Study of Heat Sources and Sinks and the Generation of Available Potential Energy in the Indian Region During the Southwest Monsoon Season

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ABSTRACT—The total vertical velocity and the heating field were evaluated for a portion of the Indian region for a typical monsoon day in July 1966 using a two-level geostrophic model. The generation of zonal and eddy available potential energy was computed with these values of heating and cooling. Both forms of available potential energy seem to be generated by diabatic processes. The generation of eddy available potential energy may be due to the released latent heat of condensation.

1. INTRODUCTION

Available potential energy is a measure of the potential energy available for conversion into kinetic energy and can be defined as that part of the total potential energy above the amount that would exist if isentropic surfaces were horizontal. The generation of available potential energy depends on the atmospheric temperature field and the distribution of atmospheric heat sources and sinks. This concept of available potential energy, introduced in the classical paper by Lorenz (1955), led to several investigations, both theoretical (Phillips 1956, Smagorinsky 1963) and observational (Wiin-Nielsen and Brown 1962, Krueger et al. 1965, and others) on estimates of generation, dissipation, and conversion of energy in the atmosphere. A critical review of these investigations has been given by Oort (1964). In the present investigation, an attempt has been made, following Wiin-Nielsen and Brown (1962), to study heat sources and sinks and the generation of available potential energy of the atmosphere over the Indian region during the southwest monsoon season. These computations have been made for 1200 GMT on July 25, 1966, because computations regarding energy conversion were already made for this synoptic situation (Rao and Rajamani 1968). Further, the rainfall on this day was a little above normal, and this day can therefore be considered as a typical monsoon day.

2. COMPUTATIONS OF HEAT SOURCES AND SINKS

From the definition of the potential temperature \( \theta = T(p/p_0)^{\gamma/c_p} \), the first law of thermodynamics may be written in the following form:

\[
H = c_p T \left( \frac{1}{\alpha} \frac{\partial \alpha}{\partial t} + \frac{1}{\alpha} \mathbf{V} \cdot \nabla \alpha + \omega \frac{\partial \ln \theta}{\partial p} \right).
\]  

Introducing the hydrostatic relationship, we can write eq

\[
\frac{1}{c_p} H_s = \frac{3}{2} \left( \frac{\partial h}{\partial t} + \mathbf{V} \cdot \nabla h \right) - \frac{3 \sigma_s P}{2 \mathcal{R} \omega_0} \tag{2}
\]

where the subscript denotes the pressure in decibars, \( H \) is the diabatic heating per unit mass per unit time, \( g \) is the acceleration of gravity, \( \mathcal{R} \) is the gas constant for dry air, \( c_p \) is the specific heat of air at constant pressure, \( h \) is the thickness in geopotential meters, \( P \) is the pressure difference (cb) between the 800- and 400-mb levels, \( \omega \) is the vertical velocity in the \( x, y, p \)-coordinate system, \( \alpha \) is the specific volume, and \( \sigma_s \), the static stability at the 600-mb level, is given by \( \sigma_s = \omega_0 (\partial \ln \theta/\partial p) \). Thus, from eq (2), we see that the diabatic heating is proportional to the difference between the adiabatic vertical velocity (the first term on the right side) and the total vertical velocity (last term).

To solve eq (2), we must first evaluate \( \omega_0 \). For this purpose, the geostrophic vorticity equation in the following form is used:

\[
\frac{\partial \xi}{\partial t} + \mathbf{V} \cdot \nabla (\xi + f) = f_0 \frac{\partial \omega}{\partial p} \tag{3}
\]

where \( \xi \) is the geostrophic relative vorticity and \( f_0 \) is the Coriolis parameter at 20° latitude. It is also assumed that the \( \omega \)-profile is parabolic with the absolute maximum at 600 mb and \( \omega = 0 \) at 1000 and 200 mb. The assumption \( \omega = 0 \) at 200 mb was made because earlier computations of \( \omega \) by the geostrophic baroclinic model (Rao and Rajamani 1970) showed very small values even at the 300-mb level. Applying eq (3) to 400- and 800-mb levels we get, by subtraction and manipulation,

