The Use of Time Series Analysis Techniques in Forecasting Meteorological Drought

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ABSTRACT

Using an exponential smoothing procedure and an autoregressive-moving average process, forecasts for the monthly Palmer Drought Severity Index were calculated. The autocorrelation and partial autocorrelation functions of severity index values were used as a starting point for the autoregressive-moving average model selection process. Of the many possible autoregressive-moving average models, the one that was selected provided the best forecasts based on the mean square error. Monthly data for the period 1929–1969 were utilized in a nonlinear least-squares computer routine to arrive at estimated parameter values for the autoregressive-moving average model. Monthly forecasts with a lead time of one month were generated using the exponential smoothing and autoregressive-moving average procedures for the period 1970–1972. These forecasts were compared with the myopic (persistence) forecasts, $X_{t+1}=X_t$. The mean square errors of the forecasts were 0.63 for the autoregressive-moving average model, 0.65 for the myopic model, and 0.79 for the exponential smoothing model. From the mean-square-error calculations, it appears that there is no statistically significant difference between the forecasts given by the Box-Jenkins and myopic models; however, the 95% confidence intervals for these two models overlap only slightly during the first part of the forecast period indicating that there may be some advantage to using the Box-Jenkins model instead of the myopic model. Both of these models are superior to the exponential smoothing model. These results demonstrate the usefulness of the relatively new autoregressive-moving average time series analysis procedures.

1. Introduction

The problems associated with drought have been plaguing farmers for centuries. Once a drought has begun, some relief can usually be obtained by the use of irrigation methods; however, efficient economic planning pertaining to the alleviation of the effects of a prolonged drought depends on the ability to adequately forecast the drought and its severity. The purpose of this paper is to examine alternative approaches to time series modeling in the forecasting of droughts. Two relatively new procedures are represented by the works of Box and Jenkins (1970) and Brown (1963).

In the past, most work on drought prediction was based on return period analysis; a probability distribution was fitted to a drought severity index which yielded the estimated length of time between droughts of a certain severity. The return period procedure does not constitute a forecast nor does it facilitate the preparation of decisions concerning controlling the effects of a prolonged drought. On the other hand, forecasts arrived at via time series analysis do lend themselves to that task. If the forecasts indicate that there is a high probability that the drought will continue at the same or higher level, an optimal strategy toward alleviating the drought can be created. The decision variables are the forecasts; hence they need to be as accurate as possible.

There are a number of possible ways to classify and measure drought severity. The system currently used by both the Environmental Data Service and the National Weather Service (both agencies are part of the U. S. Department of Commerce's National Oceanic and Atmospheric Administration) is called the Palmer Drought Index (P. D. I.). For a complete discussion of meteorological drought and the P. D. I., see Palmer (1965). The procedures used in the P. D. I. require the calculation of a moisture anomaly index (z) and a drought (or wet spell) severity factor (X). The z values in the P. D. I. are calculated from:

$$z=dk \quad \text{where} \quad d=P-\hat{P}. \tag{1}$$

In Eq. (1), P is the areal (in the P. D. I., a state climatic division) average precipitation for a particular month. The $\hat{P}$ is defined by Palmer to be "the amount of precipitation that would have maintained the water resources of an area at a level appropriate for the

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established economic activity of the area." Palmer abbreviates this statement by using "climatically appropriate for existing conditions (CAFEC)." The $\hat{P}$ values are obtained from:

$$\hat{P}=\widehat{ET}+\hat{R}+\widehat{RO}-\hat{L} \tag{2}$$

where $\widehat{ET}$, $\hat{R}$, $\widehat{RO}$, and $\hat{L}$ are CAFEC values for evapotranspiration, recharge, runoff, and the net loss of soil moisture, respectively. PE is potential evapotranspiration. Palmer calls k the climatic characteristic. In the calculations for the Central Climatic Division in Ohio, k is calculated from long-term means:

$$\bar{k}=1.5\log_{10}\left[\left(\frac{\overline{PE}+\bar{R}+\overline{RO}}{\bar{P}+\bar{L}}+2.80\right)\Big/\bar{D}\right]+0.50. \tag{3}$$

In (3), $\bar{D}$ is the mean of the absolute value of d.

