive the estimator weights for each of the partitions. The weights depend on the inverse of the total variance, and the sum of weights is normalized to unity. Table 5 lists the resulting weights for each of the estimators. As an example, when the (attenuation corrected) reflectivity at the gauge site lies within the 9–30-dBZ range, then the four estimators should use the set \( W_1 \) of weights.

We make following observations from Table 5:

1. \( Z-R \) weight decreases with increasing \( Z_h \).
2. \( R(K_{dp}) \) weight increases with increasing \( Z_h \).
3. \( R(A_h) \) weight shows least fluctuation with change in \( Z_h \), and
4. \( R(A_h, Z_{dr}) \) shows a small peak in the partition 35–40 dBZ.

We can, therefore, conclude that while the \( Z-R \) estimator is more suitable for low-intensity rain (as reflected by its largest weight in the first two partitions), the \( R(K_{dp}) \) is more suitable during high-intensity rain. For all other cases in between, the weights are distributed equally among all estimators.

We applied the weights from Table 5 to the four radar-rainfall estimates corresponding to the 12 gauge locations to obtain a new composite-weighted estimator \( R_{\text{discrete}} \). Here, we used the radar data obtained from the PPI scans from 0800 to 1600 UTC for the event of 26 May 2013. In Fig. 9a, we compare 2-h accumulations obtained from each of the four radar rain-rate estimators with the corresponding accumulations for the

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### Table 3. Measurement errors expressed in terms of \( \text{var}(R)/(\bar{R})^2 \).

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( Z-R )</th>
<th>( R(K_{dp}) )</th>
<th>( R(A_h) )</th>
<th>( R(A_h, Z_{dr}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–30</td>
<td>0.1000</td>
<td>0.6500</td>
<td>0.8300</td>
<td>0.8300</td>
</tr>
<tr>
<td>30–35</td>
<td>0.1000</td>
<td>0.2500</td>
<td>0.3600</td>
<td>0.3200</td>
</tr>
<tr>
<td>35–40</td>
<td>0.1250</td>
<td>0.1540</td>
<td>0.2050</td>
<td>0.2050</td>
</tr>
<tr>
<td>40–44</td>
<td>0.0200</td>
<td>0.0200</td>
<td>0.0450</td>
<td>0.0420</td>
</tr>
<tr>
<td>44–49</td>
<td>0.0220</td>
<td>0.0195</td>
<td>0.0470</td>
<td>0.0390</td>
</tr>
<tr>
<td>( \geq 49 )</td>
<td>0.0250</td>
<td>0.0150</td>
<td>0.0440</td>
<td>0.0410</td>
</tr>
</tbody>
</table>

### Table 4. Algorithm errors. For each of the partitions (dBZ), the error values are FSEs based on disdrometer data.

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( Z-R )</th>
<th>( R(K_{dp}) )</th>
<th>( R(A_h) )</th>
<th>( R(A_h, Z_{dr}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–30</td>
<td>0.8390</td>
<td>0.7955</td>
<td>0.9575</td>
<td>0.6790</td>
</tr>
<tr>
<td>30–35</td>
<td>0.1127</td>
<td>0.0615</td>
<td>0.0867</td>
<td>0.0164</td>
</tr>
<tr>
<td>35–40</td>
<td>0.1644</td>
<td>0.0349</td>
<td>0.0967</td>
<td>0.0335</td>
</tr>
<tr>
<td>40–44</td>
<td>0.2683</td>
<td>0.0216</td>
<td>0.0944</td>
<td>0.0613</td>
</tr>
<tr>
<td>44–49</td>
<td>0.3635</td>
<td>0.0218</td>
<td>0.0764</td>
<td>0.0934</td>
</tr>
<tr>
<td>( \geq 49 )</td>
<td>0.3980</td>
<td>0.0247</td>
<td>0.1702</td>
<td>0.2157</td>
</tr>
</tbody>
</table>
12 gauges. Figure 9b shows the corresponding composite-weighted estimator comparison. The simultaneous decrease in bias and scatter is obvious here. Finally, Fig. 10 plots the event totals for the 12 gauges against the output from our composite-weighted algorithm. The comparison brings out remarkable linearity and excellent agreement of gauge data with our algorithm. One can also note that the WY gauges, which were closer to XPOL-5 radar, show better agreement than the other gauges. In Table 6, we show the normalized bias, the mean absolute error, and the correlation coefficient for the four individual estimators derived from the 2-h accumulations corresponding to Fig. 9a, along with those for the composite-weighted algorithm from Fig. 9b. We notice that the composite-weighted algorithm outperforms all other individual estimators in terms of the three error-statistics parameters.