\[
\omega_0 = \frac{P_0}{2 f_0^2} \left\{ \frac{\partial}{\partial t} \nabla^2 h + \frac{J (z_0, \frac{y}{f_0} \nabla^2 h) + J[h, (\xi + f)_h]}{\partial p} \right\} \tag{4}
\]

where

\[
z_0 = \frac{1}{2} (z_s + z_b)
\]
The values of \( w \) computed from eq (4) are substituted in eq (2), and the value of \( H_{lc} \) is evaluated.

Procedure

When we transformed eq (2) and (4) from the differential form to a finite-difference form, we used a time increment of 12 hr, which is the interval between two consecutive upper air observations. Further, to have the heating fields valid at the map time (i.e., 1200 GMT on July 25, 1966), we prepared charts for 800- and 400-mb surfaces with data from 0000 and 1200 GMT on July 25, 1966, and 0000 GMT on July 26, 1966. Charts for 800- and 400-mb surfaces at 1200 GMT on July 25, 1966, are given in figure 1.

The gridpoint values were smoothed by Shuman's method with \( v=0.5 \) (Shuman 1957) to eliminate large-amplitude short waves. The values for 0600 and 1800 GMT have been obtained by algebraic averaging of the smoothed fields. The time differences and the Jacobians in eq (4) were then computed from these fields, and the total vertical velocity field was computed. Because the fields of total vertical velocity and heating were computed from the averaged 0600 and 1500 GMT charts, these two fields will represent an average for the period from 0600 to 1800 GMT.

3. DISCUSSION OF RESULTS

The field of total vertical velocity and the field of heating and cooling are shown in figures 2 and 3. One sees that the pattern of the heating and cooling field closely resembles the field of total vertical velocity, thereby suggesting that the dominant contribution to the heating or cooling in eq (2) is by the third term. The maximum heating of 22.3°C/day is near the head of the Bay of Bengal (20°N, 88°E) and the maximum cooling over India (approx. 6°C/day) is near Bombay (20°N, 74°E). To get the values of heating or cooling in units of joules per gram per second, we multiply the values by the conversion factor of 11.72 \( \times 10^{-6} \text{J.g}^{-1} \text{.deg}^{-1} \text{s}^{-1} \), and the value of maximum heating becomes 261.36 \( \times 10^{-6} \text{J.g}^{-1} \text{s}^{-1} \).

In a study of the normal distribution of the heat sources and sinks in the lower troposphere over the Northern Hemisphere, Asakura and Katayama (1964) get a value of 200 ly/day, or 19.0 \( \times 10^{-6} \text{J.g}^{-1} \text{s}^{-1} \), near the head of the Bay of Bengal for the month of July by the dynamical method of computations. Because this is the mean value for the month of July, our computed value of 261.36 \( \times 10^{-6} \text{J.g}^{-1} \text{s}^{-1} \) for an individual day may not be unreasonable.

The computed values of the three terms in eq (2) are given in table 1. The second term is found to be one order of magnitude smaller in most instances than the first and the third terms.

From the computed values of heating and cooling, the average value of heating of a unit mass of air for the area studied is 9.82 \( \times 10^{-5} \text{°C}/\text{s} \) or 8.5°C/day. This value is compared with the heating that will be available from the release of latent heat by the precipitation, keeping in view that, according to Jacobs (1949), "the amount of energy released to the atmosphere through condensation of water vapor can be obtained completely from data concerning precipitation amounts." From the 24-hr rainfall reported at 0300 GMT on July 26, 1966, by 70 stations in the region, the average rainfall was computed to be 2.34 cm/day. If we assume that the latent heat liberated goes to heat a column of 1-cm² area and 500-mb thickness (i.e., cloud base at 800 mb and top at 300 mb), the average heating per unit mass of air is 11.5°C/day. [The latent heat of water vapor at 0°C (2.5 kJ/g) was used.] If we assume a radiation cooling of 1.7°C/day based on the study by Kelkar and Godbole (1970), the net heating would be about 9.8°C/day. Thus, our computed value of 8.5°C/day seems to be in fair agreement with the value computed from the latent heat, thereby confirming the validity of estimates of heating by this diagnostic method.
4. ON THE GENERATION OF AVAILABLE POTENTIAL ENERGY

Since the distribution of atmospheric heat sources and sinks has been evaluated, it is now possible to compute the generation of available potential energy.