2. The data and method of analysis

Data for this study consisted of 44 years (1929–1972) of monthly z and X values for the Central Climatic Division of Ohio. Three different X values are calculated from the following equation:

$$X_{j,i}=X_{j,i-1}+z_i/3-0.103X_{j,i-1}. \tag{4}$$

In (4), i is the monthly time index; j takes on the values 1, 2, or 3, which are associated with the following X values: 1) severity index for a wet spell that is becoming established; 2) severity index for a drought that is becoming established; and 3) severity index for any wet spell or any drought that has become definitely established. The final severity index value comes from one of these three values. It is this final severity index that will be used in this study.

From (4) it can be seen that the index for the current month is computed from the last month's index and the z value for the current month. To forecast the X values for the next month requires that the z values for the next month be forecasted, i.e.,

$$X_{i+1}=0.897X_i+z_{i+1}/3. \tag{5}$$

The whole severity index approach is based on the z values. Thus, the usefulness of this index depends on good forecasts of the moisture anomaly value.

The P. D. I. is routinely calculated by the Environmental Data Service computer for all state climatic divisions in the United States. Each climatic division has a predetermined constant $\bar{k}$ value for each month.

Explicit evaluation of the behavior of the indices over time requires a model of the process which generates the X and z values. Given measurement or specification errors, the modeling of X or z will contain both a deterministic and stochastic component. Optimal forecasts depend crucially on the characterization of the stochastic component.

A brief examination of (4) and (5) suggests alternative approaches toward modeling the processes over time. One could concentrate primarily on forecasting the z's, using the forecasted values to "drive" the X relation; or, one could model the X's directly, purely on the basis of past X values. In the second case, the effect of the z's would be subsumed in the past X's. The z's proved to be of little direct help in the modeling, as the analysis of the z's suggested that their behavior over time was not significantly different from a random walk. Thus an attempt was made to model the X's based on 41 years of monthly values. The mean and variance of the X values were -0.001 and 4.98, respectively. The maximum X value was 5.14, the minimum was -6.21.

TABLE 1. Autocorrelation and partial autocorrelation functions for the drought severity index values.

Lag	ACF1 for X	PACF1 for X	ACF2 for X	PACF2 for X
1	0.88	0.88	0.00	0.00
2	0.75	−0.06	−0.03	−0.03
3	0.64	−0.03	−0.03	−0.03
4	0.53	−0.04	−0.06	−0.07
5	0.44	0.00	−0.06	−0.06
6	0.36	0.00	−0.02	−0.02
7	0.29	−0.04	−0.06	−0.07
8	0.23	0.01	−0.08	−0.09
9	0.19	0.04	−0.13	−0.15
10	0.18	0.10	−0.02	−0.05
11	0.17	0.01	−0.03	−0.06
12	0.18	0.03	0.01	−0.04
13	0.18	0.00	0.04	−0.01
14	0.17	−0.03	−0.02	−0.06
15	0.17	0.03	−0.01	−0.05
16	0.17	0.02	−0.02	−0.06
17	0.17	0.03	0.04	−0.01
18	0.17	−0.02	0.09	0.05
19	0.14	−0.07	0.06	0.03
20	0.10	−0.06	0.03	0.02
21	0.05	−0.05	−0.04	−0.04
22	0.01	0.01	−0.07	−0.07
23	−0.01	0.04	−0.09	−0.10
24	0.00	0.08	−0.05	−0.07

ACF1 for X: No differencing, variance=4.98, std. error=0.05.
ACF2 for X: First order ordinary differencing, variance=1.24, std. error=0.05.

A good starting point for model specification is an examination of the autocorrelation and partial autocorrelation functions of the X values. Under very general assumptions, the estimated correlation functions can be "compared" with a set of theoretical correlation functions; each corresponding to a specific model. A near "match" between an estimated correlogram with a theoretical correlogram yields information concerning the structure of the process under investigation. Since these estimated functions indicate the behavior of the process over various time lags, they play an important role in arriving at models which have good forecasting properties. In columns labeled ACF1 and PACF1 in Table 1 are the values of the autocorrelation function and partial autocorrelation function, respectively, for the X values. The entries in the table suggested a slowly decaying exponential process. This result is consistent with the autoregressive form of (5). The X

TABLE 2. Autocorrelations of residuals for the (2,0,2) model.