We can also conveniently adapt our proposed methodology for real-time operations. The error variances for the measurement errors are not only dependent on $K_{dp}$, $A_h$, and $Z_{dr}$ (all of which are correlated with $Z_h$ to some extent), but also on PIA and PIA$_{dp}$. This implies that the weights change with events. We can overcome this variation by deriving the weights on a near-real-time basis. In other words, after the initial data processing, we can extract the necessary outputs required to derive the weights for each radar pixel, and then compute the total error variances and the resulting weights for the composite-weighted algorithms. A parallel study is currently evaluating a similar procedure using a “varying” lookup table (Thurai et al. 2016). This study analyzes several rain events from a 2015 measurement campaign where many surface instruments were deployed 13 km south-southeast of the Colorado State University (CSU) CHILL X- and S-band radar in Greeley, Colorado.

We can further evaluate our algorithms following the procedures in Anagnostou et al. (2013) by characterizing

<table>
<thead>
<tr>
<th>$Z_h$ (mm)</th>
<th>Weights for $Z-R$</th>
<th>Weights for $R(K_{dp})$</th>
<th>Weights for $R(A_h)$</th>
<th>Weights for $R(A_h, Z_{dr})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–30</td>
<td>0.36</td>
<td>0.23</td>
<td>0.19</td>
<td>0.22 (W1)</td>
</tr>
<tr>
<td>30–35</td>
<td>0.36</td>
<td>0.24</td>
<td>0.17</td>
<td>0.23 (W2)</td>
</tr>
<tr>
<td>35–40</td>
<td>0.21</td>
<td>0.33</td>
<td>0.20</td>
<td>0.26 (W3)</td>
</tr>
<tr>
<td>40–44</td>
<td>0.08</td>
<td>0.54</td>
<td>0.16</td>
<td>0.22 (W4)</td>
</tr>
<tr>
<td>44–49</td>
<td>0.06</td>
<td>0.57</td>
<td>0.19</td>
<td>0.18 (W5)</td>
</tr>
<tr>
<td>≥49</td>
<td>0.07</td>
<td>0.70</td>
<td>0.13</td>
<td>0.11 (W6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R$ interval (mm h$^{-1}$)</th>
<th>Weights for $R(A_h, Z_h)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1–2</td>
<td>W1</td>
</tr>
<tr>
<td>2–4</td>
<td>W2</td>
</tr>
<tr>
<td>4–8</td>
<td>W3</td>
</tr>
<tr>
<td>8–15</td>
<td>W4</td>
</tr>
<tr>
<td>15–30</td>
<td>W5</td>
</tr>
<tr>
<td>≥30</td>
<td>W6</td>
</tr>
</tbody>
</table>
the performance of different rainfall estimators using statistical metrics such as relative mean error, relative root-mean-square error, Nash–Sutcliffe coefficient (or efficiency score; Nash and Sutcliffe 1970), and Heidke skill score (HSS). Anagnostou et al. (2013) computed the first three bulk error statistics for each of the algorithms as a function of several contiguous, non-overlapping PIA ranges and defined relatively immune behavior of these statistics to PIA as a criterion for algorithm evaluation.