As shown by Lorenz (1955), the generation of available potential energy depends on the volume integral of the product of deviations of temperature and heating from the area averages of these quantities. The temperature field is known, and the heating on an individual day has also been estimated. Therefore, it is possible to compute the required integral.

As in the case of the heating computation, the computation of the generation of available potential energy has been based on the same two-level geostrophic model.
The thermodynamic equation is written in the following form:

$$\frac{\partial}{\partial t} \left( \frac{\partial z}{\partial p} \right) + \mathbf{v} \cdot \nabla \left( \frac{\partial z}{\partial p} \right) + \frac{g}{\nu} \omega = -\frac{R}{\sigma c_p} \frac{H}{p}.$$  (5)

As defined by Lorenz (1955) and Wiin-Nielsen and Brown (1962), the expression for rate of change of available potential energy is

$$\frac{g}{\sigma} \int_0^{p_0} \int \frac{\partial}{\partial t} \left[ \frac{1}{2} \left( \frac{\partial z'}{\partial p} \right)^2 \right] dsp dp.$$  (6)

An area average is defined as follows:

$$(-) = \frac{1}{L} \int_L (\ ) dx.$$  (7)

and a primed quantity indicates the deviation of the quantity from its areal mean.

If we multiply eq (5) by $\partial z'/\partial p$ and integrate over a limited volume of the atmosphere, we get an equation for the rate of change of available potential energy in that volume as

$$\frac{g}{\sigma} \int_0^{p_0} \int \left\{ \frac{\partial}{\partial t} \left[ \frac{1}{2} \left( \frac{\partial z'}{\partial p} \right)^2 \right] \right\} dsp dp$$
$$= -\frac{g}{\sigma} \int_0^{p_0} \int \mathbf{v} \cdot \nabla \left( \frac{\partial z'}{\partial p} \right) dsp dp - \int_0^{p_0} \int \omega \frac{\partial z'}{\partial p} dsp dp$$
$$- \frac{R}{\sigma c_p} \int_0^{p_0} \int \frac{1}{p} \left( \frac{\partial z'}{\partial p} \right) H' dvp dp.$$  (8)

The first term on the right-hand side represents advection of the available potential energy into the volume. The second term on the right-hand side represents transformation of available potential energy into kinetic energy. The last term is the generation term of the available potential energy.

**Generation Term, G**

Since $(- \partial z'/\partial p)$ is related to temperature, the generation term [the third term on the right-hand side of eq (8)] will be positive if the atmosphere is heated in regions that are relatively warmer than others and cooled where it is relatively colder.

With our assumptions that vertical velocities are zero at $p=1000$ mb and at $p=200$ mb, the effective depth of the atmosphere becomes 800 mb or 80 cb and the generation term becomes

$$G = \frac{R}{\sigma c_p} \frac{80}{40 \times 60} \int h' H' ds = k \int h' H' ds$$  (9)

where

$$k = \frac{R}{\sigma c_p} \frac{80}{40 \times 60}$$

and where $h'$ is the areal deviation of the thickness between 800 and 400 mb. Because the heating applies to 600 mb (60 cb), $1/p$ has been set equal to 1/60. An average along the latitude circle is defined as

$$(-) = \frac{1}{L} \int_L (\ ) dx.$$  (10)

where $L$ is the length of a portion of the latitude circle. Hence, we may write

$$h' = \tilde{h}' + h'' \quad H' = \tilde{H}' + H''.$$  (11)

where double primed quantities are the deviations from the zonal average and denote eddies.