Lag	Autocorrelation	Lag	Autocorrelation
1	0.003	16	−0.006
2	−0.015	17	0.037
3	0.016	18	0.087
4	−0.007	19	0.058
5	0.002	20	0.034
6	0.040	21	−0.026
7	−0.009	22	−0.054
8	−0.035	23	−0.078
9	−0.083	24	−0.040
10	0.012	25	0.062
11	−0.001	26	0.021
12	0.027	27	−0.030
13	0.055	28	−0.018
14	−0.003	29	0.027
15	0.002	30	−0.021

values were then filtered by ∇, the first order backward differencing operator. Columns labeled by ACF2 and PACF2 show the effect of the differencing. An examination of the resulting correlograms showed that the differencing operation had essentially reduced the X values to "white noise" (random, uncorrelated series). Thus, the model that would seem to be most appropriate was the myopic (persistence) one, i.e., $X_{t+1}=X_t$. The previously mentioned work by Box and Jenkins (1970) and Brown (1963) prompted the authors to seek a more refined model. The Box-Jenkins procedure involves modeling the X's parametrically, using the class of autoregressive-moving average stochastic processes delineated in the equation:

$$\phi(B)\tilde{Z}_t=\theta(B)a_t \qquad (6)$$

where $\phi(B)$ and $\theta(B)$ are polynomials in the lag operator B. B is the backward shift operator and is defined as $BZ_t=Z_{t-1}$ and $B^mZ_t=Z_{t-m}$. The autogressive terms are contained in $\phi(B)$, the moving average terms in $\theta(B)$; a_t represents a white noise process; and $\tilde{Z}_t$ is the deviation from the time series mean value. In the Box-Jenkins procedure, preliminary estimates for $\phi(B)$ and $\theta(B)$ can be obtained from the estimated correlograms.

The Brown procedure relies on an exponential smoothing process defined by

$$S_t(X)=\alpha\sum_{k=0}^{t-1}(1-\alpha)^kX_{t-k}+(1-\alpha)^tX_0 \qquad (7)$$

where $S_t(X)$ is the smoothing function, α is a smoothing constant, and the X's are the time series observations. From (7) it can be seen that $S_t(X)$ is a linear combination of all past observations.

For further theoretical details of these approaches, the books listed in the reference section are recommended.

3. Results and conclusions

Models in the Box-Jenkins terminology are expressed by (P, D, Q) where P is the order of $\phi(B)$ and Q the order of $\theta(B)$. D is the level of differencing. The previously mentioned autocorrelation and partial autocorrelation functions of the X values suggest that a low order autoregressive-moving average representation is appropriate. This follows from the effect of first differencing on the autocorrelation function. In the Box-Jenkins scheme, models up to (2, 1, 6) and (2, 0, 5) were examined. Model adequacy was evaluated in two ways. First, by an approximate chi-square test (Box and Jenkins, 1970, Chapter 8). A model was deemed adequate (not necessarily best in terms of forecasting mean square error) if the following statistic was insignificant at the 0.05 level:

$$Q=Nr_k^2(\hat{a}) \qquad (8)$$

where N is the number of observations adjusted for degrees of freedom, k is the lag of the residual autocorrelation function, and $r_k(\hat{a})$ is the estimated residual autocorrelation. A "correct" model would theoretically have $r_k(a)=0$. The second test is based on the forecast mean square error,

$$\Sigma(X_t-\hat{X}_t)^2/N \qquad (9)$$

TABLE 3. One-month-ahead forecasts for the three models.

Origin	Actual	Box	Brown	$X_{t+1}=X_t$
1	1.48	1.81	2.40	2.08
2	1.15	1.24	2.14	1.48
3	0.87	0.93	1.74	1.15
4	1.68	0.70	1.30	0.87
5	1.68	1.46	1.50	1.68
6	1.66	1.47	1.62	1.68
7	1.55	1.43	1.68	1.66
8	1.16	1.32	1.63	1.55
9	1.36	0.96	1.36	1.16
10	1.37	1.15	1.31	1.36
11	1.19	1.17	1.29	1.37
12	1.16	1.00	1.17	1.19
13	0.67	0.98	1.09	1.16
14	1.18	0.53	0.75	0.67
15	−0.32	1.01	0.86	1.18
16	−1.40	−0.36	0.05	−0.32
17	0.22	−1.30	−1.08	−1.40
18	0.21	0.21	−0.75	0.22
19	0.34	0.24	−0.49	0.21
20	0.50	0.32	−0.22	0.34
21	0.75	0.45	0.07	0.50
22	−0.38	0.67	0.42	0.75
23	−0.61	−0.39	−0.03	−0.38
24	−0.07	−0.58	−0.44	−0.61
25	−0.48	−0.06	−0.35	−0.07
26	−0.69	−0.49	−0.52	−0.48
27	−0.75	−0.62	−0.73	−0.69
28	0.72	−0.66	−0.88	−0.75
29	1.26	0.70	−0.06	0.72
30	−0.29	1.18	0.78	1.26
31	−0.60	−0.29	0.35	−0.29
32	−0.65	−0.59	−0.11	−0.60
33	1.84	−0.60	−0.43	−0.65
34	1.89	1.72	0.87	1.84
35	3.40	1.74	1.69	1.98
36	3.77	3.05	3.07	3.40