### d. Smooth weight functions

For any of the four estimators, the weights are piecewise-constant functions of $Z_h$ values. It will be instructive to consider the possibility of further narrowing down the partitions and deriving weights for smaller $Z_h$ intervals. In fact, one may consider fitting a continuous or smooth function over the discrete-valued, piecewise-constant function of weights for each estimator. The polynomial and cubic spline fits show discontinuities at the partition boundaries. Therefore, we now consider approximating the piecewise-constant weights with smoothing splines (de Boor 2001). Figure 11 shows the discrete-valued weights for each estimator and the continuous-valued result of smoothing spline fit for each of the piecewise-constant curves. For clarity, Fig. 12a reproduces Fig. 6b, wherein the XPOL-5 rainfall estimate $R_{\text{equal}}$ uses identical weights for the four methods across all $Z_h$ values. Figure 12b shows the same comparison but uses the discrete-value weights for the composite-weighted estimator $R_{\text{discrete}}$ for the radar rainfall. Here, the weights correspond to Table 5. In Fig. 12c, we show the performance of composite-weighted estimator $R_{\text{continuous}}$ that uses continuous-valued weights obtained from the smoothing spline fit of Fig. 11. While the trend of the radar-rainfall estimates remains largely the same, there is a gradual decrease in overestimation at low-intensity regions as we change our weight selection process from Fig. 12a to Fig. 12c.

### Table 6. Key error metrics of the radar–gauge comparisons for the four estimators and the composite-weighted algorithm $R_{\text{discrete}}$, represented by normalized bias (NB), mean absolute error (MAE), and correlation coefficients (CC).

<table>
<thead>
<tr>
<th>$Z-R$</th>
<th>$R(K_{dp})$</th>
<th>$R(A_h)$</th>
<th>$R(A_h, Z_{dh})$</th>
<th>$R_{\text{discrete}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB (%)</td>
<td>10.3</td>
<td>25.1</td>
<td>25.0</td>
<td>-43.7</td>
</tr>
<tr>
<td>MAE (%)</td>
<td>42.1</td>
<td>47.3</td>
<td>53.8</td>
<td>45.2</td>
</tr>
<tr>
<td>CC</td>
<td>0.74</td>
<td>0.81</td>
<td>0.77</td>
<td>0.79</td>
</tr>
</tbody>
</table>

![Fig. 10. We compare event totals for the 12 gauges with the output of the composite-weighted algorithm. The WY gauges, which were closer to XPOL-5 radar, show better agreement than the other gauges. The gray curve presents the 1:1 line.](image1)

![Fig. 11. The discrete- and continuous-value weights for each of the estimators. Fitting a smoothing spline function over the piecewise-constant discrete-value weights gives the continuous-value weights. The vertical strips in the background show $Z_h$ partitions during which the discrete-value weights remain constant for each estimator. For smaller $Z_h$ values, we use only the $R(Z_h)$ estimator.](image2)

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7. Discussion and summary

The IFloodS field campaign provided a unique opportunity to develop and assess rainfall estimators for the mobile Iowa XPOL radars. Using radar data processing procedures developed in-house, we initially compared XPOL-5 data against 2DVD-based scattering simulations and observed excellent agreement of parameters such as $Z_h$, $Z_{dr}$, and $K_{dp}$. We then used the 2DVD data to formulate four radar rain-rate estimators, namely, $R(Z_h)$, $R(K_{dp})$, $R(A_h)$, and $R(A_h, Z_{dr})$, in the context of XPOL-5 radar. Here, our study presented the very first formulation of an $R(A_h, Z_{dr})$ estimator at X band. We compared the performance of the four algorithms with the measurements from 12 rain gauges. These analyses highlighted the need for hybrid algorithms, because none of the estimators seemed sufficient for all cases considered.

We proposed a new combined version of weighted and composite algorithms, where the four estimators are weighted differently depending on the range of $Z_h$ values. We derived the weights for six categories of $Z_h$ intervals to broadly represent light rain, stratiform rain, mixed rain type, convective rain, strong convection, and severe convection. The weights are directly related to the measurement and algorithm errors.

Our initial results indicate that the composite-weighted estimator reduces the variation observed using only one of the four algorithms. When compared with 12 gauge measurements in terms of 2-h rainfall accumulations, the results appear promising. Event totals (over an 8-h period) for the 12 gauges are particularly encouraging. Further validations over larger datasets and numerical evaluation using metrics suggested by Anagnostou et al. (2013) are required. However, it is clear that the algorithm can be easily adapted for real-time operation. In general, it is also sufficient to use only very few partitions for the weights. The continuous-valued weights can be helpful in reducing variations at low-intensity regions.