Substituting eq (11) into (9), we get

$$G = k \int h' \tilde{H}' ds + k \int h'' H'' ds$$  (12)
Table 2.—Values for generation (G) and conversion (C) of available potential energy (W·m⁻²) obtained by various investigators (from Oort 1964)

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<tr>
<td>G⁽¹⁾</td>
<td>1.94 [yr (9 mo)]</td>
<td>2.32 [yr]</td>
<td>3.30 [winter]</td>
<td>1.34 [summer]</td>
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<tr>
<td></td>
<td>2.81 [7 winter mo]</td>
<td></td>
<td>1.07 [6 summer mo]</td>
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<tr>
<td></td>
<td>1.07 [6 summer mo]</td>
<td></td>
<td>1.34 [summer]</td>
<td></td>
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<tr>
<td>G⁽²⁾</td>
<td>0.58 (13 days)</td>
<td></td>
<td>-0.94 [yr (9 mo)]</td>
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</tr>
<tr>
<td></td>
<td>-1.57 (7 winter mo)</td>
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<td>-0.77 (yr)</td>
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<td></td>
<td>-0.32 (6 summer mo)</td>
<td></td>
<td>-1.10 (winter)</td>
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<tr>
<td>C(Pₚ,Kₚ) *</td>
<td>0.25 (yr)</td>
<td>0.35 (6 winter mo)</td>
<td>0.10 (Jan. 1959)</td>
<td>-0.66 (yr)</td>
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<td></td>
<td></td>
<td></td>
<td>-0.11 (Apr. 1959)</td>
<td>-0.79 (6 winter mo)</td>
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<td>1.46 (Jan. 1959)</td>
<td>2.21 (yr)</td>
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<td>1.10 (Apr. 1959)</td>
<td>2.98 (6 winter mo)</td>
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<td>1.44 (6 summer mo)</td>
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<tr>
<td>C(Pₑ,Kₑ) †</td>
<td>3.02 (6 winter mo)</td>
<td></td>
<td>1.46 (Jan. 1959)</td>
<td>2.21 (yr)</td>
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<td></td>
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<td>1.10 (Apr. 1959)</td>
<td>2.98 (6 winter mo)</td>
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<td></td>
<td></td>
<td>1.44 (6 summer mo)</td>
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* G⁽¹⁾ = rate of conversion from mean available potential energy to mean kinetic energy by mean meridional circulations.

† C(Pₑ,Kₑ) = rate of conversion from eddy available potential energy to eddy kinetic energy by large-scale eddy convection.

C = 12.21 × 10⁻⁴kJ·m⁻²·s⁻¹

where G⁽¹⁾ denotes the generation of zonal available potential energy and G⁽²⁾ is the generation of eddy available potential energy. G⁽¹⁾ and G⁽²⁾ have been computed from the expressions in eq (12) and the values obtained are as follows:

G⁽¹⁾ = 12.21 × 10⁻⁴kJ·m⁻²·s⁻¹

= 1.221 W·m⁻²

and

G⁽²⁾ = 21.44 × 10⁻⁴kJ·m⁻²·s⁻¹

= 2.144 W·m⁻².

The positive values of G⁽¹⁾ and G⁽²⁾ show that, both meridionally and zonally, relatively warm air masses were heated and cold air masses were cooled as a result of the various diabatic processes of heating or cooling.

Table 2 presents available potential energy generation and conversion values obtained by other workers. Although our values relate to a single day and a very limited region, the value of 1.221 W·m⁻² for G⁽¹⁾ is in good agreement with the values 1.07 W·m⁻² computed by Wiin-Nielsen and Brown (1962) and 1.34 W·m⁻² computed by Krueger et al. (1965) for the summer periods.

In the case of generation of eddy available potential energy, Wiin-Nielsen and Brown (1962) and Brown (1964) have obtained negative values indicating a destruction of eddy available potential energy by diabatic effects, although Suomi and Shen (1963) found a build-up of eddy potential energy due to the influence of infrared cooling.