MSE for the Box forecasts = 0.63, 95% confidence bands ±2.92.
MSE for the Brown forecasts = 0.79, 95% confidence bands ±4.61.
MSE for the $X_{t+1}=X_t$ forecasts = 0.65, 95% confidence bands ±3.09.

where $\hat{X}_t$ is the forecast of X_t. One would tend to select models which yield low forecast mean square errors. If the "correct" model were identified using the Box-Jenkins procedure, it would necessarily yield minimum mean square error forecasts (Box and Jenkins, pp. 127–129). If, by following their procedure for specification, a model was arrived at which had a large mean square error, doubt would be cast on the model that was selected. Time series modeling by Box-Jenkins is an iterative process.

The Box-Jenkins model for the drought severity index was found to be (2, 0, 2); specifically:

$$X_t = 1.344X_{t-1} - 0.431X_{t-2} + a_t - 0.419a_{t-1} + 0.034a_{t-2}. \quad (10)$$

The T-statistics for the four parameters in (10) were 3.07, −2.16, −1.95, and 0.90 respectively. The Q statistic from (8) was 22.52 at 26 degrees of freedom. The residual autocorrelations for lags one through thirty are given in Table 2. All of these values were found to be small (compared to their estimated standard errors). Monthly drought severity index values from 1929–1969 were used in the estimation of (10). Thus the (2, 0, 2) autoregressive-moving average model was chosen because of its adequacy, its forecasts, and because of its parsimonious form (a relatively simple model). Monthly forecasts for the period 1970–1972 (36 months) using the Box-Jenkins model, the Brown model, and the myopic model are presented in Table 3.

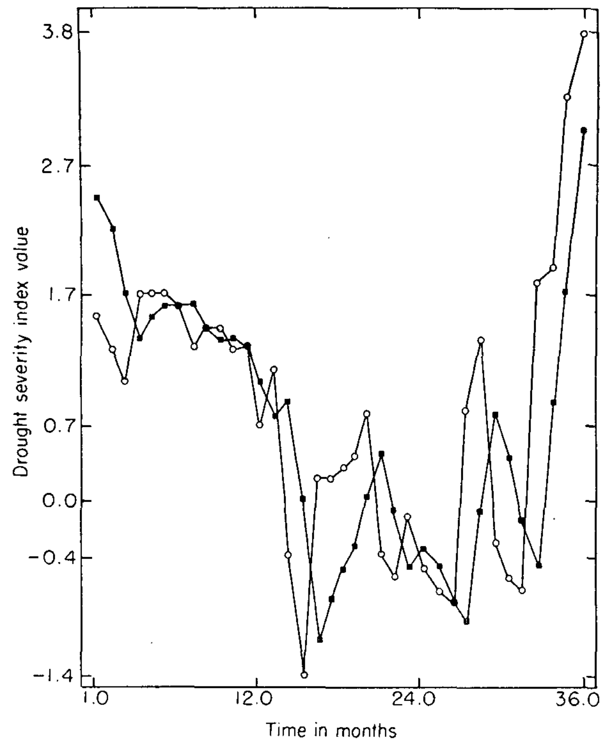

FIG. 2. Brown forecasts (blocks) versus actual severity index values (circles).