In the future, we intend to include the uncertainty due to the estimator, as reflected in the scatterplots of Fig. 2, in the measurement errors. For example, we can model the scatter spread in Fig. 2a as an error term $\varepsilon_i$ in the estimator as $R = a Z_h^b e_i = a Z_h^b \exp(0.46 A_h \beta) e_i$, and we can more accurately quantify the measurement errors. When the estimator uses more than one variable, separate error terms should be computed to account for the scatter of each of the variables. For large datasets, Monte Carlo simulations can yield better estimates of these uncertainties. Improvement in the attenuation correction algorithm may also be considered. In this regard, apart from suggestions made by Mishra et al. (2016), one may consider recommendations by Kalogiros et al. (2013, 2014) where the new parameterizations approximate the Mie scattering via corrections to the theoretical Rayleigh scattering limits.

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![Fig. 12. The time series of rain rate obtained from 2DVD SN38 data is compared to data from the corresponding range (~12 km) of XPOL-5 radar. The radar rain rate is obtained using (a) equi-weighted estimator, as in Fig. 6b; (b) composite-weighted estimator that uses discrete-value weights listed in Table 5; and (c) composite-weighted estimator that uses continuous-value weights shown in Fig. 11.](image-url)
APPENDIX

Derivation of Measurement Errors for Estimator 1

For the $Z-R$ relationship, we have

$$\frac{R}{R} = \alpha Z^\beta = \alpha Z_m^\beta \exp(0.46 A_b r \beta), \quad (A1)$$

where $\alpha$ and $\beta$ are constants, $Z_m$ is the measured (attenuated) reflectivity (in linear units), and $A_b$ is specific attenuation. At a given range $r$, the path integrated attenuation is $\text{PIA}(r) = 2 \int_0^r A_b(u) \, du$. Taking the (natural) log of both sides, it follows that

$$\ln R = \ln \alpha + \beta \ln Z_m + 0.46 \beta A_b r. \quad (A2)$$

From the Taylor series expansion (and neglecting higher-order terms) and the perturbation analysis [analogous to Eq. 8.31 of Bringi and Chandrasekar (2001)], we obtain the following by taking the finite differential of the above equation:

$$\frac{\delta R}{R} = \delta Z_m + 0.46 \beta r \delta (A_b), \quad (A3)$$

where $\delta(\cdot)$ represents the measurement uncertainty of the variable inside the parentheses. Assuming that the measurement uncertainties of $Z_m$ and $A_b$ are uncorrelated, it follows that

$$\frac{\text{var}(R)}{(R)^2} = \beta^2 \text{var}(Z_m) + (0.46 \beta r)^2 \text{var}(A_b). \quad (A4)$$

The $\text{var}(A_b)$ depends on the attenuation-correction procedure. For the ZPHI method (Testud et al. 2000), Anagnostou et al. (2004, Eq. 22) show that the $\text{var}(A_b)$ can be expressed as

$$\frac{\text{var}(A_b)}{(A_b)^2} = b_1 \text{var}(Z) + \frac{\text{var}(K_{dp})}{(K_{dp})^2}, \quad (A5)$$

where $b$ is the exponent of an $A_b$–$Z$ power law.

Additionally, if the standard deviation of the measurement error in reflectivity is 0.8 dB, then from Rinehart (1997, Eq. B.24), we obtain

$$\frac{\text{var}(Z)}{(Z)^2} = 0.04. \quad (A6)$$

Hence, substituting $\text{var}(A_b)$ from Eq. (A5) into Eq. (A4), and assuming $b = 0.8$, $\beta = 0.64$, and a standard deviation of the measurement error in $K_{dp}$ is $0.3$ km$^{-1}$, we obtain

$$\frac{\text{var}(R)}{(R)^2} = (0.016) + (A_b r)^2 \left( 0.0023 + \frac{0.0078}{K_{dp}} \right). \quad (A7)$$

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