In our case, we have obtained a positive value indicating generation of eddy available potential energy by diabatic effects. This may be due to the release of latent heat of condensation which leads to the generation of the eddy available potential energy. This feature is consistent with the view of Dutton and Johnson (1967), namely, that generation of available potential energy by the release of latent heat is important on the zonal scale and is of primary importance in the energetics of disturbances, most notably cyclones, especially in the Tropics. Also, according to Manabe and Smagorinsky (1967), in the moist model atmosphere the generation of eddy available potential energy by condensation and convection dominates in the Tropics. From figure 4.4 of Manabe and Smagorinsky (1967), one sees that the generation of eddy available potential energy due to convection and condensation lies between the values of 1 × 10⁻² and 2 × 10⁻²J·cm⁻²·mb⁻¹·day⁻¹. The value of the generation of eddy available potential energy in our study is 2.6 × 10⁻²J·cm⁻²·mb⁻¹·day⁻¹ when we convert to the units adopted by Manabe and Smagorinsky. We see good agreement in the order of magnitude between our computed values and that of Manabe and Smagorinsky.

Conversion Term, C

The second term on the right-hand side of eq (8) is the conversion term which is given by

$$C = -\int_0^{\rho_0} \int_0^\omega \frac{\partial^2 f}{\partial \rho^2} d\rho dp$$

$$= \frac{80}{40} \times \frac{2}{3} \int_0^\omega h' ds.$$  (13)

In view of our assumption that ω is parabolic with the absolute maximum at 600 mb and ω = 0 at the 1000- and 200-mb levels, the average vertical velocity for the layer 1000–200 mb will be two-thirds of the maximum value at 600 mb, and, hence, the factor 2/3 appears in eq (13).

As in the case of the generation term, the conversion term has been split up into zonal conversion and eddy conversion and the values obtained are as follows:

C⁽¹⁾ = -7.84 × 10⁻⁴kJ·m⁻²·s⁻¹

= -0.78 W·m⁻²
and
\[ C^{(0)} = -12.04 \times 10^{-4} \text{kJ.m}^{-2}.\text{s}^{-1} \]
\[ = -1.204 \text{W.m}^{-2}. \]

Thus, both zonal and eddy available potential energy are converted into zonal and eddy kinetic energy, respectively. From a comparison of the values of these quantities obtained by other workers (table 2), the values above appear to be reasonable.

**Advection Term**

To facilitate computations, we transformed the advection term in the usual way to the form of the flux term, and this was further transformed into a line integral using Gauss' theorem as follows:

\[ -\frac{g}{2\alpha} \int_{l}^{r} \int_{c} \mathbf{V} \cdot \nabla \left( \frac{\partial z'}{\partial p} \right)^2 d\alpha dp \]
\[ = -\frac{g}{2\alpha} \int_{l}^{r} \int_{c} \mathbf{V} \cdot \nabla \left( \frac{\partial z'}{\partial p} \right) d\alpha dp \]

as the other term,

\[ \int_{l}^{r} \int_{c} \left( \frac{\partial z'}{\partial p} \right)^2 \mathbf{V} d\alpha dp, \]

becomes zero due to the geostrophic assumption.

Thus, the term takes the form

\[ -\frac{g}{2\alpha} \int_{l}^{r} \int_{c} \left( \frac{\partial z'}{\partial p} \right)^2 V_{n} d\alpha dp \]

where \( V_{n} \) is the velocity along the outward directed normal. The computed value of this term is

\[ 0.1 \times 10^{-4} \text{kJ.m}^{-2}.\text{s}^{-1}. \]

The value is very small and, because this is a study for only 1 day, it may not be possible to infer anything from this result.

**5. SUMMARY**

The field of total vertical velocity has been computed for the synoptic situation of July 25, 1966, at 1200 GMT over India. The field of heating and cooling as well as the generation and conversion terms of available potential energy have been evaluated. There is a net generation of available potential energy of \( 13.77 \times 10^{-4} \text{kJ.m}^{-2}.\text{s}^{-1} \) over the region studied.

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