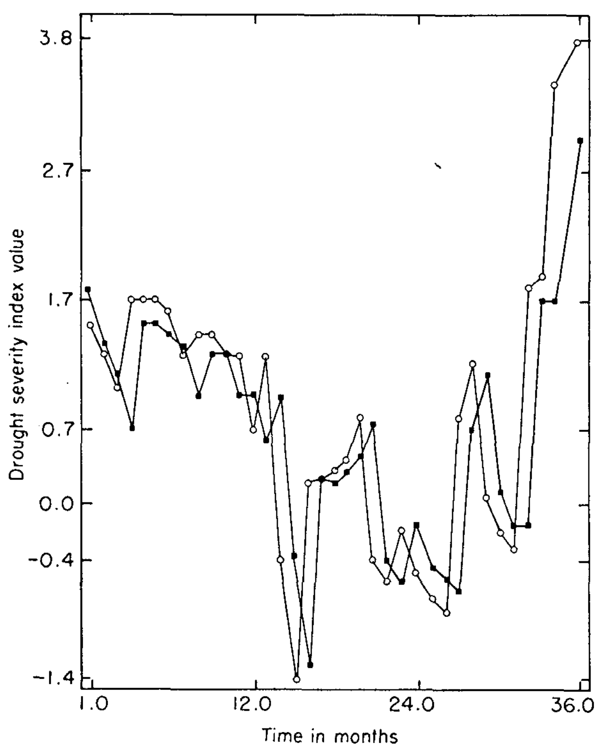

FIG. 1. Box-Jenkins forecasts (blocks) versus actual severity index values (circles).

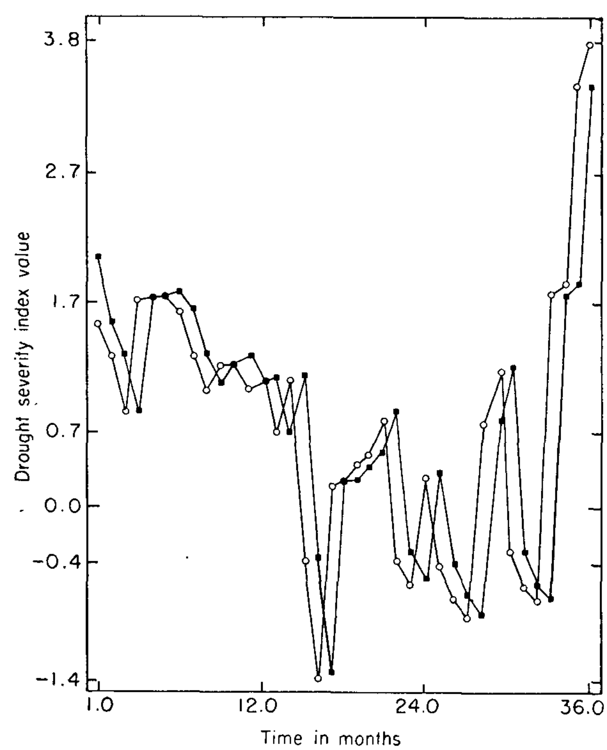

FIG. 3. Myopic forecasts (blocks) versus actual severity index values (circles).

The mean-square errors and the 95% confidence bands are also presented at the bottom of the table. The mean-square error values indicated that the Box-Jenkins and myopic models gave better forecasts than the Brown model. The superiority of the Box-Jenkins model over the myopic model is questionable since the difference in the mean square errors is so slight. To compare these two models confidence intervals were calculated on the forecast values. The confidence intervals for the Box-Jenkins and myopic models slightly overlap each other during the first year of the forecast period. From that point onward the Box-Jenkins model confidence interval lies inside the myopic model confidence interval; a fact which would suggest that there is some advantage to be gained by using the Box-Jenkins model. In some applications this advantage may not be marginal. Both the Box-Jenkins and myopic models confidence intervals lie completely within the confidence interval for the Brown model, thus indicating their superiority to the Brown exponential smoothing model. The forecasts from these three models are graphed as a function of time. These graphs are presented in Figs. 1–3. The myopic and Box-Jenkins models yield similar forecasts. The Brown forecasts differed considerably, especially from the 16th month to the 21st, the 1st–3rd months, and from the 29th through 33rd months. Both the Box-Jenkins and myopic models caught crossover points (plus to a minus and minus to a plus) with a lag of one month. This is a capability that any myopic model will have. The exponential smoothing process did not pick up the crossover points as well.

The most interesting result to come from this study was the ability of the Box-Jenkins model to perform as well as the Brown exponential smoothing model. This should be seen in light of the overwhelming acceptance of exponential smoothing techniques in forecasting. When a low order (P, D, Q) model is not identified, the Brown approach enjoys a large computational advantage. Only in a few cases, however, will Brown's procedure yield minimum mean square forecasts. For highly correlated data, the simple myopic model will generally do better than the Brown model, as was illustrated by these results.

In conclusion, these results demonstrate that the Box-Jenkins methodology has a place in the repertoire of tools available to the empirically-minded researcher. The limits of usefulness await more examination and research.

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